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A NOTE ON SOCIAL SECURITY AND PUBLIC DEBT

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Abstract

In a simple stochastic overlapping generation model, individuals work when young and retire when old, generations' productivity is affected by a serially uncorrelated random shock, and fiat money and nominal public debt are the only storable assets. In this setting, we show that social security programs featured by a constant contribution rate and budget-balance in each period, as common in the literature, are Pareto-dominated by programs allowing for budget unbalance, compensated by variations of the outstanding nominal public debt.

Keywords: Intergenerational risk sharing, social security, public debt, inflation

JEL classification: E24, E63, H55, H63

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1 Introduction

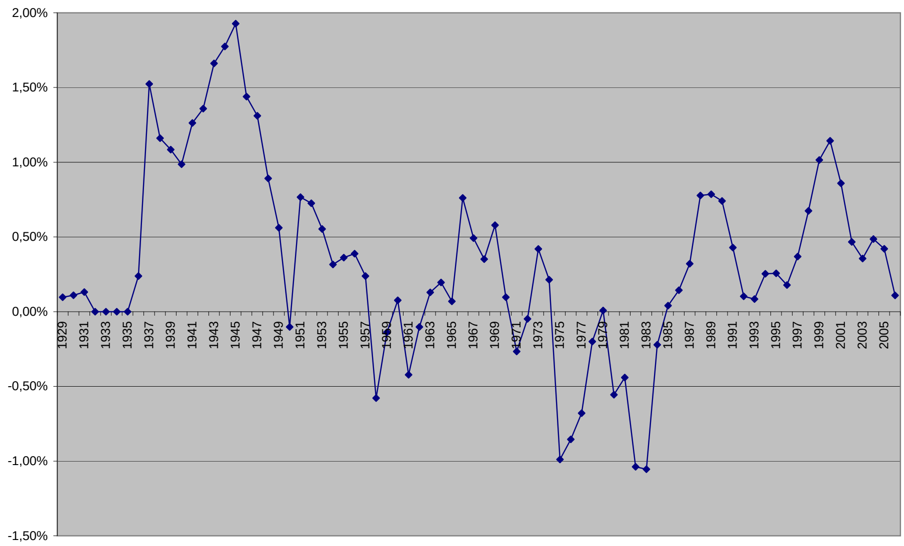
Social Security programs were introduced in many countries to mitigate the effects of economic crises. Starting with the seminal contribution by Enders and Lapan (1982), the general equilibrium literature has investigated the intergenerational risk sharing function of social security schemes. Relying on ex ante welfare analysis, the literature on stochastic OLG models has shown that social security programs may determine Pareto-improvements because of financial markets incompleteness, possibly compensated by dynamic efficiency concerns (Krueger and Kubler, 2002; Bohn, 2003; Gottardi and Kubler, 2006; Campbell and Nosbusch, 2007). A particular form of market incompleteness is determined by limitations on trading imposed by the fact that individuals belonging to future generations cannot contract with living ones, which in turn implies imperfect intergenerational risk sharing (Ball and Mankiw, 2002; Demange, 2002). Other strands of the literature on the intergenerational risk-sharing have pointed out the scope for alternative fiscal policy tools, namely public debt and capital taxation, as well as for monetary policy to improve intergenerational risk-sharing (Gordon and Varian, 1985; Gale, 1990; Smetters, 2003). The capacity of social security programs and other inter-temporal fiscal and monetary policies to improve ex ante efficiency is often limited by institutional constraints preventing policy rules to be fully state contingent (Gottardi and Kubler, 2006).

However, in real world experience, social security and other inter-temporal fiscal and monetary tools are parts of the general set of public policies. Therefore, when assessing the normative and positive capacity of (single) policies to implement or even approximate optimal intergenerational risk sharing conditions, a more comprehensive view should be taken. This paper provides an example of this approach to intertemporal fiscal policy implementing intergenerational risk sharing, starting with the positive observation that in the experience of several countries social security schemes use to be unbalanced (in surplus or deficit) from time to time and possibly for several years, partly in relationship with demographic changes and partly with aggregate economic performance (see Graph 1).

Graph 1

U.S. Social Insurance Net Savings as share of GDP between 1929 and 2006

Source: NIPA Tables, Bureau of Economic Analysis, U.S. Department of Commerce



Building on these considerations and abstracting from capital accumulation and demographic dynamics, we explore the normative scope for social security programs that do not balance the budget over time or involve only a long run budget balance. We consider a simple overlapping generations model *à la* Enders and Lapan (1982) with individuals living two periods (working when young and being retired when old) that are perfectly forward-looking. Generations are hit by productivity shocks that are distributed following a random process that is identical and uncorrelated across time. For the sake of simplicity, the only storable assets are fiat money and nominal debt issued by government. In this framework, we know that some social security transfer from workers to retirees proportional to labor income is ex ante efficient given its positive effect on intergenerational risk sharing (Enders and Lapan, 1982).

Assuming that the government balances the possible deficit or surplus of the social security budget by issuing or buying back nominal securities, we show that pension programs imposing a budget-balance rule for each period are Pareto-dominated by programs allowing for some budget unbalance (deficit and surpluses) in different periods and that intergenerational risk sharing further improves when the social security determines a surplus in the long run. Provided that a monetary equilibrium exists in steady state, in which individuals save and trade money and public debt, this result holds for any size of the pension program (say, any level of the contribution tax rate) and it is based on the improvement of the risk-sharing capacity of social security passing through the influence of the public debt on the price level.

The paper is organized as follows. The model is presented in Section 2. Fiscal policy regimes, given by government policies for social security and public debt, are analyzed in Section 3. Section 4 concludes.

2 The Model

In the economy live, at any period, two overlapping generations. Each generation is made by an infinite number of identical individuals of mass one. For the sake of simplicity, we assume no demographic dynamics. The generic individual born at time t lives two periods: in t she works earning a net income $p_t \cdot a_t \cdot l_t \cdot (1 - \tau_t)$ (where p_t is the price of the output produced, a_t is the physical productivity of labor, l_t is the individual supply of labor, and τ_t is the social security mandatory contribution); in $t+1$ she is retired and receives a social security pension equal to $p_{t+1} \cdot r_{t+1}$ (where r_{t+1} is the real pension). The utility function of the generic individual is given by

$$U_t = u(1 - l_t) + u(c_t^y) + \lambda \cdot u(c_{t+1}^o)$$

where c_t^y and c_{t+1}^o are the individual's consumption levels respectively when young and when old, $\lambda \in (0, 1]$ is the time discount rate, and $u(.)$ performs constant relative risk aversion: $-\frac{u''(c)}{u'(c)} \cdot c = \sigma$.

Each unit of labor produces $a_t \in [0, \bar{a}]$ units of perishable output. The physical productivity of labor is exogenously determined by a random variable with probability function $F(a)$, with average $E_a(a) = \int_0^{\bar{a}} a \cdot dF(a) = 1$ and finite variance, the realization is identical for the individuals of the same generation, and the process is identically and independently distributed across generations. There is no capital accumulation, and the only savings technologies are the exogenously fixed amount of fiat money, M, and the public debt possibly issued at each time t by government, B_t[1]. The real rate of return of these assets is determined by price dynamics.

Abstracting from any other role of the state in the economy, at any given time, t, a *social security regime* is determined by the vector of the contribution rate, τ_s, and the retirement benefit, r_s for each period $s \in [t, +\infty)$. Possible surpluses or

[1] The outstanding public debt could be, at some periods, negative as well, with the government lending resources to individuals. For the sake of simplicity, we assume that government bonds (as well as securities bought by government and issued by individuals) have maturity of one period.

deficits of the social security program are covered by reducing or increasing the size of the outstanding public debt and the government's budget is

$$B_{t+1} + \tau_{t+1} \cdot p_{t+1} \cdot a_{t+1} \cdot l_{t+1} = B_t + p_{t+1} \cdot r_{t+1} \tag{1}$$

The timing of the model reads as follows. The government chooses the social security regime and this is common knowledge to individuals of all generations[2]. At each time, t, young individuals choose their work effort before discovering the realization of the productivity shock. After discovering it, they competitively sell their output on the market earning a state-contingent labor income. Given the price level that is determined on the market and the expectation of price in the subsequent period, the young choose the amount of first period consumption and savings, buying fiat money by old individuals and bonds by government. At period $t+1$, the old earn a net income made by the social security benefit, by the resale of fiat money to young individuals, and by the reimbursement of government bonds. Then, old individuals inelastically demand the consumption good on the market.

3 Market Equilibria and Fiscal Policy Regimes

As first we focus on individual choices in terms of labor, consumption and savings. The individual born at time t cannot choose anything once she reaches her second period of life, $t+1$: she just offers all nominal assets bought in time t as store of value, $M + B_t$; then, with the income obtained by selling assets and by the social security, r_{t+1}, she inelastically demands all the consumption good that the young

[2]We do not consider expectations of regime-switching, our comparative statics contrasts (stationary) equilibria determined by different fiscal policy regimes.

are willing to sell, c^o_{t+1}, at market equilibrium price, p_{t+1}

$$c^o_{t+1} = \frac{M + B_t}{p_{t+1}} + r_{t+1} \tag{2}$$

that, by the normalization of each generation's population to one, is equal to the aggregate consumption of old individuals at time $t + 1$.

We assume that individuals have rational expectations about the price level. At the generic time $t+1$, the consumption good equilibrium price is determined by the equation of the aggregate demand (summing up the consumption of young, c^y_{t+1}, and old individuals) and supply in the economy

$$c^y_{t+1} + c^o_{t+1} = a_{t+1} \cdot l_{t+1}$$

where l_{t+1} is the aggregate supply of labor in the economy (equal to the young representative individual's one). By (2), the equilibrium price is a function of aggregate production, aggregate demand for consumption of young individuals, aggregate pensions, fiat money supply and outstanding debt at the beginning of the period $t + 1$ (say, B_t)

$$p_{t+1} = \frac{M + B_t}{a_{t+1} \cdot l_{t+1} - c^y_{t+1} - r_{t+1}} \tag{3}$$

By the government budget constraint (1) at time $t + 1$, the equilibrium price level can also be written as a function of current public debt level (B_{t+1}) and young individuals private savings

$$p_{t+1} = \frac{M + B_{t+1}}{a_{t+1} \cdot l_{t+1} \cdot (1 - \tau_{t+1}) - c^y_{t+1}} \tag{4}$$

Given the amount of nominal assets that old individuals bought when they were young (hence, given the inelastic supply of fiat money and public debt, the demand of those assets at time t)

$$M + B_t = p_t \cdot (a_t \cdot l_t \cdot (1 - \tau_t) - c_t^y)$$

the consumption demand of old individuals can be written as

$$c_{t+1}^o = \frac{p_t}{p_{t+1}} \cdot (a_t \cdot l_t \cdot (1 - \tau_t) - c_t^y) + r_{t+1} \tag{5}$$

As usual the choice between first- and second-period consumption relies upon the inter-temporal rate of return of (nominal) assets, here $\frac{p_t}{p_{t+1}}$, that by (3) and (4) depends just on aggregate labor and consumption choices, on the productivity shocks of times t and $t+1$, and on the structure of the social security. Following Enders and Lapan (1982, p. 652) we represent the first-period consumption choices by consumption propensity, $x_t \equiv \frac{c_t^y}{a_t \cdot l_t}$. Therefore, the ratio between prices at time t and $t+1$ can be written as

$$\frac{p_t}{p_{t+1}} = \frac{1}{1 + \pi_{t+1}} = \frac{a_{t+1} \cdot l_{t+1} \cdot (1 - x_{t+1}) - r_{t+1}}{a_t \cdot l_t \cdot (1 - \tau_t - x_t)} \tag{6}$$

where π_{t+1} is the inflation (or deflation) rate between t and $t+1$.

In the stationary equilibria, the inflation (or the price ratio) between two subsequent periods depends just on the productivity realization in those periods. In particular, we observe that the level of the public debt (and fiat money) is irrelevant. The reason is that the realization of the price in one period, given the price of the preceding one, depends just on the variation of the public debt between these two periods (say on the social security surplus or deficit) and not on the absolute value of outstanding debt. The intuition is that, given that in this economy there is no capital accumulation and individuals have a finite time horizon, the public debt does not crowd private investments out and the only way it affects the dynamic equilibrium is by its inflationary or deflationary effect, which in turn changes the rate of return of private savings.

3.1 Laissez-Faire and First-Best Equilibria

In absence of any fiscal policy (hence, of social security schemes and public debt management), at any time t, individuals choose labor supply, l_t, before observing their productivity realization, a_t. Therefore, in stationary equilibria (where past productivity is irrelevant for this choice), labor supply is identical at all times. Then, having observed the current productivity level and given the expectation of the future one, individuals adjust their consumption propensity, x_t: a *stationary equilibrium* is a vector of individual choices $\{l^*, \{x^*(a)\}_{a\in[0,\bar{a}]}\}$ and a state-contingent price ratio between current and future times

$$\frac{1}{1+\pi^*(a,a_+)} = \frac{a_+ \cdot (1 - x^*(a_+))}{a \cdot (1 - x^*(a))} \tag{7}$$

for all a and a_+ respectively the current and future productivity realizations.

Given the optimal stationary labor choice, l^*, the realization of the current productivity, a, and the rational expectation about the inflation rate, $\pi(a, a_+)$, the representative individual chooses the optimal consumption propensity function,

$\{x^*(a)\}_{a\in[0,\bar{a}]}$[3], solving the optimization program

$$\max_{\{x(a,l^*)\}_{a\in[0,\bar{a}]}} u(1-l^*) + u(a \cdot l^* \cdot x(a,l^*)) + \lambda \cdot E_{a_+}\left(u\Big(\frac{a \cdot l^* \cdot (1-x(a,l^*))}{1+\pi(a,a_+)}\Big)\right) \tag{8}$$

with

$$u'(a \cdot l^* \cdot x^*(a)) \cdot a \cdot (1-x^*(a)) = \tag{9}$$
$$= \lambda \cdot E_{a_+}(u'(a_+ \cdot l^* \cdot (1-x^*(a_+))) \cdot a_+ \cdot (1-x^*(a_+)))$$

for any a, the corresponding first order condition. The left-hand-side of (9) is constant with respect to a, therefore the consumption propensity increases (or is constant or decreases) in a if an only if the relative risk aversion of individuals is lower (or equal or higher) than 1[4] (Enders and Lapan, 1982, p. 653). Moreover, by (9), at the optimum

$$\lambda \cdot E_a\Big(\frac{u'(a \cdot l^* \cdot (1-x^*(a)))}{u'(a \cdot l^* \cdot x^*(a))}\Big) = 1 \tag{10}$$

Knowing the shape of the optimal consumption propensity as a function of the labor supply, $x^*(a,l)$, we can obtain the optimal labor supply by maximizing indi-

[3]Following the notation introduced by Enders and Lapan (1982): $x(a,l)$ is the generic consumption propensity as function of a generic labor supply; $x^*(a,l)$ is the optimal consumption propensity as function of a generic labor supply; and $x^*(a) = x^*(a,l^*)$ is the optimal consumption propensity as function of the optimal labor supply, l^*.

[4]By (9), the sign of

$$\frac{dx^*(a)}{da} = \frac{(1-\sigma)\cdot(1-x^*(a))}{x^*(a)+\sigma\cdot(1-x^*(a))} \cdot \frac{x^*(a)}{a}$$

depends on $1-\sigma$.

vidual's expected utility with respect to l

$$\max_{l} u(1-l) + E_{a,a_+}\left(u(a \cdot l \cdot x^*(a,l)) + \lambda \cdot u\left(\frac{a \cdot l \cdot (1 - x^*(a,l))}{1 + \pi(a, a_+)}\right)\right) \tag{11}$$

By the intertemporal independence of productivity levels and by the Envelope Theorem, the first order condition solving the program is

$$u'(1-l^*) = \tag{12}$$

$$= E_a(u'(a \cdot l^* \cdot x^*(a)) \cdot a \cdot x^*(a) + \lambda \cdot u'(a \cdot l^* \cdot (1 - x^*(a)) \cdot a \cdot (1 - x^*(a)))$$

Can the government improve the social welfare? In stochastic overlapping generations models, the working (future retired) generation cannot write a contract with the unborn (future working) generation to pool the risks of productivity shocks. This creates a scope for government intervention to maximize the ex ante welfare of the representative agent in the economy. Considering that productivity levels are intertemporally independent, a fully powerful and benevolent social planner would maximize the ex ante expected utility of individuals

$$\max_{l, \{x(a,l)\}_{a \in [0,\bar{a}]}} u(1-l) + E_a(u(a \cdot l \cdot x(a,l)) + \lambda \cdot u(a \cdot l \cdot (1 - x(a,l)))) \tag{13}$$

choosing l and $x(a,l)$ for all a. The first order condition of (13) with respect to l replicates the solution chosen by individuals that take the labor-leisure decision without observing the productivity shock. Conversely the first order condition with respect to the state-contingent consumption propensity is

$$u'(a \cdot l^* \cdot x^*(a)) = \lambda \cdot u'(a \cdot l^* \cdot (1 - x^*(a)))$$

for all a, that implies the *first-best intergenerational risk-sharing condition*

$$\lambda \cdot \frac{u'(a \cdot l^* \cdot (1 - x^*(a)))}{u'(a \cdot l^* \cdot x^*(a))} = 1 \tag{14}$$

for all a. The social-planner's solution can be implemented by the government through a *state-contingent social security* scheme[5], where contributions and benefits are contingent to the current productivity level (and the budget is balanced): $\tau_t = \tau(a_t)$ and $r_t = \tau(a_t) \cdot a_t \cdot l_t$.

3.2 Fiscal Policy Regimes

Real world social security schemes hardly implement state-contingent programs. However, the economic literature has pointed out that also simple tax-and-benefit rules determine ex ante Pareto-improvements (due to enhanced intergenerational risk-sharing) with respect to the laissez-faire general equilibrium (Gottardi and Kubler, 2006). In particular, in the model we consider a simple *flat-rate social security program*, characterized by a fixed contribution rate, τ, and a transfer to the old equal to tax revenues, $r = \tau \cdot a$, is welfare improving with respect to the laissez-faire equilibrium (Enders and Lapan, 1982). With a flat rate social security the price ratio (6) becomes

$$\frac{p_t}{p_{t+1}} = \frac{a_{t+1} \cdot l_{t+1} \cdot (1 - \tau - x_{t+1})}{a_t \cdot l_t \cdot (1 - \tau - x_t)} \tag{15}$$

[5] This statement is rather intuitive: at the optimum, the government chooses tax rates inducing non-monetary stationary equilibria (hence, no further individual savings, i.e. $x(a) = 1 - \tau(a)$ for all $a \in [0, \bar{a}]$). The proof is not crucial for our main argument, therefore it is omitted. However, the author is available to provide it upon request.

Of course, if the social security tax rate is sufficiently high, then the only general equilibrium is such that no one save and buys fiat money and public bonds. In the following, we will focus on social security programs compatible with monetary stationary equilibria, in which individuals save and trade financial assets among them and with the government.

The flat rate social security, by construction, balances the public budget at each time. Let us now consider a *reform* of the flat rate social security involving a linear benefit generically defined as $r = l \cdot (\tau \cdot a + \alpha \cdot (a-1) + \beta)$. We remark that - for at least one of the two parameters, α or β, different from zero - such a social security program is generically *unbalanced* in each period, given that the revenue side of the program is unchanged and pension benefits can be different from revenues. Namely, with $\alpha \neq 0$, also assuming $\beta = 0$, the social security budget is unbalanced only if $a \neq 1 = E_a(a)$; moreover, the social security budget is balanced in expectation (and, hence, in the long run) only if $\beta = 0$.

Under unbalanced social security, the price ratio (6) becomes

$$\frac{p_t}{p_{t+1}} = \frac{a_{t+1} \cdot (1 - \tau - x_{t+1}) - \alpha \cdot (a_{t+1} - 1) - \beta}{a_t \cdot (1 - \tau - x_t)} \cdot \frac{l_{t+1}}{l_t} \tag{16}$$

Now the social security program involves, in each period, the variation of the outstanding public debt: whenever $\alpha \cdot (a-1) + \beta > 0$ (or $\alpha \cdot (a-1) + \beta < 0$) the nominal public debt increases (or decreases), thus determining inflation (or deflation). The intuition is that now the government need to finance the social security budget by issuing substitutes of fiat money thus changing the price level. Under the unbalanced social security program, the first order condition with respect to the consumption propensity $x(a, l^*)$ of the individual's optimization program becomes

$$\begin{aligned} u'(a \cdot l^* \cdot x^*(a)) \cdot a \cdot (1 - \tau - x^*(a)) = \\ \lambda \cdot E_{a_+}(u'(a_+ \cdot l^* \cdot (1 - x^*(a_+))) \cdot a_+ \cdot (1 - \tau - x^*(a_+))) + \\ -\lambda \cdot E_{a_+}(u'(a_+ \cdot l^* \cdot (1 - x^*(a_+))) \cdot (a_+ - 1)) \cdot \alpha + \\ -\lambda \cdot E_{a_+}(u'(a_+ \cdot l^* \cdot (1 - x^*(a_+)))) \cdot \beta \end{aligned} \tag{17}$$

for all $a \in [0, \bar{a}]$.

To assess the optimality of the unbalanced social security program, we consider the effect on ex ante individual utility of a marginal change of α

$$E_a\Big((u'(a \cdot l^* \cdot x^*(a)) - \lambda \cdot u'(a \cdot l^* \cdot (1 - x^*(a)))) \cdot a \cdot \frac{dx^*(a)}{d\alpha}\Big) \tag{18}$$

and β

$$E_a\Big((u'(a \cdot l^* \cdot x^*(a)) - \lambda \cdot u'(a \cdot l^* \cdot (1 - x^*(a)))) \cdot a \cdot \frac{dx^*(a)}{d\beta}\Big) \tag{19}$$

in the neighborhood of the flat-rate social security program (say, of $\alpha = 0$ and $\beta = 0$). Thus, we have

Proposition 1 *For any level of the social security tax rate, compatible with a monetary stationary equilibrium, there is a pension benefit rule involving budget unbalance that Pareto-dominates the budget-balancing one. Moreover, under the optimal pension benefit rule, deficit increases with productivity.*

Proof. See the Appendix. ■

The intuition of the Proposition 1 is rather simple. An unbalanced social security program improves on the flat-rate one by introducing some flexibility in the way current production is divided between the two generations. This flexibility increases the capacity of the social security to adapt to different contingencies. The unbalanced social security affords this result by two different channel: the basic production sharing rule is determined by the fixed tax rate; then, on the basis of current productivity realization, the real pension paid to old individuals increases above revenues from social contributions, when productivity is above its average, and decreases below contribution revenues, when productivity is below its average.

An important issue to understand the nature of our result is the following: who finances the social security scheme when it is actually unbalanced? The benchmark is the balanced social security which fully operates when the productivity is equal to the average. In such a case, pension benefits are fully paid by the young as in usual PAYG schemes. When the productivity is above (or below) the average, the old obtain pension benefits higher (or lower) than tax revenues. This feature of the unbalanced social security program drives the enhancement of intergenerational risk-sharing with respect to the basic production sharing rule, underlying the flat-rate social security program. However, the increase (or reduction) of pension benefits above (or below) tax revenues, which cannot be implemented by changing the tax rate, is actually financed by a net issue (or buy-back) of government bonds. The effect of the variation of the outstanding nominal public debt is to depress (or support) the rate of return of savings when the productivity is high (or low) thus compensating via the price effect of the public debt the unbalance in the social security budget. In other terms, when productivity is high the old finance the entire increase of pension benefits by an inflation tax; conversely, when productivity is low the reduction of pension benefits finances a deflation subsidy which accrue to old individuals (holding financial assets). Moreover, the gain or losses obtained by the old via the unbalanced pension scheme, as compared to the benchmark balanced one, are perfectly compensated by the inflation tax or deflation subsidy.

Why the unbalanced social security is optimal? As it is apparent by the above discussion the reason is not related to the way the unbalanced social security program affects the net transfer that the young pay to the old. The reason is that the debt management underlying the unbalanced pension program and the way it affects the price level is an effective mechanism to stabilize the rate of return of private savings, which in turn (as it is clear by the proof of Proposition 1 in the Appendix) depresses the optimal consumption propensity for any realization of the current productivity. In the simple economy we consider, abstracting from capital accumulation, the only effective way to increase consumption of old individuals is by raising savings of young: either by taxing the young or by creating an incentive (financed by a non-distorting tax on financial assets) for them to save.

A similar rationale underlies the next result

Proposition 2 *For any level of the social security tax rate, compatible with a monetary stationary equilibrium, there is a pension benefit rule involving a long run surplus of the social security budget that Pareto-dominates the ones balancing the budget in expectation.*

Proof. See the Appendix ■

The reason why the laissez-faire equilibrium is unable to achieve a (possibly second-best) optimal intergenerational risk sharing, thus making relevant the government role in this respect, is that private savings are too much depressed. Besides the flat-rate social security, one way to improve ex ante efficiency, highlighted by Proposition 1, is to stabilize the rate of return variability. The other is to introduce a premium for savings in the form of a positive real interest rate that is paid to savings on average (Proposition 2). In the simple setting we are considering, this can be done by means of a monetary policy introducing a small and steady deflation or, equivalently, by means of an optimal debt management characterized by a steady negative growth of the outstanding nominal public debt, which in turn implies a long-run surplus of the social security program.

4 Conclusions

We considered a simple economy where, abstracting from any demographic and economic dynamics but exogenous shocks to labor productivity, individuals live two periods: the young work, save, and contribute to the social security, while the old are retired and consume the income from private savings and from social security benefit. In this setting, the laissez faire equilibrium does not implement first-best intergenerational risk-sharing, because each generation contracts with others only after discovering the state of the world. As stressed by the general equilibrium literature, social security schemes can improve the ex ante utility of all generations, though first-best intergenerational risk sharing is still not reachable given that such programs are never sufficiently flexible to be fully state-contingent (Gottardi and Kubler, 2006). In particular, in our setting, a simple social security program, taxing with a constant rate the labor income and transferring revenues to retirees, improves the ex ante utility of all individuals (Enders and Lapan, 1982). Considering the interplay of different intertemporal fiscal policies, namely social security and public debt management, and their effect on the price level (and on the rate of returns of financial assets), we showed that, in general, unbalanced social security programs, relying on a fixed contribution rate, may improve on balanced pension schemes. This result is driven by the introduction of an indirect channel of intergenerational risk sharing passing through the determination of the price level and, in particular, the stabilization and the increase of the rate of return of financial assets. Our result supports the view, already stressed by other contributions, that the scope for social security schemes should be assessed taking into consideration their effects on asset prices (Campbell and Nosbusch, 2007, p. 2267). In this perspective, our contribution adds the idea that the government may partially control these effects through other intertemporal fiscal policy tools.

Appendix

Proof of Proposition 1. (18) is equivalent to

$$E_a\Big(1-\lambda\cdot\frac{u'(a\cdot l^*\cdot(1-x^*(a)))}{u'(a\cdot l^*\cdot x^*(a))}\Big)\cdot E_a\Big(u'(a\cdot l^*\cdot x^*(a))\cdot a\cdot\frac{dx^*(a)}{d\alpha}\Big)+$$
$$-Cov\Big(\lambda\cdot\frac{u'(a\cdot l^*\cdot(1-x^*(a)))}{u'(a\cdot l^*\cdot x^*(a))},u'(a\cdot l^*\cdot x^*(a))\cdot a\cdot\frac{dx^*(a)}{d\alpha}\Big)$$

and, by (17), in the neighborhood of $\alpha=\beta=0$ equal to

$$-Cov\Big(\lambda\cdot\frac{u'(a\cdot l^*\cdot(1-x^*(a)))}{u'(a\cdot l^*\cdot x^*(a))},u'(a\cdot l^*\cdot x^*(a))\cdot a\cdot\frac{dx^*(a)}{d\alpha}\Big) \tag{20}$$

By the assumption of constant relative risk aversion, the derivative of $\frac{u'(a\cdot l^*\cdot(1-x(a)))}{u'(a\cdot l^*\cdot x(a))}$ with respect to a has the sign of $d_a x^*(a)$ (that, in turn, depends on $1-\sigma$). By the first order condition (17), it follows that the derivative of $u'(a\cdot l^*\cdot x^*(a))\cdot a\cdot\frac{dx^*(a)}{d\alpha}$ with respect to a has the sign of $u'(a\cdot l^*\cdot x^*(a))\cdot a\cdot\frac{dx^*(a)}{d\alpha}\cdot d_a x^*(a)$. Manipulating again (17), it follows that $u'(a\cdot l^*\cdot x^*(a))\cdot a\cdot\frac{dx^*(a)}{d\alpha}$ has the sign of $\lambda\cdot E_{a_+}(u'(a_+\cdot l^*\cdot(1-x^*(a_+)))\cdot(a_+-1))<0$ for any $a\in[0,\bar{a}]$. Therefore, (20) is positive, hence $d\alpha^*>0$ and the proposition follows. ■

Proof of Proposition 2. In the neighborhood of $\alpha=\beta=0$, (19) is equivalent to

$$-Cov\Big(\lambda\cdot\frac{u'(a\cdot l^*\cdot(1-x^*(a)))}{u'(a\cdot l^*\cdot x^*(a))},u'(a\cdot l^*\cdot x^*(a))\cdot a\cdot\frac{dx^*(a)}{d\beta}\Big) \tag{21}$$

By the first order condition (17), it follows that the derivative of $u'(a\cdot l^*\cdot x^*(a))\cdot a\cdot\frac{dx^*(a)}{d\beta}$ with respect to a has the sign of $u'(a\cdot l^*\cdot x^*(a))\cdot a\cdot\frac{dx^*(a)}{d\beta}\cdot d_a x^*(a)$. By (17),

$u'(a \cdot l^* \cdot x^*(a)) \cdot a \cdot \frac{dx^*(a)}{d\beta}$ has the sign of $\lambda \cdot E_{a_+}(u'(a_+ \cdot l^* \cdot (1 - x^*(a_+))) > 0$ for any $a \in [0, \bar{a}]$. Therefore, (21) is positive, hence $d\beta^* < 0$ and the proposition follows. ■

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