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AN EQUILIBRIUM MODEL OF HABITAT CONSERVATION
UNDER UNCERTAINTY AND IRREVERSIBILITY

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December 2010

“MARCO FANNO” WORKING PAPER N.122

An Equilibrium Model of Habitat Conservation under Uncertainty and Irreversibility*

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December 3, 2010

Abstract

In this paper stochastic dynamic programming is used to investigate habitat conservation by a multitude of landholders under uncertainty about the value of environmental services and irreversible development. We study land conversion under competition on the market for agricultural products when voluntary and mandatory measures are combined by the Government to induce adequate participation in a conservation plan. We analytically determine the impact of uncertainty and optimal policy conversion dynamics and discuss different policy scenarios on the basis of the relative long-run expected rate of deforestation. Finally, some numerical simulations are provided to illustrate our findings.

KEYWORDS: OPTIMAL STOPPING, DEFORESTATION, PAYMENTS FOR ENVIRONMENTAL SERVICES, NATURAL RESOURCES MANAGEMENT.

JEL CLASSIFICATION: C61, D81, Q24, Q58.

1 Introduction

As human population grows, the human-Nature conflict has become more severe and natural habitats are more exposed to conversion. On the one hand, clearing land to develop it may lead to the irreversible reduction or loss of valuable environmental services (hereafter, ES) such as biodiversity conservation, carbon sequestration, watershed control and provision of scenic beauty for recreational activities and ecotourism. On the other hand, conserving land in its pristine state has a cost opportunity in terms of foregone profits from economic activities (e.g. agriculture, commercial forestry) which can be undertaken once land has been cleared.

By balancing marginal social benefit and cost of conservation, the social planner is required to destine the available land to conservation or development which are usually two competing and mutually exclusive uses. Despite its theoretical appeal, the idea of a social planner who, having defined a socially optimal habitat conversion rule, can implement it by simply commanding the constitution of protected areas, is far from reality. In fact, since the majority of remaining ecosystems are on land privately owned, the economic and political cost of such intervention would make the adoption of command mechanisms by Governments unlikely (Langpap and Wu, 2004; Sierra and Russman, 2006). In addition, as pointed out by Folke et al.(1996, p. 1019), "*keeping humans out of nature through a protected-area strategy may buy time, but it does not address the factors in society driving the loss of biodiversity*". In other words, protecting natural ecosystems through natural reserves and other protected areas may be a significant step in the short-run to deal with severe and immediate threats but it still does not create the structure of incentives able to mitigate the conflict human-Nature in the long-run.

At least initially, Governments favoured an indirect approach in conservation policies. The main idea behind this approach was to divert, through programs such as integrated conservation and development

*We wish to thank Guido Candela for helpful comments. We are also grateful for comments and suggestions to participants at the 12th International BIOECON and 51st SIE conferences and to seminars at FEEM & IEFE - Bocconi University and at Dept of Economics Seminars, University of Stirling. The usual disclaimer applies.

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projects, community-based natural resource management or other environment-friendly commercial ventures, the allocation of labour and capital from ecosystem damaging activities toward ecosystem conserving activities (Wells et al., 1992; Ferraro and Simpson, 2002). However, despite the initial enthusiasm, effectiveness and cost-efficiency concerns have led to abandonment of this approach in favour of compensations to be paid directly to the landholders providing conservation services (see e.g. Ferraro, 2001; Ferraro and Kiss, 2002; Ferraro and Simpson, 2005). A direct approach, mainly represented by schemes like Payments for environmental services (hereafter, PES) has become increasingly common in both developed and developing countries. Under a PES program, a provider delivers to a buyer a well-defined ES (or corresponding land use) in exchange for an agreed payment.¹ Unfortunately, also the efficacy of PES programs has been questioned since their performance has not always met the established conservation targets.² In particular, lack of additionality in the conservation efforts induced by the programs has often been suspected.³ In other words, it seems that in practice landholders have been practically paid for conserving the same extent of land they would have conserved without the program. Considering the limited amount of money for conservation initiatives and the perverse effect that wasting it may have on future funding, further research is needed to increase our understanding of the economic agent's conversion decision.

The literature investigating optimal conservation decisions under irreversibility and uncertainty over the net benefits attached to conservation represents a significant branch of environmental and resource economics (Bulte et al., 2002, Kassar and Lassere, 2004; Leroux et al., 2009). A unifying aspect in this literature is the stress on the effect that irreversibility and uncertainty have on decision making. In fact, since irreversible conversion under uncertainty over future prospects may be later regretted, this decision may be postponed to benefit from option value attached to the maintained flexibility (Dixit and Pindyck, 1994). Pioneer papers such as Arrow and Fisher (1974) and Henry (1974) have been followed by several other contributions which have improved the modelling effort and solved the technical problems posed by increasingly complex model set-up.⁴ Two contributions close to ours are Bulte et al. (2002) and Leroux et al. (2009). In the first paper, the authors determine the optimal forest stock to be held by trading off profit from agriculture and the value of ES attached to forest conservation. Their analysis highlights the value of the option to postpone land clearing under irreversibility of environmental impact and uncertainty about conservation benefits. A similar problem is solved in Leroux et al. (2009) where, unlike the previous paper, the authors allow for ecological feedback and consider its impact both on the expected trend and volatility of ES value.

Both papers, however, by solving the allocative problem from a central planner perspective, miss the complexity of challenges characterizing conservation policies and the role that competition on markets for agricultural products may have on conversion decisions.

In this paper, we aim to investigate these issues by modelling conversion decisions in a decentralized economy populated by a multitude of homogenous landholders and where the Government has introduced a payment scheme for conservation. Each landholder manages a portion of total available land and may conserve or develop it by affording some conversion cost. If land is conserved, ES have a value proportional to the area conserved which randomly fluctuates following a geometric Brownian motion. If the parcel is developed, land enters as an input into the production of goods or services (coffee, rubber, soy, palm oil, timber, biofuels, cattle, etc.) and the farmer must compete with other farmers on the market. In this context, the Government introduces a land use policy which aims to balance conservation and development.

The policy is based on a PES scheme implemented through a conservation contract establishing different requirements and payments before and after land conversion has occurred. In particular, if the entire plot is conserved, then the landholder receives a certain payment whereas if he/she decides to clear it, then he/she must set aside the portion indicated in the contract (i.e. the plot may be only partially developed) and receives a different payment. Finally, we also consider the possibility that the Government may impose a

¹In this respect we follow Wunder (2005, p. 3) where a PES is defined as "(i) a voluntary transaction where (ii) a well-defined ES (or a land-use likely to secure that service) (iii) is being "bought" by a (minimum one) ES buyer (iv) from a (minimum one) ES provider (v) if and only if the ES provider secures ES provision (conditionality)".

²As reported by Ferraro (2001), this may be due to several reasons such as lack of funding, failures in institutional design, poor definition and weak enforcement of property rights and strategic behaviour by potential ES providers. See Ferraro (2008) on information failures and Smith and Shogren (2002) on specific contract design issues.

³We refer in particular to government-financed programs. On the performance of user vs. government-financed interventions see Pagiola (2008) on PSA program in Costa Rica and Wunder et al. (2008) for a comparative analysis of PES programs in developed and developing countries. See Ferraro and Pattanayak (2006) for a call on empirical monitoring of conservation programs.

⁴Among them, see for instance Conrad (1980), Clarke and Reed (1989), Reed (1993), Conrad (1997), Conrad (2000).

limit on the total forested land which can be cleared.

Under this conservation program we solve for the conversion path taking a real option approach but unlike from previous literature we internalize the role of market entry dynamics. It follows that under competition the conversion path must be determined on the basis of a long-run zero profit condition. In this respect, it becomes interesting to study the impact that different payment scheme may have on the conversion dynamic. We then allow for two different PES schemes. Under the first one, the payment rate for land unit is more generous when the entire plot is conserved while in the second scheme the opposite is proposed. Both situations may arise in reality, reflecting different sensitivity towards conservation or, more practically, the way Governments try to adapt to the economic and political framework they face.⁵

Under both schemes, we are able to determine analytically the optimal conversion rules. Not surprisingly, conversion is postponed if landholders conserving the entire plot receive a higher payment. This is due to the higher cost opportunity of conversion which is higher since it includes the payments implicitly given up converting. Interestingly, we show that, as suggested by Ferraro (2001), a landholder may conserve the entire plot even if partially compensated for the provided ES. Moreover, under this payment design, only progressive reductions in the payments trigger land clearing and, at the end, landholders may clear a surface smaller than the one targeted by the Government. With the second payment design, i.e. higher transfer if land is developed, the structure of incentives is reversed and an implicit bias toward conversion is introduced. An important mass of landholders rapidly convert land till the last plot where profit from agriculture covers the conversion cost while the rest prefers to wait and clear the plots only if payments rise. Under both PES designs, we note that, by setting a limit to the land surface that landholders may develop together, the Government may induce rush in the conversion dynamics.⁶ In fact, landholders, fearing a restriction in the exercise of the option to convert, may start a conversion run which rapidly exhausts the entire forest stock up to the fixed limit.⁷

Finally, to assess the temporal performance of the optimal conservation policy and study the impact of increasing uncertainty about future environmental benefits on conversion speed, we rearrange the optimal conversion rules in the form of regulated processes (Harrison, 1985, chp. 2) and derive the long-run average growth rate of deforestation. Hence, under different policy scenarios, we use the rate of deforestation to rank different policies on the basis of current optimal forest stock and expected total conversion time. Interestingly, we show that uncertainty about payments, even if it induces conversion postponement in the short-run, reduces expected time for total conversion in the long run.

We then use it to illustrate, through several numerical simulations, optimal forest conversion in Costa Rica.⁸ We find that when landholders conserving the whole plot are offered a higher payment with respect to the ones developing, then higher uncertainty over payments increases the long-run average rate of conversion. The opposite occurs when the policy rewards more generously farmers conserving only a portion of their plot.

The remainder of the paper is organized as follows. In Section 2 the basic set-up for the model is presented. In Section 3 we study the equilibrium in the conversion strategies under two policy scenarios. In Section 4, we discuss issues related to the PES voluntary participation and contract enforceability. Section 5 is devoted to the derivation of the long-run average rate of conversion. In Section 6 we illustrate our main findings through numerical exercises. Section 7 concludes.

⁵For instance, if forest conservation does not qualify under the CDM (Clean Development Mechanism) while reforestation does, then it may be plausible for a Government to push towards timber harvest and subsequent reforestation on some land in order to cash funding on carbon markets and finance conservation on the remaining habitat. Note that this was actually the case in the first commitment period (2008-2012) (IPCC, 2007).

⁶In Australia, the Productivity Commission reports evidence of pre-emptive clearing due to the introduction of clearing restrictions (Productivity Commission, 2004). On unintended impacts of public policy see for instance Stavins and Jaffe (1990) showing that, despite an explicit federal conservation policy, 30% of forested wetland conversion in the Mississippi Valley has been induced by federal flood-control projects. In this respect, see also Mæstad (2001) showing how timber trade restrictions may induce an increase in logging.

⁷A similar effect has been firstly noted by Bartolini (1993). In this paper, the author studies decentralized investment decision in a market where a limit on aggregate investment is present.

⁸Unlike Leroux et al. (2009) who exogenously assume a maximum annual conversion rate (2.5% in the case of Costa Rica forests), we calculate it optimally on the basis of land currently converted and information on current and future payments.

2 A Dynamic Model of Land Conversion

Consider a country where at time period $t = 0$ the total land available, L , is allocated as follows:

$$L = A_0 + F \tag{1}$$

where A_0 is the surface cultivated and F is the portion still in its pristine natural state covered by primary forest.⁹ Assume that F is divided into small and homogenous parcels of equal extent held by a multitude of identical risk-neutral agents.¹⁰ By normalizing such extent to 1 hectare, F denotes also the number of agents in the economy.¹¹ Natural habitats provide valuable environmental goods and services at each time period t .¹² Let $B(t)$ represent the per-parcel value of such goods and services and assume it randomly fluctuates according to the following geometric Brownian motion:

$$\frac{dB(t)}{B(t)} = \alpha dt + \sigma dz(t), \quad \text{with } B(0) = B \tag{2}$$

where α and σ are respectively the drift and the volatility parameters, and $dz(t)$ is the increment of a Wiener process.¹³

At each t , two competitive and mutually exclusive destinations may be given to forested land: conservation or irreversible development. Once the plot is cleared, the landholder becomes a farmer using land as an input for agricultural production (or commercial forestry).¹⁴

2.1 The Government

To induce conservation the Government offers to each agent a contract to be accepted on a voluntary basis. A compensation equal to $\eta_1 B(t)$ with $\eta_1 \in [0, 1]$ is paid at each time period t if the entire plot is conserved.¹⁵ On the contrary, if the landholder aims to develop his/her parcel, a restriction is imposed in that a portion of the total surface, $0 \leq \lambda \leq 1$, must be conserved.¹⁶ In this case, a payment equal to $\lambda \eta_2 B(t)$ with $\eta_2 \in [0, 1]$ may be offered to compensate the landholder.¹⁷ Since ES usually have the nature of public good, payment rates, η_1 and η_2 , may be interpreted as the levels of appropriability that the society is willing to

⁹As in Bulte et al. (2002) A_0 may represent the best land which has been converted to agriculture.

¹⁰At the moment for the sake of generality we refer to landholders. Later we will discuss the implications of our model with respect to property rights issues.

¹¹None of our results relies on this assumption. In fact, provided that no single agent has significant market power, we can obtain identical results by allowing each agent to own more than one unit of land. See e.g. Baldursson (1998) and Grenadier (2002).

¹²They may include biodiversity conservation, carbon sequestration, watershed control, provision of scenic beauty for recreational activities and ecotourism, timber and non-timber forest products. See e.g. Conrad (1997), Conrad (2000), Clarke and Reed (1989), Reed (1993), Bulte et al. (2002).

¹³The Brownian motion in (2) is a reasonable approximation for conservation benefits and we share this assumption with most of the existing literature. Conrad (1997, p. 98) considers a geometric Brownian motion for the amenity value as a plausible assumption to capture uncertainty over individual preferences for amenity. Bulte et al. (2002, p.152) point out that "parameter α can be positive (e.g., reflecting an increasingly important carbon sink function as atmospheric CO2 concentration rises), but it may also be negative (say, due to improvements in combinatorial chemistry that lead to a reduced need for primary genetic material)". However, this assumption neglects the direct feedback effect that conversion decisions may have on the stochastic process illustrating the dynamic of conservation benefits. See Leroux et al. (2009) for a model where such effect is accounted by letting conservation benefits follow a controlled diffusion process with both drift and volatility depending on the conversion path.

¹⁴In the following, "landholder" refers to an agent conserving land and "farmer" to an agent cultivating it.

¹⁵As pointed out by Engel et al. (2008), by internalizing external non-market values from conservation, PES schemes have attracted increasing interest as mechanisms to induce the provision of ES.

¹⁶Note that our analysis is general enough to include also the case where λ is not imposed but is endogenously set by each landholder. In fact, due for instance to financial constraints limiting the extent of the development project, the landholders may find optimal not to convert the entire plot.

¹⁷Note that our contract scheme is in line with Ferraro (2001, p. 997) where the author states that conservation practitioners "may also find that they do not need to make payments for an entire targeted ecosystem to achieve their objectives. They need to include only "just enough" of the ecosystem to make it unlikely, given current economic conditions, infrastructure, and enforcement levels, that anyone would convert the remaining area to other uses". In addition, taking a different perspective, our framework seems supported also by wildlife protection programs which rarely pay farmers more than a fraction of the losses due to wildlife (See Rondeau and Bulte, 2007).

guarantee on the value generated by conserving, i.e. $B(t)$ and $\lambda B(t)$ respectively.¹⁸ In addition, besides λ the Government fixes an upper level \bar{A} on total land conversion. This limit may preclude land development for some landholders. The number of landholders for whom conserving the entire plot may become compulsory depends on the magnitude of λ . In fact, note that λ may be low enough to allow every landholder to clear land. However, the definition of λ does not need to meet such requirement since other issues may be prioritized, i.e. habitat fragmentation, critical ecological thresholds, enforcement and transaction costs for the program implementation, etc. Thus, denoting by $\bar{N} = \frac{\bar{A}}{1-\lambda}$ the number of potential farmers involved in the conversion process, we assume $\bar{N} \leq F$.

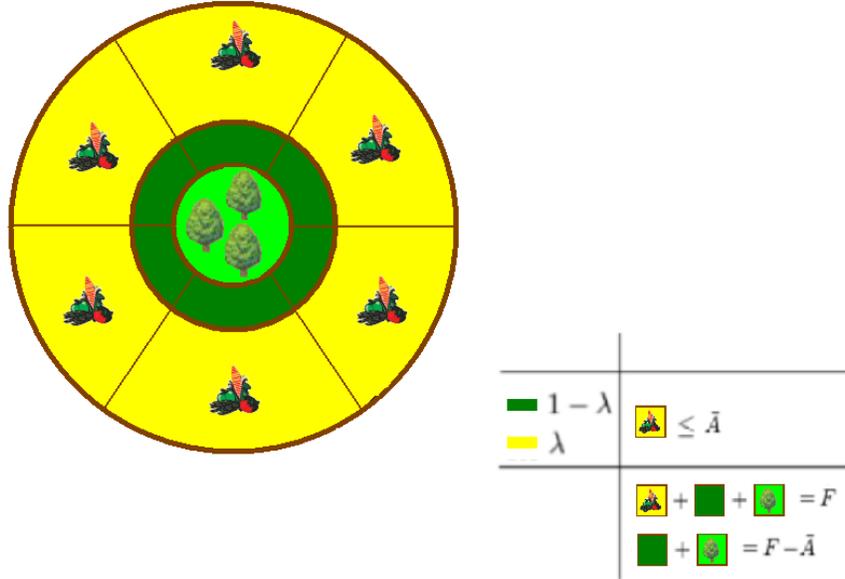


Figure 1: *Land conversion with buffer areas*

Our framework is general enough to include different conservation targets such as old-growth forests or habitat surrounding wetlands, marshes, lagoons or by the marine coastline and meet several spatial requirements. For instance, the conservation target may be represented by an area divided into homogenous parcels running along a river or around a lake or a lagoon where, to maintain a significant provision of ecosystem services, a portion of each parcel must be conserved (see figure 1). As stressed by the literature in spatial ecology, the creation of buffer areas, by managing the proximity of human economic activities, is crucial since it guarantees the efficiency of conservation measures in the targeted areas.¹⁹ In this case the conservation program may be induced by implementing a payment contract schedule differentiating for the state of land i.e. totally conserved vs. developed within the restriction enforced through environmental law. However, we are also able to consider the opposite case where the landholder may totally develop his/her plot but an upper limit is fixed on the total extent of land which can be cleared in the region.²⁰

2.2 The Landholders

Developing the parcel is an irreversible action which has a sunk cost, $(1 - \lambda)c$, including cost for clearing and settling land for agriculture.²¹ Denoting by $A(t)$ the total land developed at time t , the number of farmers

¹⁸As η_1 and η_2 are constant, payments also follow a geometric Brownian motion (easily derivable from (2)). However, this is different from the way payments are modelled in Isik and Yang (2004) where they also depend on the fluctuations in the conservation cost opportunity (profit from agriculture, changes in environmental policy, etc.).

¹⁹See for instance Hansen and Rotella (2002) and Hansen and DeFries (2007).

²⁰This could be the case for an area covered by a tropical forest (Bulte et al., 2002; Leroux et al., 2009), or a protected area where farmers located next to the site may sustainably extract natural resources (Tisdell, 1995; Wells et al., 1992).

²¹Bulte et al., (2002, p. 152) define c as "the marginal land conversion cost". It "may be negative if there is a positive one-time net benefit from logging the site that exceeds the costs of preparing the harvested site for crop production". We also

must be equal to $N(t) = \frac{A(t)}{1-\lambda}$ and since $1 - \lambda$ is fixed, the conversion dynamic must mirror the variation in the number of farmers, i.e. $dN(t) = \frac{dA(t)}{1-\lambda}$. Therefore, assuming that the extent of each plot is small enough to exclude any potential price-making consideration, we may use either $N(t)$ or $A(t)$ when evaluating the individual decision process.²² Competition on the market for agricultural products implies that at each time period t the optimal number of farmers (or the optimal total land developed) is determined by the entry zero profit condition. In addition, since the per-parcel value of services, $B(t)$, makes all agents symmetric, some random mechanism must be used to select which landholder develops first.

We assume a constant elasticity demand function for agricultural products. Since supply depends on the surface cultivated, then let demand be specified as $P_A(t) = \delta A(t)^{-\gamma}$ with $A(0) = A_0 (> 1)$, where δ is a parameter illustrating different positions of the demand and $-\gamma$ is the inverse of the demand elasticity.

Now, let's solve for the conversion process taking η_1 , η_2 and λ as exogenously given parameters. Denoting by $P_A(t)$ the marginal return as land is cleared over time, the farmer instantaneous profit function is given by:

$$\pi(A(t), B(t); \bar{A}) = (1 - \lambda)P_A(t) + \lambda\eta_2 B(t) \quad (3)$$

The discounted present value of the net benefits over an infinite horizon is:²³

$$\begin{aligned} E_0 \left[\int_0^t \eta_1 B(t) e^{-rt} dt + \int_t^\infty \pi(A(t), B(t); \bar{A}) e^{-rt} dt \right] = \\ = \frac{\eta_1 B}{r - \alpha} + E_0 \left[\int_t^\infty \Delta\pi(A(t), B(t); \bar{A}) e^{-r(t-t)} dt \right] \end{aligned} \quad (4)$$

where r is the constant risk-free interest rate,²⁴ $\Delta\pi(A(t), B(t); \bar{A}) = (1 - \lambda)P_A(t) + (\lambda\eta_2 - \eta_1)B(t)$ and t is the stochastic conversion time.²⁵

In (4) the first term represents the perpetuity paid by the Government if the parcel is conserved forever, while the second term represents the extra profit that each landholder may expect if she/he clears the land and becomes a farmer. The extra profit is, given by the crop yield sold on the market plus the difference in the payments received by the Government. As soon as the excess profit from land development equals the deforestation cost the landholder may clear the parcel. This implies that the optimal conversion timing depends only on the second term in (4).

assume, without loss of generality, that the conversion cost is proportional to the surface cleared.

²²To consider infinitesimally small agents is a standard assumption in infinite horizon models investigating dynamic industry equilibrium under competition. See for instance Jovanovic (1982), Dixit (1989), Hopenhayn (1992), Lambson (1992), Dixit and Pindyck (1994, chp. 8), Bartolini (1993), Caballero and Pindyck (1996), Dosi and Moretto (1992) and Moretto (2008).

²³See Harrison (1985, p. 44).

²⁴The introduction of risk aversion does not change the results since the analysis can be developed under a risk-neutral probability measure for $B(t)$. See Cox and Ross (1976) for further details.

²⁵Note that the expected value is taken accounting for $A(t)$ increasing over time as land is cleared.

3 The Competitive Equilibrium

Denote by $V(A(t), B(t); \bar{A})$ the value function of an infinitely living farmer.²⁶ By (4), the optimal conversion time, τ , solves the following maximization problem:²⁷

$$V(A, B; \bar{A}) = \max_{\tau} E_0 \left[\int_0^{\infty} \Delta\pi(A, B; \bar{A}) e^{-rt} dt - I_{[t=\tau]}(1-\lambda)c \right] \quad (5)$$

where $I_{[t=\tau]}$ is an indicator function and the expectation is taken considering that the total land developed A may vary over time. The indicator function states that, due to competition among farmers on the market, at the time of conversion the value from converting land must equal the cost of land clearing. In the real option literature the problem we must solve is referred to as "optimal stopping" (Dixit and Pindyck, 1994). The idea is that at any point in time the value of immediate investment (stopping) is compared with the expected value of waiting dt (continuation), given the information available at that point in time (the value of the stochastic variable B and the stock of land developed, A) and the knowledge of the two processes. If the initial size of the active farmers is $A \geq A_0$, we expect the converting process to work as follows: for a fixed number of farmers, profits in (3) move stochastically driven by B . As soon as the per-parcel value of ES reaches a critical level, say B^C , development (i.e. entry into the agricultural market) becomes feasible. This implies an increase, dA , in cultivated land and a drop in revenues from agriculture along the function $P_A(A)$. The value of services will then continue to move stochastically until the next entry occurs.

In this setting the (competitive) equilibrium bounding the profit process for each farmer can be constructed as a symmetric Nash equilibrium in entry strategies. By the infinite divisibility of F , the equilibrium can be determined by simply looking at the single landholder clearing policy which is defined ignoring the competitors' entry decisions (see Leahy 1993). Consider a short interval dt where any conversion takes place. Over this interval A is constant and the farmer holds an asset paying $\Delta\pi(A, B; \bar{A})dt$ as cash flow and $E[dV(A, B; \bar{A})]$ as capital gain. If the farmer is active then the cash flow and the expected capital gain must equal the normal return, that is $rV(A, B; \bar{A})dt = \Delta\pi(A, B; \bar{A})dt + E[dV(A, B; \bar{A})]$.

Let $V(A, B; \bar{A})$ be twice-differentiable in B , and expand $dV(A, B; \bar{A})$ using Ito's Lemma. Then, in the region where non-new conversion takes place (i.e. for $B \neq B^C$), the solution to (5) must solve the following differential equation:

$$\begin{aligned} & \frac{1}{2}\sigma^2 B^2 V_{BB}(A, B; \bar{A}) + \alpha B V_B(A, B; \bar{A}) - rV(A, B; \bar{A}) + \\ & + [(1-\lambda)\delta A^{-\gamma} + (\lambda\eta_2 - \eta_1)B] = 0 \end{aligned} \quad (6)$$

This is an ordinary differential equation since the number of farmers is constant. Using standard arguments the general solution is (see Dixit and Pindyck, 1994):

$$V(A, B; \bar{A}) = Z_1(A)B^{\beta_1} + Z_2(A)B^{\beta_2} + (1-\lambda)\frac{\delta A^{-\gamma}}{r} + (\lambda\eta_2 - \eta_1)\frac{B}{r-\alpha} \quad (7)$$

where $1 < \beta_1 < r/\alpha$, $\beta_2 < 0$ are the roots of the characteristic equation $Q(\beta) = \frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - r = 0$ and Z_1, Z_2 are two constants to be determined.

3.1 Case with $\eta_1 > \lambda\eta_2$

Suppose that a lower payment is offered for conservation once land is converted, i.e. $\eta_1 > \lambda\eta_2$.²⁸ To determine the optimal conversion threshold, $B^C(A) = B^*(A)$, the landholder must consider benefits and costs attached to conversion. According to (7), the profit accruing from the crop yield, $(1-\lambda)\frac{\delta A^{-\gamma}}{r}$, is counterbalanced by the difference in the payments, $(\lambda\eta_2 - \eta_1)\frac{B}{r-\alpha}$, received for conservation. In addition, note that as landholders convert land and become farmers profit from agriculture decreases. This negative

²⁶As we show in the appendix, the problem can be equivalently solved considering a landholder evaluating the option to develop.

²⁷In the following we will drop the time subscript for notational convenience.

²⁸Note that this may occur even if $\eta_1 < \eta_2$, i.e. the payment rate per unit of land conserved is more generous when a portion of the plot has been developed. We will discuss this case in greater detail in the next section.

effect on the value of converted land is accounted for in (7) by the second term ($Z_2(A) \leq 0$ for $A \leq \bar{A}$). In fact, since $\eta_1 > \lambda\eta_2$ then only an expected reduction in B can induce conversion. This implies that to keep $V(A, B; \bar{A})$ finite we must drop the first term by setting $Z_1 = 0$, i.e. $\lim_{B \rightarrow \infty} V(A, B; \bar{A}) = 0$. Hence, (7) reduces to:

$$V(A, B; \bar{A}) = Z_2(A)B^{\beta_2} + (1 - \lambda)\frac{\delta A^{-\gamma}}{r} + (\lambda\eta_2 - \eta_1)\frac{B}{r - \alpha} \quad (8)$$

To determine $Z_2(A)$ and $B^*(A)$ some suitable boundary conditions on (8) are required. First, development by increasing the number of competing farmers in the market keeps the value of being an active farmer below $(1 - \lambda)c$. Second, marginal rents for an active farmer must be null $B^*(A)$. These considerations can be formalized by the following proposition.

Proposition 1 *Provided that each agent rationally forecasts the future dynamics of the market for agricultural goods for land to be converted the following condition must hold*

$$V(A, B^*(A); \bar{A}) = (1 - \lambda)c \quad (9)$$

where the conversion rule is

(i) if $\hat{A} \leq \bar{A}$ then

$$B^*(A) = \frac{\beta_2}{\beta_2 - 1} (r - \alpha) \Psi \left[\left(\frac{\hat{A}}{A} \right)^\gamma - 1 \right] c \quad \text{for } A_0 < A \leq \hat{A} \quad (10)$$

(ii) if $\hat{A} > \bar{A}$ then

$$B^*(A) = \begin{cases} \frac{\beta_2}{\beta_2 - 1} (r - \alpha) \Psi \left[\left(\frac{\hat{A}}{A} \right)^\gamma - 1 \right] c, & \text{for } A_0 < A \leq A^+ \quad (\text{a}) \\ (r - \alpha) \Psi \left[\left(\frac{\hat{A}}{A} \right)^\gamma - 1 \right] c, & \text{for } A^+ < A \leq \bar{A} \quad (\text{b}) \end{cases} \quad (10 \text{ bis})$$

where $\Psi = \frac{1 - \lambda}{\eta_1 - \lambda\eta_2}$, $\hat{A} = \left(\frac{\delta}{rc} \right)^{1/\gamma}$ and $A^+ = \left[\frac{(\beta_2 - 1)\bar{A}^{-\gamma} + \hat{A}^{-\gamma}}{\beta_2} \right]^{-\frac{1}{\gamma}}$.

Proof. See appendix A.1. ■

For conversion to be optimal, the dynamic zero profit condition in (9) must hold at the threshold, $B^*(A)$. Let's analyse such condition by rearranging (9) as follows:

$$Z_2(A)B^*(A)^{\beta_2} + (1 - \lambda)\frac{\delta A^{-\gamma}}{r} + \lambda\eta_2\frac{B^*(A)}{r - \alpha} = (1 - \lambda)c + \eta_1\frac{B^*(A)}{r - \alpha}$$

This means that benefits from clearing land and becoming a farmer must match the cost opportunity of conversion, i.e. the cost for clearing and settling land plus the payment perpetuity which is implicitly given up by converting.

By Proposition 1 the whole conversion dynamics are characterized in terms of B . Since the agent's size is infinitesimal, the trigger $B^*(A)$ must be a decreasing function of A . In both figure 2 and 3, conservation is optimal in the region above the curve. In fact, in this region, B is high enough to deter conversion and each landholder conserves up to the time where B driven by (2) drops to $B^*(A)$. Then, as B crosses $B^*(A)$ from above, a discrete mass of landholders will enter the agricultural market developing (part of) their land. Since higher competition reduces profits from agriculture, entries take place until conditions for conservation are restored ($B > B^*(A)$).

Depending on \bar{A} , we obtain two different scenarios (see figure 2 and 3):

- (1) if $\hat{A} \leq \bar{A}$, the conversion process stops at \hat{A} since this is the last parcel for which conversion makes economic sense ($\frac{\delta}{r}\hat{A}^{-\gamma} - c = 0$). This in turn implies that the surface, $\bar{A} - \hat{A} \geq 0$, is conserved forever at a total cost equal to $\eta_1 \frac{B}{r-\alpha} (\bar{A} - \hat{A})$.

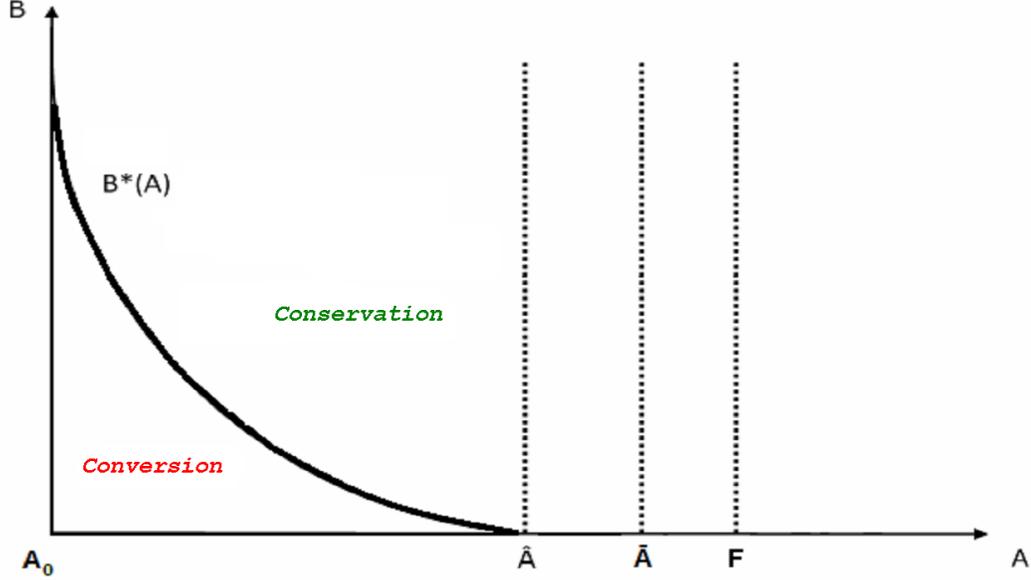


Figure 2: *Optimal conversion threshold with $\eta_1 > \lambda\eta_2$ and $\hat{A} \leq \bar{A}$*

- (2) if $\hat{A} > \bar{A}$, land is converted smoothly up to A^+ following the curve (10 bis(a)). If the surface of cultivated land falls within the interval $A^+ \leq A \leq \bar{A}$, when B hits the threshold $B^*(A)$, the landholders start a run for conversion up to \bar{A} . Unlike the previous case, here the limit imposed by the Government binds and restricts conversion on a surface, $\bar{A} - \hat{A} > 0$, where development would be profitable from the landholder's viewpoint. The intuition behind this result is immediate if we take a backward perspective. When the limit imposed by the Government \bar{A} is reached, then it must be $Z_2(\bar{A}) = 0$ since no market entry may occur. Hence, condition (9) reduces to $V(\bar{A}, B^*(\bar{A}); \bar{A}) = (1 - \lambda) \frac{\delta \bar{A}^{-\gamma}}{r} + (\lambda\eta_2 - \eta_1) \frac{B^*(\bar{A})}{r-\alpha} = (1 - \lambda)c$ from which we obtain (10bis (b)) as optimal trigger. This implies that at \bar{A} marginal rents induced by future reduction in B are not null, i.e. $V_B(\bar{A}, B; \bar{A}) < 0$, and they would be entirely captured by market incumbents. Since each single landholder realizes the benefit from marginally anticipating his entry decision, then an entry run occurs to avoid the restriction imposed by the Government. However, by rushing, the rent attached to information on market profitability, collectable by waiting, vanishes. Therefore there will be a land extent (i.e. a number of farmers), $A^+ < \bar{A}$, such that for $A < A^+$ no landholder finds it convenient to rush since the marginal advantages from a future reduction in B are lower than the option value lost.²⁹ Note also that, as A^+ is given by $B^*(A^+) = B^*(\bar{A})$, the threshold

²⁹This means the A^+ th is the last landholder for whom $V_B(A^+, B^*(A^+); \bar{A}) = 0$.

in (10bis), triggering the run, results in the traditional NPV break-even rule (see Appendix A.1).³⁰

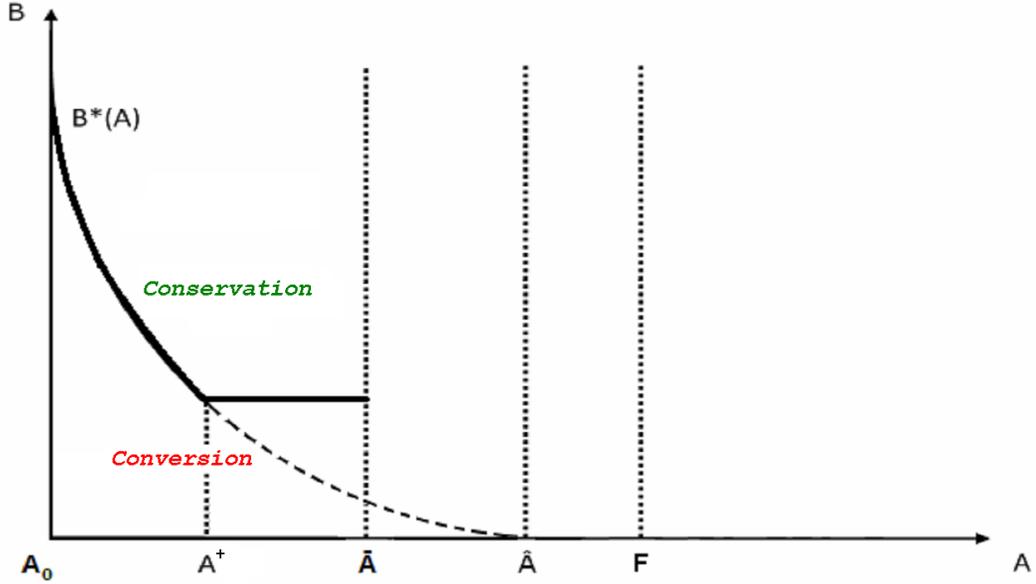


Figure 3: *Optimal conversion threshold with $\eta_1 > \lambda\eta_2$ and $\hat{A} > \bar{A}$*

A useful and interesting feature of the conversion rule in (10) is that by setting $\eta_1 = 1$, $\eta_2 = 0$ and $\lambda = 0$ then $\Psi = 1$ and (10) collapses into the conversion strategy of the social planner in Bulte et al. (2002). In our model, it is immediate to show that several combinations of the second-best tools η_1 , η_2 and λ result in $\Psi = 1$ and lead to a first-best conversion strategy.³¹

δ	c	r	γ
> 0	< 0	< 0	< 0

Table 1: *Comparative statics on \hat{A}*

As shown in table 1 the definition of the last plot, \hat{A} , which is worth converting, depends on parameters regulating the demand for agricultural goods, the interest rate and the land unit conversion cost. A higher δ illustrating a higher demand for agricultural products and/or a more rigid demand moves \hat{A} forward since higher profits support the conversion for a larger land surface. Similarly, as $c \rightarrow 0$, all the available land will be cultivated ($\hat{A} \rightarrow \bar{A}$). With a higher r future returns from agriculture become relatively lower with respect to the cost of clearing land and land conversion is less attractive.

δ	c	r	γ	
≥ 0	≤ 0	≤ 0	≤ 0	
α	σ^2	η_1	η_2	λ
≤ 0	≤ 0	≤ 0	≥ 0	$< 0_{(\frac{\eta_1}{\eta_2} > 1)}$
				$\geq 0_{(\frac{\eta_1}{\eta_2} \leq 1)}$

Table 2: *Comparative statics on $B^*(A)$*

³⁰In Bartolini (1993) a similar result is obtained. Under linear adjustment costs and stochastic returns, investment cost is constant up to the investment limit where it becomes infinite. As a reaction to this external effect, recurrent runs may occur under competition as aggregate investment approaches the ceiling. See also Moretto (2008).

³¹In other words, a competitive equilibrium evolves as maximizing solution for the expected present value of social welfare in the form of consumer surplus (Lucas and Prescott, 1971; Dixit and Pindyck, 1994, ch.9).

In table 2, we provide some comparative statics illustrating the effect that changes in the exogenous parameters have on the threshold level $B^*(A)$. Changes in an exogenous parameter, whenever increasing (decreasing) conversion benefits with respect to conservation benefits, redefine, by moving upward (downward) the boundary $B^*(A)$, the conversion and conservation regions. In this light, for instance, to a higher δ corresponds higher profits from agriculture and thus a higher $B^*(A)$ and a larger conversion region. The same effect is also produced by a relatively more inelastic demand. On the contrary, the opposite occurs as c increases since a higher conversion cost decreases net conversion benefits. With an increase in the interest rate, exercise of the option to convert should be anticipated but this effect is too weak to prevail over the effect that a higher r has on the cost opportunity of conversion. Studying the effect of volatility, σ , and of growth parameter, α , the sign of the derivatives is in line with the standard insight in the real options literature. An increase in the growth rate and volatility of B determines postponed exercise of the option to convert. This can be explained by the need to reduce the regret of taking an irreversible decision under uncertainty. Since the cost of this decision is growing at a faster rate and there is uncertainty about its magnitude, waiting to collect information about future prospects is a sensible strategy.

As expected, an increase in η_1 pushes the barrier downward since it makes it more profitable to conserve the plot and keep open the option to convert. In line with this result, the barrier responds in the opposite way to an increase in η_2 which implicitly provides an incentive to conversion. Changes in λ have a non-monotonic effect on the barrier which depends on the ratio between the two payment rates. A higher λ defines a stricter requirement on development that may push the barrier downward for two reasons. First, a lower return from agriculture since less land is cultivated which is, however, balanced by a lower cost for clearing land, and second, as $\frac{\eta_1}{\eta_2} > 1$ a higher payment on the marginal unit which the farmer is required to set aside is guaranteed if the plot is totally conserved. The case where $\frac{\eta_1}{\eta_2} \leq 1$ and the barrier shifts upward can be easily explained by inverting the second argument.

These considerations mostly hold for both (10) and (10 bis). Clearly, over the interval $A^+ < A \leq \bar{A}$ since the option multiple, $\frac{\beta_2}{\beta_2 - 1}$, drops out, the barrier $B^*(A)$ is not affected by σ . The derivative with respect to the benefit drift α maintains the sign in table 2 while the comparative statics on r reveals:

$$\frac{\partial B^*(A)}{\partial r} = \begin{cases} > 0 \text{ for } r < \alpha \left(\frac{\hat{A}}{A}\right)^\gamma \\ \leq 0 \text{ for } r \geq \alpha \left(\frac{\hat{A}}{A}\right)^\gamma \end{cases} \quad \text{for } A^+ < A \leq \bar{A}$$

Finally, since by (10bis) the same level of B triggers the entry of a positive mass of landholders, i.e. $B^*(A^+) = B^*(\bar{A})$, it is worth highlighting that the surface triggering a conversion rush is independent of the definition of η_1 , η_2 and λ . The Government policy may either speed up or slow down the conversion dynamic but it cannot alter A^+ which depends only on the choice of \bar{A} with respect to \hat{A} . Note that $\frac{\partial A^+}{\partial \bar{A}} > 0$ which reasonably means that as $\bar{A} \rightarrow \hat{A}$ the run would be triggered only by a relatively a lower level for B . In other words, since in expected terms a higher \bar{A} implies a less strict threat of being regulated, then landholders are not willing to give up information rents collectable by waiting. Not surprisingly, $\frac{\partial A^+}{\partial \hat{A}} < 0$. A lower \hat{A} implies a faster drop in the profit from agriculture as A increases and then a lower incentive for the conversion run.

3.2 Case with $\eta_1 < \lambda\eta_2$

Now, assume that $\eta_1 < \lambda\eta_2$. In this case $\eta_2 > \eta_1$ will necessarily obtain, that is, the payment rate per unit of land conserved is more generous when a portion of the plot has been developed. This could be the case for a Government which, having run out of funding for the conservation program, may be willing to sacrifice some pristine habitat in order to more generously finance conservation on a smaller scale.³² Differently, this choice may also be reasonably explained by a Government wishing to indirectly induce a switch towards certain agricultural or forestry practises by offering a more favourable payment rate. For instance, the Government may choose to prefer timber harvest and subsequent reforestation as land-use to cash funding on carbon markets and finance conservation on the remaining habitat.³³ In this case, our model allows framing of

³²Note that this scenario does not exclude to push toward a form of development which is perceived as environmental-friendly.

³³Under the CDM (Clean Development Mechanism) of the Kyoto Protocol, forest conservation/avoided deforestation efforts were not considered in the first commitment period (2008-2012) (IPCC, 2007). On the contrary, through the CDM, investment in tree planting projects has been undertaken (Santilli et al., 2005; van Vliet, 2003). In our model, this would imply an η_1 lower

the competition between these two "green" but mutually exclusive land destinations, i.e. secondary forests vs. primary forests. Finally, the Government could simply consider it fair and/or politically expedient to better reward conservation as soon as the restriction on development is binding and the real conservation cost opportunity is implicitly revealed.

As in the previous section, the optimal conversion threshold, $B^C(A) = B^{**}(A)$, must be determined by matching benefits and costs from conversion. Unlike the previous case, when developing land in addition to the profit accruing from agriculture, $(1 - \lambda) \frac{\delta A^{-\gamma}}{r}$, the landholder can earn a higher payment for ES provision since $(\lambda\eta_2 - \eta_1) \frac{B}{r - \alpha} > 0$. Hence, it makes sense to clear land as B increases. However, as above market competition has a negative effect on the value from farming which, being entry-free, lies below $(1 - \lambda)c$. This effect is accounted for the first term ($Z_1(A) \leq 0$ for $A \leq \bar{A}$) in (7) since as $\lim_{B \rightarrow 0} V(A, B; \bar{A}) = 0$ then to keep $V(A, B; \bar{A})$ finite we must set $Z_2 = 0$. It follows that (7) reduces to:

$$V(A, B; \bar{A}) = Z_1(A)B^{\beta_1} + (1 - \lambda) \frac{\delta}{r} A^{-\gamma} + (\lambda\eta_2 - \eta_1) \frac{B}{r - \alpha} \quad (11)$$

As in the previous case, we determine $Z_1(A)$ and $B^{**}(A)$ by imposing the free entry condition and zero marginal rents at $B^{**}(A)$. That is,

Proposition 2 *Provided that each agent rationally forecasts the future dynamics of the market for agricultural goods for land to be converted the following condition must hold*

$$V(A, B^{**}(A); \bar{A}) = (1 - \lambda)c \quad (12)$$

where the conversion rule is

(i) if $\hat{A} \leq \bar{A}$ then

$$B^{**}(A) = \begin{cases} 0, & \text{for } A_0 < A \leq \hat{A} & \text{(a)} \\ \frac{\beta_1}{\beta_1 - 1} (r - \alpha) \Omega \left[1 - \left(\frac{\hat{A}}{A} \right)^\gamma \right] c, & \text{for } \hat{A} < A \leq A^{++} & \text{(b)} \\ (r - \alpha) \Omega \left[1 - \left(\frac{\hat{A}}{A} \right)^\gamma \right] c, & \text{for } A^{++} < A \leq \bar{A}, & \text{(c)} \end{cases} \quad (13)$$

(ii) if $\hat{A} > \bar{A}$ then

$$B^{**}(A) = 0, \quad \text{for } A_0 < A \leq \bar{A} \quad (13 \text{ bis})$$

where $\Omega = \frac{1 - \lambda}{\lambda\eta_2 - \eta_1}$, $\hat{A} = \left(\frac{\delta}{rc} \right)^{1/\gamma}$ and $A^{++} = \left[\frac{(\beta_1 - 1)\bar{A}^{-\gamma} + \hat{A}^{-\gamma}}{\beta_1} \right]^{-\frac{1}{\gamma}}$.

Proof. See appendix A.3. ■

Equation (12) defines the dynamic zero profit condition which must hold at $B^{**}(A)$. Proposition 2 illustrates the conversion dynamic as B fluctuates according to (2). Here, unlike the previous case, the threshold $B^{**}(A)$ is an increasing continuous function of A and the conversion region is above the barrier. Development is worthwhile only if B crosses $B^{**}(A)$ from below. In the conservation region the landholder conserves as B is not high enough to trigger conversion and waits until the stochastic process B moves up to $B^{**}(A)$. At that point, a mass of landholders enters the market keeping profits low enough to push the barrier upward.

than η_2 in relative terms. Only recently, at the December 2009 United Nations Framework Convention on Climate Change (UNFCCC) meeting in Copenhagen this controversial issue discussed and finally forest conservation should now be allowed to qualify (Phelps et al., 2010). See also Fargione et al. (2008) on land clearing and biofuel carbon debt. On palm oil trees vs. primary forests see Butler et al., (2009), Fitzherber et al., (2008) and Koh and Ghazoul (2008).

Also in this case, depending on the value of \bar{A} , two different scenarios emerge (see figure 4 and 5). That is,

(1) if $\hat{A} \leq \bar{A}$ then

- a surface equal to \hat{A} is converted independently from the value taken by B . In the interval $A_0 < A \leq \hat{A}$ landholders rush as agricultural profits are so high that there is no reason to wait. Moreover, they know that no matter what value B takes, they are paid more for conserving less since $\lambda\eta_2 > \eta_1$;
- once the surface \hat{A} has been converted, landholders convert smoothly up to A^{++} according to (13b). Note that over the interval $\hat{A} < A \leq A^{++}$, as land is converted, an increasing B is required to trigger conversion. This is due to the fact that profit from agriculture does not cover the cost of clearing and settling land for cultivation. Hence, landholders convert only if the payment for conservation is high enough to cover the gap.
- if B is high enough to support conversion up to A^{++} then a run is activated and the remaining land is cleared up to the upper limit. This dynamic is due to the fact that an upper limit, \bar{A} , has been established for development. In fact, since for $A > \bar{A}$ the value attached to conversion vanishes, to cash it each landholder must run to anticipate the others. However, as in the previous case, the run dissipates the rent that the landholders earn by postponing conversion and collecting information on market profitability, i.e. at $B^{**}(A^{++}) = B^{**}(\bar{A})$, $(1 - \lambda)\frac{\delta}{r}\bar{A}^{-\gamma} + (\lambda\eta_2 - \eta_1)\frac{B^{**}(\bar{A})}{r - \alpha} = (1 - \lambda)c$. Note that also in this case the occurrence of a run is due to the external effect ($V_B(\hat{A}, B; \bar{A}) > 0$) induced by the presence of a ceiling on land development.

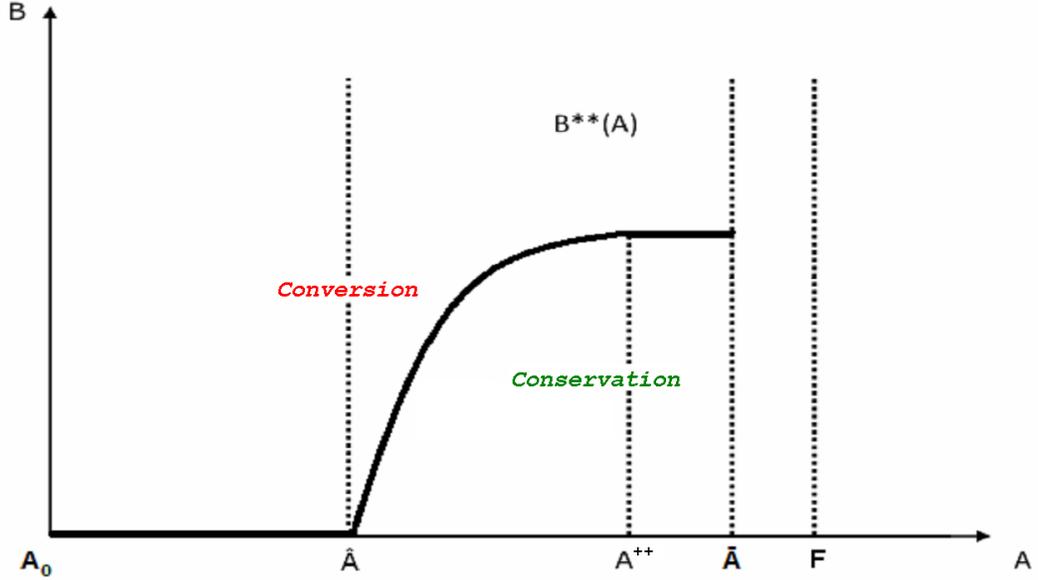


Figure 4: Optimal conversion threshold with $\eta_1 < \lambda\eta_2$ and $\hat{A} \leq \bar{A}$

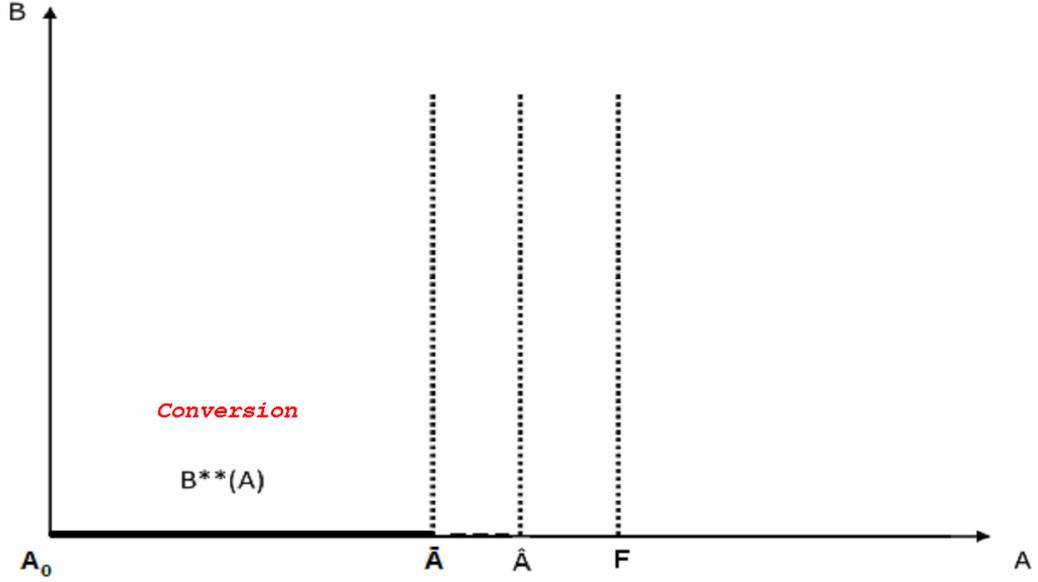


Figure 5: *Optimal conversion threshold with $\eta_1 < \lambda\eta_2$ and $\hat{A} > \bar{A}$*

- (2) If $\hat{A} > \bar{A}$ a surface equal to \bar{A} is converted for any B . As above, in the interval $A_0 < A \leq \bar{A}$, landholders rush for two reasons, namely high agricultural profits and a more generous transfer to compensate conservation. Also in this case the limit, \bar{A} , restricts profitable land conversion over the surface $\hat{A} - \bar{A} > 0$ where land would be converted for any B even without a conservation payment.

The analysis of the case where $\hat{A} > \bar{A}$ is obvious since $B^{**}(A) = 0$. Let's then focus on the case $\hat{A} \leq \bar{A}$. In (13) over the interval $A_0 < A \leq \hat{A}$, $B^{**}(A) = 0$ and then the only interesting comparative statics are those concerning \hat{A} provided in table 1 and previously discussed. For $\hat{A} < A \leq \bar{A}$ the discussion provided in section 3.1 applies as the two barriers $B^*(A)$ and $B^{**}(A)$ are symmetric and react to changes in the parameters in the opposite way. However, this implies that the above considerations on the impact of a change in the parameters are still valid.

δ	c	r	γ	
≤ 0	≥ 0	≥ 0	≥ 0	
α	σ^2	η_1	η_2	λ
≥ 0	≥ 0	≥ 0	≤ 0	< 0

Table 3: *Comparative statics on $B^{**}(A)$*

An increase in λ has a monotonic downward shifting effect on the barrier.³⁴ This makes sense since for any level of B the payment rate on the additional marginal land unit to be set aside is higher if land is developed ($\eta_2 > \eta_1$). We complete the analysis of (13) by studying the barrier $B^{**}(A)$ for $A^{++} < A \leq \bar{A}$. Since the option value multiple, $\frac{\beta_1}{\beta_1 - 1}$, drops out, the barrier is not affected by σ . We also note that unlike the results in table 3, $\frac{\partial B^{**}(A)}{\partial \alpha} \leq 0$ and:

$$\frac{\partial B^{**}(A)}{\partial r} = \begin{cases} \geq 0 & \text{for } r \geq \alpha \left(\frac{\hat{A}}{A}\right)^\gamma \\ < 0 & \text{for } r < \alpha \left(\frac{\hat{A}}{A}\right)^\gamma \end{cases} \quad \text{for } A^{++} < A \leq \bar{A}$$

³⁴Note that $\frac{\partial B^{**}(A)}{\partial \lambda} = \frac{\eta_1 - \eta_2}{\eta_2 \lambda - \eta_1} B^{**}(A) < 0$ since for $\eta_2 \lambda > \eta_1$ is $\eta_2 > \eta_1$.

Finally, also in this case, the policy parameters η_1 , η_2 and λ are neutral in the definition of A^{++} in (13 bis). This threshold depends only on \hat{A} and the ceiling \bar{A} . We find that $\frac{\partial A^{++}}{\partial \hat{A}} > 0$ and $\frac{\partial A^{++}}{\partial \bar{A}} > 0$. If the limit \bar{A} is less strict, the landholders are less willing to dissipate information rents and participate in the run only for high level of B . A lower \hat{A} implies a faster fall in the profit from agriculture as A increases and then a higher incentive for developing land as soon as B is high enough. Since this consideration is anticipated by all landholders, the run starts at a lower A^{++} .

4 Voluntary participation or contract enforceability?

Once the optimal conversion rules have been determined, we focus in this section on the issue of voluntary participation which is a crucial aspect in a PES scheme (Wunder, 2005). In this section we present the conditions under which the conservation contract is accepted on a voluntary basis. In this respect, two elements must be considered. First, the dynamic of the whole conversion process involving all the landholders who enrolled under the conservation program. Second, the restrictions on land development that the Government may wish to impose in the form of takings on landholders not entering the conservation program.³⁵

Focusing on the second element, the Government may find it desirable for the landholder to only partially develop his plot, i.e. $0 < \lambda \leq 1$. Conversely, from (9) and (12) it emerges that the landholder may consider it profitable to develop the entire plot, i.e. $\lambda = 0$. Therefore, the conservation contract may be accepted on a voluntary basis only if each landholder is better-off signing it than not. As can be easily seen, the acceptance will depend on the expectation concerning the ability of the Government to impose a $\lambda > 0$. Let's formalize this consideration assuming that no compensation is paid if a taking occurs. Since by propositions 1 and 2 the conversion is optimal at $B^C(A)$ with $C = *, **$, then an infinitely living landholder signs the contract if and only if:

$$\frac{\eta_1}{r - \alpha} B^C(A) + V(A, B^C(A); \bar{A}) \geq E_t \left[\int_t^\infty e^{-r(s-t)} (1 - \theta\lambda) \delta A(s)^{-\gamma} ds \right] \quad \text{for } C = *, ** \quad (14)$$

where $\theta \in [0, 1]$ is the probability of regulation, i.e. the restriction λ holds also for landholders not signing the contract. In (14) the LHS describes the position of a landholder within the program while on the RHS we have the expected present value for a landholder not accepting the contract and developing land at time t . Note that in the last case the conversion option is exercised as soon as the expected cost of conversion, $(1 - \theta\lambda)c$, equals the expected benefit from conversion. Rearranging (14) yields:

$$\frac{\eta_1}{r - \alpha} B^C(A) + (1 - \lambda)c \geq (1 - \theta\lambda)c \quad (15)$$

which holds if

$$\eta_1 B^C(A) - \lambda(1 - \theta)(r - \alpha)c \geq 0 \quad \text{for } C = *, **$$

where $(r - \alpha)c$ is the annualized conversion cost. Depending on the parameters this condition may not hold for some A . Note in fact that since $B^*(A)$ is a decreasing function of A , while $B^{**}(A)$ is increasing, (15) implies that:

Proposition 3 *If $\theta \in [0, 1]$ then contract acceptance can be voluntary for some but not all the landholders in the conservation program.*

Proof. Straightforward from propositions 1 and 2. ■

Segerson and Miceli (1998) show that an agreement can always be signed on a voluntary basis if the probability of future regulation is positive. By Proposition 3 we show that this result does not hold in our frame. In fact, uncertainty about future regulation does not allow capturing of all the agents who can be

³⁵Although most of the PES programs in developing countries were introduced as quid pro quo for legal restrictions on land clearing, there are no specific contract conditions preventing the landholder from clearing the area enrolled under the program (Pagiola, 2008, p. 717). In principle, sanctions may apply. For instance, in the PSA (Pagos por Servicios Ambientales) program in Costa Rica, payments received plus interest should be returned by the landholders exiting the scheme (FONAFIFO, 2007). However, in a developing country context, economic and political costs may reduce the enforcement of such sanction.

potentially regulated. A similar result is obtained by Langpap and Wu (2004) in a regulator-landowner two-period model for conservation decisions under uncertainty and irreversibility. In their paper, since contract pay-offs are uncertain and signing is an irreversible decision, under certain conditions a landholder may not accept it to stay flexible. Unlike them, we show that under the same threat of regulation a contract can be voluntarily signed by some landholders and not by others. Not surprisingly, imposing by contract constraints on land development reduces flexibility and discourages voluntary participation. Clearly, due to decreasing profit from agriculture, this holds for some landholders but not for all since entering the conservation program becomes more attractive as land is progressively cleared.³⁶

Summing up, we can conclude that voluntary participation crucially depends on the likelihood of takings but also on the magnitude of the compensation payment which a court may impose. In fact, needless to say, if takings can be compensated, then, by (14) and (15), the requirement for contract acceptance becomes more stringent and it is more difficult to sustain agreements on a voluntary basis.³⁷

5 The long-run average rate of forest conversion

In line with Ferraro (2001, p. 997) we have shown that even if the ES provided by a targeted ecosystem ($\eta_1 \leq 1$) is not entirely compensated for, we may be able to induce landholders to conserve their plot. However, we believe that this result only "statically" addresses the conservation/development dilemma. Hence, in this section we aim to study the temporal implications of the optimal conversion policy, i.e. how long it takes to clear the target surface \bar{A} , and the impact of increasing uncertainty about future environmental benefits, B , and conversion cost, c , on conversion speed under the two policy scenarios characterized above. To do so, we derive the long-run average growth rate of forest conversion through a robust linear approximation (see Appendix A.4 and A.5).³⁸

5.1 Case with $\eta_1 > \lambda\eta_2$

Let's consider the case where $\hat{A} \leq \bar{A}$. This represents the more interesting case since the analysis below remains valid also for the opposite case over the range $A < A^+$. Note in fact that for $A \geq A^+$ the long-run rate of reforestation must obviously tend to infinity due to the conversion run.

Let's now focus our attention on the long-run average growth rate of forest conversion. Rearranging (10) yields:

$$\xi = \frac{\beta_2}{\beta_2 - 1} (1 - \lambda) \frac{P_A(A)}{r} - \frac{\eta_1 - \lambda\eta_2}{r - \alpha} B \quad \text{for } \xi < \hat{\xi} \quad (16)$$

where $\hat{\xi} = \frac{\beta_2}{\beta_2 - 1} (1 - \lambda) c$.

The first term on the RHS of (16) represents the expected discounted profit from the cultivation of land conditional on the number of farmers remaining constant. The multiple $\frac{\beta_2}{\beta_2 - 1} < 1$ accounts for the presence of uncertainty and irreversibility. The second term is the expected discounted flow of payments implicitly given up by developing land net of the payments for conservation paid for setting aside λ as required by the Government. Note that ξ can be defined as a regulated process in the sense of Harrison (1985, chp. 2) with $\hat{\xi}$ as upper reflecting barrier. This implies that when a reduction of B drives ξ upward toward $\hat{\xi}$ some landholders find it profitable to convert land. New entries in the market, however, determine a drop along $P_A(A)$ which by balancing for the effect of B prevents ξ from rising above $\hat{\xi}$. Since entry is instantaneous, the rate of deforestation is infinite at $\hat{\xi}$.³⁹ Conversely, if $\xi < \hat{\xi}$ the level of B is high enough to support conservation, no entries occur and consequently the deforestation rate is null. Hence, the reflecting barrier

³⁶Note that in this respect the scenario $\eta_1 < \lambda\eta_2$ is the most problematic since switching to competitive farming is so advantageous for the first developers that they never accept the contract. This implies that (1) should be restated as $L' = A'_0 + F$ with $A'_0 > A_0$ since by (15) only a lower number of landholders may enter the program on a voluntary basis.

³⁷On compensation and land taking see Adler (2008).

³⁸See Dixit and Pindyck (1994, pp. 372-373), and Hartman and Hendrickson (2002) for a calculation of the long-run average growth rate of investment.

³⁹The fact that at $\hat{\xi}$ the rate of conversion is infinite follows from the non-differentiability of B and then of A with respect to the time t (see Harrison, 1985; Dixit, 1993).

$\hat{\xi}$ does not generate a finite rate of deforestation over time but long periods of inaction followed by short periods of rapid bursts of land conversion.

In this section, our aim is to then find a steady-state (long-run) distribution for A from which we can determine the average growth rate of forest conversion over a long period of time. Since A and B enter additively on (16) some manipulation is required to apply the well-known properties of log-normal distribution and show that ξ is log-normally distributed.⁴⁰ Denoting by $\frac{1}{dt}E(d \ln A)$ the measure of the average growth rate of forest conversion, in the appendix we prove that:

Proposition 4 *When $\eta_1 > \lambda\eta_2$, for an initial point (\tilde{B}, \tilde{A}) such that $\xi(\tilde{B}, \tilde{A}) = \hat{\xi}$ (10) can be approximated by*

$$\frac{A}{\tilde{A}} = \left(\frac{B}{\tilde{B}}\right)^{-\frac{1}{\gamma}} \left[1 - \left(\frac{\tilde{A}}{A}\right)^\gamma\right] \quad (17a)$$

and the average or expected long-run growth rate of deforestation can be approximated by:

$$\frac{1}{dt}E[d \ln A] \simeq -\frac{\alpha - \frac{1}{2}\sigma^2}{\gamma} \left[1 - \left(\frac{\tilde{A}}{A}\right)^\gamma\right] \quad \text{for } \alpha < \frac{1}{2}\sigma^2 \quad (17b)$$

where $A_0 \leq \tilde{A} < \hat{A}$ and $\hat{A} = \left(\frac{\delta}{rc}\right)^{1/\gamma}$.

Proof. See Appendix A.5. ■

Thus, if \tilde{B} is the current value of ES and, by (10), \tilde{A} is the corresponding optimal surface of converted land, the expression in (17b) is the best guess for the average rate at which the forested surface, $\tilde{A} - \hat{A}$, is cleared. Remember that if then $\hat{A} > \tilde{A}$ the deforestation rate is null since $\frac{\delta}{r}A^{-\gamma} < c$ for $\hat{A} < A \leq \tilde{A}$.

Furthermore, it is straightforward to verify that the rate in (17b) is increasing in the volatility of future payments. Although at a first glance this result may seem counterintuitive, it follows from the distribution of the log-normal process ξ with an upper reflecting barrier at $\hat{\xi}$. A higher volatility has two distinct effects. First, it pushes the barrier $\hat{\xi}$ downward; second, by increasing the positive skewness of the distribution of ξ , it raises the probability of the barrier being reached.⁴¹ Both effects induce a higher rate of deforestation in both the short-run and long-run.

We also note that a higher conversion cost c induces a lower long-run average rate of deforestation. Two effects must be recognized. The first is immediate and driven by the higher c . The second is more subtle. A higher c prevents from converting now for a certain B . Since conversion in the future will be triggered by a decreasing B the landholder can benefit from an implicit advantage by paying a lower $(\eta_1 - \lambda\eta_2)B$ which is a conversion cost opportunity.

5.2 Case with $\eta_1 < \lambda\eta_2$

Consider the interval $\hat{A} \leq \tilde{A}$.⁴² Rearranging (13) yields:

$$\varsigma = \frac{\beta_1}{\beta_1 - 1} (1 - \lambda) \frac{P_A(A)}{r} + \frac{\lambda\eta_2 - \eta_1}{r - \alpha} B \quad \text{for } \varsigma < \hat{\varsigma} \quad (18)$$

where $\hat{\varsigma} = \frac{\beta_1}{\beta_1 - 1} (1 - \lambda) c$.

The first term on the RHS of (18) is the expected discounted profit from the cultivation of land if any further conversion occurs. The multiple $\frac{\beta_1}{\beta_1 - 1} > 1$ accounts for uncertainty and irreversibility. Unlike (16), since $\eta_1 < \lambda\eta_2$ the second term stands for the expected discounted flow of payments received when developing λ , net of the payments implicitly given up. Again, ς can be characterized as a regulated process having $\hat{\varsigma}$ as upper reflecting barrier. Whenever an increase in B leads ς upward toward $\hat{\varsigma}$ new plots are cleared. This will produce an increase in the supply of agricultural goods and consequently a drop along $P_A(A)$ preventing ς from passing $\hat{\varsigma}$. It follows that to keep the surface conserved unchanged, we must have $\varsigma > \hat{\varsigma}$.

⁴⁰Technically, the log-normality is a property of the process for ξ linearized around an initial point (\tilde{B}, \tilde{A}) . See Appendix A.5 for further details.

⁴¹We show in appendix A.6 that to a higher σ corresponds a higher probability of hitting $\hat{\xi}$ and thus a higher long run average deforestation rate.

⁴²The same discussion provided in the previous section applies for $A \geq A^{++}$.

As shown in the appendix:

Proposition 5 When $\eta_1 > \lambda\eta_2$, for an initial point (\tilde{B}, \tilde{A}) such that $\varsigma(\tilde{B}, \tilde{A}) = \hat{\varsigma}$ (13) can be approximated by:

$$\frac{A}{\tilde{A}} = \left(\frac{B}{\tilde{B}}\right)^{-\frac{1}{\gamma}} [1 - (\frac{\hat{A}}{\tilde{A}})^\gamma] \quad (19a)$$

and the average or expected long-run growth rate of deforestation can be approximated by:

$$\frac{1}{dt} E [d \ln A] \simeq \frac{\alpha - \frac{1}{2}\sigma^2}{\gamma} \left[\left(\frac{\tilde{A}}{\hat{A}}\right)^\gamma - 1 \right] \quad \text{for } \alpha > \frac{1}{2}\sigma^2 \quad (19b)$$

where $\tilde{A} > \hat{A}$ and $\hat{A} = (\frac{\delta}{rc})^{1/\gamma}$.

Proof. See Appendix A.5. ■

As above, (19b) is the best guess for the average rate at which the forest stock, $\bar{A} - \tilde{A}$, is exhausted over a long-run horizon. Furthermore, the long-run average rate of deforestation (19b) is decreasing in the volatility of future payments. Again, it should be remembered that ς is a log-normal process with an upper reflecting barrier at $\hat{\varsigma}$. As volatility soars the barrier $\hat{\varsigma}$ moves upward and positive skewness in the distribution of ς increases. Whilst the first has a reducing effect on the rate, the second raises the probability of hitting $\hat{\varsigma}$ and consequently the rate of deforestation.⁴³ In addition, since the expected discounted profit from competitive farming decreases as land is converted, the former effect prevails in the long-run. Conversely, we find that the rate is increasing in c . This may seem surprising at first sight. As a higher c prevents conversion we would expect landholders to hold on the decision to develop, but postponing conversion is costly since the per-period increase in payments $(\lambda\eta_2 - \eta_1)B > 0$ would have to be given up. Since the weight of expected discounted higher payments accruing if conversion is anticipated prevails over the expected pay-off from delay, the average rate of deforestation is increasing in c . More formally, a higher c , by inducing a shift upward for the barrier, should definitely decrease the probability of hitting it. However, as c increases, then \hat{A} decreases and so does A^{++} . This means that a run will start at a lower surface A^{++} as soon as $B^{**}(\bar{A})$ has been reached. Since the run will exhaust the stock \bar{A} and drastically lower the profit from agriculture then $A^{++} - \hat{A}$ landholders will prefer to anticipate the conversion. Note that since $\frac{\partial(A^{++} - \hat{A})}{\partial \hat{A}} > 0$, even for a $B < B^{**}(\bar{A})$, they will prefer to convert to trade-off the dramatic effect on the profit due to the run with a higher profit from farming. This latter effect justifies a higher deforestation rate in the long-run.

6 The Costa Rica case study

In this section we provide a numerical example to illustrate our findings. We calibrate the model to fit the characteristics of a humid Atlantic zone of Costa Rica.⁴⁴ Parameters in our calculations take the following values:

1. The total extent of originally forested area is $\bar{A} = 320000$ hectares. On this extent, we consider a converted portion equal to $A_0 = 25000$ hectares.⁴⁵ We draw demand for agricultural products as in Bulte et al. (2002) by setting $\delta = \$6990062$ (in 1998 US\$) and $\gamma = 0.887$.
2. The annual value of ES is equal to $\tilde{B} = \$75/ha$. This value accounts only for the forest production function and does not include the regulatory function and existence values. As B is assumed to fluctuate according to (2) we investigate the impact of its drift, α , and volatility, σ , on the optimal forest stock. To do this, in our analysis α takes values 0, 0.025, and 0.05 while σ varies within the interval $[0, 0.325]$.
3. A 7% risk free interest rate is assumed ($r = 0.07$). Finally, unlike Bulte et al. (2002) where $c = 0$ we also consider different levels of costly deforestation.

⁴³See appendix A.6 for the intuition behind this result.

⁴⁴Detailed data are provided by Bulte et al. (2002, pp. 154-155). See also Conrad (1997).

⁴⁵Note that later we will set $\bar{A} = A_0 = 25000$.

6.1 Optimal forest stock and long-run average rate of deforestation under different payment schemes

6.1.1 Case with $\eta_1 > \lambda\eta_2$

For the case $\eta_1 > \lambda\eta_2$ we study the following five different policy scenarios:

	η_1	η_2	λ	Ψ
Scenario 1	1	0	0	1
Scenario 2	0.7	0	0	1.4286
Scenario 3	1	0	0.3	0.7
Scenario 4	0.7	0.5	0.3	1.2727
Scenario 5	0.7	1	0.3	1.75

Table 4: policy scenarios

and compare them for two different levels of costly deforestation, $c = 0$ and $c = 500$, respectively.⁴⁶ In particular, by these comparisons, our aim is

- **1 vs. 2:** to highlight the impact of a reduction in the compensation for ES provision on the landholders' decision to conserve. Note that scenario 1 resembles the case of a centralized economy in Bulte et al. (2002) where the planner optimally allocates land by comparing expected marginal benefit from conservation and its expected marginal cost opportunity.
- **1 vs. 3:** to study the effect of restricted land development on the optimal forest stock and on the long-run average rate of deforestation.
- **4 vs. 5:** to verify the effect of a more favourable conservation payment rate when the restriction on land development is binding (i.e. land development is privately optimal).

In the tables below we provide the optimal forest stock which should be held, $\bar{A} - \hat{A}$,⁴⁷ and the average deforestation rate (Def rate) at which such stock should be optimally exhausted in the long-run.⁴⁸ Note that in our calculations the deforestation rate may be null in two cases. First, when the optimal forest stock, $\bar{A} - \hat{A}$ is completely exhausted and second, when conditions in (17b) or (19b) do not hold. We will distinguish between them using 0 for the former and a dash for the latter.

Scenarios 1 and 2 By Tables 5, 6, 7 and 8 we show that:

- i) the optimal forest stock increases as σ soars. With higher uncertainty, conversion is triggered at a lower level for B . This implies that for any given value of \tilde{B} the corresponding optimal converted surface must be lower. Note that as anticipated by (17) the long-run rate of deforestation is increasing in σ ;
- ii) the optimal forest stock increases in α . This makes sense as long as a higher expected growth rate on the level of PES induces conversion postponement. The long-run average rate of deforestation decreases in the drift. This is not a surprising result since an increase in α , by leading ξ downward and far from $\hat{\xi}$, must reduce the average rate;
- iii) for $c = 0$ Government policies have no effect on the long-run deforestation rate. On the contrary, we observe a reducing effect when $c = 500$. However, note that if conversion is costly ($c > 0$) and $\bar{A} < \hat{A}$, the reduction in the long-run deforestation rate is also due to the presence of a threshold A^+ where the deforestation rate is infinite.⁴⁹ Note also that as may be expected, a higher land stock is conserved with costly conversion. Finally, we observe that a reduction in η_1 has a strong effect on the optimal forest stock but only a marginal effect on the deforestation rate.

⁴⁶Numerical results under other scenarios are available upon request.

⁴⁷By substituting $\tilde{B} = \$75/ha$ in equation (10) we get the optimal converted land $\tilde{A} = A(\tilde{B})$, then subtracting \tilde{A} from \bar{A} we obtain the optimal forest stock.

⁴⁸Note that variations in the long-run deforestation rate under different policy scenarios are only due to a different initial converted land stock \tilde{A} .

⁴⁹This is the case, for instance, in our calculations where for $c = 500$ we get $\hat{A} = \left(\frac{6990062}{0.07 \cdot 500}\right)^{\frac{1}{0.887}} = 9.455 \times 10^5 > 320000 = \bar{A}$.

σ	$\alpha=0.00$		$\alpha=0.025$		$\alpha=0.05$	
	\bar{A} -	Def rate	\bar{A} -	Def rate	\bar{A} -	Def rate
0	0	0	76684	-	222474	-
0,025	0	0	79996	-	223155	-
0,05	0	0	88790	-	225127	-
0,075	461	0,0032	100809	-	228196	-
0,1	23495	0,0056	114211	-	232103	-
0,125	44778	0,0088	127886	-	236574	-
0,15	64431	0,0127	141240	-	241361	-
0,175	82567	0,0173	153977	-	246265	-
0,2	99297	0,0225	165955	-	251135	-
0,225	114723	0,0285	177125	0,0004	255866	-
0,25	128945	0,0352	187484	0,0070	260389	-
0,275	142053	0,0426	197056	0,0144	264662	-
0,3	154136	0,0507	205879	0,0225	268664	-
0,325	165273	0,0595	213998	0,0314	272388	0,0032

Table 5: *Optimal forest stock and long-run average rate of deforestation in scenario 1 with $c = 0$*

σ	$\alpha=0.00$		$\alpha=0.025$		$\alpha=0.05$	
	\bar{A} -	Def rate	\bar{A} -	Def rate	\bar{A} -	Def rate
0	0	0	0	0	174200	-
0,025	0	0	0	0	175218	-
0,05	0	0	0	0	178165	-
0,075	0	0	0	0	182755	-
0,1	0	0	12349	-	188596	-
0,125	0	0	32793	-	195279	-
0,15	0	0	52758	-	202436	-
0,175	0	0	71798	-	209767	-
0,2	0	0	89706	-	217048	-
0,225	13115	0,0285	106405	0,0004	224121	-
0,25	34376	0,0352	121892	0,0070	230883	-
0,275	53973	0,0426	136201	0,0144	237270	-
0,3	72036	0,0507	149391	0,0225	243253	-
0,325	88687	0,0595	161529	0,0314	248821	0,0032

Table 6: *Optimal forest stock and long-run average rate of deforestation in scenario 2 with $c = 0$*

For instance, consider the case where $\alpha = 0$ and $\sigma = 0.225$. Under scenario 1, the optimal forest stocks held are respectively 114723ha with $c = 0$ and 215077ha with $c = 500$ while the deforestation rates are 2.85% and 2.45% respectively. Under scenario 2 with a 30% lower conservation payment rate, the forest stock increases from 13115ha with $c = 0$ to 173266ha with $c = 500$. The rate is unchanged with $c = 0$ while it reduces to 2.31% for $c = 500$. From tables 5 and 7 we note that a slightly higher expected payment growth rate, $\alpha = 0.025$, induces an increase in the forested land stock from 177125ha to 202243ha while the deforestation rate becomes practically null (from 0.04% to 0.03%). In this case by reducing η_1 to 0.7, the forest stock is

106405ha with $c = 0$ and 156461ha with $c = 500$. Conversely, there is no effect on the deforestation rate since we have respectively 0.04% and 0.03%.

Comparing results for $\alpha = 0.05$ in all the tables, the long-run average rate of deforestation is practically null. In particular, comparing table 6 and 7 we note that, even if at the cost of some hectares of forest, a second-best policy establishing a reduced payment, $\eta_1 = 0.7$, guarantees the same results in terms of deforestation rates and a forest stock however higher than the one currently available in the Atlantic region.⁵⁰

σ	$\alpha=0.00$		$\alpha=0.025$		$\alpha=0.05$	
	\bar{A} -	Def rate	\bar{A} -	Def rate	\bar{A} -	Def rate
0	138990	-	138986	-	138986	-
0,025	149250	0,0003	140887	-	139960	-
0,05	159060	0,0011	145990	-	142796	-
0,075	168411	0,0025	153098	-	147262	-
0,1	177304	0,0046	161212	-	153037	-
0,125	185739	0,0072	169705	-	159775	-
0,15	193725	0,0106	178218	-	167148	-
0,175	201268	0,0145	186547	-	174877	-
0,2	208381	0,0192	194577	-	182737	-
0,225	215077	0,0245	202243	0,0003	190556	-
0,25	221371	0,0305	209514	0,0060	198207	-
0,275	227278	0,0372	216376	0,0124	205601	-
0,3	232816	0,0446	222827	0,0196	212678	-
0,325	238004	0,0527	228876	0,0274	219400	0,0027

Table 7: *Optimal forest stock and long-run average rate of deforestation in scenario 1 with $c = 500$*

σ	$\alpha=0.00$		$\alpha=0.025$		$\alpha=0.05$	
	\bar{A} -	Def rate	\bar{A} -	Def rate	\bar{A} -	Def rate
0	76689	-	76683	-	76683	-
0,025	89299	0,0003	79011	-	77875	-
0,05	101473	0,0010	85279	-	81352	-
0,075	113184	0,0023	94061	-	86846	-
0,1	124419	0,0042	104158	-	93986	-
0,125	135168	0,0067	114813	-	102364	-
0,15	145427	0,0098	125580	-	111595	-
0,175	155193	0,0136	136202	-	121343	-
0,2	164471	0,0180	146526	-	131332	-
0,225	173267	0,0231	156461	0,0003	141346	-
0,25	181589	0,0288	165955	0,0056	151222	-
0,275	189452	0,0353	174980	0,0117	160837	-
0,3	196867	0,0424	183524	0,0185	170108	-
0,325	203852	0,0503	191587	0,0260	178978	0,0026

Table 8: *Optimal forest stock and long-run average rate of deforestation in scenario 2 with $c = 500$*

Scenarios 1 and 3 Let's focus on the role of restrictions on land development by comparing scenario 1 (tables 5, 7) and 3 (tables 9, 10) where respectively no set-aside ($\lambda = 0$) and 30% ($\lambda = 0.3$) set-aside requirements are established by the conservation contract signed by the landholder. Note that such requirement is not compensated ($\eta_2 = 0$) and thus it is similar to a taking. In this case, however, unlike a taking which is generally imposed by law, the requirement is accepted on a voluntary basis by signing the initial conservation

⁵⁰Bulte et al., (2002) report a forest stock of about 80.000 – 100.000ha.

contract.

σ	$\alpha=0.00$		$\alpha=0.025$		$\alpha=0.05$	
	\bar{A} -	Def rate	\bar{A} -	Def rate	\bar{A} -	Def rate
0	52246	-	157244	-	254765	-
0,025	71596	0,0004	159460	-	255220	-
0,05	89598	0,0014	165343	-	256539	-
0,075	106259	0,0032	173382	-	258592	-
0,1	121666	0,0056	182347	-	261206	-
0,125	135903	0,0088	191494	-	264196	-
0,15	149049	0,0127	200427	-	267398	-
0,175	161180	0,0173	208946	-	270679	-
0,2	172371	0,0225	216959	-	273936	-
0,225	182690	0,0285	224431	0,0004	277101	-
0,25	192202	0,0352	231360	0,0070	280126	-
0,275	200971	0,0426	237763	0,0144	282984	-
0,3	209053	0,0507	243664	0,0225	285661	-
0,325	216503	0,0595	249095	0,0314	288152	0,0032

Table 9: *Optimal forest stock and long-run average rate of deforestation in scenario 3 with $c = 0$*

σ	$\alpha=0.00$		$\alpha=0.025$		$\alpha=0.05$	
	\bar{A} -	Def rate	\bar{A} -	Def rate	\bar{A} -	Def rate
0	188722	-	188718	-	188718	-
0,025	196684	0,0003	190199	-	189477	-
0,05	204239	0,0012	194161	-	191683	-
0,075	211386	0,0027	199655	-	195146	-
0,1	218135	0,0049	205888	-	199608	-
0,125	224493	0,0077	212371	-	204787	-
0,15	230473	0,0111	218826	-	210423	-
0,175	236088	0,0152	225100	-	216297	-
0,2	241352	0,0201	231109	-	222235	-
0,225	246279	0,0256	236811	0,0003	228105	-
0,25	250886	0,0318	242187	0,0063	233814	-
0,275	255190	0,0387	247232	0,0130	239298	-
0,3	259206	0,0463	251950	0,0204	244516	-
0,325	262951	0,0546	256351	0,0285	249446	0,0029

Table 10: *Optimal forest stock and long-run average rate of deforestation in scenario 3 with $c = 500$*

The effect of the restriction is quite evident. In fact, by reducing the profit from agriculture it implicitly induces forest conservation. However, for the case discussed in the previous section ($\alpha = 0, \sigma = 0.225$) the introduction of such constraint has no significant effect on the forest stock and on the rate of deforestation. With $c = 0$, the forest stock increases from 114723ha to 182690ha while the deforestation rate remains the same (2.85%). For $c = 500$, the forested surface increases from 215077ha to 246279ha with a limited change in the deforestation rate (from 2.45% to 2.56%). For the same level of volatility, as expected, a higher payment growth rate ($\alpha = 0.025$) induces landholders to hold on conservation since the cost opportunity of conversion is higher.

Scenarios 4 and 5 We elicit the effect of a payment for conserved land (η_2) when conversion is optimal by comparing scenario 4 (tables 11 and 13) and scenario 5 (tables 12 and 14).

σ	$\alpha=0.00$		$\alpha=0.025$		$\alpha=0.05$	
	\bar{A} -	Def rate	\bar{A} -	Def rate	\bar{A} -	Def rate
0	0	0	663	-	192003	-
0,025	0	0	5010	-	192896	-
0,05	0	0	16552	-	195484	-
0,075	0	0	32326	-	199513	-
0,1	0	0	49915	-	204641	-
0,125	0	0	67862	-	210508	-
0,15	0	0	85390	-	216791	-
0,175	8385	0,0173	102105	-	223227	-
0,2	30341	0,0225	117826	-	229619	-
0,225	50588	0,0285	132486	0,0004	235829	-
0,25	69252	0,0352	146082	0,0070	241764	-
0,275	86457	0,0426	158644	0,0144	247372	-
0,3	102314	0,0507	170223	0,0225	252625	-
0,325	116931	0,0595	180879	0,0314	257512	0,0032

Table 11: *Optimal forest stock and long-run average rate of deforestation in scenario 4 with $c = 0$*

σ	$\alpha=0.00$		$\alpha=0.025$		$\alpha=0.05$	
	\bar{A} -	Def rate	\bar{A} -	Def rate	\bar{A} -	Def rate
0	76685	-	94892	0,0027	110689	0,0026
0,025	80181	-	95259	0,0027	110762	0,0025
0,05	83364	-	96232	0,0025	110974	0,0024
0,075	86264	-	97556	0,0022	111304	0,0023
0,1	88908	-	99024	0,0019	111724	0,0021
0,125	91322	-	100514	0,0015	112205	0,0019
0,15	93528	-	101963	0,0012	112721	0,0017
0,175	95544	-	103339	0,0008	113251	0,0014
0,2	97390	-	104629	0,0004	113779	0,0011
0,225	99080	-	105829	-	114293	0,0009
0,25	100629	-	106940	-	114786	0,0006
0,275	102051	-	107965	-	115254	0,0004
0,3	103356	-	108910	-	115695	0,0001
0,325	104555	-	109780	-	116106	-

Table 12: *Optimal forest stock and long-run average rate of deforestation in scenario 5 with $c = 0$*

In both scenarios when the entire plot is conserved the payment, η_1 , is equal to 0.7. By contract, as soon as the trigger for land conversion is met, a 30% restriction on land development ($\lambda = 0.3$) must be respected in exchange for a payment rate $\eta_2 = 0.5$ in scenario 4 and $\eta_2 = 1$ in scenario 5. We observe that an increase in the compensation for the restriction implies, ceteris paribus, a reduction in forest stock. For $\alpha = 0$, $\sigma = 0.225$ and $c = 0$, 50588ha of forest are conserved under scenario 4 while no conservation is undertaken under scenario 5. If conversion is costly ($c = 500$), the forested surface is equal to 188080ha under scenario

4 and reduces to 144113ha under scenario 5. The reduction in the deforestation rate is minimal (from 2.36% to 2.21%). The analysis above similarly applies when $\alpha = 0.025$.

σ	$\alpha=0.00$		$\alpha=0.025$		$\alpha=0.05$	
	\bar{A} -	Def rate	\bar{A} -	Def rate	\bar{A} -	Def rate
0	98233	-	98228	-	98228	-
0,025	110097	0,0003	100421	-	99351	-
0,05	121511	0,0011	106319	-	102625	-
0,075	132458	0,0024	114566	-	107792	-
0,1	142927	0,0044	124024	-	114495	-
0,125	152914	0,0069	133977	-	122345	-
0,15	162419	0,0101	144007	-	130974	-
0,175	171443	0,0139	153874	-	140064	-
0,2	179994	0,0184	163436	-	149353	-
0,225	188080	0,0236	172613	0,0003	158642	-
0,25	195713	0,0294	181359	0,0058	167776	-
0,275	202908	0,0359	189653	0,0120	176647	-
0,3	209680	0,0432	197485	0,0189	185178	-
0,325	216046	0,0511	204860	0,0265	193320	0,0026

Table 13: *Optimal forest stock and long-run average rate of deforestation in scenario 4 with $c = 500$*

σ	$\alpha=0.00$		$\alpha=0.025$		$\alpha=0.05$	
	\bar{A} -	Def rate	\bar{A} -	Def rate	\bar{A} -	Def rate
0	35851	-	35846	-	35846	-
0,025	49690,57	0,0002	38394	-	37150	-
0,05	63131,56	0,0010	45270	-	40959	-
0,075	76139,87	0,0022	54938	-	46992	-
0,1	88691,55	0,0040	66108	-	54855	-
0,125	100767,8	0,0064	77955	-	64119	-
0,15	112354,7	0,0094	89993	-	74370	-
0,175	123443,5	0,0130	101933	-	85248	-
0,2	134029,8	0,0172	113600	-	96450	-
0,225	144113,7	0,0221	124887	0,0003	107739	-
0,25	153699	0,0277	135728	0,0054	118927	-
0,275	162793,1	0,0339	146083	0,0112	129877	-
0,3	171406,3	0,0409	155933	0,0178	140487	-
0,325	179551,5	0,0486	165270	0,0251	150687	0,0025

Table 14: *Optimal forest stock and long-run average rate of deforestation in scenario 5 with $c = 500$*

6.1.2 Case with $\eta_1 < \lambda\eta_2$

In this section, the analysis is undertaken by fixing $\eta_1 = 0.4$, $\eta_2 = 1$, $\lambda = 0.5$ ($\Omega = 5$) and letting α and σ vary as above. Differently, we suppose that conversion is costly and assume $c = 1400$ in table 15 and $c = 2000$ in table 16.⁵¹ These conversion costs correspond to 11% and 16% respectively of the revenue from

⁵¹This is necessary to avoid degenerate scenarios. Note, for instance, that by (13) if $c = 0$ then $B^{**}(A) = 0$ for $A_0 < A \leq \infty$.

agriculture for $\tilde{A} = 25000$ and to 58% and 83% for $\tilde{A} = 160000$. Other parameters keep the same values as those used in the previous section. In this case, as discussed in section 3.2, the policy introduced by the Government is biased towards conversion. Note in fact that if the payment rate, η_2 , is null, then conversion would stop at \hat{A} which, with the parameters used in this section, is always lower than \bar{A} . It follows that the additional deforestation, $\bar{A} - \hat{A}$, is due to the structure of incentives designed by the Government which by paying $\lambda\eta_2 > \eta_1$ is implicitly making attractive an otherwise unprofitable land conversion ($\frac{\delta}{r}A^{-\gamma} < c$ for $\hat{A} < A \leq \bar{A}$). In table 15, we observe that landholders generally prefer land conversion to conservation. Only a limited surface may be conserved if $\alpha = 0.05$. This is mainly due, at least initially (up to \hat{A}), to positive profit from agriculture over the portion $1 - \lambda$ coupled with a more favourable payment rate ($\eta_2 > \eta_1$) on the portion λ under conservation. In this interval, deforestation occurs at an infinite rate. Conversely, as c increases since the speed of deforestation must slow down, we can obtain finite deforestation rates. Studying the impact of conversion costs we note, by comparing tables 15 and 16, that even if the policy is biased toward conversion, the payment $\lambda\eta_2 > \eta_1$ may not be high enough to cover the gap between agricultural profit and conversion costs. This in turn results in very low or null rate of deforestation. Finally, an additional effect is driven by the volatility of payments σ . Not surprisingly, as uncertainty soars we find higher forest stocks and lower rates of deforestation.

σ	$\alpha=0.00$		$\alpha=0.025$		$\alpha=0.05$	
	\tilde{A} -	Def rate	\tilde{A} -	Def rate	\tilde{A} -	Def rate
0	0	0	0	0	0	0
0,025	0	0	0	0	0	0
0,05	0	0	0	0	0	0
0,075	0	0	0	0	721	0,0032
0,1	0	0	0	0	1657	0,0030
0,125	0	0	0	0	2727	0,0027
0,15	0	0	0	0	3874	0,0023
0,175	0	0	0	0	5048	0,0020
0,2	0	0	0	0	6214	0,0016
0,225	0	0	0	0	7348	0,0012
0,25	0	0	0	0	8434	0,0009
0,275	0	0	0	0	9462	0,0005
0,3	0	0	0	0	10427	0,0002
0,325	0	0	0	0	11328	-

Table 15: *Optimal forest stock and long-run average rate of deforestation with $c = 1400$*

σ	$\alpha=0.00$		$\alpha=0.025$		$\alpha=0.05$	
	\bar{A} -	Def rate	\bar{A} -	Def rate	\bar{A} -	Def rate
0	76685	-	94892	0,0027	110689	0,0026
0,025	80181	-	95259	0,0027	110762	0,0025
0,05	83364	-	96232	0,0025	110974	0,0024
0,075	86264	-	97556	0,0022	111304	0,0023
0,1	88908	-	99024	0,0019	111724	0,0021
0,125	91322	-	100514	0,0015	112205	0,0019
0,15	93528	-	101963	0,0012	112721	0,0017
0,175	95544	-	103339	0,0008	113251	0,0014
0,2	97390	-	104629	0,0004	113779	0,0011
0,225	99080	-	105829	-	114293	0,0009
0,25	100629	-	106940	-	114786	0,0006
0,275	102051	-	107965	-	115254	0,0004
0,3	103356	-	108910	-	115695	0,0001
0,325	104555	-	109780	-	116106	-

Table 16: *Optimal forest stock and long-run average rate of deforestation with $c = 2000$*

6.2 Long-run average rate of deforestation for a given forest stock

In this section we propose a different exercise. In the previous section given a certain \tilde{B} we computed the optimal forest stock and the associated deforestation rate. However, by doing so, we did not address the dynamic long-run effect of the different conservation policies. In fact, since a history of the previous deforestation dynamic is not available, a comparison between the different rates of deforestation makes no sense. Here, since we aim to illustrate the temporal implications of a conversion path, i.e. how long it takes to clear the targeted surface \tilde{A} , we establish a common initial converted land surface, $\tilde{A} = 25000$, and let α and σ vary for different levels of conversion cost, c . Then, by providing the long-run average deforestation rate (Def rate) and the associated expected time for total conversion (Timing), we illustrate the impact of increasing growth and volatility of future payments and conversion cost on conversion speed.

α	0		0,025		0,05	
σ	Def rate	Timing	Def rate	Timing	Def rate	Timing
0	-	∞	-	∞	-	∞
0,025	0,0004	35775	-	∞	-	∞
0,05	0,0014	8944	-	∞	-	∞
0,075	0,0032	3978	-	∞	-	∞
0,1	0,0056	2241	-	∞	-	∞
0,125	0,0088	1436	-	∞	-	∞
0,15	0,0127	999	-	∞	-	∞
0,175	0,0173	736	-	∞	-	∞
0,2	0,0226	565	-	∞	-	∞
0,225	0,0285	448	0,0004	35755	-	∞
0,25	0,0352	364	0,0071	1794	-	∞
0,275	0,0426	302	0,0144	878	-	∞
0,3	0,0507	255	0,0226	565	-	∞
0,325	0,0595	218	0,0314	408	0,0032	3978

Table 17: Long-run deforestation rates and timing with $c = 0$

α	0		0,025		0,05	
σ	Def rate	Timing	Def rate	Timing	Def rate	Timing
0	-	∞	-	∞	-	∞
0,025	0,0004	36334	-	∞	-	∞
0,05	0,0014	9089	-	∞	-	∞
0,075	0,0031	4043	-	∞	-	∞
0,1	0,0056	2277	-	∞	-	∞
0,125	0,0087	1459	-	∞	-	∞
0,15	0,0125	1015	-	∞	-	∞
0,175	0,0170	748	-	∞	-	∞
0,2	0,0222	574	-	∞	-	∞
0,225	0,0281	455	0,0004	36334	-	∞
0,25	0,0347	370	0,0069	1823	-	∞
0,275	0,0420	306	0,0142	892	-	∞
0,3	0,0499	259	0,0222	574	-	∞
0,325	0,0586	221	0,0309	414	0,0031	4043

Table 18: Long-run deforestation rates and timing with $c = 200$

α	0		0,025		0,05	
σ	Def rate	Timing	Def rate	Timing	Def rate	Timing
0	-	∞	-	∞	-	∞
0,025	0,0003	37239	-	∞	-	∞
0,05	0,0014	9315	-	∞	-	∞
0,075	0,0030	4143	-	∞	-	∞
0,1	0,0054	2333	-	∞	-	∞
0,125	0,0085	1496	-	∞	-	∞
0,15	0,0122	1041	-	∞	-	∞
0,175	0,0166	766	-	∞	-	∞
0,2	0,0216	588	-	∞	-	∞
0,225	0,0274	466	0,0003	37239	-	∞
0,25	0,0338	379	0,0068	1868	-	∞
0,275	0,0409	314	0,0139	914	-	∞
0,3	0,0487	265	0,0216	588	-	∞
0,325	0,0572	227	0,0301	425	0,0030	4143

Table 19: Long-run deforestation rates and timing with $c = 500$

By comparing table 17, 18 and 19 we observe that when uncertainty increases, higher conversion cost, c , has a negligible effect in terms of delayed conversion. Conversely, with low uncertainty ($\sigma \in [0, 0.125]$) and no expected growth in the payments ($\alpha = 0$), the difference in expected total conversion time between the cases $c = 500$ and $c = 0$ is substantial (see figure 6). However, note that this effect vanishes as α rises to 2.5%. This means that with low uncertainty it is possible to deter conversion even if costless ($c = 0$) by simply guaranteeing an expected growth in the payments. This is consistent with the general stronger impact that a higher α has in terms of delayed conversion when compared to the effect due to increasing uncertainty.⁵²

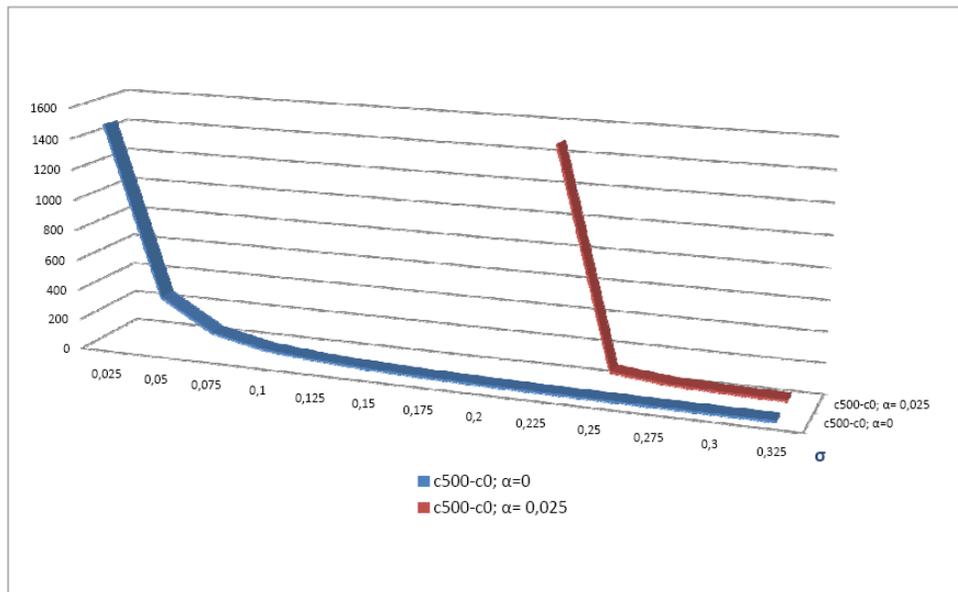


Figure 6: Difference in expected time for total conversion between $c = 500$ and $c = 0$ with $\alpha = 0$ and $\alpha = 0.025$.

⁵²Our findings seem in contrast with the calibration used in Leroux et al. (2009) where the authors assume a deforestation rate equal to 2.5 with $\alpha = 0.05$ and $\sigma = 0.1$. In fact, we show that for those values the deforestation rate should be null. A 2.5% deforestation rate would be justified only for lower α and higher σ .

7 Conclusions

In this paper we contribute to the vast literature on optimal land allocation under uncertainty and irreversible habitat conversion. We extend previous work in three respects. First, departing from the standard central planner perspective we investigate the role that competing farming may have on conversion dynamics. Profits from conversion decreasing in the number of developers may discourage conversion in particular if society is willing to reward habitat conservation as land use. Second, in this decentralized framework, we look at the conservation effort that Government land policy, through a combination of voluntary and command approaches, may stimulate.

In this regard, an interesting result is represented by the considerable amount of conservation that the Government can induce by partially compensating agents for the ES provided.

In addition, we show that the existence of a ceiling for the stock of developable land may produce perverse effects on conversion dynamics by activating a run which instantaneously exhausts the stock. Third, we believe that time matters when dynamic land allocation is analysed. Hence, we suggest the use of the optimal long-run average expected rate of conversion to assess the temporal performance of conservation policy and we show its utility by running several numerical simulations based on realistic policy scenarios.

A Appendix

A.1 Equilibrium under $\eta_1 > \lambda\eta_2$

The value function of a farmer is given by:

$$V(A, B; \bar{A}) = Z_2(A)B^{\beta_2} + \frac{(1-\lambda)\delta}{r}A^{-\gamma} + (\lambda\eta_2 - \eta_1)\frac{B}{r-\alpha} \quad (\text{A.1.1})$$

Since each agent rationally forecasts the future dynamics of the market for agricultural goods at $B^*(A)$ she/he must be indifferent between conserving and converting. That is:

$$Z_2(A)B^*(A)^{\beta_2} + \frac{(1-\lambda)\delta}{r}A^{-\gamma} + (\lambda\eta_2 - \eta_1)\frac{B^*(A)}{r-\alpha} = (1-\lambda)c \quad (\text{A.1.2})$$

In addition, the following conditions must hold (see e.g. proposition 1 in Bartolini (1993) and Grenadier (2002, p. 699)):

$$V_A(A, B^*(A); \bar{A}) = Z_2'(A)B^*(A)^{\beta_2} - (1-\lambda)\frac{\delta\gamma A^{-(\gamma+1)}}{r} = 0 \quad (\text{A.1.3})$$

and

$$\begin{aligned} \frac{\partial V(A, B^*(A); \bar{A})}{\partial A} &= V_A(A, B^*(A); \bar{A}) + V_B(A, B^*(A); \bar{A})\frac{dB^*(A)}{dA} \\ &= \left[\beta_2 Z_2(A) B^*(A)^{\beta_2-1} + \frac{\lambda\eta_2 - \eta_1}{r-\alpha} \right] \frac{dB^*(A)}{dA} = 0 \end{aligned} \quad (\text{A.1.4})$$

Finally, considering the limit on conversion, \bar{A} , imposed by the Government it follows that:

$$Z_2(\bar{A}) = 0 \quad (\text{A.1.5})$$

Condition (A.1.4) illustrates two scenarios. In the first one, each landholder exercises the option to convert at the level of $B^*(A)$ where the value, $V(A, B^*(A); \bar{A})$ is tangent to the conversion cost, $(1-\lambda)c$.⁵³ That is, $V_B(A, B^*(A); \bar{A}) = \beta_2 Z_2(A) B^*(A)^{\beta_2-1} + \frac{\lambda\eta_2 - \eta_1}{r-\alpha} = 0$. It is easy to verify that, as conjectured, $Z_2(A) < 0$. In the case $V(A, B^*(A); \bar{A})$ is smooth at the conversion threshold and $B^*(A)$ is a continuous function of A . In the second scenario, the optimal threshold $B^*(A)$ does not vary with A , i.e. $V_B(A, B^*(A); \bar{A}) \neq 0$ and $\frac{dB^*(A)}{dA} = 0$. This implies that the landholder may benefit from marginally anticipating or delaying the conversion decision. In particular, if $V_B(A, B^*(A); \bar{A}) < 0$ then the value of conversion is expected to increase as B drops. Conversely, if $V_B(A, B^*(A); \bar{A}) > 0$ then losses must be expected as B drops. However, in both cases (A.1.4) holds by imposing $\frac{dB^*(A)}{dA} = 0$.

By (A.1.4) we can split $[A_0, \bar{A}]$ into two intervals where one of the following two conditions must hold:

$$\beta_2 Z_2(A) B^*(A)^{\beta_2-1} + \frac{\lambda\eta_2 - \eta_1}{r-\alpha} = 0 \quad (\text{A.1.6})$$

$$\frac{dB^*(A)}{dA} = 0 \quad (\text{A.1.7})$$

Since $Z_2(\bar{A}) = 0$ and $\frac{\lambda\eta_2 - \eta_1}{r-\alpha} < 0$, then (A.1.6) cannot hold at $A = \bar{A}$. Therefore, (A.1.7) must hold at $A = \bar{A}$ and by (A.1.2) it follows that:

$$B^*(\bar{A}) = (r-\alpha)\Psi \left[\left(\frac{\hat{A}}{\bar{A}} \right)^\gamma - 1 \right] c \quad \text{for } A^+ \leq A \leq \bar{A} \quad (\text{A.1.8})$$

where $\hat{A} = \left(\frac{\delta}{rc} \right)^{1/\gamma}$ represents the last parcel conversion which makes economic sense. In fact, note that since $(\lambda\eta_2 - \eta_1)\frac{B}{r-\alpha} < 0$ then $\frac{\delta}{r}A^{-\gamma} \leq c$ for $A \geq \hat{A}$.

⁵³This condition holds at any reflecting barrier without any optimization being involved (Dixit, 1993).

Now let's define A^+ as the largest $A \leq \bar{A}$ that satisfies (A.1.6). This implies that for all the landholders in the range $A^+ \leq A \leq \bar{A}$, we have $\frac{dB^*(A)}{dA} = 0$ and conversion takes place at $B^*(\bar{A})$. Over the range $A < A^+$ (A.1.3) holds by definition. Hence, plugging (A.1.6) into (A.1.4) we obtain:

$$B^*(A) = \frac{\beta_2}{\beta_2 - 1} (r - \alpha) \Psi \left[\left(\frac{\hat{A}}{A} \right)^\gamma - 1 \right] c \quad \text{for } A < A^+ \quad (\text{A.1.9})$$

Finally, by the continuity of $B^*(A)$ follows that $B^*(A^+) = B^*(\bar{A})$.

Substituting:

$$\frac{\beta_2}{\beta_2 - 1} (r - \alpha) \Psi \left[\left(\frac{\hat{A}}{A^+} \right)^\gamma - 1 \right] c = (r - \alpha) \Psi \left[\left(\frac{\hat{A}}{\bar{A}} \right)^\gamma - 1 \right] c \quad (\text{A.1.10})$$

where

$$A^+ = \left[\frac{(\beta_2 - 1)\bar{A}^{-\gamma} + \hat{A}^{-\gamma}}{\beta_2} \right]^{-\frac{1}{\gamma}}$$

The conversion policy is summarized by (A.1.8) and (A.1.9). The conversion policy should be smooth until the surface $A^+ < \bar{A}$ has been converted. At A^+ landholders rush and a run takes place to convert the residual land before the limit imposed by the Government is met. By (A.1.9), $B^*(A)$ is decreasing with respect to A . This makes sense since further land conversion reduces the profit from agriculture and a landholder would convert land only if she/he expects a future reduction in B .

We must investigate two different scenarios, i.e. $\hat{A} \leq \bar{A}$ and $\hat{A} > \bar{A}$. From (A.1.10) it follows that:

$$\frac{\beta_2}{\beta_2 - 1} \left[\left(\frac{\hat{A}}{A^+} \right)^\gamma - 1 \right] = \left(\frac{\hat{A}}{\bar{A}} \right)^\gamma - 1 \quad (\text{A.1.10 bis})$$

Studying (A.1.10 bis) we can state that since $\frac{\beta_2}{\beta_2 - 1} > 0$:

- if $\hat{A} \leq \bar{A}$ then it must be $\bar{A} \leq A^+$. This implies that there is no run taking place. Land will be converted smoothly according to (A.1.8) up to \hat{A} since $\frac{\delta}{r} A^{-\gamma} \leq c$ for $A \geq \hat{A}$;
- if $\hat{A} > \bar{A}$ then it must be $A^+ < \bar{A}$. In this case, land is converted smoothly up to A^+ where landholders start a run to convert land up to \bar{A} .

A.2 Value of the option to convert

In this appendix we show that, by competition, the value of the opportunity to develop the plot by the single farmer is null at the conversion threshold. The value of the option to convert, $F(A, B; \bar{A})$, is the solution of the following differential equation (Dixit and Pindyck, 1994, ch. 8):

$$\frac{1}{2} \sigma^2 B^2 F_{BB}(A, B; \bar{A}) + \alpha B F_B(A, B; \bar{A}) - r F(A, B; \bar{A}) = 0 \quad \text{for } B > B^C(A) \quad (\text{A.2.1})$$

where $B^C(A)$ is the optimal threshold for conversion. Note that this is an ordinary differential equation, the general solution of which can be written as:

$$F(A, B; \bar{A}) = C_1(A) B^{\beta_1} + C_2(A) B^{\beta_2} \quad (\text{A.2.2})$$

where $1 < \beta_1 < r/\alpha$, $\beta_2 < 0$ are the positive and the negative root of the characteristic equation $\Psi(\beta) = \frac{1}{2} \sigma^2 \beta(\beta - 1) + \alpha\beta - r = 0$, and C_1, C_2 are two constants to be determined.

Suppose for instance that $\eta_1 > \lambda \eta_2$.⁵⁴ This implies that as B increases, the value of the option to convert should vanish as ($\lim_{B \rightarrow \infty} F(A, B; \bar{A}) = 0$) then we set $C_1 = 0$. Now, let's determine the optimal

⁵⁴It is easy to show that the results in this section would hold also for the case $\eta_1 < \eta_2 \lambda$.

conversion threshold \bar{B} and the constant $C_1(A)$. We attach to the differential equation above the following value matching and the smooth pasting conditions:

$$F(A, B^C(A); \bar{A}) = V(A, B^C(A); \bar{A}) - (1 - \lambda)c \quad (\text{A.2.3})$$

$$C_2(A)B^C(A)^{\beta_2} = Z_2(A)B^C(A)^{\beta_2} + \frac{(1 - \lambda)\delta A^{-\gamma}}{r} + \frac{(\lambda\eta_2 - \eta_1)B^C(A)}{r - \alpha} - (1 - \lambda)c$$

$$F_B(A, B^C(A); \bar{A}) = V_B(A, B^C(A); \bar{A}) \quad (\text{A.2.4})$$

$$\beta_2 C_2(A)B^C(A)^{\beta_2 - 1} = \beta_2 Z_2(A)B^C(A)^{\beta_2 - 1} + \frac{\lambda\eta_2 - \eta_1}{r - \alpha}$$

It follows that:

$$B^C(A) = B^*(A) = \frac{\beta_2}{\beta_2 - 1}(r - \alpha)\Psi\left[\frac{\delta}{r}A^{-\gamma} - c\right]$$

From (A.1.4) we obtain $V_B(A, B^*(A); \bar{A}) = \beta_2 Z_2(A)B^*(A)^{\beta_2 - 1} + \frac{\lambda\eta_2 - \eta_1}{r - \alpha} = 0$ which in turn implies $Z_2(A) = -\frac{\lambda\eta_2 - \eta_1}{r - \alpha} \frac{B^*(A)^{1 - \beta_2}}{\beta_2}$.

Rearranging (A.2.4) and substituting we have:

$$\begin{aligned} C(A) &= Z_2(A) + \frac{\lambda\eta_2 - \eta_1}{r - \alpha} \frac{B^C(A)^{1 - \beta_2}}{\beta_2} \\ &= \frac{\lambda\eta_2 - \eta_1}{(r - \alpha)\beta_2} (B^C(A)^{1 - \beta_2} - B^*(A)^{1 - \beta_2}) = 0 \end{aligned}$$

As expected the value of the option to convert is null at $B^C(A) = B^*(A)$.

A.3 Equilibrium under $\eta_1 < \lambda\eta_2$

The value function of a farmer is given by:

$$V(A, B; \bar{A}) = Z_1(A)B^{\beta_1} + \frac{(1 - \lambda)\delta}{r}A^{-\gamma} + (\lambda\eta_2 - \eta_1)\frac{B}{r - \alpha} \quad (\text{A.3.1})$$

As in section A.1 the following conditions must hold at $B^{**}(A)$

$$Z_1(A)B^{**}(A)^{\beta_1} + \frac{(1 - \lambda)\delta}{r}A^{-\gamma} + (\lambda\eta_2 - \eta_1)\frac{B^{**}(A)}{r - \alpha} = (1 - \lambda)c \quad (\text{A.3.2})$$

$$V_A(A, B^{**}(A); \bar{A}) = Z_1'(A)B^{**}(A)^{\beta_1} - \gamma \frac{(1 - \lambda)\delta A^{-(\gamma+1)}}{r} = 0 \quad (\text{A.3.3})$$

$$\begin{aligned} \frac{\partial V(A, B^{**}(A); \bar{A})}{\partial A} &= V_A(A, B^{**}(A); \bar{A}) + V_B(A, B^{**}(A); \bar{A}) \frac{dB^{**}(A)}{dA} \\ &= \left[\beta_1 Z_1(A) B^{**}(A)^{\beta_1 - 1} + \frac{\lambda\eta_2 - \eta_1}{r - \alpha} \right] \frac{dB^{**}(A)}{dA} = 0 \end{aligned} \quad (\text{A.3.4})$$

In addition, since land can be converted only up to \bar{A} then it follows that:

$$Z_1(\bar{A}) = 0 \quad (\text{A.3.5})$$

Condition (A.3.4) illustrates two scenarios. In the first one, each landholder exercises the option to convert at the level of $B^{**}(A)$ where the value, $V(A, B^{**}(A); \bar{A})$ is tangent to the conversion cost, $(1 - \lambda)c$. That is, $V_B(A, B^{**}(A); \bar{A}) = \beta_1 Z_1(A)B^{**}(A)^{\beta_1 - 1} + \frac{\lambda\eta_2 - \eta_1}{r - \alpha} = 0$. Since $\frac{\lambda\eta_2 - \eta_1}{r - \alpha} > 0$ it can be easily verified that, as conjectured, $Z_1(A) < 0$. In the case $V(A, B^{**}(A); \bar{A})$ is smooth at the conversion threshold and $B^{**}(A)$ is a continuous function of A . In the second scenario, the optimal threshold $B^{**}(A)$ does not change with

A , i.e. $V_B(A, B^{**}(A); \bar{A}) \neq 0$ and $\frac{dB^{**}(A)}{dA} = 0$. As discussed in section A.1, this implies that at the margin the landholder may benefit from anticipating or delaying the conversion decision.

By (A.3.4) the set $[A_0, \bar{A}]$ can be split into two subsets where one of the following two conditions must hold:

$$\beta_1 Z_1(A) B^{**}(A)^{\beta_1-1} + \frac{\lambda\eta_2 - \eta_1}{r - \alpha} = 0 \quad (\text{A.3.6})$$

$$\frac{dB^{**}(A)}{dA} = 0 \quad (\text{A.3.7})$$

Note that as $Z_1(\bar{A}) = 0$ and $\frac{\lambda\eta_2 - \eta_1}{r - \alpha} > 0$, then (A.3.6) cannot hold at $A = \bar{A}$. Therefore, (A.3.7) must hold at $A = \bar{A}$. From (A.3.2) it follows that:

$$B^{**}(\bar{A}) = (r - \alpha) \Omega \left[1 - \left(\frac{\hat{A}}{\bar{A}}\right)^\gamma \right] c \quad \text{for } A^{++} < A \leq \bar{A} \quad (\text{A.3.8})$$

We now define A^{++} as the largest $A \leq \bar{A}$ for which (A.3.6) holds. This means that for all the landholders in the range $A^{++} \leq A \leq \bar{A}$, we must have $\frac{dB^{**}(A)}{dA} = 0$ and conversion occurs at $B^{**}(\bar{A})$. Over the range $A < A^{++}$ (A.3.3) holds by definition. Then, substituting (A.3.6) into (A.3.4) we get:

$$B^{**}(A) = \frac{\beta_1}{\beta_1 - 1} (r - \alpha) \Omega \left[1 - \left(\frac{\hat{A}}{A}\right)^\gamma \right] c \quad \text{for } A_0 < A \leq A^{++} \quad (\text{A.3.9})$$

However, note that $B^{**}(A) < 0$ for $A < \hat{A}$. Since by (2) B cannot be negative then it must be:

$$B^{**}(A) = \begin{cases} 0, & \text{for } A_0 < A \leq \hat{A} \\ \frac{\beta_1}{\beta_1 - 1} (r - \alpha) \Omega \left[1 - \left(\frac{\hat{A}}{A}\right)^\gamma \right] c, & \text{for } \hat{A} < A \leq A^{++} \end{cases} \quad (\text{A.3.9 bis})$$

By continuity of $B^{**}(A)$ it follows that $B^{**}(A^{++}) = B^{**}(\bar{A})$. Substituting:

$$\frac{\beta_1}{\beta_1 - 1} (r - \alpha) \Omega \left[1 - \left(\frac{\hat{A}}{A^{++}}\right)^\gamma \right] c = (r - \alpha) \Omega \left[1 - \left(\frac{\hat{A}}{\bar{A}}\right)^\gamma \right] c \quad (\text{A.3.10})$$

where

$$A^+ = \left[\frac{(\beta_1 - 1)\bar{A}^{-\gamma} + \hat{A}^{-\gamma}}{\beta_1} \right]^{-\frac{1}{\gamma}}$$

From (A.3.10) it follows that:

$$\frac{\beta_1}{\beta_1 - 1} \left[1 - \left(\frac{\hat{A}}{A^{++}}\right)^\gamma \right] = 1 - \left(\frac{\hat{A}}{\bar{A}}\right)^\gamma \quad (\text{A.3.10 bis})$$

As in section (A.1), two different scenarios may arise. In fact, studying (A.3.10 bis) we observe that since $\frac{\beta_1}{\beta_1 - 1} > 0$:

- if $\hat{A} \leq \bar{A}$ then it must be $\hat{A} \leq A^{++} \leq \bar{A}$. Land will be converted according to (A.3.9 bis) up to A^{++} . At A^{++} landholders will rush to convert land up to the limit, \bar{A} , established by the Government (A.3.8);
- if $\hat{A} > \bar{A}$ then it must be $\hat{A} > A^{++} > \bar{A}$. This implies that the conversion process follows (A.3.9 bis) and land is instantaneously developed up to \bar{A} .

A.4 Long-run distributions

Let h be a linear Brownian motion with parameters μ and σ that evolves according to $dh = \mu dt + \sigma dw$. Following Harrison (1985, pp. 90-91; see also Dixit 1993, pp. 58-68) the long-run density function for h

fluctuating between a lower reflecting barrier, $a \in (-\infty, \infty)$, and an upper reflecting barrier, $b \in (-\infty, \infty)$, is represented by the following truncated exponential distribution:

$$f(h) = \begin{cases} \frac{2\mu}{\sigma^2} \frac{e^{-\frac{2\mu}{\sigma^2}h}}{e^{-\frac{2\mu}{\sigma^2}b} - e^{-\frac{2\mu}{\sigma^2}a}} & \mu \neq 0 \\ \frac{1}{b-a} & \mu = 0. \end{cases} \quad (\text{A.4.1})$$

We are interested to the limit case where $a \rightarrow -\infty$. In this case, from (A.4.1) a limiting argument gives:

$$f(h) = \begin{cases} \frac{2\mu}{\sigma^2} e^{-\frac{2\mu}{\sigma^2}(b-h)} & \mu > 0 \\ 0 & \mu \leq 0. \end{cases} \quad \text{for } -\infty < h < b \quad (\text{A.4.2})$$

Hence, the long-run average of h can be evaluated as $E[h] = \int_{\Phi} hf(h) dh$, where Φ depends on the distribution assumed. In the steady-state this yields:

$$E[h] = \int_{-\infty}^b hf(h) dh = \int_{-\infty}^b h \frac{2\mu}{\sigma^2} e^{-\frac{2\mu}{\sigma^2}(b-h)} dh = \frac{2\mu}{\sigma^2} e^{-\frac{2\mu}{\sigma^2}b} \int_{-\infty}^b h e^{\frac{2\mu}{\sigma^2}h} dh = b - \frac{2\mu}{\sigma^2} \quad (\text{A.4.3})$$

A.5 Long-run average growth rate of deforestation

Taking the logarithm of (16) we get:

$$\begin{aligned} \ln \xi &= \ln \left[\frac{\beta_2}{\beta_2 - 1} (1 - \lambda) \frac{P_A(A)}{r} - \frac{\eta_1 - \lambda\eta_2}{r - \alpha} B \right] \\ &= \ln \left[\frac{\eta_1 - \lambda\eta_2}{r - \alpha} \right] + \ln [J - B] \end{aligned} \quad (\text{A.5.1})$$

where $J = \frac{\beta_2}{\beta_2 - 1} (r - \alpha) \Psi \frac{P_A(A)}{r}$ and $J > B$. Rewriting $\ln [J - B]$ as $\ln [e^{\ln J} - e^{\ln B}]$ and expanding it by Taylor's theorem around the point $(\widetilde{\ln J}, \widetilde{\ln B})$ yields:

$$\ln [J - B] \simeq v_0 + v_1 \ln J + v_2 \ln B$$

where

$$\begin{aligned} v_0 &= \ln \left[e^{\widetilde{\ln J}} - e^{\widetilde{\ln B}} \right] - \left[\frac{\widetilde{\ln J}}{1 - e^{\widetilde{\ln B} - \widetilde{\ln J}}} + \frac{\widetilde{\ln B}}{1 - e^{-(\widetilde{\ln B} - \widetilde{\ln J})}} \right] \\ v_1 &= \frac{1}{1 - e^{\widetilde{\ln B} - \widetilde{\ln J}}}, \quad v_2 = \frac{1}{1 - e^{-(\widetilde{\ln B} - \widetilde{\ln J})}}, \quad \frac{v_2}{v_1} = \frac{1 - v_1}{v_1} < 0 \end{aligned}$$

By substituting the approximation into (A.5.1) it follows that:

$$\ln \xi \simeq \ln \frac{\eta_1 - \lambda\eta_2}{r - \alpha} + v_0 + v_1 \ln J + v_2 \ln B \quad (\text{A.5.2})$$

Now, by Ito's lemma and the considerations discussed in the paper on the competitive equilibrium, $\ln \xi$ evolves according to $d \ln \xi = v_2 d \ln B = v_2 [(\alpha - \frac{1}{2}\sigma^2)dt + \sigma dw]$ with $\ln \hat{\xi}$ as upper reflecting barrier. Setting $h = \ln \xi$, the random variable $\ln \xi$ follows a linear Brownian motion with parameter $\mu = v_2(\alpha - \frac{1}{2}\sigma^2) > 0$ and has a long-run distribution with (A.4.2) as density function.

Solving (A.5.2) with respect to $\ln A$ we obtain the long-run optimal stock of deforested land, i.e.:

$$\ln A \simeq \frac{\ln \left[\frac{\eta_1 - \lambda\eta_2}{r - \alpha} \right] + v_0 + v_1 \ln \left[\frac{\beta_2}{\beta_2 - 1} (r - \alpha) \Psi \frac{\delta}{r} \right] + v_2 \ln B - h}{\gamma v_1} \quad (\text{A.5.3})$$

From (A.5.3) by some manipulations we can show that

$$\begin{aligned}
1 &= \exp\left(\frac{v_0}{v_1}\right) \left(\frac{\frac{\eta_1 - \lambda\eta_2}{r - \alpha}}{\hat{\xi}}\right)^{\frac{1}{v_1}} \left[\frac{\beta_2}{\beta_2 - 1} (r - \alpha) \Psi \frac{\delta}{r}\right] A^{-\gamma} B^{\frac{v_2}{v_1}} \\
&= \exp\left(\frac{v_0}{v_1}\right) \left[\frac{\beta_2}{\beta_2 - 1} (r - \alpha) \Psi\right]^{-\frac{v_2}{v_1}} \frac{\delta}{r} c^{-\frac{1}{v_1}} A^{-\gamma} B^{\frac{v_2}{v_1}} \\
&= \exp\left(\frac{v_0}{v_1}\right) \left(\frac{J}{\frac{\delta}{r} A^{-\gamma}}\right)^{-\frac{v_2}{v_1}} \frac{\delta}{r} c^{-\frac{1}{v_1}} A^{-\gamma} B^{\frac{v_2}{v_1}} \\
&= \exp\left(\frac{v_0}{v_1}\right) J^{-\frac{v_2}{v_1}} \left(\frac{\delta}{r} A^{-\gamma}\right)^{\frac{1}{v_1}} B^{\frac{v_2}{v_1}} \\
&= \exp(v_0) J^{-v_2} \left(\frac{\hat{A}}{A}\right)^{\gamma} B^{v_2} \\
&= \frac{\tilde{J} - \tilde{B}}{\tilde{J}^{v_1} \tilde{B}^{v_2}} J^{-v_2} \left(\frac{\hat{A}}{A}\right)^{\gamma} B^{v_2} \\
&= \left(\frac{\hat{A}}{A}\right) \left(\frac{B}{\tilde{B}}\right)^{-\frac{1}{\gamma}} \left[1 - \left(\frac{\hat{A}}{A}\right)^{\gamma}\right]
\end{aligned}$$

and

$$\frac{A}{\tilde{A}} = \left(\frac{B}{\tilde{B}}\right)^{-\frac{1}{\gamma}} \left[1 - \left(\frac{\hat{A}}{A}\right)^{\gamma}\right]$$

Note that since $\tilde{A} < \hat{A}$ then $-\frac{1}{\gamma} \left[1 - \left(\frac{\hat{A}}{A}\right)^{\gamma}\right] < 0$.

Taking the expected value on both sides of (A.5.3) leads to:

$$E[\ln A] \simeq \frac{\ln \left[\frac{\eta_1 - \lambda\eta_2}{r - \alpha}\right] + v_0 + v_1 \ln \left[\frac{\beta_2}{\beta_2 - 1} (r - \alpha) \Psi \frac{\delta}{r}\right] + v_2 [B_0 + (\alpha - \frac{1}{2}\sigma^2)t] - E[h]}{\gamma v_1}$$

Since by (A.4.3) $E(h)$ is independent on t , differentiating with respect to t , we obtain the expected long-run rate of deforestation:

$$\begin{aligned}
\frac{1}{dt} E[d \ln A] &\simeq \frac{v_2 \alpha - \frac{1}{2}\sigma^2}{v_1 \gamma} \\
&= -\frac{\alpha - \frac{1}{2}\sigma^2}{\gamma} e^{\ln \tilde{B} - \ln \tilde{J}} \quad \text{for } \alpha < \frac{1}{2}\sigma^2
\end{aligned}$$

By the monotonicity property of the logarithm, \tilde{B} must exist such that $\ln \tilde{B} = \ln \tilde{B}$. Furthermore, by plugging \tilde{B} into (10), we can always find a surface \tilde{A} and $\tilde{J} = \frac{\beta_2}{\beta_2 - 1} (r - \alpha) \Psi \frac{P_A(\tilde{A})}{r}$ such that a linearization along $(\ln \tilde{B}, \ln \tilde{J})$ is equivalent to a linearization along $(\ln \tilde{B}, \ln \tilde{J})$, where $\ln \tilde{J} = \ln \tilde{J}$. This implies that by setting (\tilde{B}, \tilde{A}) , the long-run average rate of deforestation can be written as:

$$\begin{aligned}
\frac{1}{dt} E[d \ln A] &= -\frac{\alpha - \frac{1}{2}\sigma^2}{\gamma} \frac{\tilde{B}}{\tilde{J}} = -\frac{\alpha - \frac{1}{2}\sigma^2}{\gamma} \frac{1}{1 + \frac{\beta_2}{\beta_2 - 1} (r - \alpha) \Psi \frac{c}{\tilde{B}}} \\
&= -\frac{\alpha - \frac{1}{2}\sigma^2}{\gamma} \frac{\frac{P_A(\tilde{A})}{r} - c}{\frac{P_A(\tilde{A})}{r}} = -\frac{\alpha - \frac{1}{2}\sigma^2}{\gamma} \left(1 - \frac{c}{\frac{\delta}{r} \tilde{A}^{-\gamma}}\right)
\end{aligned}$$

where $\frac{P_A(\tilde{A})}{r} = \frac{\tilde{B}}{\frac{\beta_2}{\beta_2 - 1} (r - \alpha) \Psi} + c$ and $\tilde{A} < \hat{A}$.

Similarly, from the logarithm of (18) we derive:

$$\begin{aligned}
\ln \varsigma &= \ln \left[\frac{\beta_1}{\beta_1 - 1} (1 - \lambda) \frac{P_A}{r} + \frac{\lambda\eta_2 - \eta_1}{r - \alpha} B \right] \\
&= \ln \left[\frac{\lambda\eta_2 - \eta_1}{r - \alpha} \right] + \ln [K + B]
\end{aligned} \tag{A.5.4}$$

where $K = \frac{\beta_1}{\beta_1 - 1} (r - \alpha) \Omega \frac{F_A(A)}{r}$ and $K > -B$. Rewriting $\ln[K + B]$ as $\ln[e^{\ln K} + e^{\ln B}]$ and expanding by Taylor's theorem around the point $(\widetilde{\ln K}, \widetilde{\ln B})$ leads to:

$$\begin{aligned} \ln[K + B] &\simeq \ln \left[e^{\widetilde{\ln K}} + e^{\widetilde{\ln B}} \right] + \frac{\ln K - \widetilde{\ln K}}{1 + e^{\widetilde{\ln B} - \widetilde{\ln K}}} + \frac{\ln B - \widetilde{\ln B}}{1 + e^{-(\widetilde{\ln B} - \widetilde{\ln K})}} \\ &= w_0 + w_1 \ln K + w_2 \ln B \end{aligned}$$

where

$$\begin{aligned} w_0 &= \ln \left[e^{\widetilde{\ln K}} + e^{\widetilde{\ln B}} \right] - \left[\frac{\widetilde{\ln K}}{1 + e^{\widetilde{\ln B} - \widetilde{\ln K}}} + \frac{\widetilde{\ln B}}{1 + e^{-(\widetilde{\ln B} - \widetilde{\ln K})}} \right] \\ w_1 &= \frac{1}{1 + e^{\widetilde{\ln B} - \widetilde{\ln K}}}, \quad w_2 = \frac{1}{1 + e^{-(\widetilde{\ln B} - \widetilde{\ln K})}}, \quad \frac{w_2}{w_1} = \frac{1 - w_1}{w_1} > 0 \end{aligned}$$

By plugging the approximated value for $\ln[K + B]$ into (A.5.4) we obtain:

$$\ln \zeta \simeq \ln \left[\frac{\lambda \eta_2 - \eta_1}{r - \alpha} \right] + w_0 + w_1 \ln K + w_2 \ln B \quad (\text{A.5.5})$$

Also in this case, $\ln \zeta$ evolves according to the motion $d \ln \zeta = w_2 d \ln B = w_2 [(\alpha - \frac{1}{2} \sigma^2) dt + \sigma dw]$ and $\ln \hat{\zeta}$ is a reflecting barrier. By setting $h = \ln \zeta$, $\ln \zeta$ evolves as a linear Brownian motion with parameter $\mu = w_2 (\alpha - \frac{1}{2} \sigma^2) > 0$ and (A.4.2) as density function. Then, solving (A.5.5) with respect to $\ln A$ yields the long-run optimal stock of cleared land:

$$\ln A \simeq \frac{\ln \left[\frac{\lambda \eta_2 - \eta_1}{(r - \alpha)} \right] + w_0 + w_1 \ln \left[\frac{\beta_1}{\beta_1 - 1} (r - \alpha) \Omega \frac{\delta}{r} \right] + w_2 \ln B - h}{\gamma w_1} \quad (\text{A.5.6})$$

As above, from (A.5.6) it follows that

$$\begin{aligned} 1 &= \exp\left(\frac{w_0}{w_1}\right) \left(\frac{\lambda \eta_2 - \eta_1}{r - \alpha}\right)^{\frac{1}{w_1}} \left[\frac{\beta_1}{\beta_1 - 1} (r - \alpha) \Omega \frac{\delta}{r}\right] A^{-\gamma} B^{\frac{w_2}{w_1}} \\ &= \exp\left(\frac{w_0}{w_1}\right) \left[\frac{\beta_1}{\beta_1 - 1} (r - \alpha) \Omega\right]^{-\frac{w_2}{w_1}} \frac{\delta}{r} c^{-\frac{1}{w_1}} A^{-\gamma} B^{\frac{w_2}{w_1}} \\ &= \exp\left(\frac{w_0}{w_1}\right) \left(\frac{K}{\frac{\delta}{r} A^{-\gamma}}\right)^{-\frac{w_2}{w_1}} \frac{\delta}{r} c^{-\frac{1}{w_1}} A^{-\gamma} B^{\frac{w_2}{w_1}} \\ &= \exp\left(\frac{w_0}{w_1}\right) K^{-\frac{w_2}{w_1}} \left(\frac{\delta}{r} A^{-\gamma}\right)^{\frac{1}{w_1}} B^{\frac{w_2}{w_1}} \\ &= \exp(w_0) K^{-w_2} \left(\frac{\hat{A}}{A}\right)^{\gamma} B^{w_2} \\ &= \frac{\tilde{K} + \tilde{B}}{\tilde{K}^{w_1} \tilde{B}^{w_2}} K^{-w_2} \left(\frac{\hat{A}}{A}\right)^{\gamma} B^{w_2} \\ &= \left(\frac{\hat{A}}{A}\right) \left(\frac{B}{\tilde{B}}\right)^{-\frac{1}{\gamma}} \left[1 - \left(\frac{\hat{A}}{A}\right)^{\gamma}\right] \end{aligned}$$

and

$$\frac{\hat{A}}{\tilde{A}} = \left(\frac{B}{\tilde{B}}\right)^{-\frac{1}{\gamma}} \left[1 - \left(\frac{\hat{A}}{A}\right)^{\gamma}\right]$$

Note that since in this case $\tilde{A} > \hat{A}$ then $-\frac{1}{\gamma} \left[1 - \left(\frac{\hat{A}}{A}\right)^{\gamma}\right] > 0$.

From (A.5.6) the expected value of $\ln A$ is given by:

$$E[\ln A] \simeq \frac{\ln \left[\frac{\lambda \eta_2 - \eta_1}{r - \alpha} \right] + w_0 + w_1 \ln \left[\frac{\beta_1}{\beta_1 - 1} (r - \alpha) \Omega \frac{\delta}{r} \right] + w_2 [B_0 + (\alpha - \frac{1}{2} \sigma^2)t] - E[h]}{\gamma w_1}$$

By the usual steps:

$$\begin{aligned}\frac{1}{dt}E[d\ln A] &\simeq \frac{w_2}{w_1} \frac{\alpha - \frac{1}{2}\sigma^2}{\gamma} \\ &= \frac{\alpha - \frac{1}{2}\sigma^2}{\gamma} e^{\ln \tilde{B} - \ln \tilde{K}} \quad \text{for } \alpha > \frac{1}{2}\sigma^2\end{aligned}$$

As shown above, having fixed a pair (\tilde{B}, \tilde{A}) we can conveniently linearize along $(\ln \tilde{B}, \ln \tilde{J})$ where $\ln \tilde{B} = \ln \tilde{B}$ and $\ln \tilde{K} = \ln \tilde{K}$ with $\tilde{K} = \frac{\beta_1}{\beta_1 - 1} (r - \alpha) \frac{1 - \lambda}{\eta_2 \lambda - \eta_1} \frac{P_A(\tilde{A})}{r}$. This leads to the following expression for the long-run average rate of deforestation:

$$\begin{aligned}\frac{1}{dt}E[d\ln A] &= \frac{\alpha - \frac{1}{2}\sigma^2}{\gamma} \frac{\tilde{B}}{\tilde{K}} = \frac{\alpha - \frac{1}{2}\sigma^2}{\gamma} \frac{1}{\frac{\beta_1}{\beta_1 - 1} (r - \alpha) \Omega \frac{c}{\tilde{B}} - 1} \\ &= \frac{\alpha - \frac{1}{2}\sigma^2}{\gamma} \frac{c - \frac{P_A(\tilde{A})}{r}}{\frac{P_A(\tilde{A})}{r}} = \frac{\alpha - \frac{1}{2}\sigma^2}{\gamma} \left(\frac{c}{\frac{\delta}{r} \tilde{A}^{-\gamma}} - 1 \right)\end{aligned}$$

where $\frac{P_A(\tilde{A})}{r} = c - \frac{\tilde{B}}{\frac{\beta_1}{\beta_1 - 1} (r - \alpha) \Omega}$ and $\tilde{A} > \hat{A}$.

A.6 The impact of uncertainty on the distribution of ξ (and ς)

Rearranging (A.5.2) yields

$$\ln \xi \simeq U_\xi + v_2 \ln B \tag{A.6.1}$$

where $U_\xi = \ln \frac{\eta_1 - \lambda \eta_2}{r - \alpha} + v_0 + v_1 \ln J$.

By some manipulations:

$$\begin{aligned}e^{\ln \xi} &= e^{U_\xi + v_2 \ln B} \\ \xi &= e^{U_\xi} B^{v_2}\end{aligned} \tag{A.6.2}$$

Using Ito's lemma

$$\begin{aligned}d\xi &= e^{U_\xi} \left[v_2 B^{v_2 - 1} dB + \frac{1}{2} v_2 (v_2 - 1) B^{v_2 - 2} (dB)^2 \right] \\ &= e^{U_\xi} B^{v_2} v_2 \left\{ \left[\alpha + \frac{1}{2} (v_2 - 1) \sigma^2 \right] dt + \sigma dw \right\} \\ &= \xi v_2 \left\{ \left[\alpha + \frac{1}{2} (v_2 - 1) \sigma^2 \right] dt + \sigma dw \right\}\end{aligned}$$

Calculating first, second moment and variance for ξ we obtain:

$$\begin{aligned}E(\xi) &= \xi(0) e^{v_2 [\alpha + \frac{1}{2} (v_2 - 1) \sigma^2] t} \\ E(\xi^2) &= \xi^2(0) e^{2v_2 [\alpha + (v_2 - \frac{1}{2}) \sigma^2] t} \\ Var(\xi) &= \xi^2(0) e^{2v_2 [\alpha + \frac{1}{2} (v_2 - 1) \sigma^2] t} (e^{v_2^2 \sigma^2 t} - 1)\end{aligned}$$

Note that since $\alpha + \frac{1}{2} (v_2 - 1) \sigma^2 < 0$ and $v_2 < 0$ then $E(\xi)$ is increasing in t . Finally, by deriving $Var(\xi)$ with respect to σ it is easy to check that

$$\frac{\partial Var(\xi)}{\partial \sigma} = 2v_2 \sigma t e^{2v_2 [\alpha + \frac{1}{2} (v_2 - 1) \sigma^2] t} \xi^2(0) \left[(v_2 - 1) (e^{v_2^2 \sigma^2 t} - 1) + v_2 e^{v_2^2 \sigma^2 t} \right] > 0$$

That is, as σ soars $Var(\xi)$ increases and so does the probability of hitting $\hat{\xi}$ which in turn implies an increase in the long run average deforestation rate.

Similarly, starting from (A.5.5)

$$\ln \zeta \simeq U_\zeta + w_2 \ln B \quad (\text{A.6.3})$$

where $U_\zeta = \ln \left[\frac{\lambda \eta_2 - \eta_1}{r - \alpha} \right] + w_0 + w_1 \ln K$, one may easily check that

$$\text{Var}(\zeta) = \zeta^2(0) e^{2w_2[\alpha + \frac{1}{2}(w_2 - 1)\sigma^2]t} (e^{w_2^2 \sigma^2 t} - 1)$$

and

$$\frac{\partial \text{Var}(\zeta)}{\partial \sigma} = 2w_2 \sigma t e^{2w_2[\alpha + \frac{1}{2}(w_2 - 1)\sigma^2]t} \zeta^2(0) \left[(w_2 - 1)(e^{w_2^2 \sigma^2 t} - 1) + w_2 e^{w_2^2 \sigma^2 t} \right] < 0$$

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