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GIMME A BREAK! IDENTIFICATION AND ESTIMATION OF THE MACROECONOMIC EFFECTS OF MONETARY POLICY SHOCKS IN THE U.S.

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July 2014

"MARCO FANNO" WORKING PAPER N.181

Gimme a Break! Identification and Estimation of the Macroeconomic Effects of Monetary Policy Shocks in the U.S.*

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Abstract

We employ a novel identification scheme to quantify the macroeconomic effects of monetary policy shocks in the United States. The identification of the shocks is achieved by exploiting the instabilities in the contemporaneous coefficients of the structural VAR (SVAR) and in the covariance matrix of the reduced-form residuals. Different volatility regimes can be associated with different transmission mechanisms of the identified structural shocks. We formally test and reject the stability of our impulse responses estimated with post-WWII U.S. data by working with a break in macroeconomic volatilities occurred in the mid-1980s. We show that the impulse responses obtained with our non-recursive identification scheme are quite similar to those conditional on a standard Cholesky-SVARs estimated with pre-1984 data. In contrast, recursive vs. non-recursive identification schemes return substantially different macroeconomic reactions conditional on Great Moderation data, in particular as for inflation and a long-term interest rate. Using our non-recursive SVARs as auxiliary models to estimate a small-scale new-Keynesian model of the business cycle with an impulse response function matching approach, we show that the instabilities in the estimated VAR impulse responses are informative as for the calibration of some key-structural parameters.

Keywords: Structural break, recursive and non-recursive VARs, identification, monetary policy shocks, impulse responses.

J.E.L. classification: C32, C50, E52.

*We thank Giovanni Caggiano, Riccardo Lucchetti, Michel Normandin, Giovanni Pellegrino, and Jouko Vilmunen for their valuable comments, and participants to presentations held to the Fourth International Conference in memory of Carlo Giannini (Pavia), the Society for Computational Economics (Oslo), the Bank of Finland and the University of Padova for useful interactions. Authors' contacts:
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1 Introduction

Since the seminal contribution by Sims (1980), Structural Vector Autoregressions (SVARs henceforth) have widely been employed by macroeconomists to establish stylized facts and discriminate among competing models. A lot of effort has been devoted to study the effects of monetary policy shocks in the United States (see, among others, Christiano, Eichenbaum, and Evans (1999), (2005)). Typically, exogenous variations of the federal funds rate have been identified by estimating fixed coefficient-VARs and appealing to the Cholesky-identification scheme. In other words, researchers have exploited long samples and applied a scheme which imposes a recursive structure on the contemporaneous relationships of the macroeconomic variables of interest. The underlying assumptions behind such identification scheme are: i) some macroeconomic variables (e.g., real GDP, inflation) are 'slow moving', i.e., they are assumed to react to monetary policy shocks with a lag; ii) the systematic monetary policy component immediately reacts to macroeconomic shocks that affect the equilibrium value of such slow moving variables.

Fixed coefficient-recursive VARs are potentially quite powerful, because of the number of degrees of freedom and the fact that they do not require the econometrician to identify macroeconomic shocks other than the one of interest. However, some researchers have found evidence inconsistent with the assumption of a lower-triangular economic system (see Del Negro, Schorfheide, Smets, and Wouters (2007) as for the immediate reaction of output to a monetary policy shock, Faust, Swanson, and Wright (2004) as for the contemporaneous response of prices, Normandin and Phaneuf (2004) as for both, and Gertler and Karadi (2014) as regards the on-impact reactions of a variety of rates related to different maturities). Moreover, most DSGE macroeconomic models typically feature no lags in the monetary policy transmission mechanism (see, e.g., King (2000), Smets and Wouters (2007), Galí (2008)).[1] As a matter of fact, however, recursive schemes are still quite popular among VAR macroeconomists. The reason is simple. More often than not, the non-recursive schemes implied by DSGE models are unfeasible due to insufficient information coming from the reduced-form variance-covariance matrix.[2] Moreover, fixed coefficient-VAR are questionable in presence of

[1] Notable exceptions are Rotemberg and Woodford (1997), Christiano, Eichenbaum, and Evans (2005), Boivin and Giannoni (2006), and Altig, Christiano, Eichenbaum, and Lindé (2011).

[2] Of course, such non-recursive schemes become feasible if the econometrician imposes the full set of cross-equation restrictions due to rational expectations. In this case, however, the need of estimating a VAR is unclear, given the knowledge of the true data-generating process by the econometrician.

parameter instability. Changes in the variance of the shocks hitting the U.S. economic system are well documented (see, e.g., McConnell and Perez-Quiros (2000), Stock and Watson (2002), Sims and Zha (2006), Smets and Wouters (2007), Justiniano and Primiceri (2008), Canova, Gambetti, and Pappa (2008), and Canova (2009)). Breaks in the policy regime, possibly related to changes in the Federal Reserve's chairmanship, have also been supported by some recent empirical investigations (Clarida, Galí, and Gertler (2000), Lubik and Schorfheide (2004), Boivin and Giannoni (2006), Benati and Surico (2009), Mavroeidis (2010), and Castelnuovo and Fanelli (2013)). Variations in the parameters related to private sector's behavior have been detected by, among others, Canova (2009), Inoue and Rossi (2011), Canova and Menz (2011), Canova and Ferroni (2012), and Castelnuovo (2012). Unfortunately, in presence of breaks, VARs estimated over long samples may return a biased picture of the macroeconomic effects of monetary policy shocks.

This paper's contribution is threefold. First, we propose a novel identification strategy to assess the macroeconomic effects of monetary policy shocks in SVARs. Our strategy is based on the idea that structural, contemporaneous macroeconomic relationships may change across different heteroskedasticity regimes detected in the data. Breaks in the reduced-from variance-covariance matrix are shown to be informative as for changes in the impulse vectors of our VARs as well as variations in the size of the monetary policy shock. We present and discuss novel necessary and sufficient identification conditions that generalize those related to fixed parameter-SVARs. The structural models identified with our methodology are labelled 'SVAR-WB' (where 'WB' stands for 'with break'). Importantly, our methodology jointly tackles the two issues indicated above, i.e., the possibility of non-zero contemporaneous macroeconomic responses to monetary policy shocks and parameter instability.

We then employ our SVAR-WB to model a vector of seven U.S. macroeconomic variables for the post-WWII period, and contrast our impulse responses with those obtained with alternative identifications schemes. The identification of our break-based VAR models exploits the change in the variance of the reduced form VAR errors detected in the mid-1980s. This choice is justified by the vast literature documenting heteroskedasticity in the U.S. (McConnell and Perez-Quiros (2000), Stock and Watson (2002), Sims and Zha (2006), Smets and Wouters (2007), Justiniano and Primiceri (2008), Canova, Gambetti, and Pappa (2008), and Canova (2009)).

Finally, we estimate a small-scale new-Keynesian model of the business cycle featuring a cost-channel à la Ravenna and Walsh (2006) via impulse response function matching. Similar frameworks have successfully been employed in recent empirical analysis to describe features of the U.S. business cycle (see, among others, Clarida, Galí, and Gertler (2000), Lubik and Schorfheide (2004), Boivin and Giannoni (2006), and Benati and Surico (2009)) or to interpret some VAR-related puzzles (Carlstrom, Fuerst, and Paustian (2009)). We estimate a version of the Ravenna and Walsh (2006) model by minimizing the distance between the impulse response functions (IRFs) implied by the New-Keynesian model and the IRFs estimated with our non-recursive SVAR-WB. This exercise is conducted to shed light on the relationship between the instabilities of our VAR impulse responses (due to a structural break) and the possible time-dependence of (a subset of) the parameters of a new-Keynesian model widely employed in empirical analysis.

Our results read as follows. First, we provide formal evidence of instability in the U.S. post-WWII macroeconomic impulse responses to a monetary policy shock. Such evidence is conditional on a break in the early 1980s, which can be easily interpreted in light of the switch from the Great Inflation to the macroeconomic Great Moderation.[3] This result is robust to the employment of recursive and non-recursive identification schemes, and - as shown in a Technical Supplement available upon request - it is valid irrespective of whether the data collected in the SVARs-WB are modeled as highly persistent stationary time series or as non-stationary cointegrated time series. Digging further, we find that the VAR dynamics typically associated to the entire post-WWII U.S. history are *de facto* driven by observations related to the Great Inflation only. This result clearly applies to the 'price puzzle', which is the positive short-run reaction of inflation to a temporary monetary policy tightening. We find the 'price puzzle' to be present in the first sub-sample only, no matter what the identification scheme employed in our analysis is.

Second, we show that recursive restrictions imply responses that are extremely (and somewhat surprisingly) similar to those produced with a non-recursive scheme for the Great Inflation period. Quite differently, these two alternative identification schemes lead to substantially heterogeneous predictions on the macroeconomic effects of monetary policy shocks as for the post-1984 phase. In particular, the recessionary and deflationary effects of policy shocks are estimated to be more profound. Moreover, the hump-shaped macroeconomic path often found in recursive SVARs is not confirmed by our non-recursive framework as for durable consumption, inflation, and the federal

[3]The term 'Great Moderation' was coined by Stock and Watson (2002) to indicate the substantial reduction in the volatility of the U.S. real GDP growth rate and inflation occurred in the mid-1980s.

funds rate, and the reaction of the long-term interest rate is documented to be significantly negative. This last result confirms the one by Bagliano and Favero (1998), who also work with a non-recursive scheme and find a negative short run correlation between the federal funds rate and the long-term interest rate conditional on policy shocks in the Great Moderation period. Third, our impulse response matching exercise points to clear instabilities in the estimated structural parameters of the Ravenna and Walsh (2006) model. In particular, the influence exerted by the cost channel in affecting inflation is found to be drastically lower when moving from the pre-1984 phase to the Great Moderation. In light of the relevance of the cost channel for monetary policy design stressed by Ravenna and Walsh (2006), we believe that this finding is a good example to underline the importance of accounting for breaks when conducting impulse response function matching estimations of monetary policy DSGE models. The SVAR-WB approach proposed in this paper naturally accounts for such instabilities.

The paper is organized as follows. In Section 2 we discuss the methodology used to identify the macroeconomic effects of monetary policy in our reference SVAR-WB. In Section 3 we present and discuss a battery of results obtained with a standard vector of seven U.S. macroeconomic series. Section 4 proposes the estimation of a small-scale structural model featuring the cost-channel via impulse response function matching. Section 5 discusses the contacts with the methodological literature. Section 6 concludes.

2 The SVAR-WB: Identification analysis

We introduce a novel methodology to identify the macroeconomic effects of monetary policy shocks when the VAR error covariance matrix and the parameters governing the mapping from the VAR disturbances are subject to breaks. We also present novel necessary and sufficient identification conditions.

To fix ideas and notation, we briefly start from a reference fixed parameter-SVAR. Let $\boldsymbol{z}_t = (z_{1,t}, ..., z_{n,t})'$ be the $n \times 1$ vector of observable variables. We assume that the reference model for $\boldsymbol{z}_t$ is given by the SVAR:

$$\boldsymbol{z}_t = \boldsymbol{\Pi} \boldsymbol{w}_t + \boldsymbol{u}_t \ \ , \ \boldsymbol{u}_t = \boldsymbol{C} \boldsymbol{e}_t \ \ , \ \boldsymbol{e}_t \sim \mathrm{WN}(\boldsymbol{0}_n \ , \boldsymbol{I}_n), \quad t = 1, ..., T. \tag{1}$$

In system (1), $\boldsymbol{u}_t$ is the n-dimensional White Noise process of reduced form errors (disturbances) with covariance matrix $\boldsymbol{\Sigma}_u = E(\boldsymbol{u}_t \boldsymbol{u}_t')$, $\boldsymbol{w}_t = (\boldsymbol{z}_{t-1}', \boldsymbol{z}_{t-2}', ..., \boldsymbol{z}_{t-k}', \boldsymbol{d}_t')'$ is the vector of VAR regressors, k is the VAR lag order and $\boldsymbol{d}_t$ is a b-dimensional subvector collecting deterministic components. The reduced form parameters are contained in $\mathbf{\Pi} = (\boldsymbol{A}_1, ..., \boldsymbol{A}_k, \boldsymbol{\Psi})$ and $\mathbf{\Sigma}_u$, where $\boldsymbol{A}_j$, $j = 1, ..., k$ are $n \times n$ matrices, $\boldsymbol{e}_t$ is the n-dimensional vector of orthogonal structural shocks, $\boldsymbol{\Psi}$ collects the loadings of the deterministic components, and $\boldsymbol{C}$ is the $n \times n$ matrix which maps the structural shocks onto the VAR disturbances. We denote 'reduced form parameters' the elements in $\mathbf{\Pi}$ and $\mathbf{\Sigma}_u$, and 'structural parameters' the elements in $\boldsymbol{C}$.

Next, we assume that it is known that at time T_B, where $1 < T_B < T$, the matrix $\mathbf{\Sigma}_u$ changes. In our setup, also the matrix $\mathbf{\Pi}$ is allowed to change. The date T_B corresponds to the first observation in the second regime. We focus on the case of a single break for simplicity and consistently with the developments in the next empirical sections. However, our methodology can in principle deal with a number of break dates larger than one (for a discussion, see Bacchiocchi and Fanelli (2012)).[4]

The baseline specification in eq. (1) is replaced with

$$\boldsymbol{z}_t = \mathbf{\Pi}(t)\boldsymbol{w}_t + \boldsymbol{u}_t \ , \ \boldsymbol{u}_t = \boldsymbol{C}(t)\boldsymbol{e}_t \ \ , \ \boldsymbol{e}_t \sim \mathrm{WN}(\mathbf{0}_n \ , \ \boldsymbol{I}_n) \tag{2}$$

where $\mathbf{\Pi}(t)$ and $\mathbf{\Sigma}_u(t)$ are given by

$$\mathbf{\Pi}(t) = \mathbf{\Pi}_1 \times \mathbf{1}\left(t < T_B\right) + \mathbf{\Pi}_2 \times \mathbf{1}(t \geq T_B) \tag{3}$$

$$\mathbf{\Sigma}_u(t) = \mathbf{\Sigma}_{u,1} \times \mathbf{1}\left(t < T_B\right) + \mathbf{\Sigma}_{u,2} \times \mathbf{1}(t \geq T_B), \tag{4}$$

where $\mathbf{1}\left(\cdot\right)$ is the indicator function, $\mathbf{\Pi}_1$ and $\mathbf{\Sigma}_{u,1}$ are the matrices of reduced form parameters in the pre-break regime and $\mathbf{\Pi}_2$ and $\mathbf{\Sigma}_{u,2}$ are the reduced form parameters in the post-break regime, respectively. We temporarily leave the $\boldsymbol{C}(t)$ matrix in eq. (2) unspecified. Our main assumption is that $\mathbf{\Sigma}_{u,1} \neq \mathbf{\Sigma}_{u,2}$, i.e., we assume our data to be characterized by two volatility regimes. Differently, we allow, but not necessarily require, the condition $\mathbf{\Pi}_1 \neq \mathbf{\Pi}_2$ to be met.

One crucial hypothesis in the recent literature on the identification of SVARs through changes in volatility is that the variation in $\mathbf{\Sigma}_u$ *is not associated* with a change in $\boldsymbol{C}$, i.e. $\boldsymbol{C}(t) = \boldsymbol{C}$ for $t = 1, ..., T$, given eq. (4). Under this condition, one can uniquely identify the elements of $\boldsymbol{C}$ (up to sign changes) by exploiting the simultaneous factorization of the matrices $\mathbf{\Sigma}_{u,1}$ and $\mathbf{\Sigma}_{u,2}$:

[4] In a companion paper, Bacchiocchi and Fanelli (2012) apply the methodology proposed in this paper to a small-scale VAR which focuses on the interaction between nominal interest rate and money. Our larger-scale VAR involves macroeconomic indicators such as consumption and investment, which carry relevant information on agents' expectations. This is done to i) study the effects of monetary policy shocks identified with our methodology on those variables, and ii) tackle the non-fundamentalness issue which is likely to arise when dealing with small-scale VARs (see Forni and Gambetti (2014) and the references therein).

$$\boldsymbol{\Sigma}_{u,1} = \boldsymbol{P}\boldsymbol{P}' \quad , \quad \boldsymbol{\Sigma}_{u,2} = \boldsymbol{P}\boldsymbol{V}\boldsymbol{P}' \tag{5}$$

where $\boldsymbol{P}$ is a $n \times n$ non-singular matrix and $\boldsymbol{V}$ =diag$(v_1, ..., v_n) \neq \boldsymbol{I}_n$ is a diagonal matrix with elements $v_i > 0$, $i = 1, ..., n$. Identification can be achieved by setting $\boldsymbol{C} = \boldsymbol{P}$, where the choice $\boldsymbol{C} = \boldsymbol{P}$ is unique except for sign changes if all v_i's are distinct, see Lanne and Lütkepohl (2008, 2010). In this case, no theory-driven restriction is needed to identify $\boldsymbol{C}$. However, as previously pointed out, one has to assume the coefficients of the matrix of the contemporaneous relationships to be fixed. We relax this assumption by *allowing for changes in the* $\boldsymbol{C}$ *matrix* via the following specification:

$$\boldsymbol{C}(t) = \boldsymbol{C} + \boldsymbol{Q} \times \mathbf{1}\,(t \geq T_B) \tag{6}$$

where $\boldsymbol{Q}$ is a $n \times n$ matrix whose elements capture the changes of the coefficients of $\boldsymbol{C}$ from the pre- to the post-break regime, and the matrix $(\boldsymbol{C} + \boldsymbol{Q})$ is assumed to be invertible. The so defined SVAR gives rise to the set of restrictions

$$\boldsymbol{\Sigma}_{u,1} = \boldsymbol{C}\boldsymbol{C}' \quad , \quad t = 1, ..., T_B - 1 \tag{7}$$

$$\boldsymbol{\Sigma}_{u,2} = (\boldsymbol{C} + \boldsymbol{Q})(\boldsymbol{C} + \boldsymbol{Q})' \quad , \quad t = T_B, ..., T. \tag{8}$$

In this parametrization, the hypothesis $\boldsymbol{\Sigma}_{u,1} \neq \boldsymbol{\Sigma}_{u,2}$ implies $\boldsymbol{Q} \neq 0_{n\times n}$, i.e. the change in the covariance matrix is automatically associated with a change in the structural parameters.[5]

Eq.s (7)-(8) are not sufficient to identify the shocks of the SVAR with a break. To see this, observe that eq.s (7)-(8) provide $n(n+1)$ symmetry-induced restrictions, while $\boldsymbol{C}$ and $\boldsymbol{Q}$ contain $2n^2$ elements. In absence of further restrictions, $n(n-1)$ parameters in $\boldsymbol{C}$ and $\boldsymbol{Q}$ are unidentified. We thus consider a joint set of linear restrictions on $\boldsymbol{C}$ and $\boldsymbol{Q}$ of the form

$$\begin{pmatrix} vec(\boldsymbol{C}) \\ vec(\boldsymbol{Q}) \end{pmatrix} = \begin{pmatrix} \boldsymbol{S}_C & \boldsymbol{S}_I \\ \mathbf{0}_{n^2\times a_C} & \boldsymbol{S}_Q \end{pmatrix} \begin{pmatrix} \boldsymbol{\varphi} \\ \boldsymbol{q} \end{pmatrix} + \begin{pmatrix} \boldsymbol{s}_C \\ \boldsymbol{s}_Q \end{pmatrix}, \tag{9}$$

then the SVAR in eq. (2) can be identified.[6] In eq. (9), $\boldsymbol{\varphi}$ is a $a_C \times 1$ vector which collects the unrestricted (free) structural parameters, $\boldsymbol{S}_C$ is a $n^2 \times a_C$ known selection

[5] The converse, instead, is not generally true because it is possible to find examples in which $\boldsymbol{Q} \neq 0_{n\times n}$ but $\boldsymbol{\Sigma}_{u,1} = \boldsymbol{\Sigma}_{u,2}$. Consider, as an example, the case $\boldsymbol{C} := \begin{bmatrix} c_{11} & -c_{12} \\ c_{12} & c_{22} \end{bmatrix}$, $\boldsymbol{Q} := 2c_{12} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. It is easy to verify that $\boldsymbol{\Sigma}_{u,1} = \boldsymbol{\Sigma}_{u,2} = \begin{bmatrix} c_{11}^2 + c_{12}^2 & c_{11}c_{12} - c_{12}c_{22} \\ c_{11}c_{12} - c_{12}c_{22} & c_{12}^2 + c_{22}^2 \end{bmatrix}$.

[6] The upper triangular structure of $\boldsymbol{S} = \begin{pmatrix} \boldsymbol{S}_C & \boldsymbol{S}_I \\ \mathbf{0}_{n^2\times a_C} & \boldsymbol{S}_Q \end{pmatrix}$ is not mandatory. Indeed, it can be easily shown that given a set of generic linear restrictions, using the QR-decomposition, it can be transformed into an upper triangular structure (upper trapezoidal matrix) as in the definition of S.

matrix, $\boldsymbol{s}_C$ is a $n^2 \times 1$ known vector and $a_C \leq \frac{1}{2}n(n-1)$, $\boldsymbol{q}$ is a $a_Q \times 1$ vector which collects the unrestricted (free) elements of $\boldsymbol{Q}$, $\boldsymbol{S}_Q$ is a $n^2 \times a_Q$ known selection matrix and $\boldsymbol{s}_Q$ is a $n^2 \times 1$ known vector. Finally, $\boldsymbol{S}_I$ is a known selection matrix by which it is possible to impose cross-restrictions on the elements of $\boldsymbol{C}$ and $\boldsymbol{Q}$ (obviously $\boldsymbol{S}_I$ will be zero in the case of no cross-restrictions).

Throughout the paper we denote the system described by eq.s (2)-(4), eq. (6) and eq. (9) with the acronym 'SVAR-WB'. The next proposition derives our main result on the identification of SVARs-WB.

Proposition 1 *Consider the SVAR-WB defined by eqs. (2)-(4), eq. (6) and the restrictions in eq. (9). Assume that* $\boldsymbol{\Sigma}_{u,1} \neq \boldsymbol{\Sigma}_{u,2}$ *and that the matrices* $\boldsymbol{C}_0$ *and* $(\boldsymbol{C}_0+\boldsymbol{Q}_0)$ *are non-singular, where* $\boldsymbol{C}_0$ *and* $\boldsymbol{Q}_0$ *are the counterparts of* $\boldsymbol{C}$ *and* $\boldsymbol{Q}$ *once the vectors* $\boldsymbol{\varphi}$ *and* $\boldsymbol{q}$ *in eq. (9) have been replaced by their 'true' values* $\boldsymbol{\varphi}_0$ *and* $\boldsymbol{q}_0$*. Then, the necessary and sufficient rank condition for the SVAR-WB to be locally identified is*

$$rank\left\{(\boldsymbol{I}_2 \otimes \boldsymbol{D}_n^+)\begin{pmatrix} (\boldsymbol{C}_0 \otimes \boldsymbol{I}_n) & \boldsymbol{0}_{n^2\times n^2} \\ (\boldsymbol{C}_0+\boldsymbol{Q}) \otimes \boldsymbol{I}_n & (\boldsymbol{C}_0+\boldsymbol{Q}_0) \otimes \boldsymbol{I}_n \end{pmatrix}\begin{pmatrix} \boldsymbol{S}_C & \boldsymbol{S}_I \\ \boldsymbol{0}_{n^2\times a_C} & \boldsymbol{S}_Q \end{pmatrix}\right\} = a_C + a_Q \tag{10}$$

where $\boldsymbol{D}_n^+ = (\boldsymbol{D}_n'\boldsymbol{D}_n)^{-1}\boldsymbol{D}_n'$ *is the Moore-Penrose inverse of the duplication matrix* $\boldsymbol{D}_n$ *(i.e.* $\boldsymbol{D}_n$ *is such that* $\boldsymbol{D}_n vech(\boldsymbol{\Sigma}_{u,1}) = vec(\boldsymbol{\Sigma}_{u,1})$*), and* $\boldsymbol{v}_0 = (\boldsymbol{\varphi}_0', \boldsymbol{q}_0')'$ *is a 'regular point' (i.e. the rank condition in eq. (10) does not change within a neighborhood of* $\boldsymbol{v}_0$*). The necessary order condition is*

$$(a_C + a_Q) \leq n(n+1). \tag{11}$$

Proof. See Appendix.

■

If the SVAR-WB meets the rank condition in eq. (10), the SVAR-WB is identified.[7] In particular, the system is just-identified when the rank condition holds and the number of restrictions on $\boldsymbol{C}$ and $\boldsymbol{Q}$ is $(a_C + a_Q) = n(n+1)$, and is over-identified (with testable over-identification restrictions) when the rank condition holds with $(a_C+a_Q) < n(n+1)$. The (population) orthogonalized IRFs are given by

$$\boldsymbol{\Gamma}_{1,h} = \left[\gamma_{1,l,m,h}\right] = \boldsymbol{G}'(\boldsymbol{A}_1^*)^h \boldsymbol{G}\boldsymbol{C}_0 \ , \quad h = 0,1,2,... \text{ 'pre-break' regime} \tag{12}$$

[7] The full-column rank condition in eq. (10) can be verified ex-post by replacing $\boldsymbol{\gamma}$ and $\boldsymbol{q}$ with their maximum likelihood estimates. Alternatively, Bacchiocchi and Fanelli (2012) discuss an algorithm that can be used to check the rank condition prior to estimation.

$$\boldsymbol{\Gamma}_{2,h} = \left[\gamma_{2,l,m,h}\right] = \boldsymbol{G}'(\boldsymbol{A}_{\mathbf{2}}^{*})^{h}\boldsymbol{G}(\boldsymbol{C}_0+\boldsymbol{Q}_0) \ , \ \ h = 0,1,2,... \text{ `post-break' regime} \tag{13}$$

where $\boldsymbol{G}$ is a selection matrix of conformable dimensions and

$$\boldsymbol{A}_i^{*} = \begin{pmatrix} & \boldsymbol{\Pi}_i & \\ \boldsymbol{I}_{n(k-1)} & & \mathbf{0}_{n(k-1)\times n} \end{pmatrix} \quad , \quad i = 1,2$$

are the reduced form VAR companion matrices in the pre- and post-break regimes, respectively.[8] In this setup, the element $\gamma_{i,l,m,h} = \frac{\partial z_{l,t}}{\partial e_{m,t-h}}$ of the matrix $\boldsymbol{\Gamma}_{i,h}$ captures the response of variable $z_{l,t}$ to a structural shock $e_{m,t}$ at horizon h (h=0,1,2,...), in the volatility regime characterized by the covariance matrix $\boldsymbol{\Sigma}_{u,i}, i = 1,2$.

In presence of stationary variables, the IRFs in eq.s (12)-(13) can be estimated consistently by replacing the matrices $\boldsymbol{C}_0$, $\boldsymbol{Q}_0$ and $\boldsymbol{A}_i^{*}$, $i = 1,2$ ($\boldsymbol{A}_1^{*}$ and $\boldsymbol{A}_2^{*}$ have all their eigenvalues inside the unit circle) with their consistent estimates (see Bacchiocchi and Fanelli (2012)). The same can be done if $\boldsymbol{z}_t$ contains non-stationary cointegrated variables, under the condition that the unit roots driving the system are properly fixed as suggested in, e.g., Phillips (1998), Lütkepohl and Reimers (1992), Amisano and Giannini (1997) and Vlaar (2004), and discussed in our Technical Supplement. Confidence bands for the IRFs can be computed accordingly.

3 Empirical results

We now turn to our empirical application. We model a vector $\boldsymbol{z}_t$ of seven U.S. macroeconomic variables, i.e., non-durable personal consumption (NDCONS), durable personal consumption (DCONS), fixed-private investments (INVEST), gross domestic product (GDP), inflation (INFL), federal funds rate (FFR), and 10 year-Treasury Bill rate (10YR). The source of the data is the Federal Reserve Bank of St. Louis. The first four time series are all expressed in real and per-capita terms, and are considered in logs. The inflation rate is computed as the quarterly growth rate of the GDP deflator. The interest rates are quarterly rates. All series are expressed in percent terms.

We consider the sample 1960Q1-2008Q2. The beginning of the sample corresponds approximately to the beginning of the phase of rising inflation in the post-WWII U.S. economic history. The end of the sample is intended to limit the issues related to the recent financial crisis, which led the Federal Reserve to decrease the nominal interest rate and hit the zero lower bound, a situation which can hardly be captured by (regime-contingent) linear VARs like ours.

[8] Note that given our assumption on $\boldsymbol{\Pi_1}$ and $\boldsymbol{\Pi_2}$, $\boldsymbol{A_1^{*}}$ and $\boldsymbol{A_2^{*}}$ may be different or equal in eqs. (12)-(13).

Our aim is to identify the macroeconomic effects of a monetary policy shock. We do so by working with two SVARs-WB as represented in eq.s (2)-(9). Both these models are based on the (testable) hypothesis of a change in the error covariance matrix $\mathbf{\Sigma}_u$ at the beginning of the 1980s (see Section 3.1), and fulfill the identification conditions stated in Proposition 1. Importantly, they differ in the way the matrices ($\boldsymbol{C}$, $\boldsymbol{Q}$) are specified. The first one, labeled as 'recursive SVAR-WB', assumes a standard recursive structure of the economic system (Section 3.2). The second one, termed 'non-recursive SVAR-WB', allows (without necessarily requiring) (a) monetary policy shocks to contemporaneously affect all variables of the vector $\boldsymbol{z}_t$, and (b) all variables in $\boldsymbol{z}_t$ to exert a contemporaneous influence on the nominal interest rate (Section 3.3).[9]

3.1 Evidence of a change in the VAR error covariance matrix

We specify our reduced form VAR for $\mathbf{z}_t =(NDCONS_t, DCONS_t, INVEST_t, GDP_t, INFL_t, FFR_t, 10YR_t)$' with equation-specific constants and four lags.[10] As shown in Section 2, the SVAR-WB approach hinges upon the exploitation of a structural break. The macroeconomic literature has recently documented a dramatic fall in the variances of the main macroeconomic indicators, which has been termed 'Great Moderation'. Kim and Nelson (1999) and Stock and Watson (2002) offer support for a break in the macroeconomic volatilities around 1984. McConnell and Perez-Quiros (2000) identify 1984Q1 as the break-date of the variance of the U.S. real GDP. Boivin and Giannoni (2006) also detect a break in the coefficients of a reduced-form VAR for the U.S. economy in the early 1980s. As in Justiniano and Primiceri (2008) and Blanchard and Riggi (2013), we take such date as a break-point in our sample, i.e., $T_B =$1984Q1.

We formally test the occurrence of the break in the reduced form parameters at time $T_B =$1984Q1 through a standard LR Chow-type test. We first focus on the joint null hypothesis that the VAR reduced form parameters are constant across the two regimes 1960Q1-1983Q4 and 1984Q1-2008Q2, i.e. $(\mathbf{\Pi}_1=\mathbf{\Pi}_2) \wedge (\mathbf{\Sigma}_{u,1}=\mathbf{\Sigma}_{u,2})$ against the alternative $(\mathbf{\Pi}_1 \neq \mathbf{\Pi}_2) \vee (\mathbf{\Sigma}_{u,1} \neq \mathbf{\Sigma}_{u,2})$. The null hypothesis of stable parameters is clearly rejected. The LR statistic is equal to $LR =$-2[1213.20 - (602.89+953.90)]=687.18 and has a p-value of 0.000 (taken from the $\chi^2(231)$ distribution).[11] Obviously, also the

[9]In line with most of the literature, and for comparative purposes, we treat all modeled variables as highly persistent but covariance stationary time series. Our Technical Supplement extends the analysis of the SVAR-WB to the case of unit roots and cointegration, and shows that our qualitative results are substantially unaffected.

[10]A robustness check involving an equation-specific linear trend returned virtually identical results.

[11]Our estimates are 'quasi'-ML estimates because of the maintained assumption of a Gaussian like-

LR Chow-type test for the null $\mathbf{\Sigma}_{u,1}$=$\mathbf{\Sigma}_{u,2}$ against the alternative $\mathbf{\Sigma}_{u,1} \neq \mathbf{\Sigma}_{u,2}$ (and the implicit hypothesis $\mathbf{\Pi}_1$=$\mathbf{\Pi}_2$) leads us to strongly reject the null of stability.

Overall, even considering that the LR Chow-type tests may be over-rejective in finite samples, we can safely conclude that the sub-periods 1960Q1-1983Q4 and 1984Q1-2008Q2 represent two distinct regimes characterized by different error covariance matrices. This evidence calls for the employment of models able to deal with breaks and provide information for the identification of monetary policy shocks, a task for which our SVAR-WB approach is clearly suited.

3.2 Recursive approach

In the recursive scheme, we order the policy instrument after the remaining macroeconomic variables, the only exception being represented by the long-term interest rate, which is ordered last in $\boldsymbol{z}_t$. The ordering of the variables is the following: $\boldsymbol{z}_t = (NDCONS_t, DCONS_t, INVEST_t, GDP_t, INFL_t, FFR_t, 10YR_t)'$. As for recursive SVARs, the ordering of the variables is justified by the usual considerations regarding "slow moving" variables (those ordered before the federal funds rate) vs. "fast moving" ones (the long term interest rate).

Recursive SVAR-WB: Identification scheme. The recursive structure in the SVAR-WB is implemented by specifying $\boldsymbol{C}$ and $\boldsymbol{Q}$ lower triangular. More precisely, $\boldsymbol{C}$ and $\boldsymbol{Q}$ in eq. (6) are given by

$$\underbrace{\begin{pmatrix} u_t^{NDCONS} \\ u_t^{DCONS} \\ u_t^{INVEST} \\ u_t^{GDP} \\ u_t^{INFL} \\ u_t^{FFR} \\ u_t^{10YR} \end{pmatrix}}_{\boldsymbol{u}_t} = \left\{ \underbrace{\begin{pmatrix} * & & & & & & \\ * & * & & & & & \\ * & * & * & & & & \\ * & * & * & * & & & \\ * & * & * & * & * & & \\ * & * & * & * & * & * & \\ * & * & * & * & * & * & * \end{pmatrix}}_{\boldsymbol{C}} \right.$$

lihood. Thus, all LR tests discussed throughout the paper should be interpreted as 'quasi'-LR tests.

$$
\left. + \underbrace{\begin{pmatrix} * & & & & & & \\ * & * & & & & & \\ * & * & * & & & & \\ * & * & * & * & & & \\ * & * & * & * & * & & \\ * & * & * & * & * & * & \\ * & * & * & * & * & * & * \end{pmatrix}}_{\boldsymbol{Q}} \times \mathbf{1}\left(t \geq T_B\right) \right\} \underbrace{\begin{pmatrix} e_t^{NDCONS} \\ e_t^{DCONS} \\ e_t^{INVEST} \\ e_t^{GDP} \\ e_t^{INFL} \\ e_t^{FFR} \\ e_t^{10YR} \end{pmatrix}}_{\boldsymbol{e}_t} \tag{14}
$$

where asterisks ('$*$') denote unrestricted coefficients, empty entries correspond to zeros and the interpretation of the reduced form disturbances u_t^x and structural shocks e_t^x, $x = \{NDCONS,\ DCONS,\ INVEST,\ GDP,\ INFL,\ FFR,\ 10YR\}$ is straightforward. This specification allows the structural parameters to change across the two volatility regimes without changing the triangular - Cholesky-type - structure underlying the mapping from the structural shocks to the reduced form disturbances.

The upper panel of Table 1 reports the ML estimates of the matrices $\boldsymbol{C}$ and $\boldsymbol{Q}$ under the scheme in eq. (14). The associated IRFs to a monetary policy shock (see eq.s (12)-(13) for their population counterpart), along with 95% confidence bands, are plotted in Figure 1. Our impulse responses are rescaled to represent the macroeconomic effects of a policy shock of a 25 basis point-magnitude.[12]

Recursive SVAR-WB: Results. The top-panels of Figure 1 depict the macroeconomic reactions to a monetary policy shock conditional on the pre-1984 period. An unexpected increase in the short-term policy rate trigger conventional macroeconomic reactions (see, e.g., Christiano, Eichenbaum, and Evans, 1999, 2005). In particular, all real aggregates react negatively, persistently, and significantly to such a monetary policy tightening. The reaction of durable spending comoves positively with non-durable spending, but with a much larger sensitivity with respect to the latter, a result in line with some recent evidence by Erceg and Levin (2006), Barsky, House, and Kimball (2007), and Monacelli (2009). Investments comove with consumption and real GDP, with a sensitivity much larger than that of non-durable consumption and GDP, and slightly larger than, but very similar to, the one of durable spending. The recessionary effects of a monetary policy tightening are associated to a persistent and significant

[12] This choice implies that the differences in the contemporanous responses between regimes in all scenarios (recursive, non recursive) have to be assigned to the different impulse vectors only, i.e., no role is played by the different standard deviations of the monetary policy shocks.

deflationary phase. This follows a positive (although insignificant) short-run responses of the price level, a regularity called 'price puzzle' in the VAR literature (Eichenbaum, 1992). Finally, the long-term interest rate comoves positively with the short-term policy rate, it shows an on-impact positive and significant reaction, and a persistent decline towards its steady state value, which is reached after some quarters.

Interestingly, the post-break dynamics suggested by our recursive SVAR-WB are not as clear cut. The reactions of consumption and GDP are very imprecisely estimated, and statistically in line with flat responses. Differently, the dynamics of investments are still significant, but surrounded by a much larger uncertainty than in the pre-break scenario. The reaction of inflation shows no sign of the 'price puzzle', which appears to be, if anything, an evidence related to the inflationary events occurred in the 1970s, as also documented by a variety of authors (Hanson (2004), Boivin and Giannoni (2006), Castelnuovo and Surico (2010), Boivin, Kiley, and Mishkin (2010)). The response of the long-term rate, while still being positive and significant, turns out to be somewhat different as for its dynamics, and it is also imprecisely estimated.

Comparison with fixed coefficient-SVARs: Confidence bands. To appreciate the costs related to using the fixed-coefficient SVAR for modeling our data, Figure 2 superimposes our 95% confidence intervals (already shown in Figure 1) to the 95% intervals obtained by estimating a Cholesky-SVAR with fixed-coefficients over the period 1960Q1-2008Q2. Interestingly, the top panels in Figure 2 reveals that the pre-break period is clearly dominant in terms of estimated macroeconomic dynamics in reaction to a policy shock, i.e., the contribution of the post-1984 observations is quite marginal. In particular, the indications coming from the fixed-coefficient SVAR are basically the same as those arising from our methodology conditional on the pre-break sample, the only exception being the precision of the estimated response of the long-term rate, which is much larger when our methodology is employed. In contrast, the post-1984 responses are markedly different from a statistical standpoint from those suggested by the fixed-coefficient full-sample SVAR. Hence, SVARs estimated over the post-WWII period tend to report macroeconomic responses to a monetary policy shock which are clearly driven by the dynamics of the pre-1980s and are not necessarily representative of the dynamics of the Great Moderation period. The superimposition of impulse responses estimated with fixed-coefficient SVARs in two different samples (1960Q1-1983Q4 vs. 1960Q1-2008Q2) supports this statement (evidence available upon request).

We have established the empirical differences between a fixed-coefficient SVAR approach and our recursive SVAR-WB. Are these differences relevant from an economic policy standpoint? It is often the case that mean reactions are considered as a reference for calibration exercises. A frequent application in macroeconomics is the estimation of new-Keynesian DSGE models of the business cycle by the impulse response function matching approach, which is a limited information approach that minimizes the distance between sample and DSGE model generated impulse responses (for a formal presentation, see Canova (2007)). Would this approach deliver different point estimates for the structural models of interest? To answer this question, we move to the analysis of our mean responses.

Comparison with fixed coefficient-SVARs: Mean responses. Figure 3 plots the IRFs arising from the fixed-coefficient SVAR vs. ours. Perhaps not surprisingly in light of our discussion above, two considerations are in order. First, the reactions predicted by the fixed-coefficient SVAR and by our SVAR-WB conditional on the pre-1984 observations are extremely similar for the first two years, and clearly comove afterwords. Second, they differ from our responses conditional on the post-1984, and substantially so from a quantitative standpoint for most of our variables. The response of non durable consumption is much milder (basically halved) in most of the first eight quarters. The reaction of investments is differently shaped in the short-run, with an opposite concavity with respect to the one predicted by the fixed-coefficient SVAR. The dynamics of the gross domestic product are very different in the first ten quarters, with a much milder influence by monetary policy shocks on the business cycle with respect to the one during the great inflation phase. The short-run reaction of inflation is quite different not only in terms of magnitude, but also in terms of sign. Both the fixed-coefficient SVAR and our pre-1984 SVAR-WB suggest a positive response of inflation. Viceversa, Great Moderation data point to a standard deflationary effect of unexpected policy tightenings. Finally, the difference in terms of concavity is evident when looking at our estimated responses of the long-term interest rate.

Implications for the calibration of DSGE models: Some conjectures. Can these differences lead to different calibrations of macroeconomic models employed to conduct policy analysis? Some contributions in the literature suggest a positive answer. Boivin and Giannoni (2006) estimate the structural parameters of a small-scale new-Keynesian recursive DSGE model by matching its impulse responses to a monetary policy shock to those predicted by a trivariate recursive SVAR modeling output, inflation, and the federal funds rate. The SVAR is estimated over the Great Inflation and the Great Moderation subsamples (the break-date they select is 1979Q2). Notably, the impulse responses they get are quite similar to ours as for the variables in common. Their estimation of their DSGE model provides evidence in favor of instabilities in the policy parameters responsible for the systematic reaction of the Federal Reserve to movements in inflation and output. Moreover, they detect breaks in the parameters regarding the behavior of the private sector. Counterfactual simulations conditional on such different estimates of their structural parameters lead them to conclude that the systematic component of the U.S. monetary policy may have stabilized the economy more effectively during the Great Moderation. Boivin and Giannoni's (2006) results suggest that punctual differences in the estimated macroeconomic reactions to policy shocks may lead to different descriptions of the U.S. economy in the 1970s as opposed to the Great Moderation.[13]

Our results suggest that some of the estimations obtained via impulse response function matching by an auxiliary SVAR model taken over the entire post-WWII period might be valid for the Great Inflation period only. For instance, the structural interpretation of the 'price puzzle' has been offered empirical support by Christiano, Eichenbaum, and Evans (2005), who work with a recursive DSGE model with working capital. In such a model, the short-term interest rate is part of firms' marginal costs due to firms' need to borrow money from the banking system to pay workers before the goods market opens. Hence, due to an unexpected policy tightening, firms' marginal costs increase, so inducing entrepreneurs to adjust prices upward. If this 'cost-channel' is stronger than the standard demand channel, inflation may very well go up in response to a monetary policy shock. Christiano et al.'s (2005) model replicates this short-run correlation. In light of our results, however, such correlation is likely to be time-varying. This is important for two reasons. First, because of the relevance of the cost channel for the design of optimal monetary policy (Ravenna and Walsh (2006)). Second, because of the number of structural parameters in medium-scale models à la Christiano, Eichenbaum, and Evans (2005) whose calibration is affected by such short-run correlation. Rabanal (2007) documents the role empirically played by staggered wage setting, wage indexation, and variable capital utilization, along with firms' borrowing constraints, in replicating the VAR inflation-policy rate short-run positive correlation in Christiano et

[13] Blanchard and Riggi (2013) perform a similar exercise with a model in which oil price shocks play a meaningful role as for the dynamics of inflation and output and, consequently, monetary policy. They find instabilities in the macroeconomic reactions to oil price shocks estimated by VARs conditional on great inflation vs. great moderation data. A calibration exercise of their structural DSGE model based on impulse response-matching points to clear gains in the Federal Reserve's credibility with the advent of Paul Volcker as the Federal Reserve's chairman. Again, we take this evidence as supportive of the role played by instabilities in VAR responses in guiding the calibration of state-of-the-art DSGE models.

al.'s (2005) framework. Tillmann (2009) finds the cost-channel to be more relevant in the pre-Volcker era than during the Volcker-Greenspan regime. Our evidence suggests that the relative strength of the cost-channel in transmitting the effects of a monetary policy shock to inflation may very well be time-varying. An empirical exercise dealing with the estimation of a DSGE model of the business cycle via the impulse response matching approach is proposed in Section 4.

3.3 Non-recursive approach

Are the results obtained so far identification scheme-dependent? To answer this question, we move to the alternative identification scheme, i.e., the non-recursive SVAR-WB.

Non-recursive SVAR-WB: Identification scheme. We work with a 'full' $\boldsymbol{C}$ matrix and a diagonal $\boldsymbol{Q}$ matrix in eq.s (2)-(9). As a consequence, *all shocks hitting the modeled economy are allowed to affect all variables of the VAR system contemporaneously.* This is an interesting case for macroeconomists, in that the matrix regulating the contemporaneous relationships among the variables in the vector allows all shocks to move all variables with no lag. As anticipated in the Introduction, this is a common assumption in the DSGE literature, which focuses on microfounded structural model whose parameters are structural and that can therefore be meaningfully employed to conduct policy-relevant exercises. We remark that the 'full' specification for $\boldsymbol{C}$ is unfeasible (unidentified) in the context of fixed-parameters SVARs.

We specify the restrictions on $\boldsymbol{C}$ and $\boldsymbol{Q}$ such that eq. (6) collapses to

$$\underbrace{\begin{pmatrix} u_t^{NDCONS} \\ u_t^{DCONS} \\ u_t^{INVEST} \\ u_t^{GDP} \\ u_t^{INFL} \\ u_t^{FFR} \\ u_t^{10YR} \end{pmatrix}}_{\boldsymbol{u}_t} = \left\{ \underbrace{\begin{pmatrix} * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \end{pmatrix}}_{\boldsymbol{C}} \right.$$

$$
+\underbrace{\begin{pmatrix} * & & & & & & \\ & * & & & & & \\ & & * & & & & \\ & & & * & & & \\ & & & & * & & \\ & & & & & * & \\ & & & & & & * \end{pmatrix}}_{\boldsymbol{Q}} \times \mathbf{1}\left(t \geq T_B\right) \Bigg\} \underbrace{\begin{pmatrix} e_t^{NDCONS} \\ e_t^{DCONS} \\ e_t^{INVEST} \\ e_t^{GDP} \\ e_t^{INFL} \\ e_t^{FFR} \\ e_t^{10YR} \end{pmatrix}}_{\boldsymbol{e}_t}. \tag{15}
$$

Given the break in the covariance matrix, in our setup the $\boldsymbol{Q}$ matrix must be different from zero, implying a break on the simultaneous coefficients too. In particular, we allow for variations in the instantaneous impact of e_t^x on u_t^x as implied by the diagonal structure of the $\boldsymbol{Q}$ matrix in eq. (15). This structure allows us to keep a 'full' structure for the $\boldsymbol{C}$ matrix while satisfying the identification condition in eq. (10). Moreover, the non-zero diagonal elements on the $\boldsymbol{Q}$ matrix enable us to capture variations possibly occurring at the structural shock-level. The lower panel of Table 1 reports the ML estimates of the matrices $\boldsymbol{C}$ and $\boldsymbol{Q}$ under the non-recursive scheme in eq. (15).

Non-recursive vs. recursive SVARs WB: Confidence bands, comparisons. Figure 4 overimposes the IRFs obtained with our non-recursive identification scheme to the IRFs implied by the recursive SVAR-WB already shown in Figure 1, having defined the monetary policy shocks as in Section 3.2. The top panels, focusing on the pre-break period, show a surprisingly similarity between the dynamics predicted by the Cholesky-type responses and those produced with our non-recursive scheme in terms of signs, magnitudes, persistence, and precision of the estimates. As a matter of fact, this result suggests that the zero-restrictions implied by the recursive scheme do not necessarily have relevant consequences as for the dynamics of the economic system during the first sub-sample. In fact, when allowing for non-zero on impact responses by all variables, such responses are close to zero as for our four real variables. Differently, inflation and the two nominal rates react positively and significantly in the short-run. Again, such positive responses do not imply different estimated dynamics with respect to those predicted by the estimated recursive SVAR-WB. Hence, even if not from a strictly statistical standpoint, our results are overall supportive as for the restrictions imposed by the recursive identification scheme over the Great Inflation period.

A very different picture emerges when moving to the after-1984 estimated dynamics. First, our non-recursive SVAR-WB predicts negative and significant reactions of all our four real variables. Second, inflation responds negatively on impact. Third, the long-term interest rate reacts negatively and persistently. Fourth, the uncertainty surrounding our dynamic responses in the post-1984 sample is larger than the one affecting the responses conditional on the Great Inflation sample. This may be due to a more uncertain on-impact reaction, which is dynamically propagated via the lagged coefficients of our VARs, or to more uncertain estimates of the VAR coefficients, or both. Most importantly, compared to the recursive case, our non-recursive SVAR-WB suggests that policy shocks had real effects also during the Great Moderation, induced a deflation and a negative reaction of the long-term interest rate during the Great Moderation. Our results suggest that the choice of the identification scheme is quite relevant as for post-1984 U.S. macroeconomic data.

Non-recursive vs. recursive SVARs WB: Mean responses, comparisons. Figure 5 plots the point estimates of our IRFs. Not surprisingly (in light of the information carried by Figure 4), the dynamics of the 1960s and 1970s in response to a policy shock appear to be almost identically described by our recursive vs. non-recursive identification schemes. Quite a different picture emerges when analyzing the Great Moderation. Our non-recursive scheme predicts a more severe recession (even when discarding the on-impact responses, the area below the zero lines is clearly much wider conditional on our non-recursive SVAR-WB as for the four real variables under investigation). Also the deflationary phase is estimated to be deeper. The federal funds rate does not follow a hump-shaped path in going back to its long-run equilibrium value. Instead, it drops quickly, 'overshoots' its steady state level, and takes negative values for a few quarters before going back to its steady state. As already pointed out, probably the most striking difference concerns the long-term interest rate, which reacts negatively according to our non-recursive SVAR-WB. This finding is in line with Bagliano and Favero (1998), who also exploit a non-recursive scheme to identify a monetary policy shock in their macro-financial VAR and find the reaction of the long-term bill rate to be negative and significant in the 1980s and 1990s. Possibly, this result is related to the gained credibility by the Federal Reserve since the advent of Paul Volker as Chair of the Board of Governors of the Federal Reserve System, which has also been followed by an increase in the evolution of the communication pursued by the Federal Reserve which is likely to have improved the ability to manage long-term inflation expectations (for a review on the role of communication in managing inflation expectations, see English, Lopez-Salido, and Tetlow (2013)). Intriguingly, this suggests that the recession predicted by our model should not necessarily be imputed to the larger costs paid by households and entrepreneurs when borrowing long-term.

3.4 Plausibility of our non-recursive identification scheme: A discussion

A discussion on the monetary policy shocks identified with our agnostic non-recursive identification procedure is in order. Our methodology does not impose any zero-restrictions to identify monetary policy shocks. Monetary policy shocks are often identified in VAR analysis by assuming that they do not exert an immediate impact on quantities as well as prices. Differently, demand and supply shocks are allowed to have an immediate impact on the policy instrument, typically a short-term interest rate. Restrictions of this type have been popularized by, among others, Christiano, Eichenbaum, and Evans (1999, 2005). Admittedly, if variables like consumption, investment, output, and inflation are genuinely "slow-moving" and react to monetary policy shocks with a delay up to a quarter, missing the imposition of such zero-restrictions may result in a lack of relevant information in our non-recursive identification scheme. However, we note that the zero-restrictions imposing a lag in the response of macroeconomic aggregates are not undisputed in the literature. In fact, they are not consistent with micro-founded models relying on standard assumptions on the timing of the formation of rational expectations, which allows immediate effects of monetary policy shocks on the components of aggregate demand and inflation (see, e.g., Smets and Wouters (2007), Galí (2008)). Moreover, as anticipated in our Introduction, recent contributions have found support for an immediate response of output and inflation to monetary policy shocks (see, Del Negro, Schorfheide, Smets, and Wouters (2007) as for output, and Faust, Swanson, and Wright (2004) as regards inflation, Normandin and Phaneuf (2004) as for both aggregates). Moreover, interest rates other than the federal funds rate are likely to react to monetary policy shocks within a quarter (Bagliano and Favero (1998), Gertler and Karadi (2014)). Hence, it seems of interest to work with identification schemes alternative to the recursive one, which is what we do it this paper.

It is important to check the sensibility of our estimates of the monetary policy shocks. Figure 6 contrasts our estimates of the policy shocks conditional on our non-recursive model with four alternative measures of policy shocks, i.e., the one obtained with our recursive framework admitting a break, the one obtained with a standard recursive SVAR à la Christiano et al. (1999, 2005) which assumes no breaks in the VAR coefficients, the monetary policy shocks estimated by Smets and Wouters (2007), and a measure of policy shocks proposed by Romer and Romer (2004). Smets and Wouters (2007) model monetary policy shocks as stochastic deviations from a Taylor rule in an estimated medium-scale micro-founded DSGE framework featuring a variety of nominal and real frictions. Such framework has become a reference for researchers in central banks and policy institutions. Romer and Romer (2004) identify policy innovations in two steps. First, they use a narrative approach to identify changes in the Federal Reserve's target interest rate occurring during FOMC's meetings. Then, they regress this measure of policy changes on the Fed's real-time forecasts of past, current, and future inflation, output growth, and unemployment. In doing so, they isolate the innovations of these policy changes that are orthogonal to the information set possessed by the Federal Reserve, i.e., the monetary policy shocks.

A look at these five measures of policy shocks confirms that our methodology has the potential to meaningfully isolate exogenous variations in the federal funds rate. These five series clearly comove, with their local peaks typically anticipating recessions or occurring in correspondence to economic downturns. The correlation between the estimates of the policy shocks obtained with our non-recursive model and the recursive one (both admitting a break in 1984Q1) reads 0.68, while that of the former with the estimates provided by a fixed coefficient-recursive SVAR, the model by Smets and Wouters (2007), and Romer and Romer's (2004) approach reads 0.69, 0.49, and 0.33, respectively. Granger-causality tests based on bivariate VARs modeling the policy shocks estimated with our non-recursive identification scheme and, alternatively, one of the other four measures of policy shocks clearly reject any anticipatory effects in any direction. Again, this is consistent with the fact that these measures of policy shocks, while being quantitatively somewhat different, tend to comove and carry common information regarding the exogenous variations of the federal funds rate in the post-WWII U.S. period. We see this validation check as supportive for our non-recursive identification proposal, at least as far as the identification of monetary policy shocks is concerned.[14]

3.5 Recursive vs. non-recursive schemes: Comparison

A natural question at this point is: Which identification scheme should we trust more? By construction, the likelihood of our estimated recursive and non-recursive SVARs-WB is the same. This is so because both SVARs-WB discussed in the previous Sections are just-identified in absence of additional restrictions on $\boldsymbol{C}$ and $\boldsymbol{Q}$.

[14]This aim of this Section is to show that the monetary policy shocks obtained with our methodology appear to be sensible when contrasted with other, more conventional measure of policy innovations. For a comparison of the different macroeconomic effects associated to monetary policy shocks identified with a number of approaches recently pursued by the literature, see Coibion (2012).

However, a closer inspection of the results in Table 1 reveals that not all estimated coefficients in $\boldsymbol{C}$ and $\boldsymbol{Q}$ are significant. Thus, we consider 'constrained' formulations of the two SVARs-WB by setting the non-significant elements in $\widehat{\boldsymbol{C}}$ and $\widehat{\boldsymbol{Q}}$ to zero. Then, we are in the condition of selecting the best fitting model, i.e., the constrained model associated to the lower LR test (higher p-value). In other words, we use standard LR tests for the zero over-identifying restrictions as a 'metric' to assess the estimated recursive and non-recursive SVARs-WB.

The constrained versions of the two estimated SVARs-WB are reported in Table 1, along with the corresponding LR tests. The LR test built on the restricted version of our recursive SVAR-WB model vs. the baseline, unrestricted one returns a p-value of 0.07. As for our non-recursive SVAR-WB model, the p-value associated to the likelihood-ratio reads 0.93. The value of the likelihood function of the non-recursive SVAR-WB turns out to be considerably larger than the one of the recursive SVAR-WB, i.e., 1550.25 as for the former vs. 1540.12 as for the latter. While not representing decisive evidence, these results suggest that the fit of our non-recursive SVAR-WB is at least as good as the one associated to a more standard recursive model.

4 DSGE model estimation with IRF matching and non-recursive VARs: An example

SVAR impulse responses to a monetary policy shock are often used to calibrate structural DSGE models of the business cycle (see, for instance, Rotemberg and Woodford (1997), Christiano, Eichenbaum, and Evans (2005), Boivin and Giannoni (2006), Altig, Christiano, Eichenbaum, and Lindé (2011)). While being somewhat prone to identification-related issues (Canova and Sala (2009)), this procedure enables the macroeconometrician to exploit the SVAR impulse responses to a given (set of) shock(s) to calibrate the whole economic structure without having to make any reference to other shocks, which are then left unmodeled. The underlying working hypothesis is that of stable impulse responses to the identified shock. As previously shown, however, the assumption of stability of the macroeconomic responses to a monetary policy shock in the entire U.S. post-WWII history is questionable.

We investigate to what extent instabilities in the impulse responses to a monetary policy shock identified with our SVAR-WB may translate into instabilities in the structural parameters of a small-scale DSGE model for the U.S. economy. We do so by appealing to the non-recursive version of the estimated SVAR-WBs. As observed in our Introduction, most DSGE models of the business cycle feature (a) macroeconomic shocks (other than monetary policy shocks) that may affect the nominal interest rate without delays, and (b) no delays in the transmission of the monetary policy shock to the economy. A few examples of models having these features include Clarida, Galí, and Gertler (2000), Lubik and Schorfheide (2004), Smets and Wouters (2007), Benati (2008), Canova (2009), Benati and Surico (2009), Christiano, Motto, and Rostagno (2013). Importantly, our unrestricted non-recursive SVAR-WB features the same contemporaneous structure as the determinate VAR solution implied by these models. Moreover, the statistical argument discussed in Section 3.5 lends support to our non-recursive identification scheme. We then exploit the impulse responses obtained with our non-recursive SVAR-WB (that accounts for the instabilities we found in the responses to a monetary policy shock in the pre-1984 vs. post-1984 subsamples) to calibrate a structural AD/AS model for the U.S. economy.

We consider a DSGE model featuring a cost-channel à la Ravenna and Walsh (2006). The model reads as follows:

$$
\begin{aligned}
\pi_t &= \beta[\xi_\pi E_t \pi_{t+1} + (1-\xi_\pi)\pi_{t-1}] + \kappa(x_t + \alpha R_t) + \varepsilon_t^\pi && (16)\\
x_t &= \xi_x E_t x_{t+1} + (1-\xi_x)x_{t-1} - \tau(R_t - E_t\pi_{t+1}) + \varepsilon_t^x && (17)\\
R_t &= (1-\phi_i)(\phi_\pi \pi_t + \phi_x x_t) + \phi_i R_{t-1} + \varepsilon_t^i && (18)
\end{aligned}
$$

The new-Keynesian Phillips curve in eq. (16) suggests that current inflation π_t evolves as a function of expected and past inflation, the output gap x_t, the nominal interest rate R_t, and a 'supply-shock' ε_t^π. The relevance of expected inflation, as opposed to past inflation, is regulated by the parameter ξ_π. Under the restriction $\alpha = 0$, the slope of the Phillips curve is identified by the parameter κ. The presence of the policy rate in the Phillips curve has recently been microfounded by Christiano, Eichenbaum, and Evans (2005) and Ravenna and Walsh (2006). They assume that a share of firms in a given economy may need to borrow resources to pay workers' wages before the final goods market opens. Hence, the policy rate enters these firms' marginal costs as a proxy of the interest rate paid on their loans. In eq. (16), this occurs when $\alpha > 0$. Christiano, Eichenbaum, and Evans (2005) and Ravenna and Walsh (2006) interpret α as the share of firms acceding the financial markets, an interpretation which puts a unitary bound on the economically meaningful value of α. An alternative interpretation refers to the pass-through from the policy decisions to the interest rate $R_t^l = \alpha R_t$ charged by commercial banks on loans to the private sector, with α possibly larger than one (Chowdhury, Hoffmann, and Schabert (2006)). This interpretation leaves the data more 'free' to speak as for the value of α (admittedly, at the cost of giving such parameter a reduced-form flavor).

Moving to the dynamic IS equation (17), the evolution of the output gap x_t is dictated by its expected/past realizations, whose relevance for the current output gap is regulated by the parameter ξ_x, the ex-ante real interest rate $R_t - E_t\pi_{t+1}$, whose role is affected by the intertemporal elasticity of substitution τ, and the 'demand shock' ε_t^x. Finally, the policy rule (18) postulates a systematic reaction of the monetary policymakers to movements in inflation and the output gap respectively captured by the parameters ϕ_π and ϕ_x. Such reaction is allowed to be smooth, the degree of interest rate smoothing being ϕ_i. Movements of the interest rate can also be induced by monetary policy shocks, here captured by the stochastic element ε_t^i. The distribution of the random processes $\varepsilon_t^j \sim N(0, \sigma_j^2)$, $j = (\pi, x, R)$.

The choice of considering a model with the cost-channel is motivated as follows. The cost channel has been seen as a possible rationale for the 'price puzzle', i.e., the positive short-run conditional correlation between inflation and short-term interest rates often obtained by VAR analysis when simulating the effects of a monetary policy shock (Barth and Ramey (2001), Chowdhury, Hoffmann, and Schabert (2006)). Ravenna and Walsh (2006) show that the presence of the cost-channel in an otherwise standard new-Keynesian framework may have important consequences for the optimal monetary policy design. Our non-recursive VARs suggest that the 'price puzzle' evidence may be conditional on observations coming from the 1970s, a finding in line with previous contributions (Boivin and Giannoni (2006), Castelnuovo and Surico (2010)). It is then of interest to understand to what extent this instability may have consequences for the estimated strength of the cost-channel for the U.S. economy.

Our non-recursive SVAR-WB is estimated with seven observables. We focus on the impulse responses of inflation, output, and the policy rate.[15] The extra-information contained in the SVAR-WB is valuable in this context, in that it is likely to augment the precision of the estimated monetary policy shocks.[16] Let $\widetilde{\boldsymbol{\theta}} = (\beta, \xi_\pi, \kappa, \alpha, \xi_x, \tau, \phi_\pi, \phi_x, \phi_i)'$

[15] A common assumption is that monetary policy shocks are neutral in the long run, i.e., they exert no effects on potential output. Therefore, the impulse response of output and the one of the output gap are exactly the same.

[16] A related exercise is conducted by Boivin and Giannoni (2006), who perform a robustness check of their main results by estimating monetary policy shocks with a Factor-Augmented VAR.

be the vector of structural parameters of the model (16)-(18). We fix four parameters before estimation. As it is customary in the literature dealing with quarterly data, we set the discount factor $\beta = 0.99$. After some experimentation, we verified that the parameters of the policy rule are somewhat difficult to estimate precisely, possibly due to known identification issues affecting the impulse response function matching methodology (see Canova and Sala (2009) for a formal presentation of this issue, and Christiano, Eichenbaum, and Evans (2005) for a similar calibration choice in their indirect inference exercise regarding their medium-scale DSGE model). Hence, we set the following values for our policy parameters: $\phi_\pi = 2$, $\phi_x = 0.1$ and $\phi_i = 0.8$. This calibration reflects a variety of estimates in the literature (e.g,. Clarida, Galí, and Gertler (2000), Lubik and Schorfheide (2004), Benati and Surico (2009)). The assumption of an aggressive reaction to movements in inflation is justified by our willingness of avoiding the case of multiple equilibria, which is more likely when the cost channel is present in the economic system (Surico (2008)).[17]

The vector of parameters whose value is picked up by impulse response matching is the following: $\boldsymbol{\theta} = (\xi_\pi, \kappa, \alpha, \xi_x, \tau)'$. Let $\boldsymbol{\gamma}(\theta)$ be the vector collecting the stacked set of the impulse responses of inflation, output, and the policy rate to a monetary policy shock (normalized so to induce a 25 basis point on-impact increase of the policy rate) over the first 20 quarters after the shock.[18] Let $\widehat{\boldsymbol{\gamma}}_i$, $i = 1, 2$ be the set of impulse responses estimated by our non-recursive SVAR-WB in the heteroskedasticity regime i, where in our case $i = 1$ corresponds to the 1960Q1-1983Q4 sample, and $i = 2$ to the 1984Q1-2008Q2 sample. Referring to the notation used in eq.s (12)-(13), the vector $\widehat{\boldsymbol{\gamma}}_i$ is obtained by selecting, for $i = 1, 2$, a proper set of elements from the estimated matrices of IRFs $\hat{\boldsymbol{\Gamma}}_{i,0}$, $\hat{\boldsymbol{\Gamma}}_{i,1}$, ..., $\hat{\boldsymbol{\Gamma}}_{i,20}$. The impulse response function matching estimator of $\boldsymbol{\theta}$ in the heteroskedasticity regime i is therefore given by

$$\hat{\boldsymbol{\theta}}= arg\min\ Loss(\boldsymbol{\theta}), \quad Loss(\boldsymbol{\theta})=[\widehat{\boldsymbol{\gamma}}_i - \boldsymbol{\gamma}(\boldsymbol{\theta})]'\,\hat{\boldsymbol{W}}_{T,i}^{-1}[\widehat{\boldsymbol{\gamma}}_i - \boldsymbol{\gamma}(\boldsymbol{\theta})] \tag{19}$$

where $\hat{\boldsymbol{W}}_{T,i}$ is a diagonal matrix with the sample variances of the estimated $\widehat{\boldsymbol{\gamma}}_i$ along

[17] Clarida, Galí, and Gertler (2000) document an increase in the systematic reaction of U.S. policymakers to movements in inflation, and interpret the conquest of the U.S. inflation in light of such increase. Similar evidence is provided by, among others, Lubik and Schorfheide (2004), Boivin and Giannoni (2006), Benati and Surico (2009), and Castelnuovo and Fanelli (2013). Our approach focuses on the instability of the parameters of the private sector, first and foremost the cost-channel parameter. It should therefore be seen as complementary to those of the above mentioned studies.

[18] Our loss function, which hinges upon the information exclusively coming from scaled impulse responses to a monetary policy shock, does not provide us with any information to identify the standard deviations of the structural shocks of our DSGE model, which are then left unidentified.

the main diagonal (see Christiano, Eichenbaum, and Evans (2005) and Canova (2007)).

Table 2 reports our estimates of $\boldsymbol{\theta}$ in the two regimes. The second column (first column with estimated parameters) collects the estimated parameters for the pre-1984 period. The IS curve is found to be of hybrid form, i.e., past realizations of output (weighted by $1-\hat{\xi}_x$) contribute to explain the current stance of the output gap, possibly due to habit formation in consumption (Furher (2000)). The point estimates of the elasticity of intertemporal substitution τ is 0.03, a value approximately in line with most of the estimates in Fuhrer and Rudebusch (2004). The new-Keynesian Phillips curve is also found to have a one lead-one lag structure (suggested by the point estimate $\hat{\xi}_\pi = 0.53$), an evidence consistent with the idea that part of the U.S. inflation persistence in the 1960s and 1970s might be attributed to price indexation (Benati (2008)). The slope of the Phillips curve is found to be somewhat low. Interestingly, the cost-channel parameter α is estimated to be around three, a value suggesting an upward pressure exerted by the policy tightenings on firms' marginal costs. Such large value of α is due to the 'price puzzle' in our VAR impulse responses, which forces our DSGE model to generate a positive short-run conditional correlation between inflation and the policy rate.[19]

Admittedly, the value of the cost-channel parameter is not estimated precisely. We then re-estimate the model by imposing the constraint $\alpha = 0$. Some instabilities may be detected in the estimated parameters, above all as for the intertemporal elasticity of substitution and price indexation. More importantly, the 'distance' measure associated to the constrained estimates, which refers to the departures of the DSGE model's impulse responses with respect to those of our auxiliary VARs, turns out to be larger than the one associated to the unconstrained model. Moreover, and not surprisingly given the transmission of the monetary policy shock in our model, the constrained framework is incapable to replicate the VAR 'price puzzle'.[20]

A different picture emerges when moving to the 1984Q1-2008Q2 sample. First, the persistence of output is estimated to be larger (the weight assigned to expected output gap is lower). Second, the intertemporal elasticity of substitution is found to be three times as large as the one of the pre-1984 sample. Third, past inflation is found to play no role in explaining current inflation. Fourth, the slope of the Phillips curve is larger,

[19] Ravenna and Walsh (2006) perform a battery of GMM estimations of their new-Keynesian Phillips curve with the cost-channel. They also find values for α larger then one, though not statistically different from one at standard confidence levels.

[20] The comparison between the impulse responses of our VARs vs. DSGE model, not shown for the sake of brevity, is available upon request.

and clearly significant. Fifth, the impact of the cost-channel on inflation dynamics is quantified to be zero. This last finding makes it crystal clear how instabilities in the IRF may importantly translate into instabilities in key-parameters of the structural model.

Our results suggest that the relevance of the cost-channel in the U.S. economic framework is likely to be time-varying and larger in the pre-1984 phase. A similar evidence in favor of a substantially reduced importance of the working capital requirements for inflation is provided by Barth and Ramey (2001) and Tillmann (2009), who interpret it as the consequence of financial innovations and deregulation occurred at the beginning of the 1980s. In light of the different trade-offs a central bank is called to face conditional on a different relative importance of the cost channel with respect to the standard demand channel, this result provides valuable information on how to assess optimal monetary policy in the U.S. during the Great Inflation vs. the Great Moderation (Ravenna and Walsh (2006)). Importantly, the instability of the cost-channel parameter suggests that optimal monetary policy may very well be time-dependent.

5 Relation to the methodological literature

The approach used in this paper is related to some recent works by Normandin and Phaneuf (2004), Lanne and Lütkepohl (2008, 2010) and Lanne, Lütkepohl, and Maciejowska (2010) on the identification of SVARs subject to different volatility regimes and, more generally, to the contributions of Rigobon and Sack (2003, 2004). Similarly to these authors, we exploit the presence of breaks in the VAR covariance matrix to identify the structural shocks. Differently from these authors, however, we remove the assumption that structural breaks affect only the error covariance matrix and leave the impulse vectors unchanged.[21] In our setup, a change in the VAR covariance matrix is automatically associated with a change in the structural parameters, hence the identification analysis of the macroeconomic effects of monetary policy shocks involves a mix of volatility-driven and theory-driven restrictions. The SVAR-WB approach does not identify one structural model but different structural models in different heteroskedasticity regimes. Hence, unlike the above mentioned contributions, we allow the patterns of IRFs to differ across regimes. Moreover, our SVAR-WB is designed to deal with a few structural breaks that are best thought of as permanent and not as stochastically recur-

[21] Uhlig (2005) terms 'impulse vector' the column vector of the matrix of the contemporaneous relationships among the variables of the VAR which captures the on impact response of such variables to an identified structural shock.

ring (reversible) events. Differently, Lanne, Lütkepohl, and Maciejowska (2010) model the changes in the error covariance matrix through an underlying Markov switching process. Our SVAR-WB does not belong to the class of 'fully' time-varying VARs recently employed to model the evolution of the correlation among macroeconomic U.S. variables by Cogley and Sargent (2005a), Cogley and Sargent (2005b), Primiceri (2005), Sims and Zha (2006), and Canova, Gambetti, and Pappa (2008), among others. With respect to these authors, we use identification schemes that allow for non-recursive contemporaneous relationships.[22]

Our methodology enriches the set of available strategies to identify a monetary policy shock without appealing to a recursive scheme. An agnostic identification procedure consistent with a full impulse vector as for monetary policy shocks is represented by sign restrictions. Faust (1998), Canova and de Nicoló (2002) and Uhlig (2005) (among others) show how to deal with a set of restrictions imposed on moments generated by the estimated VAR (correlations, impulse responses) to identify a structural shock of interest. Sign restrictions have been shown to be quite powerful to discriminating among competing classes of structural models (Canova and Paustian (2011)) and identify the effects of structural shocks in general. However, the distinction between model uncertainty and parameter uncertainty has to be carefully drawn when computing dynamic responses to identified shocks (Fry and Pagan (2011)). Romer and Romer (2004) identify monetary policy shocks in a model-free fashion by performing a careful reading of the minutes reporting the discussions and monetary policy decisions by the FOMC. After identifying the series of the changes in the policy rate, they regress such series over a set of macroeconomic forecasts readily available to policymakers, therefore purging the identified series from the component systematically reacting to economic conditions. Kliem and Kriwoluzky (2013) employ the shocks identified by Romer and Romer (2004) as instruments in a "proxy-VAR" approach à la Stock and Watson (2012). A similar approach is pursued by Gertler and Karadi (2014), who identify policy shocks which include shocks to forward guidance by appealing to high-frequency data on policy surprises on interest rates. Faust, Swanson, and Wright (2004) use the prices of federal funds future to identify monetary policy shocks. Del Negro and Schorfheide (2004) and Del Negro, Schorfheide, Smets, and Wouters (2007) employ structural, non-recursive DSGE models to form priors for the estimation of Bayesian VARs. Our approach is complementary to those listed above, in that it requires the econometrician neither to

[22]For a recent paper dealing with overidentified, non-recursive, time-varying coefficients SVARs, see Canova and Forero (2012).

specify a set of restrictions or moments to be met for the identification of the shock to be in place, nor an auxiliary DSGE model, nor to undertake the reading and interpretation of minutes revealing information on policy decisions. Differently, it lets the data speak (conditional on some choices on the short-run relationships, performed in a quite flexible context) on the impulse vectors of interest. Moreover, it naturally deals with structural breaks, something that these alternative identification schemes are not necessarily designed to deal with. Admittedly, given the agnostic flavor of our approach, this may come at the cost of confounding the effects of a monetary policy shock with those of a different structural shock. As shown in Section 3.4, however, the measure of monetary policy shocks obtained with our SVARs-WB turns out to be quite correlated with other measures present in the literature (among others, Romer and Romer's (2004) and Smets and Wouters' (2007)). At the very least, our IRFs offer a documentation of conditional responses of the U.S. economy to a shock which is consistent with an exogenous movement of the federal funds rate orthogonal to the rest of the system.

6 Conclusions

We have shown how to identify structural shocks in non-recursive Structural Vector Autoregressions (SVARs) featuring instabilities in the covariance matrix of the residuals and in the coefficients of the matrix responsible for the contemporaneous relationships of the modeled variables, denoted with SVAR-WB. After presenting our methodology in detail, we have exploited it to identify the effects of monetary policy shocks in the U.S. economy using post-WWII quarterly data. We have shown that a non-recursive SVAR-WB implies impulse responses very similar to those coming from a standard Cholesky-type recursive SVAR when pre-1984 data are considered. Differently, non-recursive vs. recursive schemes tell different stories on the dynamics occurred during the Great Moderation. We have provided statistical support in favor of non-recursive schemes, which are consistent with the SVAR representation of the large majority of the DSGE models employed by central banks and academic scholars to perform their empirical analysis. Then, we have estimated a structural model of the business cycle featuring the cost channel via impulse response function matching by appealing to the responses associated to our non-recursive SVARs. Our results show that instabilities in the impulse responses to a monetary policy shock may have clear consequences for model calibration exercises. In particular, we have found the relevance of the cost-channel to be much larger in the 1970s than during the Great Moderation, a result possibly related to the financial liberalization that took place in the U.S. in the late 1970s/early 1980s. In light of the policy relevance of the relative importance of demand vs. supply channels of the transmission of policy shocks (Ravenna and Walsh (2006)), our results suggest that instabilities in SVAR impulse responses may be taken as a signal of the possible time-dependence of optimal policy design.

Our effort lines up with previous contributions by Rigobon and Sack (2003, 2004), Normandin and Phaneuf (2004), Lanne and Lütkepohl (2008, 2010), and Lanne, Lütkepohl, and Maciejowska (2010) in showing that instabilities may represent relevant sources of information to identify structural shocks. Our analysis adds to the above mentioned contributions the idea that the transmission mechanism of the shocks may change across volatility regimes. Hence, also the parameters relating the reduced form disturbances and structural shocks may change. As stressed above, the detection of such instabilities is likely to be informative to calibrate models constructed for policy design. We see the identification of shocks and time-dependent macroeconomic reactions to such shocks due to breaks as a promising and policy-relevant avenue for future research.

7 Appendix: Proof of Proposition 1

We define the vectors $\boldsymbol{\alpha} = (vec(\boldsymbol{C})', vec(\boldsymbol{Q})')'$ and $\boldsymbol{v} = (\boldsymbol{\gamma}', \mathbf{q}')'$, and the $n(n+1)$-dimensional vector $\boldsymbol{\sigma}_{+}^{*} = (\boldsymbol{\sigma}'_{+1}, \boldsymbol{\sigma}'_{+2})'$, where $\boldsymbol{\sigma}_{+1} = vech(\boldsymbol{\Sigma}_{u,1})$, $\boldsymbol{\sigma}_{+2} = vech(\boldsymbol{\Sigma}_{u,2})$ and $vech(\cdot)$ is the column-stacking operator which selects only the diagonal and lower triangle elements of a symmetric matrix. The mapping between the reduced form and the structural parameters in eq.s (7)-(8) can be written in the form

$$\begin{aligned} \boldsymbol{\sigma}_{+1} &= vech(\boldsymbol{C}\boldsymbol{C}') \\ \boldsymbol{\sigma}_{+2} &= vech[(\boldsymbol{C}+\boldsymbol{Q})(\boldsymbol{C}+\boldsymbol{Q})']. \end{aligned}$$

The condition $\boldsymbol{\sigma}_{+1} \neq \boldsymbol{\sigma}_{+2}$ ($\boldsymbol{\Sigma}_{u,1} \neq \boldsymbol{\Sigma}_{u,2}$) implies $\boldsymbol{Q} \neq 0_{n\times n}$. Then, following Rothenberg (1971), necessary and sufficient condition for local identification is that the $n(n+1) \times (a_C + a_Q)$ Jacobian matrix $\frac{\partial \boldsymbol{\sigma}_{+}^{*}}{\partial \boldsymbol{v}'}$ has full column-rank evaluated at $\boldsymbol{v}_0 = (\boldsymbol{\gamma}'_0, \mathbf{q}'_0)'$. By the chain rule we have

$$\frac{\partial \boldsymbol{\sigma}_{+}^{*}}{\partial \boldsymbol{v}'} = \frac{\partial \boldsymbol{\sigma}_{+}^{*}}{\partial \boldsymbol{\alpha}'} \times \frac{\partial \boldsymbol{\alpha}}{\partial \boldsymbol{v}'}$$

where in particular

$$\frac{\partial \boldsymbol{\sigma}_{+}^{*}}{\partial \boldsymbol{\alpha}'} = \underset{n(n+1)\times 2n^2}{\begin{pmatrix} \frac{\partial \boldsymbol{\sigma}_{+1}}{\partial vec(\boldsymbol{C})'} & \frac{\partial \boldsymbol{\sigma}_{+1}}{\partial vec(\boldsymbol{Q})'} \\ \frac{\partial \boldsymbol{\sigma}_{+2}}{\partial vec(\boldsymbol{C})'} & \frac{\partial \boldsymbol{\sigma}_{+2}}{\partial vec(\boldsymbol{Q})'} \end{pmatrix}} \quad , \quad \frac{\partial \boldsymbol{\alpha}}{\partial \boldsymbol{v}'} = \boldsymbol{S} = \underset{2n^2\times(a_C+a_Q)}{\begin{pmatrix} \boldsymbol{S}_C & \boldsymbol{S}_I \\ \mathbf{0}_{n^2\times a_C} & \boldsymbol{S}_Q \end{pmatrix}}. \tag{20}$$

By using the properties of the matrices $\boldsymbol{K}_n$, $\boldsymbol{N}_n$ and $\boldsymbol{D}_n^{+}$ (Magnus and Neudecker (2007)) and applying standard derivative rules, the four blocks of the matrix $\frac{\partial \boldsymbol{\sigma}_{+}^{*}}{\partial \boldsymbol{\alpha}'}$ on the left of eq. (20) are given by:

$$\frac{\partial \boldsymbol{\sigma}_{+1}}{\partial vec(\boldsymbol{C})'} = \frac{\partial vech(\boldsymbol{C}\boldsymbol{C}')}{\partial vec(\boldsymbol{C})'} = \boldsymbol{D}_n^{+}\frac{\partial vec(\boldsymbol{C}\boldsymbol{C}')}{\partial vec(\boldsymbol{C})'} = 2\ \boldsymbol{D}_n^{+}\boldsymbol{N}_n(\boldsymbol{C}\otimes\boldsymbol{I}_n) = 2\ \boldsymbol{D}_n^{+}(\boldsymbol{C}\otimes\boldsymbol{I}_n);$$

$$\frac{\partial \sigma_{+1}}{\partial vec(\boldsymbol{Q})'} = 0_{\frac{1}{2}n(n+1)\times n^2};$$

$$\frac{\partial \sigma_{+2}}{\partial vec(\boldsymbol{C})'} = \frac{\partial vech[(\boldsymbol{C}\boldsymbol{C}' + \boldsymbol{C}\boldsymbol{Q}' + \boldsymbol{Q}\boldsymbol{C}' + \boldsymbol{Q}\boldsymbol{Q}']}{\partial vec(\boldsymbol{C})'} = \frac{\partial vech(\boldsymbol{C}\boldsymbol{C}')}{\partial vec(\boldsymbol{C})'} + \frac{\partial vech(\boldsymbol{C}\boldsymbol{Q}')}{\partial vec(\boldsymbol{C})'} + \frac{\partial vech(\boldsymbol{Q}\boldsymbol{C}')}{\partial vec(\boldsymbol{C})'}$$

$$= 2\ \boldsymbol{D}_n^{+}(\boldsymbol{C}\otimes\boldsymbol{I}_n) + \boldsymbol{D}_n^{+}(\boldsymbol{Q}\otimes\boldsymbol{I}_n) + \boldsymbol{D}_n^{+}(\boldsymbol{I}_n\otimes\boldsymbol{Q})\boldsymbol{K}_n = 2\ \boldsymbol{D}_n^{+}(\boldsymbol{C}\otimes\boldsymbol{I}_n) + 2\boldsymbol{D}_n^{+}(\boldsymbol{Q}\otimes\boldsymbol{I}_n);$$

$$\frac{\partial \boldsymbol{\sigma}_{+2}}{\partial vec(\boldsymbol{Q})'} = \frac{\partial vech[(\boldsymbol{C}\boldsymbol{C}' + \boldsymbol{C}\boldsymbol{Q}' + \boldsymbol{Q}\boldsymbol{C}' + \boldsymbol{Q}\boldsymbol{Q}']}{\partial vec(\boldsymbol{Q})'} = 2\ \boldsymbol{D}_n^{+}(\boldsymbol{C}\otimes\boldsymbol{I}_n) + 2\boldsymbol{D}_n^{+}(\boldsymbol{Q}\otimes\boldsymbol{I}_n).$$

The Jacobian is therefore given by

$$\frac{\partial \boldsymbol{\sigma}_+^*}{\partial \boldsymbol{v}'} = \begin{pmatrix} 2\ \boldsymbol{D}_n^+(\boldsymbol{C}\otimes\boldsymbol{I}_n) & \boldsymbol{0}_{\frac{1}{2}n(n+1)\times n^2} \\ 2\ \boldsymbol{D}_n^+(\boldsymbol{C}\otimes\boldsymbol{I}_n)+2\boldsymbol{D}_n^+(\boldsymbol{Q}\otimes\boldsymbol{I}_n) & 2\ \boldsymbol{D}_n^+(\boldsymbol{C}\otimes\boldsymbol{I}_n)+2\boldsymbol{D}_n^+(\boldsymbol{Q}\otimes\boldsymbol{I}_n) \end{pmatrix}\boldsymbol{S}$$

$$= 2\underbrace{\begin{pmatrix} \boldsymbol{D}_n^+ & \boldsymbol{0}_{\frac{1}{2}n(n+1)\times n^2} \\ \boldsymbol{0}_{\frac{1}{2}n(n+1)\times n^2} & \boldsymbol{D}_n^+ \end{pmatrix}}_{n(n+1)\times 2n^2}\underbrace{\begin{pmatrix} \boldsymbol{C}\otimes\boldsymbol{I}_n & \boldsymbol{0}_{n^2\times n^2} \\ (\boldsymbol{C}+\boldsymbol{Q})\otimes\boldsymbol{I}_n & (\boldsymbol{C}+\boldsymbol{Q})\otimes\boldsymbol{I}_n \end{pmatrix}}_{2n^2\times 2n^2}\underbrace{\begin{pmatrix} \boldsymbol{S}_{\boldsymbol{C}} & \boldsymbol{S}_I \\ \boldsymbol{0}_{n^2\times a_{\boldsymbol{C}}} & \boldsymbol{S}_Q \end{pmatrix}}_{2n^2\times(a_{\boldsymbol{C}}+a_Q)}$$

$$= 2(\boldsymbol{I}_n\otimes\boldsymbol{D}_n^+)\begin{pmatrix} \boldsymbol{C}\otimes\boldsymbol{I}_n & \boldsymbol{0}_{n^2\times n^2} \\ (\boldsymbol{C}+\boldsymbol{Q})\otimes\boldsymbol{I}_n & (\boldsymbol{C}+\boldsymbol{Q})\otimes\boldsymbol{I}_n \end{pmatrix}\begin{pmatrix} \boldsymbol{S}_{\boldsymbol{C}} & \boldsymbol{S}_I \\ \boldsymbol{0}_{n^2\times a_{\boldsymbol{C}}} & \boldsymbol{S}_Q \end{pmatrix}.$$

Aside from the multiplicative scalar 2, the $n(n+1)\times(a_{\boldsymbol{C}}+a_Q)$ matrix above, evaluated at the point $\boldsymbol{v}_0 = (\boldsymbol{\gamma}_0', \mathbf{q}_0')'$ ($\boldsymbol{C}_0$ and $\boldsymbol{Q}_0$) is the same as the matrix in eq. (10). The necessary order condition is obviously given by the condition $(a_C + a_Q) \leq n\,(n+1)$.■

References

ALTIG, D., L. J. CHRISTIANO, M. EICHENBAUM, AND J. LINDÉ (2011): "Firm-Specific Capital, Nominal Rigidities and the Business Cycle," *Review of Economic Dynamics*, 14(2), 225–247.

AMISANO, G., AND C. GIANNINI (1997): *Topics in Structural VAR Econometrics*. 2nd edn, Springer, Berlin.

BACCHIOCCHI, E., AND L. FANELLI (2012): "Identification in Structural Vector Autoregressive Models with Structural Changes with an Application to U.S. Monetary Policy," University of Milan, Department of Economics, Business and Statistics, Working Paper No. 57.

BAGLIANO, F. C., AND C. A. FAVERO (1998): "Measuring monetary policy with VAR models: An evaluation," *European Economic Review*, 42, 1069–1112.

BARSKY, R., C. HOUSE, AND M. KIMBALL (2007): "Sticky price models and durable goods," *American Economic Review*, 97, 984–998.

BARTH, M., AND V. RAMEY (2001): "The Cost Channel of Monetary Transmission," *NBER Macroeconomics Annual*, 16, 199–240.

BENATI, L. (2008): "Investigating Inflation Persistence Across Monetary Regimes," *Quarterly Journal of Economics*, 123(3), 1005–1060.

BENATI, L., AND P. SURICO (2009): "VAR Analysis and the Great Moderation," *American Economic Review*, 99(4), 1636–1652.

BLANCHARD, O., AND M. RIGGI (2013): "Why Are the 2000s so Different from the 1970s? A Structural Interpretation of Changes in the Macroeconomic Effects of Oil Prices," *Journal of the European Economic Association*, 11(5), 1032–1052.

BOIVIN, J., AND M. GIANNONI (2006): "Has Monetary Policy Become More Effective?," *Review of Economics and Statistics*, 88(3), 445–462.

BOIVIN, J., M. T. KILEY, AND F. S. MISHKIN (2010): "How Has the Monetary Transmission Mechanism Evolved over Time?," in B.M. Friedman and M. Woodford M. (eds): Handbook of Monetary Economics, Elsevier, Chapter 8, 369–422.

CANOVA, F. (2007): Methods for Applied Macroeconomic Research, Princeton University Press, Princeton, New Jersey.

CANOVA, F. (2009): "What Explains the Great Moderation in the US? A Structural Analysis," *Journal of the European Economic Association*, 7(4), 697–721.

CANOVA, F., AND G. DE NICOLÓ (2002): "Monetary Disturbances Matter for Business Fluctuations in the G-7," *Journal of Monetary Economics*, 49, 1131–1159.

CANOVA, F., AND F. FERRONI (2012): "The dynamics of US inflation: Can monetary policy explain the changes?," *Journal of Econometrics*, 167(1), 47–60.

CANOVA, F., AND F. J. P. FORERO (2012): "Estimating overidentified, nonrecursive, time-varying coefficients structural VARs," Universitat Pompeu Fabra, mimeo.

Canova, F., L. Gambetti, and E. Pappa (2008): “The Structural Dynamics of US Output and Inflation: What Explains the Changes?,” *Journal of Money, Credit and Banking*, 40(23), 369–388.

Canova, F., and T. Menz (2011): “Does Money Have a Role in Shaping Domestic Business Cycles? An International Investigation,” *Journal of Money, Credit and Banking*, 43(4), 577–607.

Canova, F., and M. Paustian (2011): “Business cycle measurement with some theory,” *Journal of Monetary Economics*, 58, 345–361.

Canova, F., and L. Sala (2009): “Back to Square One: Identification Issues in DSGE Models,” *Journal of Monetary Economics*, 56(4), 431–449.

Carlstrom, C., T. Fuerst, and M. Paustian (2009): “Monetary Policy Shocks, Choleski Identification, and DNK Models,” *Journal of Monetary Economics*, 56(7), 1014–1021.

Castelnuovo, E. (2012): “Estimating the Evolution of Money’s Role in the U.S. Monetary Business Cycle,” *Journal of Money, Credit and Banking*, 44(1), 23–52.

Castelnuovo, E., and L. Fanelli (2013): “Monetary Policy Indeterminacy and Identification Failures in the U.S.: Results from a Robust Test,” University of Padova "Marco Fanno" Working Paper No. 163.

Castelnuovo, E., and P. Surico (2010): “Monetary Policy Shifts, Inflation Expectations and the Price Puzzle,” *Economic Journal*, 120(549), 1262–1283.

Chowdhury, I., M. Hoffmann, and A. Schabert (2006): “Inflation Dynamics and the Cost Channel of Monetary Transmission,” *European Economic Review*, 50, 995–1016.

Christiano, L., M. Eichenbaum, and C. Evans (2005): “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy,” *Journal of Political Economy*, 113(1), 1–45.

Christiano, L., R. Motto, and M. Rostagno (2013): “Risk Shocks,” *American Economic Review*, forthcoming.

Christiano, L. J., M. Eichenbaum, and C. Evans (1999): “Monetary Policy Shocks: What Have We Learned and to What End?,” In: J.B. Taylor and M. Woodford (eds.): Handbook of Macroeconomics, Elsevier Science, 65–148.

Clarida, R., J. Galí, and M. Gertler (2000): “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory,” *Quarterly Journal of Economics*, 115, 147–180.

Cogley, T., and T. Sargent (2005a): “Drifts and Volatilities: Monetary Policies and Outcomes in the Post War U.S.,” *Review of Economic Dynamics*, 8, 262–302.

——— (2005b): “The Conquest of U.S. Inflation: Learning and Robustness to Model Uncertainty,” *Review of Economic Dynamics*, 8, 528–563.

Coibion, O. (2012): “Are the effects of monetary policy shocks big or small?,” *American Economic Journal: Macroeconomics*, 4(2), 1–32.

Del Negro, M., and F. Schorfheide (2004): "Priors for General Equilibrium Models for VARs," *International Economic Review*, 45, 643–673.

Del Negro, M., F. Schorfheide, F. Smets, and R. Wouters (2007): "On the Fit of New-Keynesian Models," *Journal of Business and Economic Statistics*, 25(2), 124–162.

Eichenbaum, M. (1992): "Comment on "Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy" by C.A. Sims," *European Economic Review*, 36(5), 1001–1011.

English, W. B., J. D. Lopez-Salido, and R. J. Tetlow (2013): "The Federal Reserve's Framework for Monetary Policy – Recent Changes and New Questions," Finance and Economics Discussion Series, Federal Reserve Board, Paper No. 2013-76.

Erceg, C., and A. Levin (2006): "Optimal monetary policy with durable consumption goods," *Journal of Monetary Economics*, 53, 1341–1359.

Faust, J. (1998): "The robustness of identified VAR conclusions about money," *Carnegie Rochester Conference Series on Public Policy*, 49, 207–244.

Faust, J., E. T. Swanson, and J. H. Wright (2004): "Identifying VARs based on high frequency futures data," *Journal of Monetary Economics*, 51, 1107–1131.

Forni, M., and L. Gambetti (2014): "Testing for Sufficient Information in Structural VARs," *Journal of Monetary Economics*, forthcoming.

Fry, R., and A. Pagan (2011): "Sign Restrictions in Structural Vector Autoregressions: A Critical Review," *Journal of Economic Literature*, 49(4), 938–960.

Fuhrer, J., and G. Rudebusch (2004): "Estimating the Euler Equation for Output," *Journal of Monetary Economics*, 51(6), 1133–1153.

Furher, J. C. (2000): "Habit Formation in Consumption and its Implications for Monetary-Policy Models," *American Economic Review*, 90(3), 367–390.

Galí, J. (2008): *Monetary Policy, Inflation and the Business Cycle*. Princeton University Press. Princeton, New Jersey.

Gertler, M., and P. Karadi (2014): "Monetary policy surprises, credit costs and economic activity," NBER Working Paper No. 20224.

Hanson, M. (2004): "The "Price Puzzle" Reconsidered," *Journal of Monetary Economics*, 51, 1385–1413.

Inoue, A., and B. Rossi (2011): "Identifying the Sources of Instabilities in Macroeconomic Fluctuations," *Review of Economics and Statistics*, 93(4), 1186–1204.

Justiniano, A., and G. Primiceri (2008): "The Time-Varying Volatility of Macroeconomic Fluctuations," *American Economic Review*, 98(3), 604–641.

Kim, C., and C. Nelson (1999): "Has the U.S. Economy Become More Stable? A Bayesian Approach Based on a Markov-Switching Model of the Business Cycle," *The Review of Economics and Statistics*, 81, 608–616.

King, R. G. (2000): "The New IS-LM Model: Language, Logic and Limits," *Federal Reserve Bank of Richmond Economic Quarterly*, 86(3), 45–103.

Kliem, M., and A. Kriwoluzky (2013): "Reconciling narrative monetary policy disturbances with structural VAR model shocks?," *Economics Letters*, 121(2), 247–251.

Lanne, M., and H. Lütkepohl (2008): "Identifying monetary policy shocks via changes in volatility," *Journal of Money, Credit and Banking*, 40, 1131–1149.

——— (2010): "Structural vector autoregressions with non normal residuals," *Journal of Business and Economic Statistics*, 28, 159–168.

Lanne, M. L., H. Lütkepohl, and K. Maciejowska (2010): "Structural vector autoregressions with Markov switching," *Journal of Economic Dynamics and Control*, 34, 21–131.

Lubik, T., and F. Schorfheide (2004): "Testing for Indeterminacy: An Application to U.S. Monetary Policy," *American Economic Review*, 94(1), 190–217.

Lütkepohl, H., and H. Reimers (1992): "Impulse response analysis of cointegrated systems with an application to German Money demand," *Journal of Economic Dynamics and Control*, 16, 53–78.

Magnus, J., and H. Neudecker (2007): "Matrix differential calculus with applications in statistics and econometrics," John Wiley and Sons, Inc.

Mavroeidis, S. (2010): "Monetary Policy Rules and Macroeconomic Stability: Some New Evidence," *American Economic Review*, 100(1), 491–503.

McConnell, M., and G. Perez-Quiros (2000): "Output Fluctuations in the United States: What Has Changed Since the Early 1980s?," *American Economic Review*, 90, 1464–1476.

Monacelli, T. (2009): "New-Keynesian models, durable goods, and collateral constraints," *Journal of Monetary Economics*, 56(2), 242–254.

Normandin, M., and L. Phaneuf (2004): "Monetary policy shocks: Testing identification conditions under time-varying conditional volatility," *Journal of Monetary Economics*, 51, 1217–1243.

Phillips, P. (1998): "Impulse response and forecast error variance asymptotics in nonstationary VARs," *Journal of Econometrics*, 83, 21–56.

Primiceri, G. (2005): "Time-Varying Structural Vector Autoregressions and Monetary Policy," *Review of Economic Studies*, 72, 821–852.

Rabanal, P. (2007): "Does Inflation Increase After a Monetary Policy Tightening? Answers Based on an Estimated DSGE Model," *Journal of Economic Dynamics and Control*, 31, 906–937.

Ravenna, F., and C. Walsh (2006): "Optimal Monetary Policy with the Cost Channel," *Journal of Monetary Economics*, 53, 199–216.

RIGOBON, R., AND B. SACK (2003): “Measuring the Reaction of Monetary Policy to the Stock Market,” *Quarterly Journal of Economics*, 118, 639–669.

RIGOBON, R., AND B. SACK (2004): “The impact of monetary policy on asset prices,” *Journal of Monetary Economics*, 51, 1553–1575.

ROMER, C., AND D. ROMER (2004): “A New Measure of Monetary Policy Shocks: Derivation and Implications,” *American Economic Review*, 94(4), 1055–1084.

ROTEMBERG, J., AND M. WOODFORD (1997): “An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy,” NBER Macroeconomics Annual, 297–346.

ROTHENBERG, T. (1971): “Identification in parametric models,” *Econometrica*, 39, 577–591.

SIMS, C. (1980): “Macroeconomics and Reality,” *Econometrica*, 48(1), 1–48.

SIMS, C., AND T. ZHA (2006): “Were There Regime Switches in U.S. Monetary Policy?,” *American Economic Review*, 96(1), 54–81.

SMETS, F., AND R. WOUTERS (2007): “Shocks and Frictions in US Business Cycle: A Bayesian DSGE Approach,” *American Economic Review*, 97(3), 586–606.

STOCK, J. H., AND M. W. WATSON (2002): “Has the Business Cycle Changed and Why?,” in M Gertler and K. Rogoff (eds.): NBER Macroeconomic Annual, MIT Press, Cambridge, MA.

STOCK, J. H., AND M. W. WATSON (2012): “Disentangling the Channels of the 2007-2009 Recession,” *Brookings Papers on Economic Activities*, forthcoming.

SURICO, P. (2008): “The Cost Channel of Monetary Policy and Indeterminacy,” *Macroeconomic Dynamics*, 12, 724–735.

TILLMANN, P. (2009): “The time-varying cost channel of monetary transmission,” *Journal of International Money and Finance*, 28, 941–953.

UHLIG, H. (2005): “What are the Effects of Monetary Policy? Results from an Agnostic Identification Procedure,” *Journal of Monetary Economics*, 52, 381–419.

VLAAR, P. (2004): “On the asymptotic distribution of impulse response functions with long-run restrictions,” *Econometric Theory*, 20, 891–903.

						Unconstrained recursive SVAR-WB							
			$\hat{C}$							$\hat{Q}$			
0.40	0	0	0	0	0	0	**-0.10**	0	0	0	0	0	0
0.94	**1.98**	0	0	0	0	0	**-0.85**	-0.31	0	0	0	0	0
0.46	**0.82**	**1.37**	0	0	0	0	-0.21	**-0.51**	**-0.47**	0	0	0	0
0.35	**0.34**	**0.10**	**0.45**	0	0	0	-0.11	**-0.23**	0.09	**-0.20**	0	0	0
-0.08	0.02	0.04	-0.04	**0.23**	0	0	0.04	**-0.05**	**-0.05**	0.03	**-0.09**	0	0
0.03	0.02	**0.04**	**-0.06**	0.02	**0.20**	0	-0.02	-0.03	-0.03	**0.08**	-0.01	**-0.13**	0
0.03	0.01	**0.03**	-0.01	-0.01	**0.05**	**0.07**	-0.01	0.00	-0.02	**0.03**	**0.03**	**-0.03**	-0.01
log-lik = 1556.80													

						Constrained recursive SVAR-WB							
			$\hat{C}$							$\hat{Q}$			
0.39	0	0	0	0	0	0	**-0.07**	0	0	0	0	0	0
0.78	**1.84**	0	0	0	0	0	**-0.59**	0	0	0	0	0	0
0.32	**0.76**	**1.38**	0	0	0	0	0	**-0.45**	**-0.49**	0	0	0	0
0.28	**0.33**	**0.17**	**0.45**	0	0	0	0	**-0.20**	0	**-0.20**	0	0	0
-0.05	0	0	0	**0.23**	0	0	0	**-0.03**	0	0	**-0.09**	0	0
0.01	0	0	**-0.03**	**0.02**	**0.20**	0	0	0	0	**0.06**	0	**-0.13**	0
0.02	0	**0.02**	0	0	**0.05**	**0.06**	0	0	0	**0.01**	**0.03**	**-0.03**	0
log-lik = 1540.12				LR test:			$\chi\,(23)$ = **33.36**			p-value=**0.08**			

						Unconstrained non-recursive SVAR-WB							
			$\hat{C}$							$\hat{Q}$			
0.27	**0.11**	0.00	0.05	**-0.17**	0.04	**-0.21**	**-0.31**	0	0	0	0	0	0
0.16	**1.67**	**0.33**	**1.02**	0.31	**0.60**	**-0.58**	0	**-2.61**	0	0	0	0	0
0.29	**0.43**	**1.46**	**0.36**	0.24	**0.20**	**-0.27**	0	0	**-0.82**	0	0	0	0
0.14	0.09	**0.12**	**0.52**	0.05	0.04	**-0.36**	0	0	0	**-0.52**	0	0	0
0.08	-0.03	0.00	-0.06	0.21	0.05	**0.07**	0	0	0	0	**-0.27**	0	0
0.00	-0.02	**0.05**	**-0.03**	-0.02	**0.20**	-0.02	0	0	0	0	0	**-0.22**	0
-0.01	0.00	**0.04**	**-0.04**	0.00	**0.04**	**-0.07**	0	0	0	0	0	0	**0.04**
log-lik = 1556.80													

						Constrained non-recursive SVAR-WB							
			$\hat{C}$							$\hat{Q}$			
0.32	**0.17**	0	0	**-0.10**	0	**-0.13**	**-0.10**	0	0	0	0	0	0
0	**1.39**	**0.53**	**1.33**	0	**0.32**	**-0.69**	0	**-1.80**	0	0	0	0	0
0	**0.37**	**1.34**	0	**0.34**	0	**-0.64**	0	0	**-0.71**	0	0	0	0
0.27	0	**0.23**	**0.46**	0	0	**-0.27**	0	0	0	**-0.53**	0	0	0
0	0	0	0	**0.21**	**0.10**	**0.09**	0	0	0	0	**-0.15**	0	0
0	0	**0.04**	**-0.03**	**-0.04**	**0.20**	0	0	0	0	0	0	**-0.16**	0
0	0	0	**-0.02**	0	**0.05**	**-0.07**	0	0	0	0	0	0	**0.03**
log-lik = 1550.25				LR test:			$\chi\,(22)$ = **13.10**			p-value=**0.94**			

Table 1: **Estimated parameters for the recursive and non-recursive SVAR-WB,** $T_B = 1984Q1$. Estimated values obtained via (Full Information) ML. Significant coefficients at 10% critical level are reported in bold.

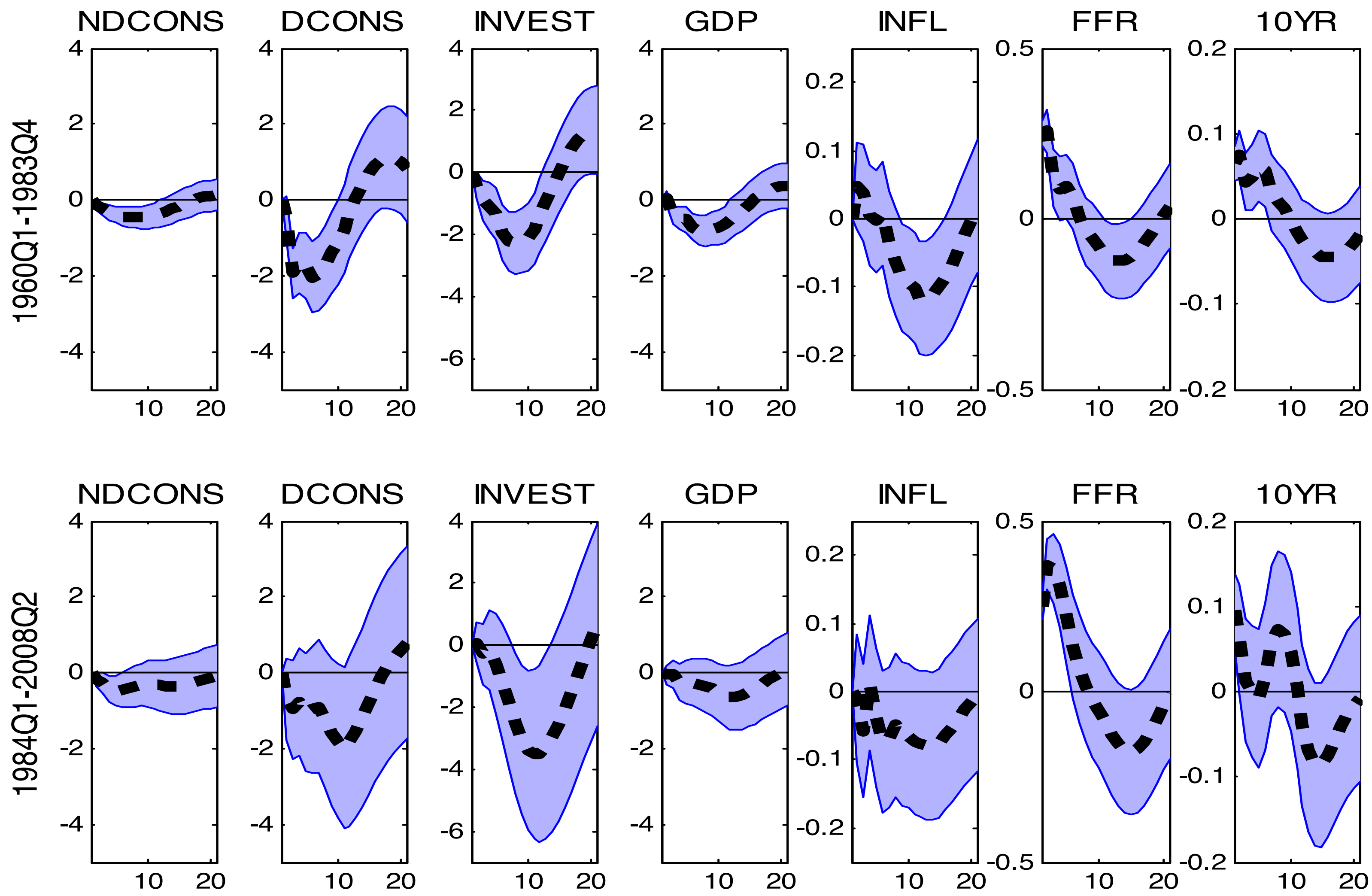

Figure 1: **SVAR-WB, Impulse response functions: Great Inflation versus Great Moderation.** Dashed-black lines: Point Estimates. Shaded-areas: 95-per cent confidence interval. Monetary policy shock identified with a Cholesky-identification scheme. Ordering of the variables in the VAR: Non durable consumption, durable consumption, investment, gpd, inflation, federal funds rate, 10 year-Treasury Bill rate.

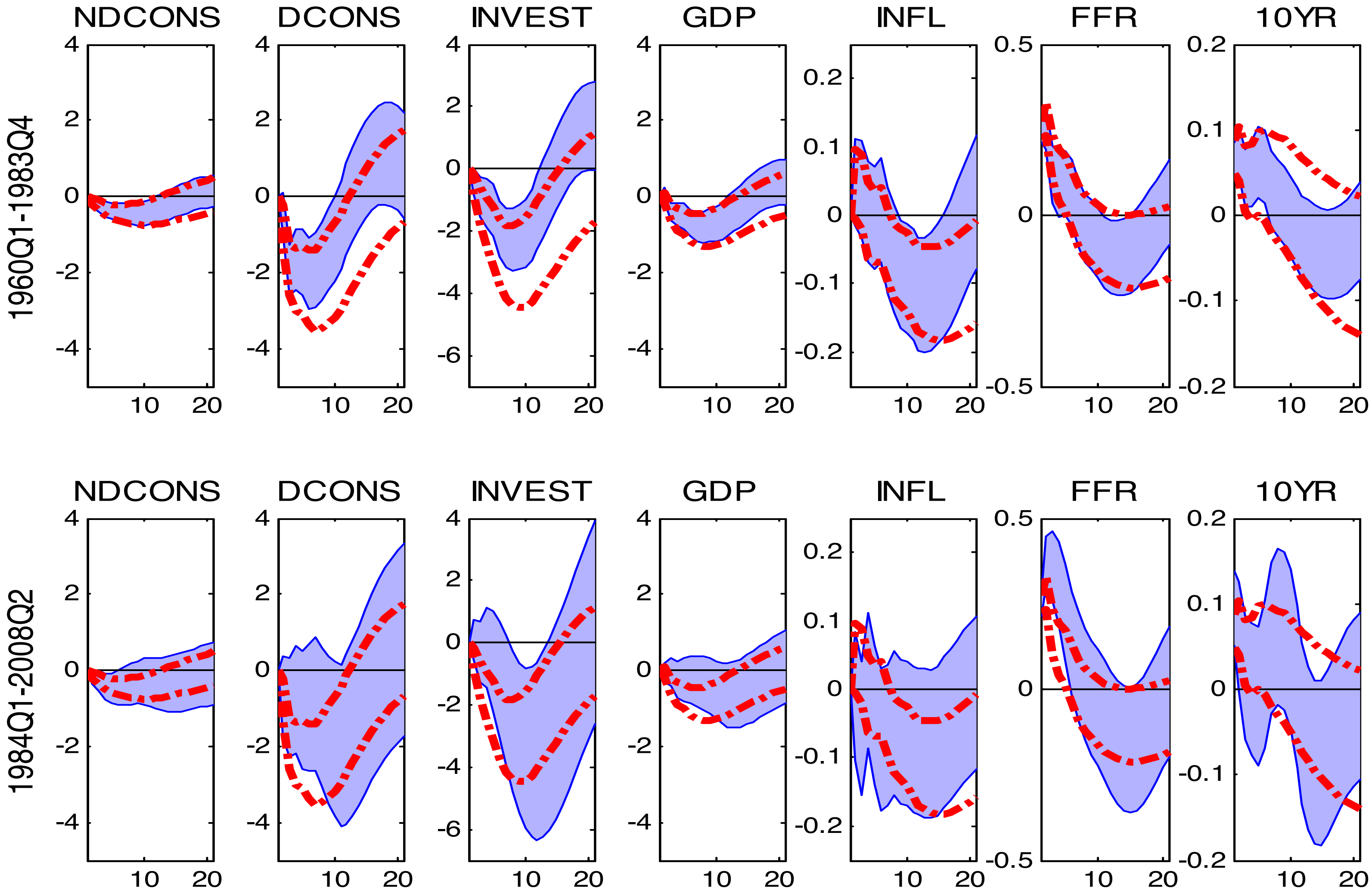

Figure 2: **Impulse response functions: Fixed-coefficient SVAR vs. SVAR-WB Impulse Responses.** Red dash-dotted lines: 95 per cent confidence intervals obained with a full-sample, fixed-coefficient VAR approach. Shaded-areas: 95-per cent confidence intervals conditional on our SVAR-WB approach, break-point: 1984Q1. Monetary policy shock identified with a Cholesky-identification scheme. Ordering of the variables in the VAR: Non durable consumption, durable consumption, investment, gpd, inflation, federal funds rate, 10 year-Treasury Bill rate.

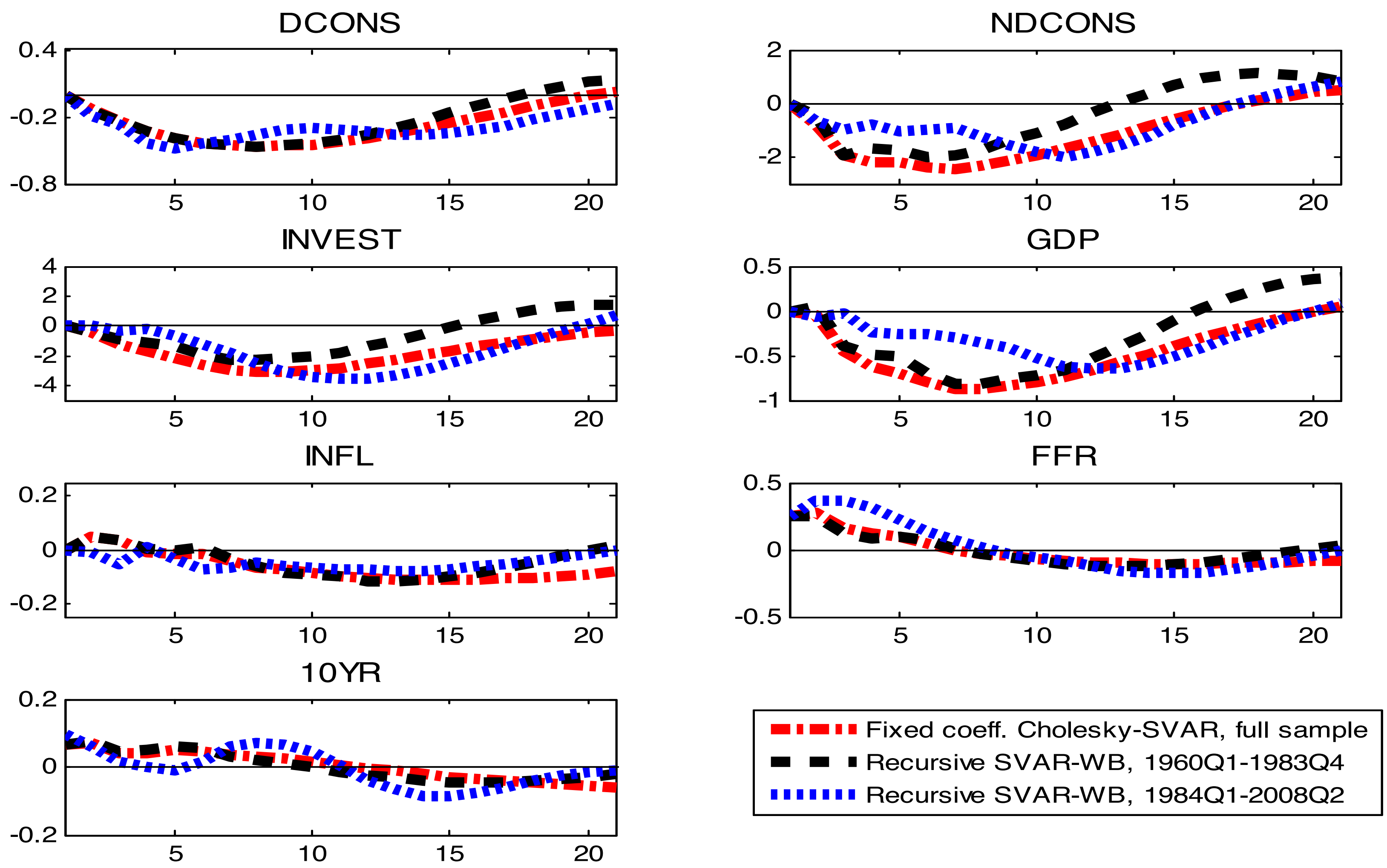

Figure 3: **Impulse response functions: Fixed-coefficient SVAR vs. SVAR-WB Impulse Responses: Point Estimates.** Red dash-dotted lines: 95 per cent confidence intervals obained with a full-sample, fixed-coefficient VAR approach. Blue dotted and black dashed lines: Reactions estimated with the Bacchiocchi-Castelnuovo-Fanelli approach proposed in this paper, which allows for a break in 1984Q1. Monetary policy shock identified with a Cholesky-identification scheme. Ordering of the variables in the VAR: Non durable consumption, durable consumption, investment, gpd, inflation, federal funds rate, 10 year-Treasury Bill rate.

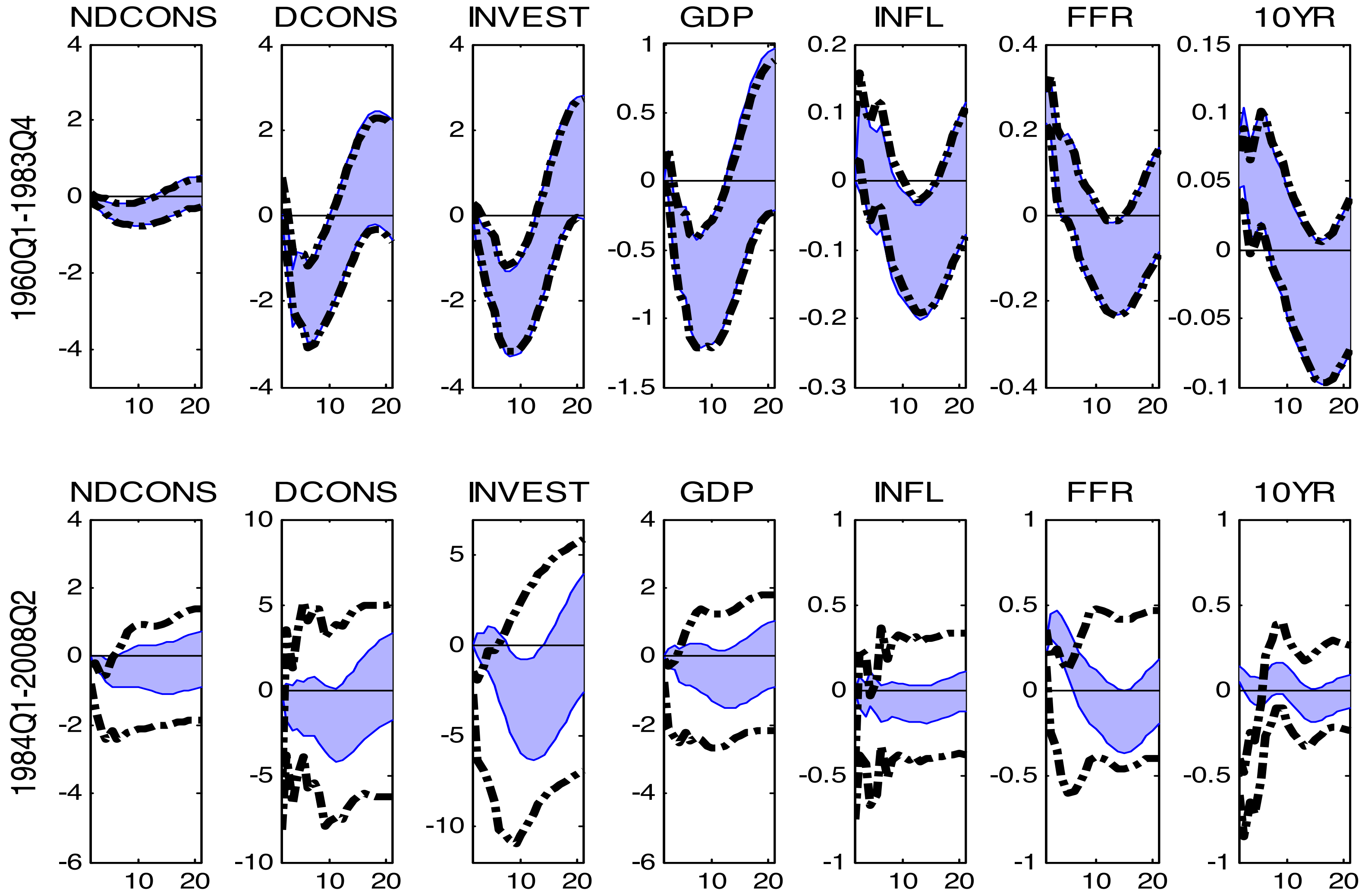

Figure 4: **Recursive- vs- Non-Recursive SVARs-WB.** Shaded areas: Recursive-VAR-WB 95 per cent confidence intervals. Black dashed-dotted lines: Non-recursive VAR-WB 95 per cent confidence intervals. VAR estimated with equation-specific constants and four lags. Ordering of the variables in the VAR: Non durable consumption, durable consumption, investment, gpd, inflation, federal funds rate, 10 year-Treasury Bill rate.

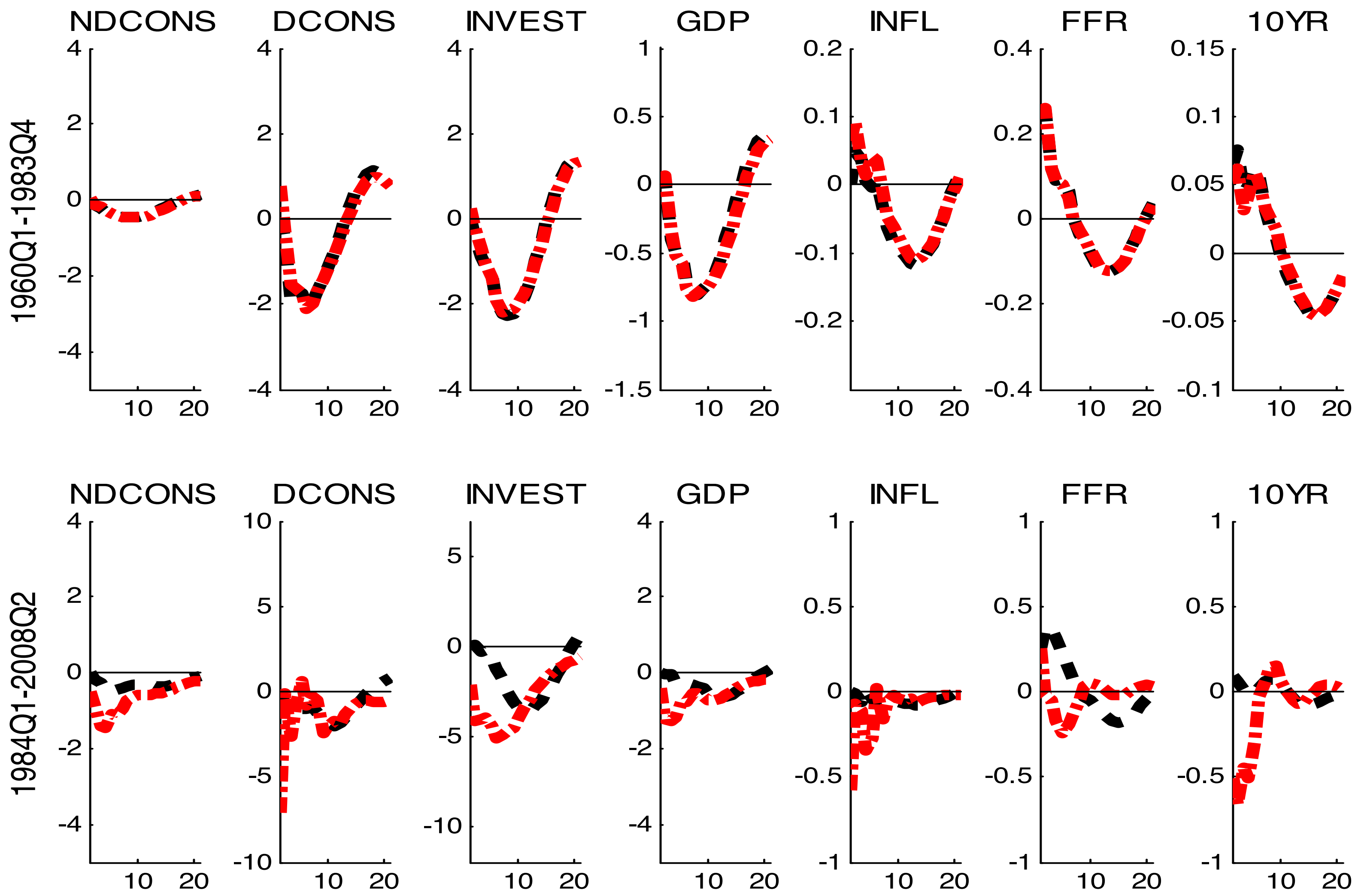

Figure 5: **Recursive- vs- non-recursive SVAR-WB: Impulse responses, point estimates.** Black dashed lines: Recursive SVAR.WB. Red dash-dotted lines: Non-recursive SVAR-WB. VAR estimated with equation-specific constants and four lags. Ordering of the variables in the VAR: Ordering of the variables in the VAR: Non durable consumption, durable consumption, investment, gpd, inflation, federal funds rate, 10 year-Treasury Bill rate.

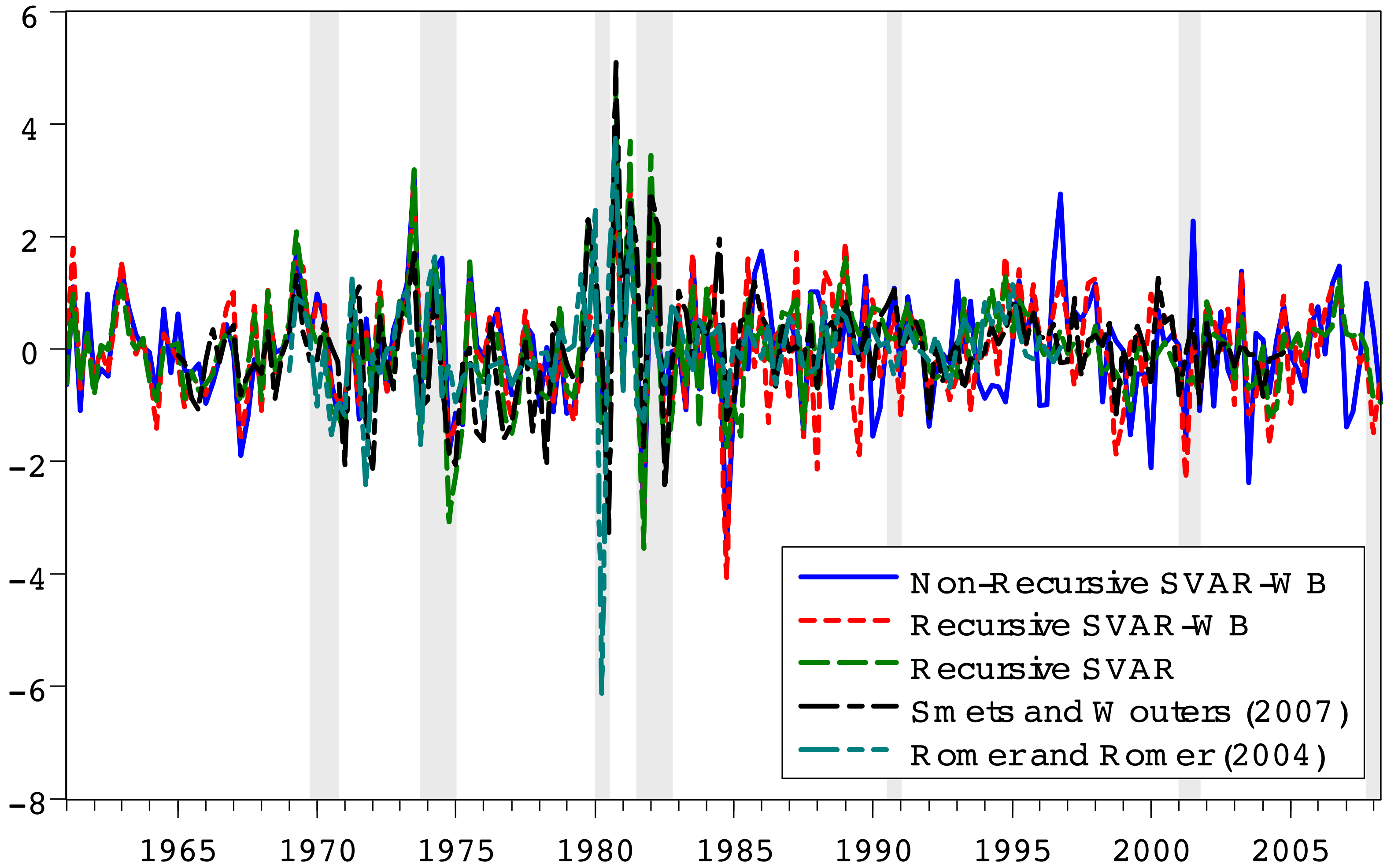

Figure 6: **Estimated policy shocks: Comparison.** Blue, solid line: Shocks conditional on our "BCF Non-Recursive VAR" model, which imposes no-zero restrictions. Red, dashed line: Shocks conditional on our "BCF CVAR" framework, which imposes a standard lower-triangular matrix on the contemporaneous relationships in the system. Green, dashed line (with longer dashes than the red ones): Shocks conditional on a standard fixed coefficient-Cholesky VAR. Black, dash-dotte line: Shocks estimated by Smets and Wouters (2007). Light green, dashed line: Shocks estimated by Romer and Romer (2004) (average of monthly cumulative values). The BCF models allow for a break in 1984Q1. Vertical gray bars: NBER recessions. The measure of policy shocks by Romer and Romer (2004) is taken from Coibion (2012). All series displayed in this Figure are standardized.

	Estimates		
Parameter	*1960Q1-1983Q4*	*1960Q1-1983Q4*	*1984Q1-2008Q2*
ξ_x	0.50 (0.01)	0.42 (0.02)	0.09 (0.16)
τ	0.03 (0.001)	0.09 (0.01)	0.73 (0.22)
ξ_π	0.53 (0.02)	0.27 (0.05)	1.00 (0.18)
κ	0.01 (0.01)	0.01 (0.01)	0.11 (0.02)
α	3.08 (2.07)	0^*	0.00 (1.80)
Distance	133.39	137.82	20.09

Table 2: **Structural DSGE model: IRF matching estimates of the structural parameters by using the non-recursive SVAR-WB.** Estimated values obtained via a minimum-distance estimator. Loss function constructed by considering the impulse responses to a monetary policy shock produced with the structural DSGE model and those produced with our non-recursive VAR-WB. Standard errors in parenthesis. The "Distance" measure reported in the last row refers to the sum of the squared deviations of the impulse responses of the model with respect to those of the SVAR-WB, weighted by the latter's variances. Calibrated values indicated with a star. [0,1] bounds for the structural parameters considered in the optimization, with the exception of the cost-channel parameter (last raw) for which we considered a [0,5] bound.

Technical Supplement of the paper:

"Gimme a Break! Identification and Estimation of the Macroeconomic Effects of Monetary Policy Shocks in the U.S."

by Bacchiocchi, E., Castelnuovo, E., Fanelli, L.

Analysis with non-stationary cointegrated variables

The analysis conducted in our paper is based on variables modeled in levels. As shown by Phillips (1998), when the data generating process is characterized by unit roots, the impulse response functions (IRFs) estimated from SVARs are no longer consistent at all horizons and inference is of non-standard type. Several solutions have been provided to account for non-stationary time series, see e.g. Lütkepohl and Reimers (1992), Amisano and Giannini (1997), Phillips (1998) and Vlaar (2004). Gonzalo and Ng (2001) and Pagan and Pesaran (2008) are examples in which the identification of nonstationary SVARs can be designed by exploiting permanent/transitory decompositions of the shocks.[1]

In this Technical Supplement, we extend the analysis of our recursive and non-recursive SVARS-WB discussed in the paper to the case in which the non-stationarity of the data and the possible presence of unit roots is explicitly taken into account. Our objective is to envisage whether the results obtained in the paper with VAR specifications based on the levels of the variables remain qualitatively unchanged when the unit roots and cointegration restrictions suggested by the data are incorporated into the system.

Consider the vector $\boldsymbol{z}_t$:=$(NDCONS_t, DCONS_t, INVEST_t, GDP_t, INFL_t, FFR_t, 10YR_t)'$ analyzed in Section 3 of the paper, and assume that the characteristic equation associated with the VAR in levels, $\det(\boldsymbol{A}(s))$=0, $\boldsymbol{A}(s)$:=$\boldsymbol{I}_n - \boldsymbol{A}_1 s - \ldots - \boldsymbol{A}_k s^k$, has exactly $n-r$ unit roots equal to s=1, with $0 \leq r < n$ (recall that n:=7)[2] Then consider the

[1] Alternatively, it is generally possible to treat the system as a highly persistent one and avoid pre-testing by constructing IRFs using the local-to-unity asymptotic theory as in, e.g., Pesavento and Rossi (2006).

[2] We recall that NDCONS stands for non-durable personal consumption, DCONS for durable per-

cointegrated Vector Equilibrium Correction (VEqC) counterpart of the system:

$$\Delta \boldsymbol{z}_t = \boldsymbol{\alpha}\boldsymbol{\beta}'\boldsymbol{z}_{t-1} + \boldsymbol{\Upsilon}_1\Delta\boldsymbol{z}_{t-1} + \ldots + \boldsymbol{\Upsilon}_{k-1}\Delta\boldsymbol{z}_{t-k+1} + \boldsymbol{\Psi}\boldsymbol{d}_t + \boldsymbol{u}_t \tag{1}$$

where $\Delta\boldsymbol{z}_t := \boldsymbol{z}_t - \boldsymbol{z}_{t-1}$, $\boldsymbol{\alpha}\boldsymbol{\beta}' = -(\boldsymbol{I}_n - \sum_{i=1}^{k}\boldsymbol{A}_i)$, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $n\times r$ matrices of full column-rank $r < n$, and $\boldsymbol{\Upsilon}_j = -\sum_{i=j+1}^{k}\boldsymbol{A}_i$, $j = 1, ..., k-1$, see Johansen (1996). Lütkepohl and Reimers (1992) and Amisano and Giannini (1997) exploit the mapping between the VAR parameters in levels and the parameters of its cointegrated VEqC counterpart to account for the unit-roots/cointegration restrictions and incorporate them in the computation of the IRFs, coming back to the (constrained) VAR in levels. Conditional on the reduced form matrices $\boldsymbol{A}_i$, $i = 1, ..., k$ embedding the unit-roots/cointegration restrictions, inference on the IRFs can be conducted in the usual way.

We follow a different route, inspired by a well-known result of cointegration analysis, exploited, among others, by e.g. Campbell and Shiller (1987) and King, Plosser, Stock, and Watson (1991). The idea is to use a reparameterization of the cointegrated VEqC in eq. (1) in terms of a stationary VAR system which embodies the identified cointegrating relationships, without any loss of information. We obtain a stationary VAR system for a vector of transformed variables which feature the cointegration relationships, and which can be used in the 'conventional' way. Obviously, the IRFs computed from the transformed system must be interpreted accordingly.

For fixed cointegration rank, r, and fixed (identified) cointegration matrix, $\boldsymbol{\beta} := \boldsymbol{\beta}^0$, the VEqC in eq. (1) can be equivalently represented as a stable VAR system of the form

$$\boldsymbol{z}_t^* = \boldsymbol{B}_1\boldsymbol{z}_{t-1}^* + \boldsymbol{B}_2\boldsymbol{z}_{t-2}^* + \ldots + \breve{\boldsymbol{B}}_k\boldsymbol{z}_{t-k}^* + \boldsymbol{\Psi}^0\boldsymbol{d}_t + \boldsymbol{u}_t^0 \tag{2}$$

where the $n \times 1$ vector $\boldsymbol{z}_t^*$ is defined by

$$\boldsymbol{z}_t^* := \begin{pmatrix} \boldsymbol{\beta}^{0\prime}\boldsymbol{z}_t \\ \boldsymbol{\tau}^{0\prime}\Delta\boldsymbol{z}_t \end{pmatrix} \qquad \begin{matrix} r \times 1 \\ (n-r)\times 1 \end{matrix}\ , \tag{3}$$

$\boldsymbol{\tau}^0$ is a $n \times (n-r)$ matrix such that $\det(\boldsymbol{\tau}^{0\prime}\boldsymbol{\beta}_\perp^0) \neq 0$, $\boldsymbol{\beta}_\perp^0$ is the orthogonal complement of $\boldsymbol{\beta}^0$, $\boldsymbol{B}_i$, $i = 1, ..., k-1$ and $\breve{\boldsymbol{B}}_k$ are matrices of parameters which depend on ($\boldsymbol{\alpha}$, $\boldsymbol{\Upsilon}_1$, ..., $\boldsymbol{\Upsilon}_{k-1}$) and $(\boldsymbol{\beta}^0, \boldsymbol{\tau}^0)'$, the matrix $\breve{\boldsymbol{B}}_k$ is restricted such that $\breve{\boldsymbol{B}}_k := (\boldsymbol{B}_{1,k} : \boldsymbol{0}_{n\times(n-r)})$, $\boldsymbol{\Psi}^0$ depends on $\boldsymbol{\Psi}$ and $(\boldsymbol{\beta}^0, \boldsymbol{\tau}^0)'$, and $\boldsymbol{u}_t^0 := (\boldsymbol{\beta}^0, \boldsymbol{\tau}^0)'\boldsymbol{u}_t$ has covariance matrix $\boldsymbol{\Sigma}_{u^0} := (\boldsymbol{\beta}^0, \boldsymbol{\tau}^0)'\boldsymbol{\Sigma}_u(\boldsymbol{\beta}^0, \boldsymbol{\tau}^0)$.[3]

sonal consumption, INVEST for fixed-private investment, GDP for gross domestic product, INFL for inflation, FFR for the federal funds rate, and 10YR for the 10 year-Treasury Bill rate. The source of the data is the Federal Reserve Bank of St. Louis.

[3]A formal proof of the equivalence between the representation in eq. (1) and the representation in eq.s (2)-(3) of the VEqC may be found in Paruolo (2003).

The system in eq.s (2)-(3) is stationary by construction and can be regarded as a reparameterization of the original cointegrated VEqC in eq. (1). Obviously, the system can be opportunely rotated, in the sense that the ordering of the elements in $\boldsymbol{z}_t^*$ must not necessarily be the one suggested by eq. (3).

The VAR in eq.s (2)-(3) can be used in alternative to the system (1) and treated as reduced form of a SVAR for the variables in $\boldsymbol{z}_t^*$. It can be used advantageously in empirical analysis when: (i) the researcher has a strong a priori about the number and form of cointegrating relations and, at the same time, the specified cointegration matrix $\boldsymbol{\beta}^0$ does not involve the estimation of unknown coefficients (e.g. when the cointegrating vectors in $\boldsymbol{\beta}^0$ are of the type 1, -1, 0, etc.); (ii) the VAR dimension, n, is large relative to the sample length, T, hence the finite sample performances of the cointegration rank test and of the test for overidentification restrictions on $\boldsymbol{\beta}$:=$\boldsymbol{\beta}^0$ may be poor (beyond the use of bootstrap methods). In our framework, $\boldsymbol{z}_t^*$ will typically contain linear combinations of variables in $\boldsymbol{z}_t$ of the form $(10YR_t - FFR_t)$ ~I(0), $(DCONS_t - GDP_t)$ ~I(0), etc., and a selection of variables in first differences, e.g. $\Delta INVEST_t$, ΔGDP_t, etc., see below. The simplest way to check for the data adequacy of the chosen specifications for $\boldsymbol{\beta}^0$ and $\boldsymbol{\tau}^0$ is to test for the stationarity of the VAR system (2)-(3). Of course, the variables in $\boldsymbol{z}_t^*$ collect by construction linear combinations of the variables in the original vector $\boldsymbol{z}_t$ or their first differences, hence the analysis based on the 'transformed' VAR system (2)-(3) must be interpreted accordingly.

The important thing to remark here is that changes in the parameters $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\Upsilon}_1, ..., \boldsymbol{\Upsilon}_{k-1}$, $\boldsymbol{\Psi}$ and $\boldsymbol{\Sigma}_u$ of the VEqC in eq. (1) reflect in changes in the parameters $\boldsymbol{B}_i$, $i = 1, ..., k-1$, $\breve{\boldsymbol{B}}_k$, $\boldsymbol{\Psi}^0$ and $\boldsymbol{\Sigma}_{u^0}$ associated with the 'transformed' VAR system (2)-(3)

Fixed the value of the cointegration rank r (and thus the number of unit roots $n - r$), and specified the matrices $\boldsymbol{\beta}^0$ and $\boldsymbol{\tau}^0$ such that only known elements are involved, the system (2)-(3) serves as the 'restricted model', i.e. the model under the null of absence of structural breaks in the parameters $\boldsymbol{B}_i$, $i = 1, ..., k-1$, $\breve{\boldsymbol{B}}_k$, $\boldsymbol{\Psi}^0$ and $\boldsymbol{\Sigma}_{u^0}$. This 'no-change' model will be estimated on the entire sample 1960Q1-2008Q2. Our testable hypothesis on the stochastic trends is that the cointegration rank in the 'no-change' model is equal to r=2, and that the two cointegrating vectors in $\boldsymbol{\beta}^0$ correspond to the nominal interest rates spread $(10YR_t - FFR_t)$ and the 'great-ratio' $(DCONS_t - GDP_t)$, respectively. This hypothesis, other than being inspired by economic considerations, seems to be supported by the graphs plotted in Figure TS1 and the formal tests presented below.[4] The matrices $\boldsymbol{\beta}^0$ and $\boldsymbol{\tau}^0$ are therefore given by

$$\boldsymbol{\beta}^{0\prime}:=\begin{pmatrix} 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \quad ; \quad \boldsymbol{\tau}^{0\prime}:=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \tag{4}$$

and the $\boldsymbol{z}_t^*$ vector in eq. (3) contains the following elements: $\boldsymbol{z}_t^*:=(\Delta NDCONS_t, DCONS_t - GDP_t, \Delta INVEST_t, \Delta GDP_t, \Delta INFL_t, \Delta FFR_t, 10YR_t - FFR_t)'$. We estimate the no-change VAR system for $\boldsymbol{z}_t^*$ on the period 1960Q1-2008Q2 with four lags and a constant, in line with what we have done in the paper with the VAR system for $\boldsymbol{z}_t$. The results are reported in Table TS1, which summarizes the value of the log-likelihood function, the LR Trace cointegration rank test and the estimated largest eigenvalue in modulus of the associated companion matrix. No unit roots should be detected in the VAR for $\boldsymbol{z}_t^*$ if the chosen r and the specified $\boldsymbol{\beta}^0$ and $\boldsymbol{\tau}^0$ in eq. (4) capture the common stochastic trends driving the system.

Table TS1 here

The results in Table TS1 provide support to our hypothesis about the common stochastic trends: the estimated 'no-change' model based on cointegration rank r=2 and the $\boldsymbol{\beta}^0$ and $\boldsymbol{\tau}^0$ in eq. (4) is stationary according to the LR Trace cointegration rank test. The estimated largest root of the VAR companion matrix, equal to 0.86, leaves no doubts about the stationarity of $\boldsymbol{z}_t^*$.

Next, we move to the cointegrated VEqC model under the alternative hypothesis of a single break in the parameters at time T_B:=1984Q1, namely our 'unrestricted' or 'one-break' model. Following Hansen (2003), in this case the VEqC system can be represented in the form

$$\Delta \boldsymbol{z}_t = \boldsymbol{\alpha}(t)\boldsymbol{\beta}'(t)\boldsymbol{z}_{t-1} + \boldsymbol{\Upsilon}_1(t)\Delta\boldsymbol{z}_{t-1} + \ldots + \boldsymbol{\Upsilon}_{k-1}(t)\Delta\boldsymbol{z}_{t-k+1} + \boldsymbol{\Psi}(t)\boldsymbol{d}_t + \boldsymbol{u}_t \tag{5}$$

[4] King, Plosser, Stock, and Watson (1991), document the stationarity of the 'great-ratio' $(INVEST_t - GDP_t)$ (see their Table 1) but their sample covers the period 1949Q1(1954Q1)-1988Q4.

where

$$\boldsymbol{\alpha}(t)\boldsymbol{\beta}'(t) := \boldsymbol{\alpha}_1\boldsymbol{\beta}_1'\mathbf{1}\left(t < T_B\right) + \boldsymbol{\alpha}_2\boldsymbol{\beta}_2'\mathbf{1}\left(t \geq T_B\right) \tag{6}$$

$$\boldsymbol{\Upsilon}_i(t) := \boldsymbol{\Upsilon}_{1,i}\mathbf{1}\left(t < T_B\right) + \ \boldsymbol{\Upsilon}_{2,i}\mathbf{1}\left(t \geq T_B\right),\ i = 1, ..., k-1$$

$$\boldsymbol{\Psi}(t) := \boldsymbol{\Psi}_1\mathbf{1}\left(t < T_B\right) + \ \boldsymbol{\Psi}_2\mathbf{1}\left(t \geq T_B\right)$$

$$\boldsymbol{\Sigma}_u(t) := \boldsymbol{\Sigma}_{u,1}\mathbf{1}\left(t < T_B\right) + \ \boldsymbol{\Sigma}_{u,2}\mathbf{1}\left(t \geq T_B\right). \tag{7}$$

and the matrices $\boldsymbol{\alpha}_i$ and $\boldsymbol{\beta}_i$ are $n \times r_i$ and have full column rank r_i, i=1,2. We assume that the common stochastic trends and cointegrating relationships are invariant across the two volatility regimes, i.e. r_1=r_2=2 and $\boldsymbol{\beta}_1$=$\boldsymbol{\beta}_2$:=$\boldsymbol{\beta}^0$, where $\boldsymbol{\beta}^0$ is given in eq. (4). In other words, we allow changes in the error covariance matrix the, possibly, in the 'short-run' adjustment coefficients at time T_B:=1984Q1 but not in the cointegration relationships. In this case, the VEqC in eq.s (5)-(7) is reparameterized in the form

$$\boldsymbol{z}_t^* = \boldsymbol{B}_1(t)\boldsymbol{z}_{t-1}^* + \boldsymbol{B}_2(t)\boldsymbol{z}_{t-2}^* + ... + \breve{\boldsymbol{B}}_k(t)\boldsymbol{z}_{t-k}^* + \boldsymbol{\Psi}^0(t)\boldsymbol{d}_t + \boldsymbol{u}_t^0 \tag{8}$$

where $\boldsymbol{z}_t^*$ is still defined as in eq. (3) and

$$\boldsymbol{B}_i(t) := \boldsymbol{B}_{1,i}\mathbf{1}\left(t < T_B\right) + \ \boldsymbol{B}_{2,i}\mathbf{1}\left(t \geq T_B\right),\ i = 1, ..., k-1 \tag{9}$$

$$\breve{\boldsymbol{B}}_k(t) := (\boldsymbol{B}_{1,k}:\mathbf{0}_{n\times(n-r)})\mathbf{1}\left(t < T_B\right) + \ (\boldsymbol{B}_{2,k}:\mathbf{0}_{n\times(n-r)})\mathbf{1}\left(t \geq T_B\right) \tag{10}$$

$$\boldsymbol{\Psi}^0(t) := \boldsymbol{\Psi}_1^0\mathbf{1}\left(t < T_B\right) + \ \boldsymbol{\Psi}_2^0\mathbf{1}\left(t \geq T_B\right)$$

$$\boldsymbol{\Sigma}_{u^0}(t) := \boldsymbol{\Sigma}_{u^0,1}\mathbf{1}\left(t < T_B\right) + \ \boldsymbol{\Sigma}_{u^0,2}\mathbf{1}\left(t \geq T_B\right). \tag{11}$$

Table TS2 and Table TS3 here

Table TS2 reports the values of the log-likelihood of the estimated reduced form VAR model in eq. (8) on the first regime 1960Q1-1983Q4, the results of the LR Trace test for cointegration rank and the estimated highest eigenvalues of the associated companion matrix. Likewise, Table TS3 reports the same information for the model estimated on the period 1984Q1-2008Q2. It can be noticed that while the hypothesis of stationarity is strongly supported by the data on the period 1960Q1-1983Q4, the LR Trace cointegration test leaves room for a scenario where the VAR for $\boldsymbol{z}_t^*$ is characterized by two unit roots (r=5) on the period 1984Q1-2008Q2, albeit the evidence is not clear-cut. The estimated largest root of the VAR companion matrix in Table TS3 is 0.94, a value which is still compatible with the case of an highly persistent but stationary VAR. By combining this observation with the low power (i.e. the ability to rejects unit roots) displayed by the cointegration rank test in highly persistent but stationary VARs, and in light of the weak evidence in favour of unit roots provided by the computed cointegration rank test, we conclude that the system for $\boldsymbol{z}_t^*$ can be assumed stationary (albeit highly persistent) also on the period 1984Q1-2008Q2.

Given the results in tables TS1-TS3, we have all the ingredients to test the null of absence of a break at time T_B:=1984Q1 along the lines of Section 3 of the paper. In this case we run the test by controlling for the number of unit roots, and standard asymptotic theory can be invoked. We first focus on the null hypothesis that all VAR reduced form parameters $\boldsymbol{B}:=(\boldsymbol{B}_1, ..., \breve{\boldsymbol{B}}_k)$, $\boldsymbol{\Psi}^0$ and $\boldsymbol{\Sigma}_{u^0}$ are constant across the two regimes, against the alternative in eqs. (9)-(10) and (11). The results suggest that the null of constant parameters is strongly rejected because the LR test is equal to LR:=-2[(-938.37) - (-493.44-105.75)]=678.36 and has a p-value of 0.000 (taken from the $\chi^2(196)$ distribution). Obviously, also the LR Chow-type test for $\Sigma_{u^0,1}=\Sigma_{u^0,2}$ (based on the implicit assumption $(\boldsymbol{B}_{1,1}, ..., \breve{\boldsymbol{B}}_{k,1}, \boldsymbol{\Psi}_1^0)=(\boldsymbol{B}_{1,2}, ..., \breve{\boldsymbol{B}}_{k,2}, \boldsymbol{\Psi}_2^0)=(\boldsymbol{B}_1, ..., \breve{\boldsymbol{B}}_k, \boldsymbol{\Psi}^0)$) leads to a strong rejection of the null of stable parameters. We reject the fixed-coefficient hypothesis and conclude that even imposing the unit roots/cointegration restrictions the sub-periods 1960Q1-1983Q4 and 1984Q1-2008Q2 represent two distinct regimes of volatility.

The identification methodology discussed in the paper and the recursive and non-recursive SVARs-WB can be thus applied to the U.S. data to tackle the effects of monetary policy shocks. The analysis is based on the (stationary) SVAR-WB:

$$\boldsymbol{z}_t^* = \boldsymbol{B}_1(t)\boldsymbol{z}_{t-1}^* + \boldsymbol{B}_2(t)\boldsymbol{z}_{t-2}^* + ... + \breve{\boldsymbol{B}}_k(t)\boldsymbol{z}_{t-k}^* + \boldsymbol{\Psi}^0(t)\boldsymbol{d}_t + \boldsymbol{u}_t^0 \ , \ \boldsymbol{u}_t^0 = \boldsymbol{C}(t)\boldsymbol{e}_t \quad , \ \boldsymbol{e}_t \sim \mathrm{WN}(\boldsymbol{0}_n \ , \boldsymbol{I}_n) \tag{12}$$

where the reduced form parameters are governed by eq.s (9)-(11), $\boldsymbol{C}(t)$ is given by

$$\boldsymbol{C}(t) := \boldsymbol{C} + \boldsymbol{Q} \times \mathbf{1}\,(t \geq T_B)$$

and $\boldsymbol{C}$ and $\boldsymbol{Q}$ can be restricted as discussed thoroughly in the paper. The corresponding identification conditions are provided by Proposition 1 of the paper. The important remark here is that the SVAR-WB for $\boldsymbol{z}_t^*$ in eq. (12) is not directly comparable to the SVAR-WB for $\boldsymbol{z}_t$ discussed in the paper. Indeed, $\boldsymbol{z}_t \neq \boldsymbol{z}_t^*$, hence the the IRFs are not directly comparable from a quantitative point of view.

The ML estimates of $\boldsymbol{C}$ and $\boldsymbol{Q}$ under the recursive and non-recursive schemes are reported in Table TS4, while the implied IRFs are plotted in Figure TS2 (recursive scheme) and Figure TS3 (non-recursive scheme). The patterns of the estimated IRFs under the two identification schemes allow us to substantially confirm, from a qualitative point of view, the stories discussed in Section 3 of the paper.

Table TS4 here

References

Amisano, G., and C. Giannini (1997): *Topics in Structural VAR Econometrics.* 2nd edn, Springer, Berlin.

Campbell, J. Y., and R. J. Shiller (1987): "Cointegration and tests of present value models," *Journal of Political Economy*, 95, 1062–1088.

Gonzalo, J., and S. Ng (2001): "A systematic framework for analyzing the dynamic effects of permanent and transitory shocks," *Journal of Economic Dynamics and Control*, 25, 1527–1546.

Hansen, P. (2003): "Structural changes in the cointegrated vector autoregressive model," *Journal of Econometrics*, 114, 261–295.

Johansen, S. (1996): *Likelihood-based inference in cointegrated vector auto-regressive models.* Oxford University Press, Oxford.

King, R., C. Plosser, J. Stock, and M. Watson (1991): "Stochastic trends and economic fluctuations," *American Economic Review*, 81, 819–840.

Lütkepohl, H., and H. Reimers (1992): "Impulse response analysis of cointegrated systems with an application to German Money demand," *Journal of Economic Dynamics and Control*, 16, 53–78.

Pagan, A., and M. H. Pesaran (2008): "Econometric analysis of structural systems with permanent and transitory shocks," *Journal of Economic Dynamics and Control*, 32, 3376–3395.

Paruolo, P. (2003): "Common dynamics in I(1) systems," Working Paper No. 2003/33, Università dell'Insubria, Varese, 1–21, Working Paper No. 2003/33, Università dell'Insubria, Varese.

Pesavento, E., and B. Rossi (2006): "Small-sample confidence intervals for multivariate impulse response functions at long horizons," *Journal of Applied Econometrics*, 21, 1135–1155.

Phillips, P. (1998): "Impulse response and forecast error variance asymptotics in nonstationary VARs," *Journal of Econometrics*, 83, 21–56.

Vlaar, P. (2004): "On the asymptotic distribution of impulse response functions with long-run restrictions," *Econometric Theory*, 20, 891–903.

Appendix: Tables and Figures

Estimated 'no-change' reduced form VAR for z_t^*			
Estimation sample: 1960Q1 - 2008Q2			
Log-lik=-938.37			
Coint. rank $r \leq$	Trace test	p-value	
0	375.13	0.000	
1	283.03	0.000	$\lambda_{\max}$(Comp.)=0.86
2	194.61	0.000	
3	128.34	0.000	
4	70.95	0.000	
5	37.32	0.000	
6	11.72	0.001	

Table TS1. **Estimated VAR for z_t^* on the period 1960Q1-2008Q2**, U.S. data. $\lambda_{\max}(\cdot)$ denotes the largest eigenvalue in modulus of the matrix in the argument. 'Comp' is the associated companion matrix.

Estimated 'one-break' ($T_B := 1984Q1$) reduced form VAR for z_t^*: first sub-period			
Estimation sample: 1960Q1 - 1983Q4			
Log-lik=-493.44			
Coint. rank $r \leq$	Trace test	p-value	
0	231.04	0.000	
1	170.34	0.000	$\lambda_{\max}$(Comp.)=0.87
2	119.37	0.000	
3	76.86	0.000	
4	43.33	0.000	
5	23.54	0.002	
6	9.17	0.002	

Table TS2. **Estimated VAR for z_t^* on the period 1960Q1-1983Q4**, U.S. data. $\lambda_{\max}(\cdot)$ denotes the largest eigenvalue in modulus of the matrix in the argument. 'Comp' is the associated companion matrix.

Estimated 'one-break' (T_B:=1984Q1) reduced form VAR for z_t^*: second sub-period			
Estimation sample: 1984Q1 - 2008Q2			
Log-lik=-105.75			
Coint. rank $r \leq$	Trace test	p-value	
0	209.06	0.000	
1	136.38	0.000	$\lambda_{\max}$(Comp.)=0.94
2	85.68	0.000	
3	56.55	0.005	
4	33.85	0.015	
5	11.97	0.160	
6	2.06	0.151	

Table TS3. **Estimated VAR for z_t^* on the period 1984Q1-2008Q2**, U.S. data. $\lambda_{\max}(\cdot)$ denotes the largest eigenvalue in modulus of the matrix in the argument. 'Comp' is the associated companion matrix.

Unconstrained recursive SVAR-WB													
			$\hat{C}$							$\hat{Q}$			
0.41	0	0	0	0	0	0	**-0.09**	0	0	0	0	0	0
0.39	**1.84**	0	0	0	0	0	**-0.49**	-0.07	0	0	0	0	0
0.37	**0.85**	**1.45**	0	0	0	0	-0.17	**-0.57**	**-0.47**	0	0	0	0
0.34	**0.18**	**0.27**	**0.50**	0	0	0	**-0.14**	**-0.12**	-0.07	**-0.22**	0	0	0
-0.07	0.04	0.03	-0.02	**0.24**	0	0	0.01	**-0.05**	-0.04	0.01	**-0.10**	0	0
0.05	**0.05**	0.02	**-0.06**	0.03	**0.20**	0	-0.04	**-0.06**	-0.01	**0.08**	-0.01	**-0.13**	0
-0.02	-0.02	0.02	**0.05**	**-0.03**	**-0.15**	**0.07**	0.01	0.03	0.00	**-0.04**	**0.05**	**0.11**	**-0.14**
log-lik = 1471.38													
Unconstrained non-recursive SVAR-WB													
			$\hat{C}$							$\hat{Q}$			
0.25	**0.09**	**0.22**	**0.20**	-0.03	**0.07**	-0.04	**-0.20**	0	0	0	0	0	0
-0.63	**1.66**	**0.47**	0.15	0.06	0.18	-0.07	0	**-3.24**	0	0	0	0	0
-0.95	**0.19**	**1.36**	0.23	0.13	0.12	-0.01	0	0	**-1.14**	0	0	0	0
-0.15	**0.10**	**0.26**	**0.56**	**0.24**	-0.02	-0.05	0	0	0	**-0.48**	0	0	0
0.00	0.01	0.03	**-0.13**	**0.21**	**-0.04**	0.04	0	0	0	0	**-0.28**	0	0
0.01	**0.02**	0.02	-0.01	**0.02**	**0.17**	**0.07**	0	0	0	0	0	**-0.15**	0
-0.02	-0.01	0.01	0.01	-0.01	**-0.09**	**-0.12**	0	0	0	0	0	0	**0.10**
log-lik = 1471.38													

Table TS4. **Estimated simultaneous parameters for the recursive and non-recursive SVAR-WB for z_t^*, $T_B = 1984Q1$.** Estimated values obtained via (Full Information) ML. Significant coefficients at 10% critical level are reported in bold.

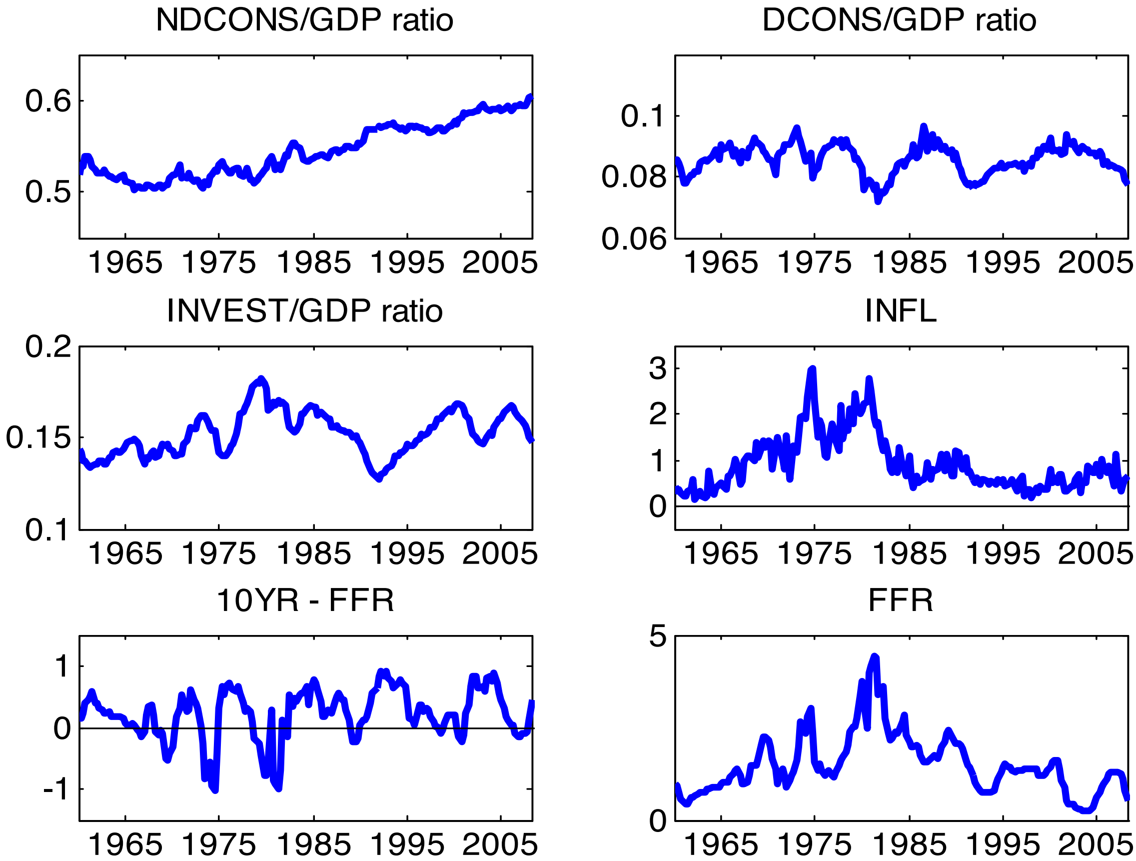

Figure TS1: **Time series** plot of some 'odds-ratios', the nominal interest rates spread and the federal funds rate over the 1960Q1-2008Q2 period. Ratios displayed by considering our variables in levels.

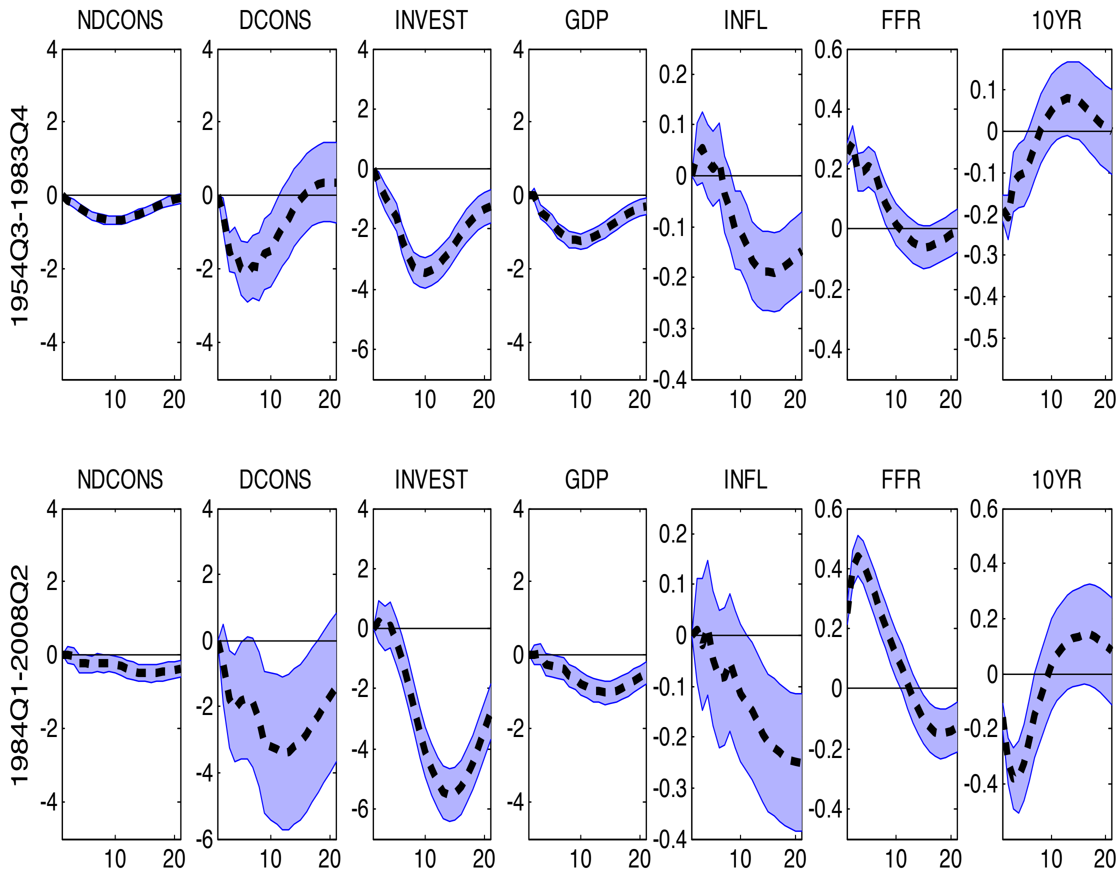

Figure TS2: **Cointegrated SVAR-WB, Impulse response functions: Great Inflation versus Great Moderation**. Dashed-black lines: Point estimates. Shaded-areas: 95-per cent confidence interval. Monetary policy shock identified with a Cholesky-identification scheme. Ordering of the variables in the VAR: Non durable consumption, durable consumption, investment, gdp, inflation, federal funds rate, 10 year-Treasury Bill rate.

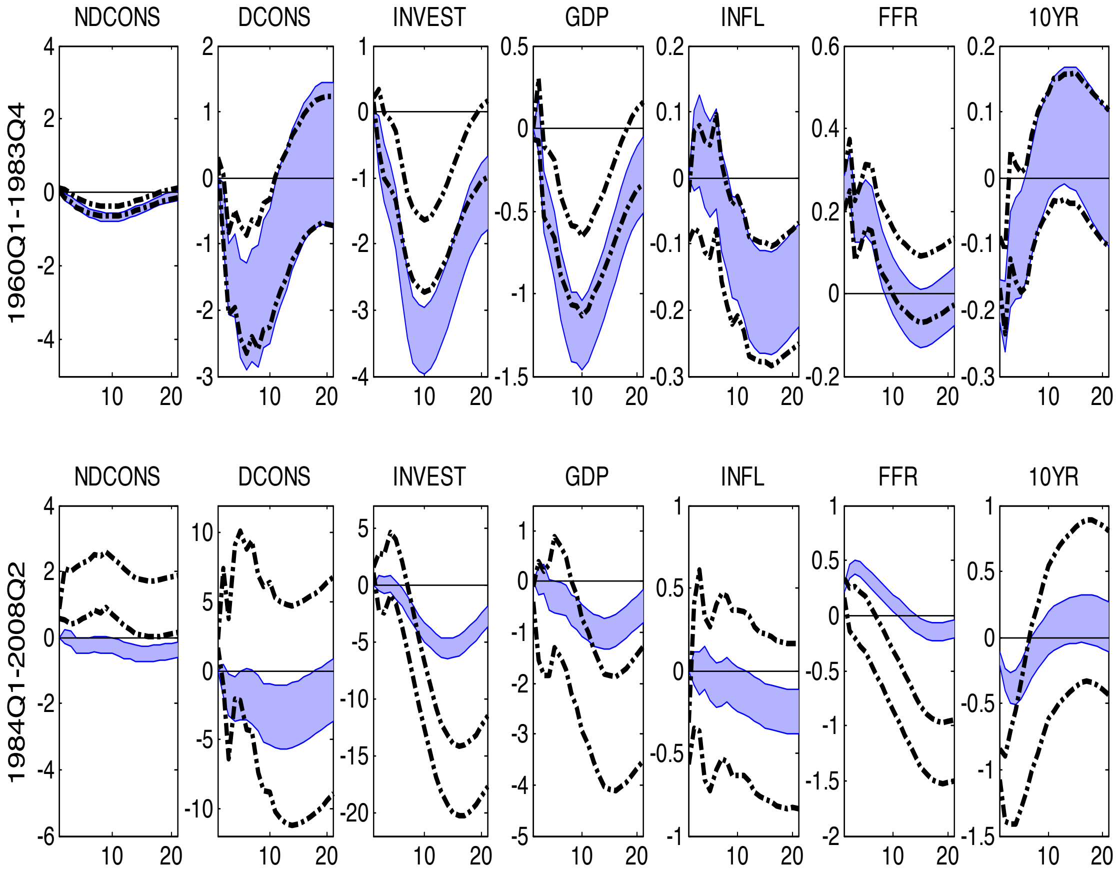

Figure TS3: **Cointegrated Recursive- vs- Non-Recursive SVARs-WB**. Shaded areas: Recursive-VAR-WB 95 per cent confidence intervals. Black dashed-dotted lines: Non-recursive VAR-WB 95 per cent confidence intervals. VAR estimated with equation-specific constants and four lags. Ordering of the variables in the VAR: Non durable consumption, durable consumption, investment, gpd, inflation, federal funds rate, 10 year-Treasury Bill rate.

Appendix: Further results

We present here some Figures omitted from the paper for the sake of brevity.

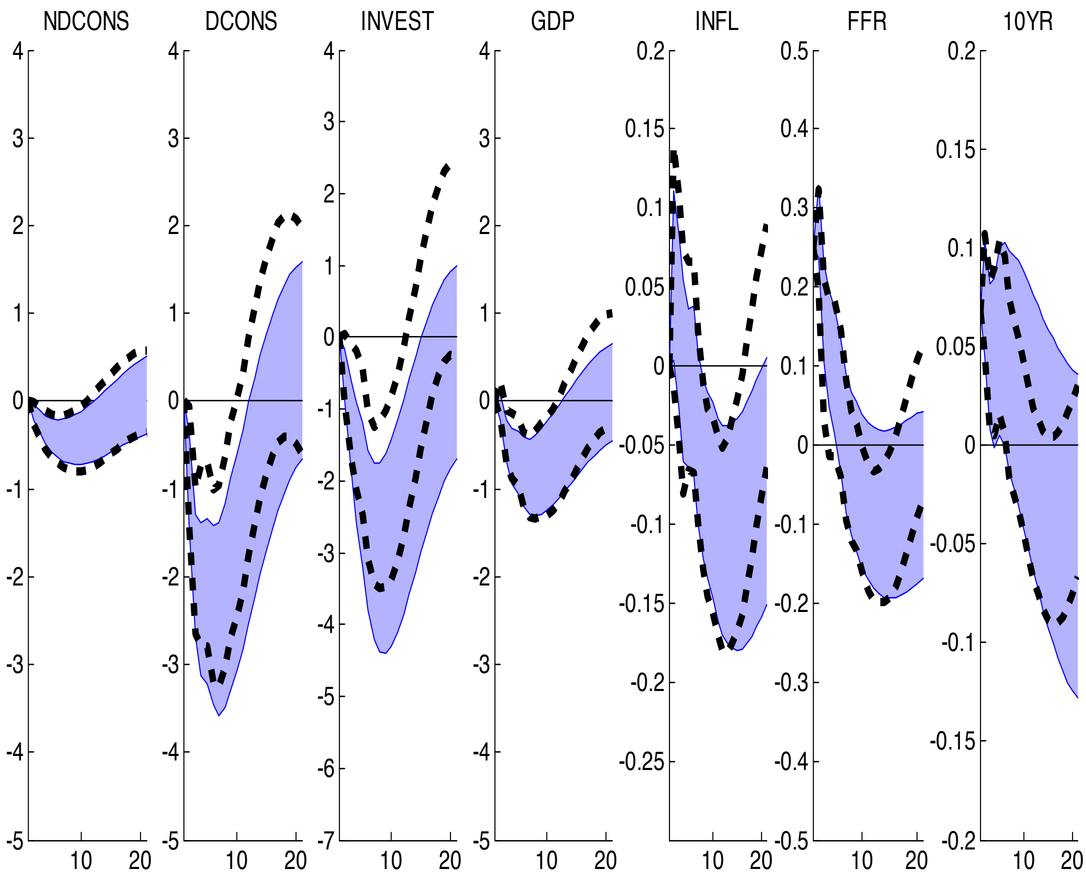

Figure TS4: **Cholesky-VARs with fixed-coefficients: Full- vs. pre-Great Moderation samples**. Shaded areas: 95 per cent confidence intervals, VAR estimated with 1954Q3-2008Q2 data. Black dashed lines: VAR estimated with 1954Q3-1983Q4 data. VAR estimated with equation-specific constants and four lags. Ordering of the variables in the VAR: Non durable consumption, durable consumption, investment, gpd, inflation, federal funds rate, 10 year-Treasury Bill rate.

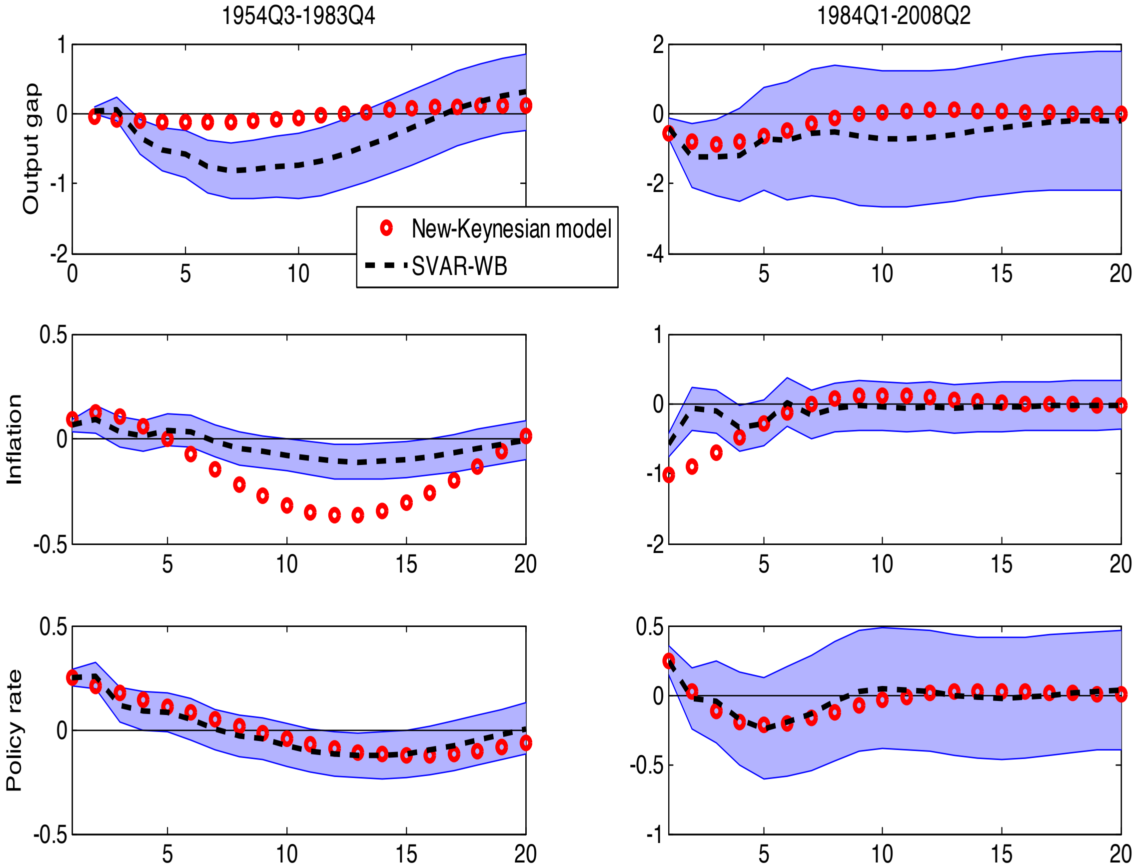

Figure TS5: **SVAR-WB vs. DSGE impulse responses to a monetary policy shock.** DSGE model calibrated via indirect inference by taking the non-recursive SVAR-WB as auxiliary model. Sample: 1960Q1-2008Q2, break in 1984Q1. Shaded areas: 95 percent confidence intervals associated to the impulse responses of our non-recursive SVAR-WB.