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MULTI-JUMPS

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Multi-jumps*

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Abstract

We provide clear-cut evidence for economically and statistically significant multivariate jumps (*multi-jumps*) occurring simultaneously in stock prices by using a novel nonparametric test based on smoothed estimators of integrated variances. Detecting multi-jumps in a panel of liquid stocks is more statistically powerful and economically informative than the detection of univariate jumps in the market index. On the contrary of index jumps, multi-jumps can indeed be associated with sudden and large increases of the variance risk-premium, and possess a statistically significant forecasting power for future volatility and correlations which implies a sizable deterioration in the diversification potential of asset allocation.

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1 Introduction

Figure 1 shows the intraday log-returns of four financial stocks (see Table 6) on December, 11^{th} 2007. In that day, a FOMC meeting was taking place, ending with the decision of lowering the target for federal funds rate of 25 basis points, due to "slowing economic growth reflecting the intensification of the housing correction" and "financial strains"[1]. The four financial companies collapsed all together in the afternoon, with a contemporaneous log-return of approximately -3% which is clearly visible in the figure. The figure also shows an evident increase, after the collapse, of both the stocks' volatility and their correlation. Moreover, the VIX index rose that day to 23.59 from 20.74 (+13.7%).

In the continuous time literature, a price movement of 3% (when the local volatility is less than 0.5%, thus of more than six standard deviations in volatility units) is typically modeled as a *jump*, that is a discontinuous variation of the price process. There are three possible routes to the detection of collective events like that in Figure 1 in the data: i) detection of a jump in a portfolio which includes the stocks (e.g., the equity index); ii) detection of jumps in individual stocks; iii) direct detection of the multivariate jump (or *multi-jump* as we call it in this paper). Surprisingly, a lot of effort has been devoted to *i*) and *ii*), both theoretically and empirically, but almost none to *iii*). In this paper, we introduce a formal test for the detection of multi-jumps, we argue that the third option is actually the most effective and we show that it reveals additional economic information which could not be revealed by the first two.

Multi-jumps are crucial events for asset allocation and risk management, as recognized by the financial literature. For example, Longin and Solnik (2001) show that correlations increase after a collective crash in the market, dampening the diversification potential of portfolio managers, and Das and Uppal (2004) use multivariate jumps to model systemic risk and its impact on portfolio choice. Bollerslev et al. (2008) use multi-jumps (common

[1]FOMC press release, December, 11^{th} 2007, available at
HTTP://WWW.FEDERALRESERVE.GOV/NEWSEVENTS/PRESS/MONETARY/20071211A.HTM

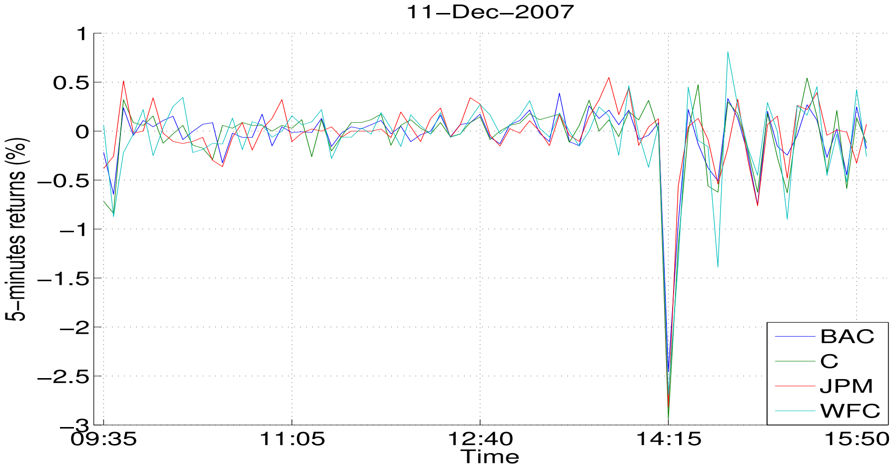

Figure 1: Intraday price changes (log-returns over 5 minutes) of Bank of America (BAC), Citigroup (C), JP Morgan (JPM) and Wells Fargo (WFC) on 11 December 2007. The four banking stocks collapse altogether around 14.15, while a FOMC meeting was taking place. We label this event a *multi-jump*. After the collapse, both volatility and correlation among stocks increases.

jumps, in their terminology) to explain jumps in the aggregated market index and discuss that, for asset allocation, it is more important to be able to detect jumps occurring simultaneously among a large number of assets, since the effect of co-jumps in a pair of assets is negligible in a huge portfolio; Gilder et al. (2014) also study the relation between common jumps and jumps in the market portfolio, and relate common jumps and news. If rare, dramatic multi-jumps can be interpreted as *systemic* events carrying market-wide information on economic fundamentals, their occurrence is also likely to affect the aggregate attitude to risk and thus have an impact on risk premia. For example, Bollerslev and Todorov (2011) empirically supported the view that risk compensation due to large jumps is quite large and time-varying, while Drechsler and Yaron (2011) and Drechsler (2013) highlight the importance of transient non-Gaussian shocks to fundamentals in explaining the magnitude of risk premia. In this paper, we complement this evidence by showing that multi-jumps can be associated with large increases in the variance risk premium.

Despite the statistical, economic and financial importance of multi-jumps, the financial econometrics literature is still missing a formal test to be used as an effective tool for their detection. A vast literature[2] concentrated on univariate jump tests. Progress on developing tests for common jumps in a pair of asset prices was started by Barndorff-Nielsen and Shephard (2003). They propose a way to separate out the continuous and co-jump parts of quadratic covariation of a pair of asset prices. Mancini and Gobbi (2012) developed an alternative threshold-based estimator of continuous covariation. Jacod and Todorov (2009) proposed two tests for co-jumps, their approach relying on functionals which depend, asymptotically, on co-jumps only. Finally, Bibinger and Winkelmann (2013) develop a co-jumps test using spectral methods. However, these methodologies apply to the case $N = 2$ only and their generalization to the case $N > 2$ is non-trivial. Bollerslev et al. (2008) propose a test for common jumps in a large panel ($N \to \infty$) which is based on the pairwise cross-product of intraday returns. In empirical work, detection of multivariate jumps is typically achieved with a simple co-exceedance rule (see, e.g., Gilder et al., 2014), according to which the multi-jump test is the intersection of univariate tests.

We fill this gap in the literature by introducing a novel testing procedure for multi-jumps which naturally applies to the case $N \geq 2$, with N finite. The proposed approach builds on the comparison of two types of suitably introduced smoothed power variations. High values of the test-statistics (which is asymptotically $\chi^2(N)$ under the null) signal the presence of a multi-jump among at least M stocks, with $M \leq N$. The smoothing procedure depends on a bandwidth which can be used to approximately select the desired M, with higher bandwidth values corresponding to higher M. We propose an automated bandwidth selection procedure which can be tuned to get the desired M.

Using simulations of realistic price processes which accommodate for the most relevant empirical features and which are implemented at the $5-minutes$ frequency (thus making the testing procedure virtually immune from distortions due to the presence of microstructure noise), we show that the proposed procedure i) has desirable size properties; ii) is

[2] Barndorff-Nielsen and Shephard (2006); Lee and Mykland (2008); Jiang and Oomen (2008); Aït-Sahalia and Jacod (2009) and Christensen et al. (2014), among others.

more powerful and better sized than the Jacod and Todorov (2009) test, which needs a much higher frequency (that is, many more data) to become effective; iii) is remarkably powerful in detecting multi-jumps and iv) strongly outperforms the co-exceedance rule in terms of power.

Results on real data are also encouraging. When applied to 16 liquid US stocks in the period 2003-2012, the test reveals the significant presence of multi-jumps. Not surprisingly, the multi-jumps occurrence rate becomes smaller with larger bandwidth, that is when we increase the minimal order M of stocks jumping jointly. However, multi-jumps with large M (high bandwidth) are rare but important events, which can be always associated with relevant market-wide economic news. This allows to interpret them as *systemic* events affecting the market on a whole.

Importantly, detection of multi-jumps in the stocks reveals additional information with respect to that conveyed by univariate jumps in the index. Indeed, while *theoretically* a multi-jump in the constituents should always correspond to a jump in the index, *empirically* this is not necessarily true since the multi-jumps could have different directions (even if empirical evidence reported in Section 5 documents that this is a quite unlikely event: multi-jumps have typically the same direction) or they could occur in a small subset of stocks, such that the jump in the index could be rather small and hard to detect. These considerations are confirmed by the data: roughly a half of detected multi-jumps in our sample cannot be associated with jumps in the index, unveiling information that univariate jumps could not reveal.

The additional information conveyed by multi-jumps is economically significant. We show that multi-jumps are strongly correlated with large increases in the variance risk premium, while univariate jumps on the index are not. This result is in line with recent theoretical literature, mentioned above, underscoring the impact of jumps in fundamentals on changes in aggregate risk aversion, and the empirical result in Todorov (2010), who makes use of a parametric model to show that price jumps are linked to the variation in the variance risk-premium. When multi-jumps are used, the association with changes in the variance risk-premium becomes clear-cut also in our fully non-parametric setting. This further indicates that multi-jumps are particularly suitable to test for systemic events, while questioning the usage of index jumps via univariate statistics to this purpose.

To further verify the potential empirical impact of multi-jumps, we show that they have substantial predictive power for volatility and correlations. Both stock correlations and volatilities are found to significantly increase after the occurrence of a multi-jump, thus confirming, on a formal statistical ground, the anedoctical evidence in Figure 1. In particular, the impact of multi-jumps on the correlation coefficient between a given pair of stocks is quite strong, especially when compared to the impact of idiosyncratic co-jumps between the same pair. These results have compelling implications for asset allocation. A risk-averse investor who allocates her wealth in a portfolio of stocks and a risk-free asset is harmed by the presence of multi-jumps in two ways. The first, which could be dealt with the model developed by Das and Uppal (2004), is the change in the optimal allocation strategy due to the presence of multi-jumps with respect to the case without multi-jumps. The second, which we quantify here, is the impact of multi-jump on the covariance matrix of the stocks, which implies an additional utility loss due to the increase in the portfolio variance and the worsening of the diversification potential. The latter effects would induce a less risky, that is less invested in stocks, optimal allocation strategy than that recommended by traditional models.

The remainder of the paper is organized as follows. Section 2 describes the continuous-time jump-diffusion model adopted in the paper. Section 3 explains the formal testing procedure and provides asymptotic results. Section 4 presents results on simulated price dynamics. Section 5 applies the test to real data and contains the empirical results and their implications for asset allocation. Section 6 concludes.

2 Model

Denote the log-prices of an N-dimensional vector of assets by $X = (X^{(i)})_{i=1,\dots,N}$. We assume that stock prices evolve continuously on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F})_{t\in[0,T]}, \mathcal{P})$ satisfying the usual conditions, and we assume the following dynamics for X, accommodating for continuous (through Brownian motion) and discontinuous (through jumps) shocks.

Assumption 1. *X is an N-dimensional Ito semimartingale following:*

$$dX_t = a_t dt + \mathbf{\Sigma}_t dW_t + dJ_t$$

where a_t (in $\mathbb{R}^N$) and $\mathbf{\Sigma}_t$ (in $\mathbb{R}^{N\times M}$) are càdlàg adapted processes, W_t is standard multivariate Brownian motion in $\mathbb{R}^M$ and J_t is a finite activity jump process of the form $J_t^{(i)} = \sum_{k=1}^{N_t^{(i)}} \gamma^{(i)}_{\tau_k^{(i)}}$, $i = 1,\dots,N$, and $N_t^{(i)}$ is a non-explosive counting process. Moreover, we assume that the jump sizes are such that, $\forall k = 1,\dots,$ we have $\mathcal{P}\left(\gamma^{(i)}_{\tau_k^{(i)}} = 0\right) = 0$, $i = 1,\dots,N$.

The model, which is very general and encompasses virtually all parametric models typically used in financial applications, allows each component of X to include idiosyncratic jumps (that occur only for a single stock) as well as common jumps among stocks. Define the process

$$\Delta X_t = X_t - X_{t^-}, \tag{1}$$

and, as an example, consider the case $N = 3$. The common jumps between $X^{(1)}$ and $X^{(2)}$ satisfy

$$\Delta X_t^{(1)} \Delta X_t^{(2)} = \gamma_t^{1(2)} \gamma_t^{2(1)} \Delta N_t^{12} + \gamma_t^{1(23)} \gamma_t^{2(13)} \Delta N_t^{123},$$

where N^{12} and N^{123} are independent counting processes, while common jumps among all the three processes[3] satisfy

$$\Delta X_t^{(1)} \Delta X_t^{(2)} \Delta X_t^{(3)} = \gamma_t^{1(23)} \gamma_t^{2(13)} \gamma_t^{3(21)} \Delta N_t^{123}.$$

The inference procedure is designed to test the null

$$\sum_{0 \le t \le T} \Delta X_t^{(1)} \Delta X_t^{(2)} \Delta X_t^{(3)} = 0$$

against the alternative

$$\sum_{0 \le t \le T} \Delta X_t^{(1)} \Delta X_t^{(2)} \Delta X_t^{(3)} \neq 0.$$

Note that the presence of a multi-jump among three assets implies the presence of co-jumps between each pair of them. However, the presence of co-jumps between each pair of assets does not necessarily imply the presence of a multi-jump among them.

We do not explicitly include in the model market microstructure contaminations, since the proposed method is thought to be applied at moderately low frequencies (e.g., five minutes) where the impact of microstructure noise should be negligible. The theory could however be easily extended to include market microstructure noise by adapting our return smoothing technique to preaveraged estimators robust to both jumps and market microstructure noise, as in Podolskij and Vetter (2009) and Hautsch and Podolskij (2013).

The theory could also be extended for infinite activity jumps (see, e.g., Aït-Sahalia and Jacod, 2012 and the references therein), since the test procedure developed below is based on smoothed estimators of integrated variances which have been shown to be consistent even in the presence of this kind of shock, see Mancini (2009) and Mancini and Gobbi (2012).

[3]To underscore the methodological contribution of this paper, we use the word *co-jump* when the common jump is between two assets, and *multi-jumps* when the common jump is among three or more assets.

3 Multi-jumps inference

Assume to record X in the interval $[0,T]$, with T fixed, in the form of $n+1$ equally spaced observations[4] and denote by $\Delta = T/n$. Define the evenly sampled logarithmic returns as

$$\Delta_j X = X_{j\Delta} - X_{(j-1)\Delta}, \qquad j = 1, \ldots, n. \tag{2}$$

In order to formulate the statistical properties of the test, define the following sets:

$$\begin{aligned} \Omega_T^{MJ,N} &= \{\omega \in \Omega \mid \text{the process} \prod_{j=1}^{N} \left(\Delta X^{(j)}\right)_t \text{ is not identically } 0\} \\ \overline{\Omega}_T^N &= \Omega \setminus \Omega_T^{MJ,N}. \end{aligned}$$

The set $\Omega_T^{MJ,N}$ contains trajectories with common multi-jumps among *all* N assets in $[0,T]$. The complementary set $\overline{\Omega}_T^N$ contains trajectories without multi-jumps in N stocks; it could however contain jumps and multi-jumps up to $N-1$ stocks. Testing for multi-jumps is equivalent to testing the following:

$$\mathcal{H}_0 : \ \left((X_t(\omega))_{t\in[0,T]} \in \overline{\Omega}_T^N\right) \text{ vs. } \mathcal{H}_1 : \ \left((X_t(\omega))_{t\in[0,T]} \in \Omega_T^{MJ,N}\right). \tag{3}$$

Inference is based on the definition of two newly defined integrated variance estimators which constitute a generalization, particularly suitable to our application, of the truncated realized variance estimator of Mancini (2009). To this purpose we need a definition of a kernel and a bandwidth.

Assumption 2. *A kernel is a function $K(\cdot) : \mathbb{R} \to \mathbb{R}$, which is differentiable with bounded first derivative almost everywhere in $\mathbb{R}$, and such that $K(0) = 1$, $0 \le K(\cdot) \le 1$ and $\lim_{x\to\infty} K(|x|) = 0$. The bandwidth process is a sequence $H_{t,n}$ of processes in $\mathbb{R}^N$*

[4] This requirement can be easily generalized to non-equally spaced observations, if we set $\bar{\Delta} = \max_{i=1,\ldots,n}(t_i - t_{i-1})$, where t_i are observation times, and require $\bar{\Delta} \to 0$, see Remark i) of Theorem 4 in Mancini (2009).

which can be written as $H_{t,n} = h_n \xi_{t,n}$*, where* h_n *is a sequence such that*

$$\lim_{n\to\infty} h_n = 0, \qquad\qquad \lim_{n\to\infty} \frac{1}{h_n}\sqrt{\frac{\log n}{n}} = 0, \tag{4}$$

and $\xi_{t,n}$ *is a vector of* N *positive adapted stochastic process on* $[0,T]$ *which are all a.s. bounded with a strictly positive lower bound.*

The bandwidth is written in the form $h_n\xi_{t,n}$ to allow for data-dependent and time-varying bandwidth. Indeed, in our application $\xi_{t,n}$ is the local variance estimated by the observations themselves, see Eq. (31). We call h_n the bandwidth parameter, and provide an automated criterion for its selection in Section B.1 in the Appendix.

We now define two novel jump-robust integrated variance estimators, which are both called *Smoothed Realized Variance.* The first one takes the form

$$\mathsf{SRV}(X^{(i)}) := \sum_{j=1}^{n} \left|\Delta_j X^{(i)}\right|^2 \cdot K\left(\frac{\Delta_j X^{(i)}}{H^{(i)}_{j\Delta,n}}\right), \tag{5}$$

where $X^{(i)}, H^{(i)}$ are the i-th components of the vectors X, H and $K(\cdot)$ and $H_{t,n}$ are the kernel and bandwidth defined in Assumption 2. This estimator coincides with the estimator in Mancini (2009) when $K(x) = I_{\{|x|\leq H_{t,n}\}}$, but allows for a different choice of the kernel. The intuition is however similar to that of Mancini (2009): "smoothed" squared returns $\left|\Delta_j X^{(i)}\right|^2 \cdot K\left(\Delta_j X^{(i)}/H^{(i)}_{j\Delta,n}\right)$ are close to squared returns $\left|\Delta_j X^{(i)}\right|^2$ when they are small; smoothed squared returns are instead small when returns are large, where the extent of "largeness" is gauged by the bandwidth $H_{j\Delta,n}$. Asymptotically, this procedure annihilates the jumps. The estimator of Mancini (2009) is the most draconian in this respect, since using the indicator function implies that smoothed returns are zero when returns are larger than $H_{j\Delta,n}$ (dubbed *threshold* in Mancini's terminology). The advantage of replacing the indicator function with a smooth kernel is that it provides an estimator which depends smoothly on the bandwidth: This stabilizes the procedure in small samples (by making it less prone to type I and II errors due to erroneous bandwidth selection) and also eases bandwidth selection.

The following theorem (proof in Appendix A) shows that $\mathsf{SRV}(X^{(i)})$ in Eq. (5) is a jump-robust consistent estimator of integrated variance.

Theorem 3.1. *Let the process X satisfy Assumption 1, and the kernel and bandwidth satisfy Assumption 2. Then, as $n \to \infty$ we have*

$$\mathsf{SRV}(X^{(i)}) \xrightarrow{p} \int_0^T (\sigma^{(i)})_u^2 du, \tag{6}$$

where $\sigma^{(i)}$ is the volatility of $X_t^{(i)}$.

The following remark introduce a correction to improve the estimator performance in small samples.

Remark 1. ***(Small Sample Correction)*** *In order to improve the finite samples unbiasedness of the estimator defined in Eq. (5), it is advisable to normalize it as follows:*

$$\frac{\sum_{j=1}^{n} \left|\Delta_j X^{(i)}\right|^2 \cdot K\left(\frac{\Delta_j X^{(i)}}{H_{j\Delta,n}^{(i)}}\right)}{\Delta \sum_{j=1}^{n} K\left(\frac{\Delta_j X^{(i)}}{H_{j\Delta,n}^{(i)}}\right)} \xrightarrow{p} \int_0^T (\sigma^{(i)})_u^2 du,$$

since $\Delta \sum_{j=1}^n K\left(\frac{\Delta_j X^{(i)}}{H_{j\Delta,n}^{(i)}}\right) \xrightarrow{p} 1$.

The second estimator takes the form:

$$\widetilde{\mathsf{SRV}}^N(X^{(i)}) := \sum_{j=1}^{n} \left|\Delta_j X^{(i)}\right|^2 \cdot \left(K\left(\frac{\Delta_j X^{(i)}}{H_{j\Delta,n}^{(i)}}\right) + \prod_{k=1}^{N}\left(1 - K\left(\frac{\Delta_j X^{(k)}}{H_{j\Delta,n}^{(k)}}\right)\right)\right). \tag{7}$$

Returns in Eq. (7) are smoothed as in Eq. (5), but they are also kept similar to the original returns if *all* multivariate returns are big. Thus, even if, when $n \to \infty$, both smoothing procedures are meant to annihilate jumps, the smoothing in Eq. (7) will let multi-jump survive. This intuition is formalized in the following theorem (proof in

Appendix A), which represents the base for inference and testing.

Theorem 3.2. *Let the process X satisfy Assumption 1, and the kernel and bandwidth satisfy Assumption 2. Then, as $n \to \infty$,*

$$\widetilde{\mathsf{SRV}}^N(X^{(i)}) \xrightarrow{p} \begin{cases} \int_0^T (\sigma^{(i)})_u^2 du + \sum_{\Delta X_t^{(1)} \ldots \Delta X_t^{(N)} \neq 0} \left(\Delta X_t^{(i)}\right)^2 & on\ \Omega_T^{MJ,N} \\ \int_0^T (\sigma^{(i)})_u^2 du, & on\ \overline{\Omega}_T^N \end{cases} ; \tag{8}$$

where $\sigma^{(i)}$ is the volatility of $X_t^{(i)}$.

Theorems 3.1 and 3.2 introduce a natural estimator for the multi-jumps on each series. By the light of Remark 1 the jump size of stock i corresponding to a multi-jump among all stocks is naturally derived in the following remark.

Remark 2. ***(Multi-jump Size Estimation)***

$$\frac{\widetilde{\mathsf{SRV}}^N(X^{(i)}) - \mathsf{SRV}(X^{(i)})}{\Delta \sum_{j=1}^{n} K\left(\frac{\Delta_j X^{(i)}}{H_{j\Delta,n}^{(i)}}\right)} \xrightarrow{p} \begin{cases} \sum_{\Delta X_t^{(1)} \ldots \Delta X_t^{(N)} \neq 0} \left(\Delta X_t^{(i)}\right)^2 & on\ \Omega_T^{MJ,N} \\ 0, & on\ \overline{\Omega}_T^N \end{cases} . \tag{9}$$

In order to define the test statistics, we follow Podolskij and Ziggel (2010) and define a iid $N \times n$ matrix of draws $(\eta_j^i)_{1 \leq i \leq N, 1 \leq j \leq n}$, defined on the canonical extension $(\Omega', \mathcal{F}', (\mathcal{F}')_{t \in [0,T]}, \mathcal{P}')$ of the original probability space $(\Omega, \mathcal{F}, (\mathcal{F})_{t \in [0,T]}, \mathcal{P})$ and independent from $\mathcal{F}$. We assume that $\mathbf{E}\left[\eta_j^i\right] = 1$ and $\mathbf{Var}\left[\eta_j^i\right] = V_\eta < \infty$. Define:

$$\widetilde{\mathsf{SV}}(X^{(i)}) := \sum_{j=1}^{n} \left|\Delta_j X^{(i)}\right|^2 \cdot K\left(\frac{\Delta_j X^{(i)}}{H_{j\Delta,n}}\right) \cdot \eta_j^i, \quad i = 1, \ldots, N, \tag{10}$$

and

$$\mathsf{SQ}(X^{(i)}) := \sum_{j=1}^{n} \left|\Delta_j X^{(i)}\right|^4 \cdot K^2\left(\frac{\Delta_j X^{(i)}}{H_{j\Delta,n}}\right), \quad i = 1, \ldots, N. \tag{11}$$

The test statistics is then defined as

$$\mathsf{S}_{\mathrm{n,N}} := \frac{1}{V_\eta} \sum_{i=1}^{N} \frac{\left(\widetilde{\mathsf{SV}}(X^{(i)}) - \widetilde{\mathsf{SRV}}^{N}(X^{(i)})\right)^2}{\mathsf{SQ}(X^{(i)})} \tag{12}$$

and its asymptotics is described in the following Theorem (proof in Appendix A).

Theorem 3.3. *Under Assumption 1 and 2, if $(\eta^i_j)_{1\le i\le N, 1\le j\le n}$ are pairwise independent, as $n \to \infty$, it holds:*

$$\begin{cases} \mathsf{S}_{\mathrm{n,N}} \xrightarrow{d} \chi^2(N), & \text{on } \overline{\Omega}^N_T \\ \mathsf{S}_{\mathrm{n,N}} \xrightarrow{p} +\infty & \text{on } \Omega^{MJ,N}_T \end{cases}; \tag{13}$$

where $\chi^2(N)$ denotes the χ-square distribution with N degrees of freedom.

Theorem 3.3 implies that the statistic $\mathsf{S}_{\mathrm{n,N}}$ can be used for testing for the presence of multi-jumps. Under $\mathcal{H}_0$, the value of $\mathsf{S}_{\mathrm{n,N}}$ will be distributed as a χ^2 with N degrees of freedom. Under $\mathcal{H}_1$, that is in the presence of multi-jumps, it will diverge as the number of observations n increases.

Notice that the test defined in Eq. (12) is of computational order N, in the sense that the computational burden increases linearly with N. In particular, the test does not require the estimation of the covariance between stocks (which would increase the computational burden as N^2).

Following the suggestions of Podolskij and Ziggel (2010) for the univariate jump test, the random variables η^i_j are allowed to take the values $\{1+\tau, 1-\tau\}$ with equal probability, so that $V_\eta = \tau^2$. In both Monte Carlo and empirical exercises we use $\tau = 0.05$.

4 Simulation study

In order to simulate the dynamics of realistic prices, we simulate a multivariate model that accommodates correlated prices, stochastic volatility, leverage effect, intraday effects, idiosyncratic jumps and multi-jumps. All these components have been shown to be present in the dynamics of high-frequency financial prices, and we use realistic parameter values. We do not include market microstructure noise (see the discussion above). The dynamics of the continuous parts of each component are given by the same stochastic differential equations driven by correlated Brownian motions:

$$\begin{aligned} dX_t^{(i)} &= \mu\, dt + \gamma_t \sigma_t^{(i)} dW_t^{(i)} + dJ_t^{(i)} \\ d\log(\sigma_t^{(i)})^2 &= (\alpha - \beta \log(\sigma_t^{(i)})^2)\, dt + \eta d\widetilde{W}_t^{(i)}, \end{aligned} \tag{14}$$

where $i = 1, \ldots, 16$, $W^{(i)}$ and $\widetilde{W}^{(i)}$ are standard Brownian motions with corr $\left(dW^{(i)}, d\widetilde{W}^{(i)}\right) = \widetilde{\rho}$; $\sigma_t^{(i)}$ are stochastic volatility factors and γ_t represent intraday effects. The Brownian motions $W^{(i)}$ driving the price dynamics can be correlated, as specified below. The pure jump parts of $X^{(i)}$ are different compound Poisson processes.

The parameters of the model are taken to be as estimated by Andersen et al. (2002) on S&P500 prices: $\mu = 0.0304$, $\alpha = -0.012$, $\beta = 0.0145$, $\eta = 0.1153$, $\widetilde{\rho} = -0.6127$; where the parameters are expressed in daily units and returns are in percentage. The intraday effects are given by:

$$\gamma_t = \frac{1}{0.1033}(0.1271 \cdot t^2 - 0.1260 \cdot t + 0.1239),$$

as estimated on S&P500 intraday returns. In our simulations, we always have $t \in [0, 1]$, with initial values for prices and volatility taken from the last simulated day.

The model (14) is discretized with the Euler scheme, using discretization step of $\Delta = \frac{1}{80}$ which roughly corresponds to 5-minutes returns for a trading day of 6.5 hours ($n = 80$). We generate samples of 1,000 days with different specifications for the jump processes $dJ_t^{(i)}$.

4.1 Two assets

We start with the case $N = 2$. The two assets are correlated, with

$$\mathrm{corr}\left(dW^{(1)}, dW^{(2)}\right) = \rho$$

with $\rho = 0.5$.

We generate different samples, subdivided into five categories. Jumps, when present, come in the form of *big jumps*, with a size of $8\sqrt{1/80}$, or *small jumps*, with a size of $4\sqrt{1/80}$ (the average volatility in simulations being around 1). In the first category (*continuous processes*), there are no generated jumps. In the second category (*one big jump*) there are no co-jumps, but we generate a single big jump in the first component $X^{(1)}$, located randomly within the day. In the third category (*big idiosyncratic jumps*), both $X^{(1)}$ and $X^{(2)}$ have big jumps, but they are idiosyncratic in the sense that they never occur in the same time interval. The first three categories thus fall under the null. In the fourth and firth categories (*big co-jumps* and *small co-jumps*), which are the alternatives, $X^{(1)}$ and $X^{(2)}$ contain one big-big and small-small co-jump respectively.

In this set of simulations, the $\mathsf{S}_{\mathrm{n,N}}$ statistics are implemented using different bandwidth parameters h_n (see Section B.1 in the Appendix), namely $h_n = 5$ and $h_n = 6.5$ (see Figure 9). For comparison, we also implement two co-jump tests proposed by Jacod and Todorov (2009): Φ_n^j, which is used to test the null hypothesis of the presence of co-jumps, and Φ_n^d, which is used to test the null of absence of co-jumps. The tests are described in Section B.2 in the Appendix.

Table 1 analyzes the size properties for the three considered tests. Notice that the size of $\mathsf{S}_{\mathrm{n,N}}$ and Φ_d^n should be computed when co-jumps are absent, while the size of Φ_n^j should be evaluated when co-jumps are present. To underscore the dependence of the $\mathsf{S}_{\mathrm{n,N}}$ on the bandwidth, we denote it by $\mathsf{S}_{n,N}(h_n)$. In the absence of jumps, both $\mathsf{S}_{80,2}(5)$ and $\mathsf{S}_{80,2}(6.5)$ have practically undistorted size at all relevant critical levels. However, when jumps are added, size distortions appear, more strongly with lower h_n, and the distortions are larger in the presence of two idiosyncratic jumps. In the case with $h_n = 6.5$, however, size distortions are reasonable: The simulated distribution of $\mathsf{S}_{80,2}(6.5)$, in the most challenging case with big idiosyncratic jumps, is compared to its asymptotic limit in Figure 2, top panel. On the other hand, the size of both Φ_n^j and Φ_n^d is quite distorted. This is not surprising, since also in Jacod and Todorov (2009) these tests have been shown to need a much larger value of n to work properly.

Table 2 analyzes the power of the three tests. All the tests perform equally well when co-jumps are big. When co-jumps are small they are obviously more difficult to detect. The power of the $\mathsf{S}_{80,2}(6.5)$ increases with smaller h_n, paralleling the corresponding larger size distortions.

These results suggest that bandwidth selection can be used to trade-off size and power. Higher bandwidth correspond to more reliable size but less power. In the case we are testing for multi-jumps with larger N, this can be particularly useful, as we discuss below.

4.2 Four assets

We next proceed to simulate a system with $N = 4$. Continuous dynamics of all the components is simulated as in equation (14), without jumps. The Brownian motions, driving the first pair of components, are positively correlated: $corr(W^{(1)}, W^{(2)}) = 0.5$. The second pair of components are negatively correlated: $corr(W^{(3)}, W^{(4)}) = -0.5$. Correlations between the other pairs is null: $corr(W^{(1)}, W^{(3)}) = corr(W^{(1)}, W^{(4)}) = 0$.

In this set of simulations, we consider five cases:

1. *Case 1:* all components of X are continuous.

2. *Case 2:* all components of X contain a single big jump, but the four jumps occur in different time intervals.

Table 1: Compares the **size** of competing tests in the case $N = 2$. Different processes for the null are considered. The sampling frequency is $n = 80$, corresponding to 5-minute intraday observations.

confidence interval →	90%	95%	99%	99.9%
continuous processes				
$\mathsf{S}_{80,2}(6.5)$	10.2	4.6	0.9	0.0
$\mathsf{S}_{80,2}(5)$	14.1	8.1	1.9	0.2
one big jump				
$\mathsf{S}_{80,2}(6.5)$	17.0	9.8	3.7	1.7
$\mathsf{S}_{80,2}(5)$	31.8	22.8	13.8	9.0
big idiosyncratic jumps				
$\mathsf{S}_{80,2}(6.5)$	15.7	9.2	5.3	2.1
$\mathsf{S}_{80,2}(5)$	44.8	37.3	28.2	19.9
Φ^d	64.3	49.1	32.7	19.3
big co-jump				
Φ^j	14.7	12.5	10.8	10.2
small co-jump				
Φ^j	42.2	41.2	41.0	41.0

3. *Case 3:* there is a single multi-jump in the first triplet of the components of X and jumps are big.

4. *Case 4:* there is a multi-jump among the four processes and all jumps are small.

5. *Case 5:* there is a multi-jump among the four processes and all jumps are big.

Thus, Cases 1,2,3 represent the null and Cases 4,5 represent the alternative. The $\mathsf{S}_{\mathrm{n,N}}$-tests are implemented with bandwidth parameters $h_n = 2.5, 3.5, 5.5$ (see Figure 9).

Table 2: Compares the **power** of competing tests in the case $N = 2$. Different processes for the alternative are considerers. The sampling frequency is $n = 80$, corresponding to 5-minute intraday observations.

confidence interval →	90%	95%	99%	99.9%
	big idiosyncratic jumps			
Φ^j	100.0	100.0	100.0	100.0
	big co-jump			
$\mathsf{S}_{80,2}(6.5)$	95.5	94.7	94.1	92.7
$\mathsf{S}_{80,2}(5)$	96.8	95.8	95.5	95.4
Φ^d	99.5	98.9	98.7	98.4
	small co-jump			
$\mathsf{S}_{80,2}(6.5)$	32.2	25.7	13.8	7.3
$\mathsf{S}_{80,2}(5)$	57.5	51.7	42.0	33.7
Φ^d	98.4	96.7	93.7	89.0

Table 3 shows the results for all Cases. With continuous processes and idiosyncratic jumps, the size distortions (increasing with smaller h_n, as before) are negligible. They are instead quite strong against a multi-jump among $M = 3$ stocks with $h_n = 2.5, 3.5$. The automated bandwidth selection indicates a value of $h_n = 5.5$ in this case, and this indeed provides a reasonable size (the distribution of the test in this case is compared to the asymptotic limit in the middle panel of Figure 2). Power is practically unaffected by the bandwidth if the multi-jump is composed of big jumps; while it decreases with higher bandwidth if multi-jumps are small.

Again, the bandwidth parameter can thus be used to trade-off size and power. Reasonable size can always be achieved, but at the obvious cost of less power. This opens an interesting possibility for the econometrician. The null (no multi-jumps across N assets) and the alternative described in Section 3 refer to asymptotic situations. In practice, we

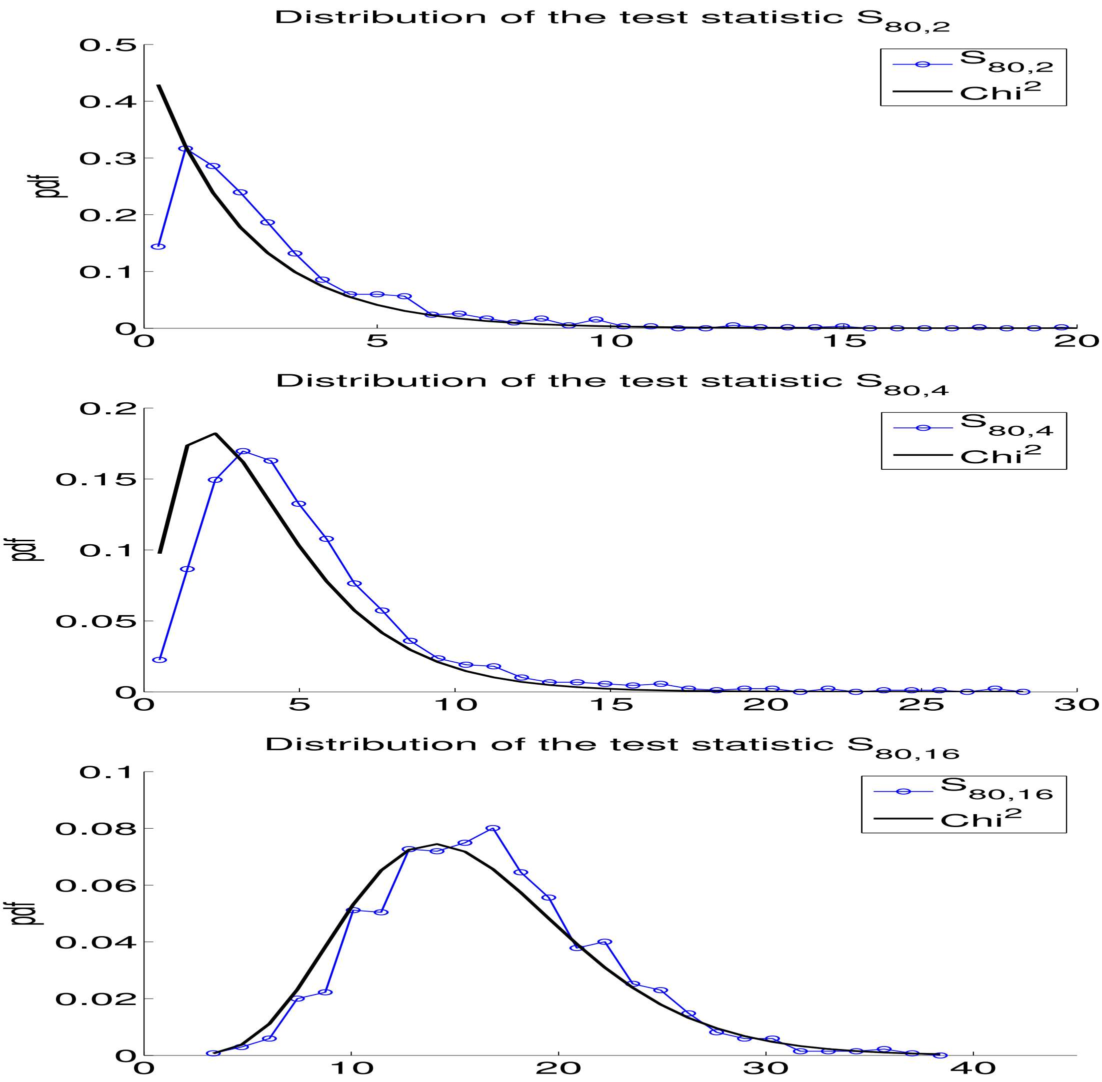

Figure 2: Shows the simulated distribution of the proposed multi-jump tests together with its asymptotic distribution under the null (which is$\chi^2(N)$); the tests are $\mathsf{S}_{80,2}(6.5)$ for $N = 2$ in the case with big idiosyncratic jumps (top panel), $\mathsf{S}_{80,4}(5.5)$ for $N = 4$ in Case 3 (center panel) and $\mathsf{S}_{80,16}(4.5)$ for $N = 16$ in Case 3 (bottom panel).

could consider several alternatives when computing the test on N assets: multi-jump in N assets, in $N-1$ assets, in $N-2$ assets and so on. The bandwidth parameter can be used to disentangle these cases. For example, looking at Table 3, we see that with $h_n = 5.5$ we would disentangle a multi-jump in 4 stocks by a multi-jump in 3 stocks; with $h_n = 3.5$ we would also detect multi-jumps in three stocks, and with $h_n = 2.5$ we would also detect multi-jumps in two stocks. In empirical work, the choice of h_n could depend on the specific research objectives.

Table 3: Shows the **size** and **power** of the proposed test in the case $N = 4$. Different processes under the null and the alternative are considered. The sampling frequency $n = 80$ corresponds to 5-minute intraday observations.

confidence interval →	90%	95%	99%	99.9%
Case 1: continuous processes				
$\mathsf{S}_{80,4}(2.5)$	12.3	6.2	1.6	0.1
$\mathsf{S}_{80,4}(3.5)$	9.9	4.1	0.6	0.1
$\mathsf{S}_{80,4}(5.5)$	11.1	5.1	0.7	0.0
Case 2: big idiosyncratic jumps				
$\mathsf{S}_{80,4}(2.5)$	14.0	7.7	3.1	1.4
$\mathsf{S}_{80,4}(3.5)$	8.6	4.3	0.4	0.0
$\mathsf{S}_{80,4}(5.5)$	5.7	1.8	0.1	0.0
Case 3: multi-jump in $N = 3$ stocks				
$\mathsf{S}_{80,4}(2.5)$	71.8	68.6	63.6	61.3
$\mathsf{S}_{80,4}(3.5)$	55.8	50.9	45.1	41.1
$\mathsf{S}_{80,4}(5.5)$	14.0	9.3	3.6	2.1
Case 4: small multi-jump				
$\mathsf{S}_{80,4}(2.5)$	94.2	92.9	90.4	87.2
$\mathsf{S}_{80,4}(3.5)$	53.2	46.9	37.5	30.4
$\mathsf{S}_{80,4}(5.5)$	10.2	4.8	0.9	0.2
Case 5: big multi-jump				
$\mathsf{S}_{80,4}(2.5)$	98.7	98.7	98.6	98.6
$\mathsf{S}_{80,4}(3.5)$	98.8	98.7	98.6	98.6
$\mathsf{S}_{80,4}(5.5)$	98.2	98.0	97.5	96.7

4.3 Many assets

We finally simulate a large number of stocks, that is $N = 16$ as in the empirical application below. Continuous parts of all components follow equation (14). The Brownian motions driving the system are correlated with the average daily correlation matrix estimated on the stock returns used in the empirical application in Section 5. We now consider the following settings:

1. *Case 1:* all components of X are continuous.
2. *Case 2:* there is a multi-jump in $M = 4$ components of X with big jumps.
3. *Case 3:* there is a multi-jump in $M = 15$ components of X with big jumps.
4. *Case 4:* there is a multi-jump in $M = N = 16$ components of X and all jumps are small.
5. *Case 5:* there is a multi-jump in $M = N = 16$ components of X and all jumps are big.

In this setting, Cases 1,2,3 represent the null and Cases 4,5 two possible alternatives. Here we implement $\mathsf{S}_{\mathrm{n,N}}$-test with $h_n = 1, 2, 4.5$ (see Figure 9). The bandwidth $h_n = 4.5$ is selected by our automated bandwidth selection method under a null with $M = 15$ multi-jumps.

Table 4 shows the results. For all the considered bandwidth, size is reasonable in the case of continuous processes and moderate multi-jump ($M = 4$), but becomes distorted in the case $M = 15$ unless we use the automatically selected value $h_n = 4.5$. This result is in line with the simulation evidence presented above; size is more reliable with higher bandwidth, while power is instead higher with lower bandwidth. The econometrician should then choose $h_n = 4.5$ if he is interested in multi-jumps among 16 jumps *only*. This will somewhat sacrifice power. If instead one is interested in multi-jumps across fewer stocks, a smaller h_n can be used; for example, with $h_n = 2$ we are still robust against moderate multi-jumps across $M = 4$ stocks, but we would detect most of the multi-jumps with $M = 15$ too (and, with decreasing power, with $M = 14, 13, \ldots$) also when their magnitude is modest. This feature of the test is actually very appealing, especially with a very large N, and indeed in our empirical application we take advantage of it by using $h_n = 2$.

Table 4: Shows the **size** and **power** of the proposed test in the case $N = 16$. Different processes under the null and the alternative are considered. The sampling frequency $n = 80$ corresponds to 5-minute intraday observations.

confidence interval →	90%	95%	99%	99.9%
Case 1: continuous processes				
$\mathsf{S}_{80,16}(4.5)$	9.1	3.6	0.2	0.0
$\mathsf{S}_{80,16}(2)$	8.2	3.8	1.2	0.2
$\mathsf{S}_{80,16}(1)$	11.0	5.6	2.4	1.1
Case 2: multi-jump in $N = 4$ assets				
$\mathsf{S}_{80,16}(4.5)$	7.5	3.8	0.4	0.0
$\mathsf{S}_{80,16}(2)$	8.6	4.6	0.7	0.1
$\mathsf{S}_{80,16}(1)$	11.5	6.8	2.6	1.7
Case 3: multi-jump in $N = 15$ assets				
$\mathsf{S}_{80,16}(4.5)$	10.4	5.9	2.0	1.2
$\mathsf{S}_{80,16}(2)$	85.4	83.4	80.9	78.7
$\mathsf{S}_{80,16}(1)$	95.1	94.5	93.6	93.2
Case 4: small multi-jump				
$\mathsf{S}_{80,16}(4.5)$	7.9	5.0	1.1	0.0
$\mathsf{S}_{80,16}(2)$	55.5	51.4	47.9	44.0
$\mathsf{S}_{80,16}(1)$	82.3	81.4	80.0	79.6
Case 5: big multi-jump				
$\mathsf{S}_{80,16}(4.5)$	75.7	72.1	66.7	61.4
$\mathsf{S}_{80,16}(2)$	99.0	99.0	98.9	98.9
$\mathsf{S}_{80,16}(1)$	98.9	98.9	98.9	98.9

Summarizing, our simulation experiments indicate that the bandwidth parameter, which trades off size and power, can always be set (with an automated procedure) to get correct size and reasonable power. Moreover, by tuning the bandwidth parameter the test can be sensibly used to detect multi-jumps with a given maximum order M, up to the total number N of assets employed.

4.4 Comparison with univariate tests

An alternative way to test for multi-jumps is the intersection of univariate test, also named co-exceedance rule, as in Gilder et al. (2014). In this section we show, with simulated data, that the new multi-jump test proposed in this paper is much more powerful than the intersection of univariate tests.

We state the co-exceedance rule as follows: reject the absence of multi-jumps in the N-dimensional price vector if the absence of jumps is rejected (based on a univariate jump test) for each component. We compare three univariate jump tests: the CPR test of Corsi et al. (2010), the BNS test of Barndorff-Nielsen and Shephard (2006) and the ABD test of Andersen, Bollerslev and Dobrev (2007), all described in Appendix B.2.

We simulate 1,000 paths of $N = 16$ stocks, with the continuous part as in subsection 4.3. Each path contains a single multi-jump across the 16 stocks, with jump sizes normally distributed with mean being equal to $8\sqrt{\Delta}$ and standard deviation $2\sqrt{\Delta}$. Hence, jump sizes are sufficiently large on average, but show dispersion such that some of the univariate jumps might be smaller (we recall that the continuous daily variance hovers around 1).

Table 5 shows size and power for univariate jump tests and multi-jump tests based on the $\mathsf{S}_{\mathrm{n,N}}$ statistics and the co-exceedance rule. For univariate tests, we confirm the findings in the literature (see Dumitru and Urga, 2012 for a wider comparison). The most powerful test is ABD, but at the cost of a distorted size. CPR and BNS are correctly sized, but CPR has higher power, thus striking a superior balance. For this reason, we mainly use CPR for detecting univariate jumps in the empirical application in Section 5.

For multi-jump test, the $\mathsf{S}_{\mathrm{n,N}}$ proposed here is much more powerful than the co-exceedance rule. The intersection of CPR would miss nearly 75% true multi-jumps at the 95% confidence level; the intersection of ABD misses only 34% at the same confidence level, but just because its size is distorted. This is not totally surprising: the co-exceedance rule with large N is too stringent unless the confidence interval used is small enough, at the

Table 5: Shows the **size** and **power** of the proposed test in comparison with the co-exceedance rule, for the case $N = 16$, and for univariate jump tests. The size is computed under the assumption of continuous processes. The sampling frequency $n = 80$ corresponds to 5-minute intraday observations.

confidence interval →	90%	95%	99%	99.9%
Multi-jump tests: power				
$\mathsf{S}_{80,16}(2)$	98.6	98.5	98.5	98.4
$\bigcap_{i=1}^{16} CPR$	32.7	25.1	9.1	1.8
$\bigcap_{i=1}^{16} BNS$	14.2	7.9	1.5	0.0
$\bigcap_{i=1}^{16} ABD$	71.8	66.0	53.7	37.1
Univariate tests on individual stocks: power				
CPR	91.2	89.0	84.4	73.9
BNS	86.0	81.9	71.5	56.1
ABD	97.0	96.6	95.1	92.8
Multi-jump tests: size				
$\mathsf{S}_{80,16}(2)$	7.7	2.6	0.3	0.0
Univariate tests on individual stocks: size				
CPR	9.2	5.3	2.1	0.0
BNS	8.6	4.8	1.9	0.0
ABD	17.3	10.8	3.0	0.8

cost of increasing spurious detection of univariate jumps. The size of the $\mathsf{S}_{n,N}$, computed on a multi-variate process without jumps, is again reasonably correct. The size of the intersection tests cannot be reported since the distribution under the null is unknown.

It is also interesting to note that the $\mathsf{S}_{n,N}$ can be conveniently used as a preliminary tool in a two-steps procedure to detect days with jumps in the first step, and then complemented by univariate tests, applied singularly to each stock, in the second step. Moreover, $\mathsf{S}_{n,N}$ can be much more effective in detecting univariate jumps (which could pass through standard univariate tests, as also shown by Bollerslev et al., 2008) when they are synchronous. Indeed, the power of $\mathsf{S}_{\mathrm{n,N}}$ declines at a much slower rate than the power of univariate tests when increasing the confidence interval. After all, many jumps are better seen than only one.

These considerations can be important for empirical studies, since jumps (and multi-jumps) are rare events. For example, with $N = 2,000$ days, testing against a jump arrival rate of 2% per day using CPR at 99% confidence interval, we expect (based on figures in Table 5) to detect 35.8 true jumps (out of 40) and 20 spurious jumps, which would jeopardize empirical work based on these measures. For this reason, very large confidence intervals (such as 99.9% or 99.99%) are tipically used in practice. With confidence intervals so selective, the $\mathsf{S}_{\mathrm{n,N}}$ could be an effective companion tool for jump detection, which is certainly more effective if these jumps are actually multi-jumps. This can be especially important when detecting jumps in large portfolios, like the stock index, as we further discuss in Section 5.

To summarize, the results in this subsection show that the co-exceedance rule is not accurate, even when it is based on relatively powerful univariate tests, while the multi-jump $\mathsf{S}_{\mathrm{n,N}}$ test proposed here is powerful and accurate, thus indicating a strong preference for the latter in empirical work. We point out that such a feature would be crucial in a number of applications, for instance when dealing with the identification of systemic multi-jumps.

5 Empirical application

The data set we use for the empirical application is the collection of $N = 16$ blue chip stocks quoted on the New York Stock Exchange and belonging to four different economic sectors, and of the S&P500 index. The stocks and the corresponding ticker are listed in Table 6. The data were recovered from the TickData One Minute Equity Data (OMED) dataset, from 2 January 2003 to 29 June 2012, for a total of $2,392$ trading days. The data

Table 6: Reports the list of the sixteen blue chip stocks used in the empirical application, their ticker and the estimated β computed with respect to the S&P500 index and used in the asset allocation exercise in Section 5.5.

Company	Ticker	β
Bank of America	BAC	1.77
Citigroup Inc.	C	1.70
JPMorgan Chase & Co.	JPM	1.60
Wells Fargo & Company	WFC	1.52
Boeing	BA	0.95
Caterpillar Inc.	CAT	1.18
FedEx Corporation	FDX	1.02
Honeywell International Inc.	HON	1.03
Hewlett-Packard Company	HPQ	0.79
International Business Machines Corp.	IBM	0.93
AT&T Inc.	T	1.05
Texas Instruments Incorporated	TXN	0.82
Kraft Foods Inc.	KFT	0.57
PepsiCo, Inc.	PEP	0.55
The Procter & Gamble Company	PG	0.56
Time Warner Inc.	TWX	1.05

went trough a standard filtering procedure. TickData one-minute equity data are adjusted for corporate actions such as mergers and acquisitions or symbol changes. Moreover, the underlying tick data used to build 1-minute time series are first controlled for cancelled trades, or records not temporally aligned with previous/subsequent data; then filtered to identify *bad ticks* which are corrected using validation with third-party sources.

In our empirical application, we use returns at the 5-minutes frequency, corresponding to $n = 77$ intraday returns. This frequency represents a tradeoff between achieving enough statistical power and avoiding distortions which could potentially arise from microstructure noise.

Table 7: Reports the frequencies of rejections (in percentage) for the null hypothesis of absence of co-jumps between stock pairs (that is, the percentage of days with co-jumps), tested with the proposed multi-jump test $\mathsf{S}_{77,2}(6.5)$ at 0.1% confidence level. The global average of rejections among pairs is 1.33%.

	C	JPM	WFC	BA	CAT	FDX	HON	IBM	HPQ	TWX	T	KFT	PEP	PG	TXN
BAC	1.55	1.92	1.80	0.84	0.88	1.17	1.17	0.96	1.09	0.88	1.09	1.59	1.17	0.79	0.92
C		1.46	1.63	0.54	1.00	0.92	1.17	1.17	1.05	0.67	1.09	1.34	0.79	0.67	0.84
JPM			2.17	1.05	1.09	1.63	1.25	1.59	1.21	1.13	1.17	1.80	1.42	1.05	0.92
WFC				1.25	1.21	1.09	2.01	1.59	1.34	1.00	1.30	1.76	1.05	1.17	0.84
BA					1.51	1.92	2.22	1.34	0.84	1.05	1.55	2.63	1.09	1.30	1.30
CAT						1.67	1.76	1.13	1.21	1.30	1.51	2.09	0.88	1.05	0.79
FDX							1.71	1.42	1.42	1.09	1.76	1.92	1.17	1.09	0.96
HON								1.46	1.30	1.13	1.96	2.63	1.34	1.84	1.63
IBM									1.63	1.21	1.84	2.05	1.42	1.09	0.96
HPQ										0.75	1.25	1.55	1.05	0.84	0.92
TWX											0.92	1.55	0.59	1.00	0.67
T												2.38	1.55	1.55	1.17
KFT													2.34	2.01	1.71
PEP														1.55	1.05
PG															1.25

5.1 Multi-jumps in the market

We start by applying the co-jumps test ($N = 2$) for all the 120 pairs throughout all the sample. Table 7 reports the percentage of rejections of the null at the 99.9% confidence level for all pairs. Co-jumps are significant events. On average (among pairs), we detect co-jumps in 1.33% of days. The low probability of co-jumps is in line with other existing empirical work (see, e.g., Table III of Lahaye et al., 2011 for stock indexes and FX rates). The co-jumps are distributed quite uniformly among stock pairs. The maximum amount of rejections is obtained between HON and KFT (2.63%), while the minimum is observed between C and BA (0.54%).

We then detect multi-jumps among all 16 stocks using a confidence interval $1 - \alpha$ such that the expected number of spurious detection in the sample is 0.1 asymptotically, that is $\alpha = 4.18 \cdot 10^{-5}$. We are thus looking for solid rejection of the null, that is strong signals and virtually no false positives. We use bandwidth parameters h_n between 1 and 3. As documented in the simulation study, the higher bandwidth $h_n = 3$ corresponds to more correct size against the null of absence of multi-jumps in *all* the 16 stocks, meaning that the null would include the case of multi-jump between $M = 15$ stocks. This is

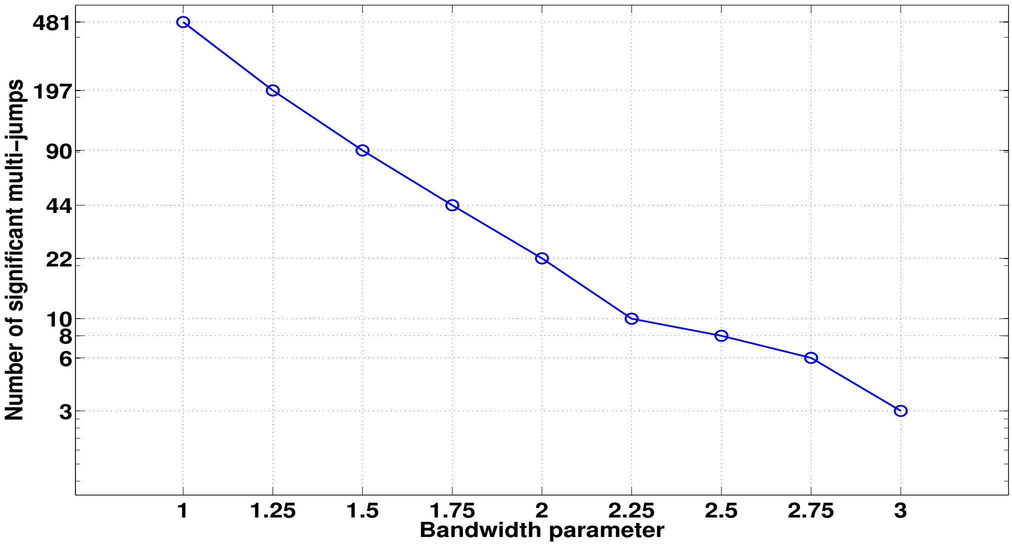

Figure 3: Reports the number of multi-jumps detected by the test introduced in Section 3. The test outcome is reported for different bandwidth parameters. The smaller bandwidth $h_n = 1$ corresponds to the detection of at least $M \simeq 5$ multi-jumps. The larger bandwidth $h_n = 3$ corresponds to the detection of at least $M \simeq 15$ multi-jumps.

certainly too stringent for empirical analysis. Table 4 shows instead that, with the lower bandwidth $h_n = 1$, the test is reasonably sized against the contemporaneous presence of $M = 4$ multi-jumps (at least). Thus, we interpret the rejection of the null with $1 \leq h_n \leq 3$ as a signal for the presence of a significant multi-jumps in at least M stocks, with $M \approx 5$ for $h_n = 1$ and $M \approx 13$ for $h_n = 2$ (see Figure 9). In the case $h_n = 2$, therefore, the multi-jump would involve all the four economic sectors.

Figure 3 reports the number of detected multi-jumps corresponding to different bandwidths. Their number vary from 481 (20.1% of the sample) at $h_n = 1$ to just 3 (0.13% of the sample) at $h_n = 3$. Thus, multi-jumps are largely statistically significant in our sample, but multi-jumps across *many* stocks are rare events.

However, these rare events are strongly economically significant. Table 8 reports the dates of the 22 multi-jumps detected with $h_n = 2$, and associates macroeconomic/financial information to each date; it also reports the corresponding VIX daily changes, SP500 percentage change and percentage volume (aggregated over all 16 stocks) changes. We can

Table 8: Multi-jump dates (when the test is implemented with $h_n = 2$, that is with approximately more than $M \simeq 13$ multi-jumps) are listed together with i) multi-jump direction, ii) percentage change in S&P500, iii) percentage volume change, iv) VIX difference and v) economic/financial events occurred on those days.

date	Multi-jump direction	SP500 change (%)	Volume change (%)	VIX change	Economic/Financial events
25-Jun-2003	negative	-0.83	$+1.81$	$+0.06$	FOMC meeting cuts federal fund rate of 25bps
18-Apr-2006	positive	$+1.71$	$+35.19$	-1.18	Release of minutes of FOMC meeting of 27-28 Mar
08-Aug-2006	negative	-0.34	$+29.23$	-0.00	FOMC keeps its target for the federal funds rates
18-Sep-2007	positive	$+2.92$	$+30.98$	-6.13	FOMC lowers its target for the federal funds rates by 50 bps
25-Feb-2008	positive	$+1.38$	-5.74	-1.03	FED Term Auction Facility
16-Jul-2008	positive	$+2.51$	$+4.70$	-3.44	Release of minutes of FOMC meeting of 24-25 Jun
29-Sep-2008	negative	-8.81	$+15.37$	$+11.98$	FOMC meeting unscheduled
10-Feb-2009	negative	-4.91	$+29.58$	$+3.03$	U.S. Treasury Secretary Geithner announces a Financial Stability Plan
17-Feb-2009	both	-4.56	$+20.10$	$+5.73$	27-28 FOMC minutes released on Feb 18
23-Feb-2010	negative	-1.21	$+14.80$	$+1.43$	FED releases minutes of its discount rate meeting on January 25, 2010.
06-May-2010	negative	-3.24	$+49.28$	$+7.89$	The Flash Crash
28-May-2010	negative	-1.24	-7.66	$+2.39$	FED announces three small auctions through the Term Deposit Facility
01-Sep-2010	positive	$+2.95$	$+13.44$	-2.16	Release of minutes of FOMC meeting of 27-28 Mar (Aug 31)
23-Jun-2011	positive	-0.28	$+32.84$	$+0.77$	FOMC meetings (21 and 22 June)
01-Jul-2011	positive	$+1.44$	-34.12	-0.65	Arab Spring starts
01-Aug-2011	negative	-0.41	-2.19	-1.59	Unscheduled FOMC meeting
01-Sep-2011	positive	-1.19	-18.67	$+0.20$	Release of minutes of FOMC meeting of 27-28 Mar (Aug 30)
31-Oct-2011	negative	-2.47	-7.49	$+5.43$	FOMC committee scheduled for 1-2 November
23-Nov-2011	negative	-2.21	-3.50	$+2.01$	Release of the minutes of the FOMC committee of 1-2 November
28-Nov-2011	positive	$+2.92$	$+50.79$	-2.34	FOMC meeting unscheduled
03-Apr-2012	negative	-0.40	$+7.99$	$+0.02$	13 March FOMC minutes released
14-Jun-2012	positive	$+1.08$	-3.91	-2.59	Federal Reserve Board issues enforcement actions

see that almost all the multi-jumps in the Table can be easily associated with impactful economic news, mainly related to FED activity, more prominently FOMC meetings, but also important financial and global news. Moreover, the traded volume tends to be considerably higher (than the previous day) on days in which a multi-jump occurs. The VIX index tends to move, in multi-jump days, in an opposite direction with respect to the market, as also noticed in Todorov and Tauchen (2010). Below we provide formal statistical evidence of a significant increase of the variance premium associated with multi-jumps.

Multi-jumps are also typically, but not always, associated with jumps in the S&P 500 stock index. We use three tests for detecting jumps in the stock index: the ABD test, the BNS test and the CPR test (see Appendix B.2 for their description) at the 99.9% confidence interval. The left panel of Table 9 reports the percentage of cases in which, in a day with a multi-jump, we also detect a jump in the index. We can see that testing for jumps in the index results in a significant information loss with respect to testing for multi-jumps. The test with the highest overlap is ABD, which however is also the test with largest size distortions (that is, with supposedly more false positives).

The fact that decreasing the bandwidth parameter we have less overlap between multi-jumps and jumps in the index is not surprising: jumps in the index are easier to detect in the presence of multi-jumps among more constituents. The fact that jumps in the stock index are not detected in *all* multi-jump days deserves further investigation. This could be due to a *subset effect* (only the 16 stocks considered here jumped, but not the other index constituents) or to a *power effect* (if the univariate tests on the index are less powerful than the multi-jump test). To shed light on this issue, we also compute the univariate jump tests on the equally weighted portfolio of the sixteen stocks (right panel of Table 9), thus eliminating the subset effect. We can indeed observe a slight increase of the performance of CPR and ABD tests, but not such to fill the gap with the multi-jump test. The performance of BNS on the equally weighted portfolio is even worst. This result demonstrates that the power effect is dominant: a multi-jump in the 16 stocks certainly implies a jump in their portfolio, which however the univariate tests are often unable to detect. The problem is very severe for the BNS test, whose performance is particularly poor. These results altogether suggests that it is significantly more powerful to test for multi-jumps among stocks than for jumps in a portfolio. The next sections also show that the additional information carried by the multi-jump test, which cannot be revealed by univariate jump tests, is economically significant.

Finally, most jumps in the index can be associated to multi-jumps in the stocks: using CPR at 99.9% confidence interval, we find 157 jumps in the index (6.56% of the sample).

	Multi-jumps and jumps in the S&P 500 index		
	$h_n = 2$	$h_n = 1.5$	$h_n = 1$
CPR	59.1%	43.2%	33.3%
BNS	40.9%	25.0%	16.7%
ABD	81.8%	63.6%	58.9%

	Multi-jumps and jumps in the equally weighted portfolio		
	$h_n = 2$	$h_n = 1.5$	$h_n = 1$
CPR	68.2%	54.5%	40.0%
BNS	36.4%	22.7%	13.3%
ABD	90.9%	75.0%	64.4%

Table 9: Reports the percentage of days with a detected multi-jumps (according to the bandwidth parameters $h_n = 2, 1.5, 1$) in which we also detect a jump in the S&P500 index (left panel) and in the equally weighted portfolio of the 16 stocks (right panel) according to three different jump tests at the 99.9% confidence interval. Testing for a jump in the portfolio is less powerful than testing for multi-jumps among constituents.

Of these jumps, 77 correspond to days in which there is a multi-jump with $h_n \geq 1$. Thus, jumps in the index can be typically (but not always) associated with multi-jumps in its most liquid constituents. The remaining jumps in the index could be explained by jumps in a subset of constituents with not enough overlap with the stocks considered here, or by size distortions larger than what predicted by our simulated data.

5.2 Jumps, multi-jumps and the variance risk premium

This section shows the relevance of detecting multi-jumps in the data (with respect to jumps in the index) by associating their occurrence to changes in the variance risk premium. The variance risk premium on day t for a given maturity τ is defined as:

$$VRP_t = QV^{\mathcal{Q}}(t, t+\tau) - QV(t, t+\tau) \tag{15}$$

where $QV^{\mathcal{Q}}(t, t+\tau)$ is the *risk-neutral* quadratic variation of the stock index between times t and $t+\tau$, and $QV(t, t+\tau)$ is the actual quadratic variation in the same interval. Bollerslev et al. (2008) highlight the empirical potential of the variance risk premium by showing that VRP_t carries significant forecasting power for future returns; see also Carr and Wu (2009); Bollerslev and Todorov (2011). We use $\tau = 1$ month and we estimate VRP_t in our sample using:

$$\widehat{VRP_t} = VIX^2_{t,t+30} - \widetilde{RV}_{t,t+30} \tag{16}$$

where $VIX_{t,t+30}$ is the 30-days VIX index computed by CBOE, that is the model-free implied volatility (Jiang and Tian, 2005), and $\widetilde{RV}_{t,t+30}$ is the forecasted realized variance in the same period obtained with the regression:

$$\log RV_{t,t+30} = \alpha_1 + \alpha_2 \log RV_{t-30,t-1} + \alpha_3 \log RV_{t-90,t-1} + \varepsilon_t, \tag{17}$$

where ε_t is iid noise and

$$RV_{t,t+h} = 252 \cdot \psi \cdot \sum_{t \le t' \le t+h} \mathsf{RV}_{t'},$$

with $\mathsf{RV}_{t'}$ being the 5-minutes open-to-close realized variance on day t, properly rescaled by 252 (to convert it to yearly units) and by the constant ψ, which is the ratio between the sum of squared close-to-close S&P500 daily returns and the average of RV in the sample, and which is meant to take into account the contribution of overnight returns to the total variance.

The time series of the estimated variance risk premium in our sample is shown in Figure 4. As expected from the theory, it is almost always positive. We associate it to jumps in the stock market index (S&P500) and multi-jumps in the sample of sixteen stocks, by using the following regression models, in which the variance risk premium is driven by an autoregressive process and by dummy variables for jumps,

$$\widehat{VRP}_t = \gamma_0 + \gamma_1 \widehat{VRP}_{t-1} + \gamma_{\mathsf{J}} \widetilde{\mathsf{J}}_t + \gamma_{MJ} \widetilde{MJ}_t + \gamma^-_{MJ} \widetilde{MJ}_t \cdot I_{\{r_t^{SP} < 0\}} + \epsilon_t, \tag{18}$$

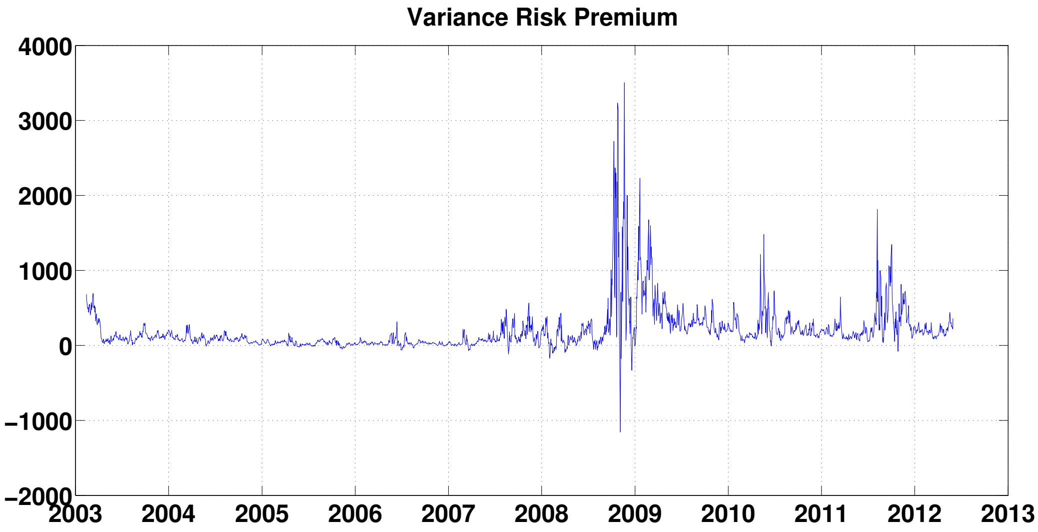

Figure 4: Reports the daily estimated variance risk premium, computed as in equation (16), for the available sample.

where t denotes the day, $\widetilde{\mathrm{J}}_t$ is an indicator function signaling jump in S&P500 index (we use the CPR test at 99.9% confidence level), $\widetilde{MJ}_t$ is an indicator function for the presence of a multi-jump (with $h_n = 2,\ 1.75$ and 1.5), $I_{\{r_t^{SP}<0\}}$ is an indicator function for *negative* close-to-close return on S&P500 on day t and ϵ_t are iid shocks with zero mean and finite variance.

We also run the same regression with *lagged* dummy variables, that is with $\widetilde{\mathrm{J}}_t \to \widetilde{\mathrm{J}}_{t-1}$, $\widetilde{MJ}_t \to \widetilde{MJ}_{t-1}$ and $I_{\{r_t^{SP}<0\}} \to I_{\{r_{t-1}^{SP}<0\}}$, to examine the predictive power of multi-jumps on the variance risk premium. Estimation results, adjusted with the standard Newey and West correction, are presented in Table 10 for various restrictions and multi-jump test bandwidth parameters.

We find that the constant γ_0 and the auto-regressive coefficient γ_1 are strongly significant. More importantly, our results show that the occurrence of jumps in the index is insignificant (or mildly negative) when regressed together with variance premium, and thus cannot be associated to it. On the contrary, multi-jumps are significant and have a *strong* impact, especially when they contribute to a market downturn. When the dummies are lagged, results are less strong but the signs do not change. The average impact

Table 10: Estimates (Newey-West corrected) of model (18) with different restrictions and different choices of the multi-jump dummy. T-statistics are in parenthesis. Top panel: regression with contemporaneous dummies. Bottom panel: regression with lagged dummies. One star denotes 90%, two star 95% and three stars 99% significance.

Contemporaneous regressions

γ_0	31.7***	31***	30.3***	30.2***	31.5***	30.7***	30.1***	28.8***	31.3***	31.3***
	(5.18)	(5.2)	(5.17)	(5.17)	(5.18)	(5.16)	(5.1)	(4.97)	(5.15)	(5.14)
γ_1	0.836***	0.835***	0.833***	0.832***	0.835***	0.835***	0.832***	0.83***	0.835***	0.835***
	(24)	(24.1)	(23.8)	(23.6)	(24)	(24.1)	(23.7)	(23.7)	(24)	(24)
γ_J	-0.41				-8.15				-8.52	-4.67
	(-0.0491)				(-0.881)				(-0.98)	(-0.534)
γ_{MJ}		89.3			93.9*					-96***
($h_n = 2$)		(1.64)			(1.68)					(-2.94)
γ_{MJ}			104***							
($h_n = 1.75$)			(2.75)							
γ_{MJ}				60.6*						
($h_n = 1.5$)				(1.86)						
γ_{MJ}^{-}						184***			188***	282***
($h_n = 2$)						(2.75)			(2.78)	(3.85)
γ_{MJ}^{-}							193***			
($h_n = 1.75$)							(3.82)			
γ_{MJ}^{-}								176***		
($h_n = 1.5$)								(3.81)		

Lagged regressions

γ_0	32.2***	31.5***	31.6***	30.5***	32.2***	31.5***	31.7***	31.4***	32.2***	32.2***
	(5.3)	(5.27)	(5.25)	(5.2)	(5.3)	(5.27)	(5.27)	(5.25)	(5.31)	(5.31)
γ_1	0.835***	0.835***	0.835***	0.831***	0.835***	0.835***	0.836***	0.834***	0.834***	0.834***
	(24)	(24)	(24)	(23.5)	(24)	(23.9)	(23.9)	(23.6)	(23.9)	(23.9)
γ_J	-7.44				-10.8*				-10.5*	-9.66*
	(-1.48)				(-1.79)				(-1.92)	(-1.71)
γ_{MJ}		34.5			40.6					-22.5***
($h_n = 2$)		(0.804)			(0.91)					(-2.58)
γ_{MJ}			4.07							
($h_n = 1.75$)			(0.0728)							
γ_{MJ}				51.8						
($h_n = 1.5$)				(1.34)						
γ_{MJ}^{-}						67			71.7	93.8
($h_n = 2$)						(1.07)			(1.13)	(1.52)
γ_{MJ}^{-}							2.49			
($h_n = 1.75$)							(0.0303)			
γ_{MJ}^{-}								29.4		
($h_n = 1.5$)								(0.608)		

of contemporaneous multi-jumps ranges from 60.6 to 104 points, and from 176 to 193 points when associated with negative S&P500 returns. This is substantial, since the average variance risk premium in our sample is equal to 193.6. This means that variance premium almost *doubles* (on average) in days with a multi-jump and a downturn of the market. The effect of downward multi-jump is so strong that it subsides the effect of jumps and overall multi-jump, the latter becoming significantly negative in the encompassing regression (last column of Table 10) indicating that positive multi-jump have the opposite effect on the variance risk premium.

Summarizing, multi-jumps, and precisely negative ones, can be associated with a significant increase in the variance risk premium, while index jumps cannot. The inability of jumps in capturing variance risk premia changes might depend on the documented inadequacy of nonparametric univariate tests in capturing economically significant jumps. From a theoretical point of view, our finding corroborates the view that non-Gaussian shocks to fundamentals (sometimes referred to as *disasters*) have a substantial impact on risk premia, see e.g. Barro (2006); Gabaix (2012) and Drechsler and Yaron (2011); Drechsler (2013) for economic models directly focusing on the relation between jumps in fundamentals and the variance risk premium. From this theoretical perspective, the absence of correlation between jumps in the stock market index and variance risk premium changes is rather puzzling. Indeed, also Todorov (2010) shows a strong link between index jump measures and variance risk premium changes through the estimation of a parametric model.[5] This paper documents that this puzzle is due to the relatively low power of univariate jump tests to detect systemic market events affecting fundamental value, and that this shortcoming can be overcome by testing for multi-jumps in a (not very large, but economically representative) stock panel.

[5] In his preliminary analysis, Todorov (2010) also reports a significant correlation between the variance risk premium premium and squared jump size, measured as the difference between Realized Variance (RV) and Tripower Variation (TV). We also find a significant correlation coefficient of 0.3927 between the two measures (we estimate squared jump size as the difference between RV and Threshold Bipower Variation), however this correlation disappears after testing for jumps. It is important to note that, in our sample, the estimated squared jump size is also correlated with RV (the coefficient is 0.4312), thus his observed correlation with VRP could be spuriously induced by the correlation between VRP and RV.

5.3 Forecasting variance

We assess the impact of multi-jumps on future variance by estimating the following regression model, which is labelled $HAR - J - MJ$:

$$\begin{aligned} \log V_t^{(i)} = \; & \beta_0^{(i)} + \beta_d^{(i)} \log V_{t-1}^{(i)} + \beta_w^{(i)} \log V_{t-5:t-1}^{(i)} + \beta_m^{(i)} \log V_{t-22:t-1}^{(i)} \\ & + \beta_J^{(i)} \widetilde{J}_{t-1}^{(i)} + \beta_{MJ}^{(i)} \widetilde{MJ}_{t-1} + \epsilon_t^{(i)}, \qquad (i = 1, \ldots, 16+1) \end{aligned} \tag{19}$$

where t denotes the day, $V_t^{(i)}$ is a realized volatility measure of stock i (we use threshold bipower variation, see Appendix B.3),

$$\log V_{t-h:t-1}^{(i)} = \frac{1}{h} \sum_{t'=t-h}^{t-1} \log V_{t'}^{(i)},$$

$\widetilde{J}_{t-1}^{(i)}$ is an indicator function signaling jump detection on day $t-1$ for the stock i (we use the CPR test at 99.9% confidence level), $\widetilde{MJ}_{t-1}$ is an indicator function for the presence of a multi-jump (we use $h_n = 1.75$ in this exercise, which is the case in which 44 multi-jumps are detected; results with different bandwidths, not reported here, deliver similar results) and $\epsilon_t^{(i)}$ are potentially correlated iid shocks with zero mean and finite variance. The explanatory variable $\widetilde{MJ}$ is the same across all stocks.

The model is a natural multivariate generalization of the univariate models adopted in Andersen, Bollerslev and Diebold (2007), Corsi et al. (2010) and Busch et al. (2011), which are all based on the Corsi (2009) HAR model. The usual hard-to-beat long range dependence in volatility delivered by the HAR model is complemented by an *idiosyncratic* component (the jumps in the single stocks) and a *systemic* component (multi-jumps).[6] The idiosyncratic components can be associated to news, as shown in the empirical work of Lee (2012), Evans (2011) and Gilder et al. (2014). We estimate the model for $i = 1, \ldots, 17$, where the $17th$ asset is the S&P500 index, using two-steps FGLS (see Appendix C). We

[6] Despite our multi-jump indicator variable is extracted from 16 equities only, we believe the indicator has a systemic interpretation given the large size and liquidity of the stocks in our dataset, and the fact they span at least four economic sectors.

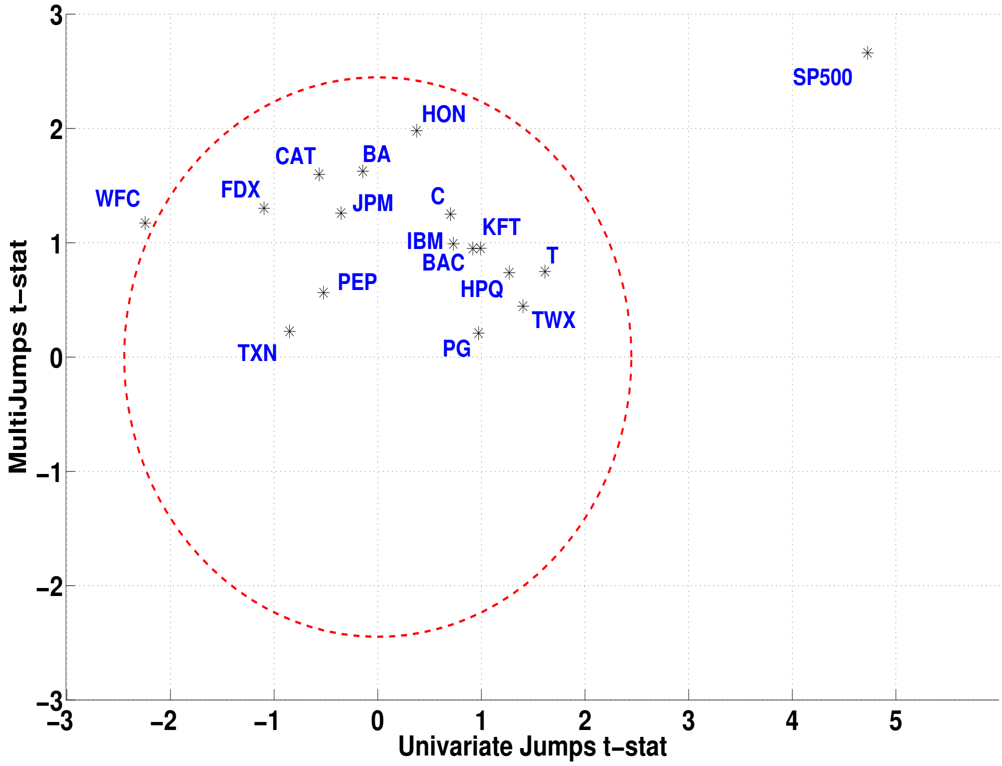

Figure 5: Reports a scatter plot of two $t - stats$: on the bottom axis, the $t - stats$ associated with the significance test for the impact of the multi-jumps dummy on the volatility; on the left axis, the $t - stats$ associated with the significance test of the idiosyncratic jump dummy on the volatility. The graph includes results for both the sixteen stocks and the stock index. The scatter plot shows that multi-jumps affect both stock and index volatility, while only idiosyncratic jumps in the index have a significant impact on the index volatility.

concentrate on estimates of the parameters $\beta_J^{(i)}$ and $\beta_{MJ}^{(i)}$, for $i = 1, \ldots, 17$. We also estimate the models with the restrictions $\beta_J^{(i)} = 0$ or $\beta_{MJ}^{(i)} = 0$.

Estimated coefficients for the three estimated models are reported in table 11, while Figure 5 shows visually the t-statistics of the estimated coefficients $\widehat{\beta}_J^{(i)}$ (when $\beta_{MJ}^{(i)} = 0$) together with those of the coefficients $\widehat{\beta}_{MJ}^{(i)} = 0$ (when $\beta_J^{(i)} = 0$). Idiosyncratic jumps display a significant impact on volatility for the S&P500 index (as found in Corsi et al., 2010) but have no forecasting power for individual stocks. We instead estimate a positive impact of multi-jumps on both stocks and the index, which is not negligible in magnitude: The average estimated coefficient is 0.0693 to be compared to average $\log V_t^{(i)}$ in our

Table 11: Estimated jump and multi-jump coefficients (with t-stats in parenthesis) for the model (19). The first column is for the model with $\beta_{MJ}^{(i)} = 0$; the second column is for the model with $\beta_J^{(i)} = 0$; the third and the fourth column are for the unrestricted model. One star denotes 90%, two star 95% and three stars 99% significance.

asset	$\widehat{\beta}_J^{(i)}$ ($\beta_{MJ}^{(i)} = 0$)	$\widehat{\beta}_{MJ}^{(i)}$ ($\beta_J^{(i)} = 0$)	$\widehat{\beta}_J^{(i)}$ unrestricted	$\widehat{\beta}_{MJ}^{(i)}$ unrestricted
BAC	0.0194	0.0811	0.0125	0.0790
	(0.9166)	(0.9491)	(0.5204)	(0.0790)
C	0.0156	0.0835	0.0020	0.0832
	(0.7024)	(1.2495)	(0.0731)	(0.0832)
JPM	-0.0067	0.0832	-0.0330	0.0900
	(-0.3534)	(1.2591)	(-1.3903)	(0.0900)
WFC	-0.0371^{**}	0.0802	-0.0370^{*}	0.0876
	(-2.2456)	(1.1716)	(-1.7792)	(0.0876)
BA	-0.0031	0.1144	0.0244	0.1109
	(-0.1450)	(1.6258)	(0.8671)	(0.1109)
CAT	-0.0121	0.1158	0.0117	0.1142
	(-0.5653)	(1.5979)	(0.4193)	(0.1142)
FDX	-0.0254	0.0933	-0.0357	0.0975
	(-1.0980)	(1.3017)	(-1.2369)	(0.0975)
HON	0.0072	0.1296^{**}	0.0285	0.1224^{*}
	(0.3758)	(1.9799)	(1.0790)	(0.1224)
IBM	0.0172	0.0669	0.0337	0.0603
	(0.7294)	(0.9901)	(1.3911)	(0.0603)
HPQ	0.0294	0.0568	0.0514	0.0490
	(1.2688)	(0.7376)	(1.5821)	(0.0490)
TXN	-0.0165	0.0147	-0.0147	0.0167
	(-0.8506)	(0.2251)	(-0.5784)	(0.0167)
T	0.0322	0.0456	-0.0089	0.0470
	(1.6140)	(0.7477)	(-0.3366)	(0.0470)
KFT	0.0257	0.0678	0.0392	0.0668
	(0.9924)	(0.9519)	(1.3616)	(0.0668)
PEP	-0.0121	0.0353	-0.0486^{*}	0.0438
	(-0.5238)	(0.5626)	(-1.8916)	(0.0438)
PG	0.0214	0.0141	0.0060	0.0129
	(0.9731)	(0.2089)	(0.1938)	(0.0129)
TWX	0.0254	0.0269	-0.0026	0.0272
	(1.4019)	(0.4450)	(-0.1143)	(0.0272)
SP500	0.1261^{***}	0.1676^{***}	0.1147^{***}	0.1334
	(4.7277)	(2.6606)	(4.2596)	(0.1334)

sample, which is 0.3348, thus implying a +20.7% increase on average. Individually, these coefficients are not statistically significant (probably due to the scarcity of multi-jumps in the data), however they are all positive, while the coefficients on individual jumps are both positive and negative.

These findings complement the evidence of Corsi et al. (2010) on the impact of jumps on volatility forecasting. Jumps in stocks do not seem to possess forecasting power on future stock volatility, unless they occur in multiple stocks, signaling a systemic event instead of an idiosyncratic event. When this happens, we typically see a jump in the stock index, and this explains why jumps are instead very significant in forecasting the index future variance. However, as shown in the previous section, this is not accompanied by an increase of the variance risk premium (which is instead observed when a multi-jump occurs).

5.4 Forecasting correlations

We assess the impact of multi-jumps on future stock correlations by estimating the following regression model, which is labelled $corrHAR - CJ - MJ$:

$$\begin{aligned} \widetilde{\rho}_t^{(i,j)} = \; & \beta_0^{(i,j)} + \beta_d^{(i,j)} \widetilde{\rho}_{t-1}^{(i,j)} + \beta_w^{(i,j)} \widetilde{\rho}_{t-5:t-1}^{(i,j)} + \beta_m^{(i,j)} \widetilde{\rho}_{t-22:t-1}^{(i,j)} \\ & + \beta_{CJ}^{(i,j)} \widetilde{CJ}_{t-1} + \beta_{MJ}^{(i,j)} \widetilde{MJ}_{t-1} + \epsilon_t^{(i,j)}, \qquad (20) \\ & (i = 1, \ldots, 16, \; j = i+1, \ldots, 16) \end{aligned}$$

where, similarly to model (19), t indexes the day, $\widetilde{\rho}_t^{(i)}$ is the Fisher transformation of realized correlations between stock i and stock $j \neq i$ (Appendix B describes how correlations are measured by means of intraday data),

$$\widetilde{\rho}_{t-h:t-1}^{(i)} = \frac{1}{h} \sum_{t'=t-h}^{t-1} \widetilde{\rho}_{t'}^{(i)},$$

$\widetilde{CJ}_{t-1}^{(i)}$ is an indicator function signaling co-jump detection on day $t-1$ between stock i and stock j (we use our multi-jump test with $N = 2$ at 99.9% confidence level and $h_n = 6.5$), $\widetilde{MJ}_{t-1}$ is the same multi-jump indicator described below Eq. (19), and $\epsilon_t^{(i,j)}$ are potentially correlated iid shocks with zero mean and finite variance. We estimate the model with 16 stocks, so that the dimension of the model is 120. As before, we estimate the model for 120 pairs using two-steps FGLS (see Appendix C) and we concentrate on estimates of the parameters $\beta_{CJ}^{(i,j)}$ and $\beta_{MJ}^{(i,j)}$. Again, the model is made by two parts: the first meant to capture the highly significant serial dependence in correlations, the second one to capture a *idiosyncratic* component (the co-jumps among pairs) and a *systemic* component (multi-jumps in the market). Lahaye et al. (2011) indeed complement the evidence on the relation between jumps and news by showing that also co-jumps between pairs are typically associated to macroeconomic releases.

We first estimate the $corrHAR - CJ - MJ$ with the restriction $\beta_{MJ}^{(i,j)} = 0$. Figure 6 reports the histograms of the $\widehat{\beta}_{CJ}^{(i,j)}$ estimated coefficients in this case, together with their t-statistics. We find that co-jumps have very small positive impact on future correlations, at least when judged individually; the implied average correlation change $\Delta\widetilde{\rho}$ after a co-jump is -0.00015 and never significant; the distribution of the t-statistics is consistent with a standard normal distribution. In a related study, Clements and Liao (2013) instead found, using a procedure similar to that proposed by Bollerslev et al. (2008), that the occurrence of common jumps between the stocks are unrelated to the level of volatility or correlation. However, their empirical findings suggest that correlations decrease immediately after a co-jump. Our results cannot confirm this finding.

We then drop the co-jumps from model (20), and we estimate it with the restriction $\beta_{CJ}^{(i,j)} = 0$. The distribution of estimated coefficients $\widehat{\beta}_{MJ}^{(i,j)}$ are reported in Figure 7, together with the associated t-statistics. Multi-jumps are instead very significant in explaining future correlations. The average coefficient, that is the average implied correlation change after a multi-jump, is 0.0827, with a robust t-statistics of 3.89 and also substantial in absolute terms since the average correlation, in our sample, is 0.2955: This implies

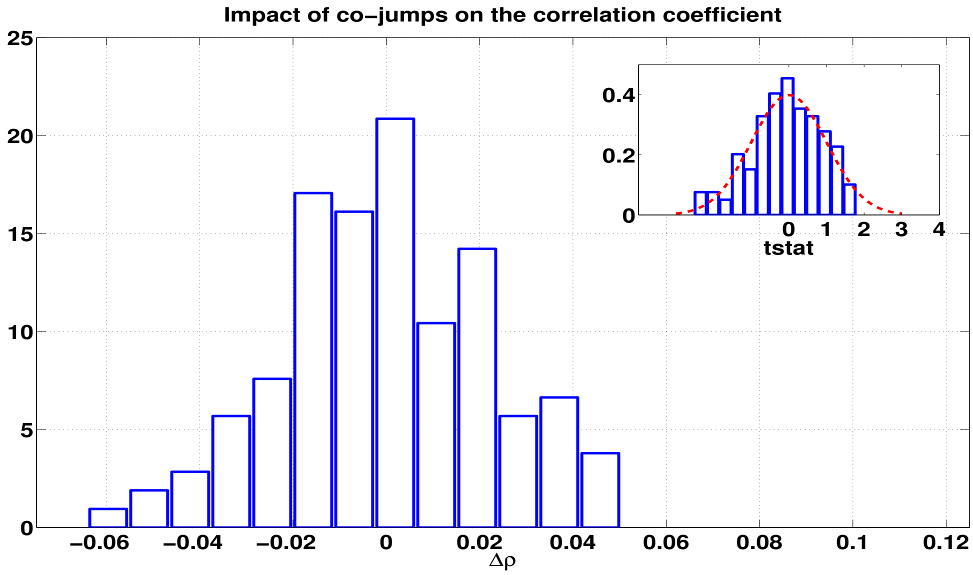

Figure 6: Reports a frequency histogram of the estimated impact of the co-jump coefficients $\widehat{\beta}_{CJ}^{(i,j)}$ on the correlation among stock pairs. These have been obtained from the model (20) under the restriction $\beta_{MJ}^{(i,j)} = 0$. We remind that the coefficients $\widehat{\beta}_{CJ}^{(i,j)}$ represent the change in correlation when a co-jump takes place. The inset plots the frequency histogram of the associated t-statistics.

a correlation increase, on average, of +28%. In this case the distribution of t-statistics is significantly shifted to the right and individual coefficients, reported in Table 12, are often significant on their own.

The results are confirmed when we estimate the unrestricted model. In this case, the strong significance of multi-jumps is confirmed, while co-jumps are still not significant. Multi-jump can again be viewed, by the light of results in Longin and Solnik (2001) who report an increase in market correlations in turmoil periods, as the statistical counterpart of systemic economic events.

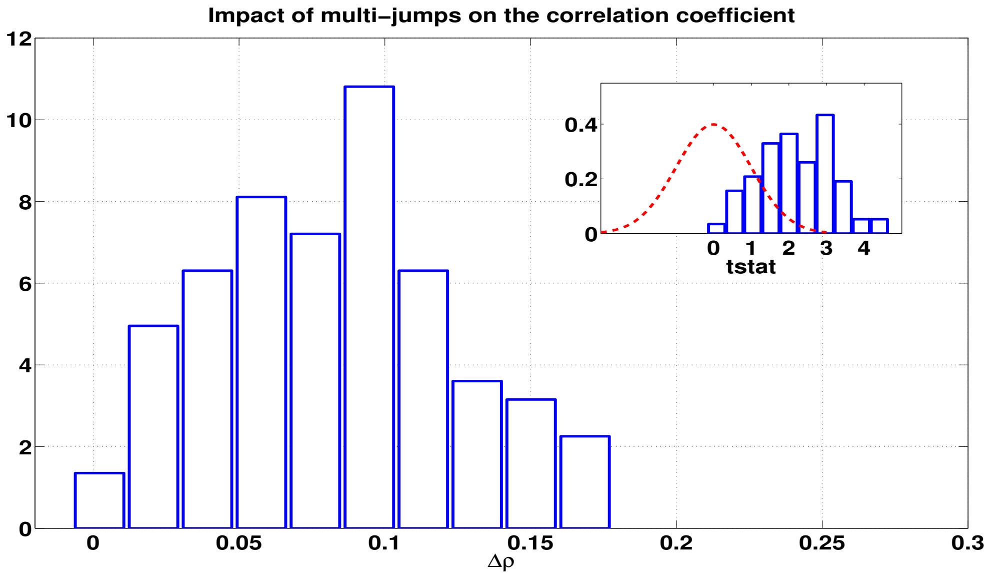

Figure 7: Reports a frequency histogram of the estimated impact of multi-jump coefficients $\widehat{\beta}_{MJ}^{(i,j)}$ on correlation among stock pairs. The average coefficient is 0.0827 with a t-stat of 3.89. These have been obtained from the model (20) under the restriction $\beta_{CJ}^{(i,j)} = 0$. We remind that the coefficients $\widehat{\beta}_{MJ}^{(i,j)}$ represent the change in correlation when a multi-jump takes place. The inset plots the frequency histogram of the associated t-statistics.

Table 12: Reports the estimated $\widehat{\beta}_{MJ}^{(i,j)}$ for the corrHAR-CJ-MJ model (20) with the restriction $\beta_{CJ}^{(i,j)} = 0$. One star denotes 90%, two star 95% and three stars 99% significance.

	C	JPM	WFC	BA	CAT	FDX	HON	IBM	HPQ	TXN	T	KFT	PEP	PG	TWX
BAC	0.025	0.068	0.093*	0.061**	0.102***	0.115***	0.104***	0.073**	0.075**	0.067**	0.053*	0.021	0.022	0.029	0.105***
C		−0.007	0.018	0.022	0.047	0.055	0.028	0.044	0.026	0.047	0.055*	−0.004	0.011	0.028	0.088*
JPM			0.072	0.038	0.085*	0.092**	0.064	0.046	0.035	0.057	0.071**	0.037	0.015	0.051	0.117**
WFC				0.087***	0.121***	0.145***	0.084**	0.105***	0.053	0.069*	0.088***	0.048	0.030	0.058*	0.122***
BA					0.152***	0.136***	0.139***	0.124***	0.096***	0.091***	0.156***	0.101***	0.109***	0.086***	0.185***
CAT						0.152***	0.131**	0.134***	0.096**	0.111***	0.105***	0.048	0.044	0.058*	0.162***
FDX							0.123***	0.094**	0.099**	0.120***	0.101***	0.075**	0.089***	0.056	0.145***
HON								0.152***	0.118***	0.114***	0.118**	0.046	0.056	0.074**	0.196***
IBM									0.086**	0.099***	0.091***	0.055	0.102***	0.075*	0.183***
HPQ										0.070*	0.049	0.056**	0.094***	0.100***	0.126***
TXN											0.097***	0.081***	0.079***	0.059*	0.135***
T												0.043	0.077**	0.095**	0.176***
KFT													0.057	0.038	0.077*
PEP														0.019	0.106***
PG															0.143***

5.5 Impact on asset allocation

We now provide an economic assessment of the impact of multi-jumps on asset allocation. We have documented an increase in the volatility of individual stocks and in the correlation among them following the occurrence of a multi-jump. These empirical regularities imply two adverse effects on the utility of an agent allocating her wealth in stocks: it makes her portfolio more volatile, and also reduces the potential benefits of diversification. In this subsection we estimate the impact, in terms of utility loss, due to the increase of volatility and correlations after a multi-jump.

To this purpose, we consider an investor that allocates her wealth among the 16 assets considered in our empirical exercise and a risk-free asset. We assume the agent preferences can be fully described by the mean and the variance of her portfolio. We denote by $\mathbf{w}$ the 16×1 vector of relative weights invested in the stocks, while $1 - \mathbf{w}'\mathbf{1}$ is portfolio fraction invested in the risk-free asset whose return is r; we denote by $\mathbf{1}$ a 16×1 vector of ones. The investor optimal allocation is derived from the maximization of the mean-variance utility

$$\max_{\mathbf{w}} \mathbf{w}' \left(\boldsymbol{\mu} - r\right) - \frac{\gamma}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \tag{21}$$

where $\boldsymbol{\Sigma}$ is the 16×16 covariance matrix of the stocks, $\boldsymbol{\mu}$ is the 16×1 vector of expected returns and γ is the risk aversion coefficient. The well-known solution to this problem is

$$\mathbf{w} = \frac{1}{\gamma}\boldsymbol{\Sigma}^{-1}\left(\boldsymbol{\mu} - r\right). \tag{22}$$

We assume that expected returns are in keeping with the equilibrium paradigm. We consider a simple CAPM model:

$$\boldsymbol{\mu} = r + \boldsymbol{\beta}\left(\mu_{mkt} - r\right), \tag{23}$$

where μ_{mkt} is the expected return on the market portfolio and $\boldsymbol{\beta}$ is the vector of betas, estimated using daily returns (the market portfolio proxy being the S&P500 index) and reported in Table 6. We set $\mu_{mkt} = 7.25\%$ and $r = 4.65\%$, corresponding to the 1950-2014 observed annual return on S&P500 and 3-months T-bill respectively. We assume that the investor observes an initial covariance matrix $\mathbf{\Sigma}$ equal to the average daily covariance matrix estimated on data, and allocates her wealth according to the risk aversion γ, the covariance matrix $\mathbf{\Sigma}$ and the vector of equilibrium returns given by Eq. (23), by using formula (22).

According to our previous results we assume that, after a multi-jump, the covariance matrix changes from $\mathbf{\Sigma}$ to $\mathbf{\Sigma}^{MJ}$. The new matrix $\mathbf{\Sigma}^{MJ}$ is computed adding to $\mathbf{\Sigma}$ the average impact on log-volatility and correlations implied by the models (19), (20) and reported in Tables 11 and 12 (we use the values of the regression without idiosyncratic jumps and co-jumps, respectively). Thus, the change from $\mathbf{\Sigma}$ to $\mathbf{\Sigma}^{MJ}$ represents the typical volatility and correlations inflation due to the occurrence of a multi-jump.

A multi-jump would thus lead to a change in the agent optimal allocation. Therefore, if the agent is not aware of the impact of the multi-jump on volatility and correlations, she faces a potential loss, since her allocation becomes sub-optimal and the occurrence of the multi-jump endanger the diversification benefits of her portfolio since the average correlation increases. If she decides to re-allocate, though, she could still face a utility loss since the correlation increase could anyway reduce the diversification potential of the new optimal portfolio.

In order to determine the impact of multi-jumps on utility, we quantify the utility loss in terms of the *certainty equivalent return* (CEQ), which we compute as

$$CEQ = \mu_P - \frac{\gamma}{2}\sigma_P^2$$

where μ_P and σ_P are the mean and the standard deviation of the investor's portfolio, see e.g. DeMiguel et al. (2009) for a similar comparison strategy. We consider two different cases. The first case corresponds to an agent who does not rebalance the portfolio after the multi-jump arrival. We quantify the loss associated with the covariance matrix change due to multi-jumps, which we compute as

$$\Delta CEQ_1 = \frac{\gamma}{2}\mathbf{w}'\left(\mathbf{\Sigma}^{MJ} - \mathbf{\Sigma}\right)\mathbf{w}.$$

In this case we evaluate the loss due to both the increase in portfolio variance[7] and the reduction in diversification benefits of the portfolio held by the investor.

In the second case, which corresponds to an investor which rebalances the portfolio after the multi-jump arrival, we compute the loss due to the effect of the multi-jumps both in the covariance and in the portfolio allocation. The second loss reads

$$\Delta CEQ_2 = \left(\mathbf{w} - \mathbf{w}^{\mathbf{MJ}}\right)'(\boldsymbol{\mu} - r) - \frac{\gamma}{2}\left(\mathbf{w}'\mathbf{\Sigma}\mathbf{w} - \mathbf{w}^{MJ\prime}\mathbf{\Sigma}^{MJ}\mathbf{w}^{MJ}\right),$$

where $\mathbf{w}^{MJ}$ is the optimal portfolio after the arrival of the multi-jump. We ignore here the impact of transaction costs for rebalancing. Thus, while the change in utility without rebalancing is associated only with the risk component of the CEQ, when rebalancing the CEQ changes both for a change in the risk of the optimal portfolios as well as for changes in the expected return.

Table 13 reports the results for various levels of risk aversion, while Figure 8 shows the loss in terms of certainty equivalent of the two strategies, again for various levels of risk aversion. Results indicate a substantial impact, in terms of expected utility, due to the occurrence of multi-jumps, ranging from 42 basis points when the investor holds a levered position and is invested roughly 200% in the stocks (borrowing at the risk-free rate), to 4 basis points for an investor who invests about 20% of her wealth in the stocks, this choice depending on her level of risk aversion. These figures refer to the case in which the investor does not rebalance her portfolio; however, if the portfolio is rebalanced (transaction costs ignored) the loss in terms of certainty equivalent is very similar, ranging from 31 to 3 basis points in the two cases mentioned above. The impact is larger with lower risk aversion

[7] We do not consider here the possible impact of multi-jumps on expected returns, and thus we do not consider potential pricing issues associated with a multi-jump factor.

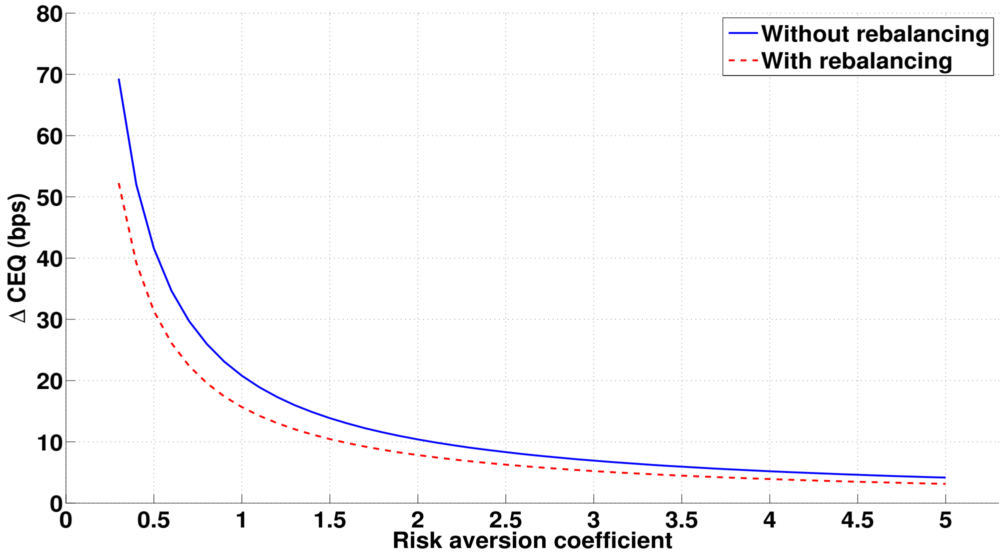

Figure 8: Average certainty equivalent return loss due to the occurrence of a multi-jump, expressed in basis points, as a function of the risk aversion coefficient used. The solid line expresses the loss in the case in which the investor does not rebalance the portfolio after the multi-jump; the dashed line instead is in the case of rebalancing. Rebalancing transaction costs are ignored.

(see Figure 8) since in this case the investor is willing to hold more risky assets, and thus her utility is more impacted by the arrival of the multi-jump.

The standard error on certainty equivalent losses depends on the statistical uncertainty of the impact of multi-jumps on correlations and volatility. The standard errors reported in parenthesis in Table 13 are computed with a parametric bootstrap procedure which draws correlation and volatility impacts on $\mathbf{\Sigma}^{\mathbf{MJ}}$ from the asymptotic distribution implied by the estimates in Tables 11 and 12. The corresponding t-stats do not depend on the risk aversion coefficient γ, and are estimated to be 7.71 for the case without rebalancing and 6.20 for the case with rebalancing. The negative impact of a multi-jump on the investor's utility is thus largely statistically significant in both cases.

The results of our simple exercise are derived in a static setting in which there is only a change in the covariance matrix at a determined point in time. Clearly, in order to thoroughly study the impact of multi-jumps on asset allocation strategies, we would need a fully specified dynamic model in which the presence of multi-jumps is internalized.

Table 13: Reports the results of the asset allocation exercise. The first column reports the risk aversion coefficient γ. Columns 2-5 report the corresponding total investment in stocks $\mathbf{w}'\mathbf{1}$, the portfolio mean μ_P, standard deviation σ_P and certainty equivalent return CEQ, all expressed in percentage form. Columns 6-7 report the new portfolio standard deviation σ_P^{MJ} and the certainty equivalent loss ΔCEQ_1 (in basis points, with standard error in parenthesis) for an investor who does not rebalance the portfolio after the occurrence of a multi-jump (the portfolio mean then does not change). Columns 8-10 report the new portfolio mean $\mu_{P'}^{MJ}$, the new portfolio standard deviation $\sigma_{P'}^{MJ}$ and the certainty equivalent loss ΔCEQ_2 (in basis points, with standard error in parenthesis) for an investor who rebalances the portfolio after the occurrence of a multi-jump. Three stars indicate significance at 99%.

	before the multi-jump				after the multi-jump				
					without rebalancing		with rebalancing		
γ	$\mathbf{w}'\mathbf{1}$ (%)	μ_P (%)	σ_P (%)	CEQ (%)	σ_P^{MJ} (%)	ΔCEQ_1 (bps)	$\mu_{P'}^{MJ}$ (%)	$\sigma_{P'}^{MJ}$ (%)	ΔCEQ_2 (bps)
0.5	199.2	10.07	32.92	7.36	35.36	41.6***	9.44	30.96	31.4***
						(5.39)			(5.06)
1	99.6	7.36	16.46	6.00	17.68	20.8***	7.05	15.48	15.7***
						(2.70)			(2.53)
2	49.8	6.00	8.23	5.33	8.84	10.4***	5.85	7.74	7.8***
						(1.35)			(1.26)
5	19.9	5.19	3.29	4.92	3.54	4.2***	5.13	3.10	3.1***
						(0.54)			(0.51)
10	10.0	4.92	1.65	4.79	1.77	2.1***	4.89	1.55	1.6***
						(0.27)			(0.25)
∞	0.0	4.65	0.00	4.65	0.00	0.0	4.65	0.00	0.0

However, our results suggest that, all the other things remaining equal, an investor aware of the presence of multi-jumps would be more conservative and hold a less risky portfolio with respect to an agent which is unaware of their presence. Indeed, the utility loss due to the presence of multi-jumps increases with variance, so the only way to offset the negative impact on expected utility of multi-jumps is to decrease the variance of the optimal portfolio.

These considerations can be complemented with the theoretical results of Das and Uppal

(2004), who develop a simple model in which an agent, endowed with power utility, allocates wealth among assets whose returns are subject to Normal shocks and also to an additional single Poisson shock which affects all of them simultaneously. Their common Poisson shock can be interpreted as a multi-jump. In this case, the optimal allocation rule is found to be different from that of an agent who ignores jumps (which is still given by Eq. 22) and is such that an investor aware of jumps will invest less in the stocks and hold less variance with respect to an investor who ignores jumps. Further, they also document that the impact of jumps increases with the leverage of the position. They, however, do not assume that the covariance matrix changes after a jump, as we do here. Our calculations show that considering the additional impact on the covariance matrix would strengthen their predicted effects on the optimal allocation strategy, thus inducing an optimal strategy which is even more conservative that what predicted by their model.

We can then conclude that the impact of multi-jumps on the covariance matrix is not only statistically significant, as shown in Sections 5.3 and 5.4, but also economically significant since it will impact negatively, and non negligibly, the utility of a risk averse investor, which is induced to a more conservative asset allocation strategy when multi-jumps and their impact on the covariance matrix are fully included in the model.

6 Concluding remarks

While the recent literature produced an abundant number of theoretical and empirical contributions about the presence of jumps in financial prices and their importance in financial models, little effort has been devoted to multivariate jumps, and this effort was almost exclusively devoted to the case of two assets. However, jumps are (asymptotically) big movements, so that their detection is much easier when they occur together, and this happens with a small but sizable frequency in the stock market.

This paper is thus meant to fill this gap in the literature by introducing a novel test for multi-jump detection for an arbitrary number of stocks. The test is found to be superior to alternatives also in the case $N = 2$, but it delivers its best results when N is large. The test does not need restrictive modeling assumptions, and can naturally trade off size and power via bandwidth selection, for which we propose an automated procedure.

Using a data set of liquid constituents of S&P500, we provide clear-cut evidence on the presence of multi-jumps in the market. Multi-jumps among several stocks are found to be rare but economically and statistically significant events. We show that they are correlated with big increases in the variance risk premium (which almost doubles in day in which there is a downward multi-jump) and we document an increase in both stock volatilities (+20% on average) and stock correlations (of stronger magnitude, +28% on average) in days following their occurrence. These findings have a substantial and statistically significant impact on asset allocation, which is quantifiable in a loss of roughly 20 bps per year for a mean-variance investor with unit risk aversion. Multi-jumps would then determine a more conservative asset allocation strategy when their impact on the covariance matrix is fully included in the allocation model.

Importantly, testing for multi-jumps in a modest panel of stocks is shown to be much more informative than testing for univariate jumps in the stocks and the equity index. Jumps in the index, indeed, cannot be associated to changes to the variance risk premium despite the growing theoretical and empirical evidence suggesting that this should be the case, and jumps and co-jumps in individual stocks cannot be associated to increases in their volatility and correlations. We conclude that the test introduced here should replace univariate tests when looking for systemic market events which can affect market variables such as second moments and risk premia. We thus believe that the tool introduced in this paper could become of fundamental help in assessing the financial role, both theoretical and empirical, of jumps in the market.

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A Mathematical proofs

Proof of Theorem 3.1

Define Threshold Realized Variance as in Mancini (2009):

$$\mathsf{TRV}^N(X^{(i)}) = \sum_{j=1}^{n} \left(\Delta_j X^{(i)}\right)^2 I_{\{|\Delta_j X^{(i)}| \le H^{(i)}_{j\Delta,n}\}}, \qquad i = 1, \ldots, N. \tag{24}$$

where I_A is the indicator function of the set A. Now, using the fact that under the assumptions $\frac{\Delta_j X^{(i)}}{H_{j\Delta,n}} = O_p\left(\frac{1}{n^{1/2}h_n}\right) \to 0$ (the stochastic bandwidth can be dealt as in Theorem 2.3 of Corsi et al., 2010), write:

$$\begin{aligned}
\mathsf{SRV}(X^{(i)}) - \mathsf{TRV}^N(X^{(i)}) &= \sum_{j=1}^{n} \left(\Delta_j X^{(i)}\right)^2 \left(K\left(\frac{\Delta_j X^{(i)}}{H^{(i)}_{j\Delta,n}}\right) - I_{\{|\Delta_j X^{(i)}| \le H^{(i)}_{j\Delta,n}\}}\right) \\
&= \sum_{\left|\frac{\Delta_j X^{(i)}}{H_{j\Delta,n}}\right| \le 1} \left(\Delta_j X^{(i)}\right)^2 \left(K\left(\frac{\Delta_j X^{(i)}}{H^{(i)}_{j\Delta,n}}\right) - 1\right) \\
&\quad + \sum_{\left|\frac{\Delta_j X^{(i)}}{H_{j\Delta,n}}\right| > 1} \left(\Delta_j X^{(i)}\right)^2 K\left(\frac{\Delta_j X^{(i)}}{H^{(i)}_{j\Delta,n}}\right) \\
(\textit{mean value theorem}) &= \sum_{\left|\frac{\Delta_j X^{(i)}}{H_{j\Delta,n}}\right| \le 1} \left(\Delta_j X^{(i)}\right)^2 K'\left(\frac{\xi_j}{H^{(i)}_{j\Delta,n}}\right) \frac{\Delta_j X^{(i)}}{H^{(i)}_{j\Delta,n}} \\
&\quad + \sum_{j=1}^{n} \left(\Delta_j X^{(i)}\right)^2 K\left(\frac{\Delta_j X^{(i)}}{H^{(i)}_{j\Delta,n}}\right) I_{\{|\Delta_j X^{(i)}| > H^{(i)}_{j\Delta,n}\}},
\end{aligned}$$

for a suitable random sequence ξ_j such that $|\xi_j| \le \left|\Delta_j X^{(i)}\right|$. By the boundedness of K' and the bandwidth process and results in Mancini (2009), the absolute value of the first term is dominated by

$$C \cdot \mathsf{TRV}^N(X^{(i)}) \frac{\sqrt{\Delta \log \frac{1}{\Delta}}}{h_n} \xrightarrow{p} 0,$$

where C is a suitable constant. For the second term, in the case of finite activity Mancini (2009) proved that when $\left|\Delta_j X^{(i)}\right| > H^{(i)}_{j\Delta,n}$ only the terms where jumps occurred are left in $\Delta_j X^{(i)}$, so that $\frac{|\Delta_j X^{(i)}|}{H^{(i)}_{j\Delta,n}} \xrightarrow{p} \infty$ and by the continuous mapping theorem $K\left(\frac{\Delta_j X^{(i)}}{H^{(i)}_{j\Delta,n}}\right) \xrightarrow{p} 0$. Using Corollary 2 in Mancini (2009) we get the desired result (see Mancini and Gobbi, 2012 for a generalizations to the semi-martingale case). ■

Proof of Theorem 3.2

Write:

$$\widetilde{\mathsf{SRV}}^N(X^{(i)}) - \mathsf{TRV}^N(X^{(i)}) = \mathsf{SRV}(X^{(i)}) - \mathsf{TRV}^N(X^{(i)}) + \underbrace{\sum_{j=1}^{n} \left(\Delta_j X^{(i)}\right)^2 \left(\prod_{k=1}^{N} \left(1 - K\left(\frac{\Delta_j X^{(k)}}{H^{(k)}_{j\Delta,n}}\right)\right)\right)}_{A_n},$$

where the first term vanishes in probability asymptotically given Theorem 3.1. For the second term, write:

$$\begin{aligned} A_n = & \sum_{\forall k \in 1,\dots N: \left|\frac{\Delta_j X^{(k)}}{H^{(k)}_{j\Delta,n}}\right| > 1} \left(\Delta_j X^{(i)}\right)^2 \left(\prod_{k=1}^{N} \left(1 - K\left(\frac{\Delta_j X^{(k)}}{H^{(k)}_{j\Delta,n}}\right)\right)\right) \\ & + \sum_{\exists k \in 1,\dots N: \left|\frac{\Delta_j X^{(k)}}{H^{(k)}_{j\Delta,n}}\right| \leq 1} \left(\Delta_j X^{(i)}\right)^2 \left(\prod_{k=1}^{N} \left(1 - K\left(\frac{\Delta_j X^{(k)}}{H^{(k)}_{j\Delta,n}}\right)\right)\right) \\ := & A_{n,1} + A_{n,2} \end{aligned}$$

The term $A_{n,2}$ vanishes in probability since, for the k's such that $\left|\frac{\Delta_j X^{(k)}}{H^{(k)}_{j\Delta,n}}\right| \leq 1$, we can write, using the mean value theorem:

$$1 - K\left(\frac{\Delta_j X^{(k)}}{H^{(k)}_{j\Delta,n}}\right) = K'(\xi^{(k)}_j)\frac{\Delta_j X^{(k)}}{H^{(k)}_{j\Delta,n}} \to 0,$$

while for the remaining k's,

$$1 - K\left(\frac{\Delta_j X^{(k')}}{H^{(k')}_{j\Delta,n}}\right) \xrightarrow{p} 1$$

as before. Thus we remain with $A_{n,1}$. Consider first the case with the indicator kernel:

$$\widetilde{A}_{n,1} := \sum_{j=1}^{n} \left(\Delta_j X^{(i)}\right)^2 \prod_{k=1}^{N} I_{\{|\Delta_j X^{(k)}| > H^{(k)}_{j\Delta,n}\}}$$

which tends to 0 on $\overline{\Omega}^N_T$ (Mancini and Gobbi, 2012, straightforward generalization of Theorem 4.2). On $\Omega^{MJ,N}_T$ $\widetilde{A}_{n,1}$ tends to $\sum_{\Delta X^{(1)}_t \dots \Delta X^{(N)}_t \neq 0} \left(\Delta X^{(i)}_t\right)^2$. Indeed, $\widetilde{A}_{n,1}$ differs from the realized variance of the process $X^{(i)\prime}$ by a finite number of asymptotically vanishing terms, where $X^{(i)\prime}$ is defined by the sum of continuous part of $X^{(i)}$ and the process of multi-jumps:

$$X^{(i)\prime} = X^{(i)} - \sum_{t \leq T} \Delta X^{(i)}_t I_{\{\Delta X^{(i)}_t \neq 0 \bigcap \Delta X^{(1)}_t \dots \Delta X^{(N)}_t = 0\}}. \tag{25}$$

The proof for $A_{n,1}$ then follows by the continuous mapping theorem as for Theorem 3.1. ■

Proof of Theorem 3.3

Given Theorems 3.1 and 3.2, it is sufficient to show that a vector of statistics $S'(X^{(1)}),...,S'(X^{(N)})$, where

$$S'(X^{(i)}) = \frac{\sum_{j=1}^{n} \left|\Delta_j X_c^{(i)}\right|^2 (1-\eta_j^i)}{\sqrt{V_\eta \, \mathsf{SQ}(X^{(i)})}}, \tag{26}$$

where X_c is the purely continuous part of X, converges stably in law to multivariate normal distribution with identity covariance matrix. In this case, we can indeed use Theorem 2 in Podolskij and Ziggel (2007) to get the desired result. To this purpose, it is enough to prove that for generic constants $c_1,...,c_N$ the linear combination

$$c_1 S'(X^{(1)}) + ... + c_N S'(X^{(N)}) \tag{27}$$

converges to a random normal variable with zero mean and variance $(c_1^2 + ... + c_N^2)$. In order to simplify the notation the proof is carried out in the case $N = 2$.

Denote by $\mathbf{E}^*[\cdot]$, $\mathbf{Var}^*[\cdot]$ and $\mathbf{Cov}^*[\cdot]$ respectively expectation, variance and covariance conditional on the observed values of X. We have:

$$\mathbf{E}^*\left[\sum_{j=1}^{n} \left|\Delta_j X_c^{(i)}\right|^2 (1-\eta_j^i)\right] = \left(1 - \mathbf{E}^*\left[\eta_1^i\right]\right) \sum_{j=1}^{n} \left|\Delta_j X_c^{(i)}\right|^2 = 0, \tag{28}$$

for any i, and

$$\begin{aligned}\mathbf{Var}^*\left[\sum_{j=1}^{n} \left|\Delta_j X_c^{(i)}\right|^2 (1-\eta_j^i)\right] &= \left(\mathbf{E}^*\left[(\eta_1^i)^2\right] - (\mathbf{E}^*\left[\eta_1^i\right])^2\right) \sum_{j=1}^{n} \left|\Delta_j X_c^{(i)}\right|^4 \\ &= \mathbf{Var}^*\left[\eta_1^i\right] \sum_{j=1}^{n} \left|\Delta_j X_c^{(i)}\right|^4;\end{aligned} \tag{29}$$

$$\begin{aligned}&\mathbf{Cov}^*\left[\sum_{j=1}^{n} \left|\Delta_j X_c^{(i)}\right|^2 (1-\eta_j^i), \sum_{j=1}^{n} \left|\Delta_j X_c^{(k)}\right|^2 (1-\eta_j^k)\right] \\ &= \mathbf{Cov}^*\left[\sum_{j=1}^{n} \left|\Delta_j X_c^{(i)}\right|^2 \eta_j^i, \sum_{j=1}^{n} \left|\Delta_j X_c^{(k)}\right|^2 \eta_j^k\right] \\ &= \left(\mathbf{E}^*\left[\eta_j^i \eta_j^k\right] - \mathbf{E}^*\left[\eta_j^i\right] \mathbf{E}^*\left[\eta_j^k\right]\right) \left(\sum_{j=1}^{n} \left|\Delta_j X_c^{(i)}\right|^2\right) \left(\sum_{j=1}^{n} \left|\Delta_j X_c^{(k)}\right|^2\right) \\ &= 0,\end{aligned} \tag{30}$$

where the last implication follows from the independence of η_j^i from η_j^k when $i \neq k$. The rest follows as in Podolskij and Ziggel (2010). ■

B Quadratic variation measures and implementation

B.1 Kernel and bandwidth selection

Numerical experiments show that the test is more stable when the kernel is smooth, and that the kernel shape is not crucial. In simulations and empirical work we use a (normalized) Gaussian kernel

$$K(x) = e^{-x^2/2}.$$

The bandwidth process is expressed as a function of the local variance, as follows:

$$H_{t,n} = h_n \cdot \widehat{\sigma}_t^{(i)} \sqrt{\frac{T}{n}}, \tag{31}$$

where $\widehat{\sigma}_t^{(i)}$ is a point estimator of $\sigma_t^{(i)}$, obtained as in Corsi et al. (2010). The bandwidth parameter h_n is found by an automated procedure based on simulations. Since we normalize to local variance, we replicate the null hypothesis by simulating N iid standard Normal variates correlated as in the data (we use average the average correlation matrix in the sample), and then add a given number M of multi-jumps (with $M < N$) of desired size. For each set of replications, we then find h_n such that the average size of of the test, computed on replications, is 5% at 95% confidence intervals. Such a bandwidth should be used for testing, approximately, for at least $M+1$ multi-jumps. Figure 9 shows the result for different N and M. Matlab© code for automated bandwidth selection, and for the multi-jump test, is available from our web pages.

We can see that the number of multi-jumps under the null affects the bandwidth, but only slightly; the effect is stronger when the number of multi-jumps under the null approaches N. A conservative choice should then be $h_n = 6.5$ (for $N = 2$) or $h_n = 4.5$ (for $N = 16$). For all N, the choice $h_n \simeq 1$ is the less conservative. For $N = 16$, the range should then be $1 \leq h_n \leq 3$ depending on which kind of signal one is interested in. This is actually beneficial to the testing procedure since when a multi-jump occurs some of the associated jumps might be small, or could happen with a small lag.

Motivated by these results, in Section 5 we use: $h_n = 6.5$ when testing for co-jumps ($N = 2$); $1.5 \leq h_n \leq 3$ when testing for many multi-jumps in the $N = 16$ case; $h_n = 2$ (with $N = 16$) in the regressions with variance premia, volatility and correlations.

Bandwidth Selection rule

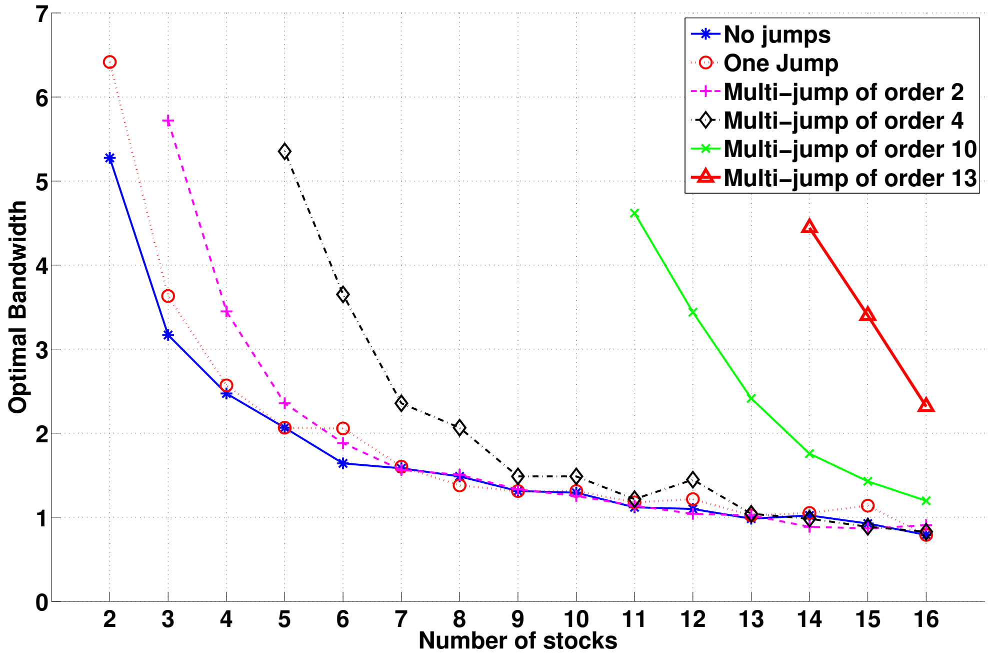

Figure 9: Shows the optimal bandwidth parameter h_n obtained by calibrating the correct size on simulated experiments in which the null hypothesis is without jumps or contaminated by one jumps, or multi-jump of order 2, 4, 10 and 13, for various values of the number of stocks. The table indicates, for example, that the choice with $h_n \simeq 6.5(4.5)$ is the most conservative with $N = 2$ ($N = 16$), and $h_n \simeq 1$ is the less conservative.

B.2 Jump tests

Here we discuss the implementation of the univariate jump tests and the Jacod and Todorov co-jump test used in the Monte Carlo experiments and in the empirical applications.

We start with univariate test, for which we use three tests: BNS, CPR and ABD. The null hypothesis for all the three tests is the absence of jumps. The BNS test introduced by Barndorff-Nielsen and Shephard (2006) is based on the comparison of the realized variance and bipower variation, which are respectively non-robust and robust to jumps measures of the integrated variance. For a 1-dimensional process X the BNS test statistic has the following form:

$$\Delta^{-1/2} \frac{1 - \frac{\mathsf{MV}(X;[1,1])}{\mathsf{RV}(X)}}{\sqrt{(\pi/4 + \pi - 5)\max(1, \frac{\mathsf{MV}(X;[4/3,4/3,4/3])}{\mathsf{MV}(X;[1,1])})}} \Rightarrow \mathcal{N}(0,1), \tag{32}$$

where the convergence in distribution holds under the null,

$$\mathsf{MV}(X;[r_1,r_2,...,r_m]) = \left(\prod_{j=1}^{m}(\mu_{r_j})^{-1}\right)(\Delta)^{1-\frac{r_1+...+r_m}{2}} \cdot \sum_{i=1}^{n-m+1}\prod_{j=1}^{m}|\Delta_{i+j}X|^{r_j}\,, \tag{33}$$

$$\mathsf{RV}(X) = \mathsf{MV}(X;[2,0,...,0]). \tag{34}$$

The CPR test, introduced by Corsi et al. (2010), is similar to BNS, however it uses threshold bipower variation instead of simple bipower variation. Moreover, the special finite sample correction is applied to the test statistic in order to improve the size of the test in finite samples. The test statistics has the following form:

$$\Delta^{-1/2}\frac{(\mathsf{RV}(X)-\mathsf{C\text{-}TMV}(X;[1,1]))\,\mathsf{RV}(X)^{-1}}{\sqrt{(\pi/4+\pi-5)\max\left(1,\frac{\mathsf{C\text{-}TMV}(X;[4/3,4/3,4/3])}{\mathsf{C\text{-}TMV}^2(X;[1,1])}\right)}} \Rightarrow \mathcal{N}(0,1), \tag{35}$$

where the convergence in distribution holds under the null, and $\mathsf{C\text{-}TMV}(X;\mathbf{r})$ is the corrected version of threshold multipower estimator. The correction consists in replacing returns $\Delta_j X > H_j$ by their expectations under the assumption $\Delta_j X \sim \mathcal{N}(0,\sigma_j^2)$:

$$\mathsf{C\text{-}TMV}(\mathbf{r}) = \left(\prod_{j=1}^{m}(\mu_{r_j})^{-1}\right)(\Delta)^{1-\frac{r_1+...+r_m}{2}} \cdot \sum_{i=1}^{n-m+1}\prod_{j=0}^{m-1} Z_r(\Delta X_{i+j},H_{i+j}), \tag{36}$$

where the function $Z_r(x,y)$ is:

$$Z_r(x,y) = \begin{cases} |x|^r & if\ x^2 \le y \\ \dfrac{1}{2N(-c_\vartheta)\sqrt{\pi}}\left(\dfrac{2}{c_\vartheta^2}y\right)^{\frac{\gamma}{2}}\Gamma\left(\dfrac{r+1}{2},\dfrac{c_\vartheta^2}{2}\right) & if\ x^2 > y \end{cases} \tag{37}$$

To implement the CPR test we always use the threshold threshold defined according to (31) with the constant $h_n = 3$.

The ABD test a modification of the Lee and Mykland (2008) test which considers the set of all intraday standardized returns:

$$z_i = \Delta_i X/\sqrt{V_i}, \tag{38}$$

where V_i is the estimate of spot volatility at time corresponding to the intraday return number i, $i = 1,...,n$. Under the null each z_i is asymptotically standard normal. Hence, one can test the absence of jumps by comparing all standardized intraday returns with the normal critical values. In order to guarantee that the daily first type error does not exceed a given level α, the size of each intraday test must be equal to $\beta = 1-(1-\alpha)^{\Delta}$.

The alternative co-jump test of Jacod and Todorov (2009) used in Section 4.1 is computed as follows. For a 2-dimensional function $f(x_1, x_2)$, define power variation by:

$$V(f, k\Delta) = \sum_{i=1}^{[n/k]} f(X_{ik\Delta} - X_{(i-1)k\Delta}), \tag{39}$$

where $k \geq 2$ is an integer. Let

$$f(x_1, x_2) = (x_1 x_2)^2, \;\; g_1(x_1, x_2) = (x_1)^4, \;\; g_2(x_1, x_2) = (x_2)^4, \tag{40}$$

and consider statistics:

$$\Phi_n^j = \frac{V(f, k\Delta)}{V(f, \Delta)}, \quad \Phi_n^d = \frac{V(f, \Delta)}{\sqrt{V(g_1, \Delta)V(g_2, \Delta)}}. \tag{41}$$

These two statistics are studied in the subset Ω_T^c of Ω in which neither X_1 nor X_2 is purely continuous. The test thus needs preliminary tests for jumps in the two series to be implemented. Denote by $\Omega_T^{jj} = \Omega_T^N \setminus \Omega_T^c$, the set of trajectories on which there are idiosyncratic jumps in X_1 and X_2 but no cojumps. Jacod and Todorov (2009) show that $\Phi_n^j \xrightarrow{p} 1$ on $\overline{\Omega}_T^N$, while $\Phi_n^d \xrightarrow{p} 0$ on Ω_T^{jj}. Hence Φ_n^j is used to test the null hypothesis of the presence of co-jumps, and Φ_n^d to test the null of absence of co-jumps. The variance of the tests depend on the covariances of the two series. For detail on constructing the critical regions and the choice of k, see Jacod and Todorov (2009).

B.3 Quadratic variation measures

In order to measure the continuous covariations among asset prices we use an approach based on the polarization of bipower variation, as in Barndorff-Nielsen and Shephard (2003), adapted for threshold bipower variations (Corsi et al., 2010). Let $Cov_t^{(i,j)}$ denote a measure of continuous quadratic covariation of two stocks i and j at day t, and $V_t^{(i)} = Cov_t^{(i,i)}$ be the measure of continuous integrated variance of stock i at day t. We first denote, for $j = 1, \ldots, n$ by

$$\not\Delta_j X = (\Delta_j X) I_{\{|\Delta_j X| \leq \theta_j\}}$$

where θ_j is a threshold computed as in Corsi et al. (2010), and by $\not X$ the cumulated price obtained with truncated returns $\not\Delta_j X$. We then set

$$Cov_t^{(i,j)} = \frac{1}{4}(\mathsf{BV}(\not X^{(i)} + \not X^{(j)}) - \mathsf{BV}(\not X^{(i)} - \not X^{(j)})), \tag{42}$$

where

$$\mathsf{BV}(X) = (\mu_1)^{-2} \sum_{j=1}^{\lfloor T/\Delta \rfloor -1} |\Delta_j X||\Delta_{j+1} X|, \tag{43}$$

is threshold bipower variation, with $\mu_1 = \sqrt{2/\pi}$; $Cov_t^{(i,j)}$ is a consistent and jump-robust estimator of the continuous part of the covariation process of the log-price processes $X^{(i)}$ and $X^{(j)}$.

The intraday measure of correlation between two stocks is defined by

$$Corr_t^{(i,j)} = \frac{Cov_t^{(i,j)}}{\sqrt{V_t^{(i)} V_t^{(j)}}}. \tag{44}$$

It can be mapped into the whole real line by the use of Fisher transformation

$$\widetilde{\rho}_t^{(i,j)} = \log \frac{1 + Corr_t^{(i,j)}}{1 - Corr_t^{(i,j)}}, \tag{45}$$

which is used in regression analysis in Section 5.

C SURE representation and estimation

This section describes the estimation and testing approach adopted in the empirical analyses to verify the joint significance of multi-jumps coefficients. Given that our purpose is to test restrictions across equations, a simultaneous equation system must be considered. The most appropriate setting is that of Seemingly Unrelated Regression Equations (SURE) which reads as

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_m \end{bmatrix} = \begin{bmatrix} X_1 & 0 & \dots & 0 \\ 0 & X_2 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & X_m \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_m \end{bmatrix}, \tag{46}$$

where Y_j is a $T-$dimensional vector containing the sample data for the $j-th$ dependent variable, X_j is the $T \times K_j$ matrix of explanatory variables in the $j-th$ equation, while β_j is the associated K_j-dimensional vector of regression coefficients. In addition, the error terms are assumed to be homoskedastic, serially uncorrelated, not cross-correlated, but are contemporaneously correlated, implying that $\mathbb{E}[\epsilon\epsilon'] = \Omega \otimes I_T$, Ω being the covariance matrix such that $\mathbb{E}[\epsilon_t \epsilon_t']$ with $\epsilon_t = [\epsilon_{1,t}, \epsilon_{2,t}, \dots, \epsilon_{m,t}]$. In our case, the dependent variable Y_j contains either the sample data of the log-realized volatility for one stock, $Y_j = \log V_t^{(i)}$, or the sample data for of the Fisher transformation of realized correlations between two stocks, $Y_j = \tilde{\rho}_t^{(i,l)}$. In the first case $m = 16$, while in the latter $m = 120$. Moreover, X_j might contain HAR terms as well as jump, co-jump and multi-jump indicator variables. Note that, the matrices $X_j, \quad j = 1, 2, \ldots m$ are specific of each dependent variables, and thus of each equation, but include both equation-specific elements, the HAR terms, jump and co-jump variables, as well as common elements, when the multi-jump indicator variable is taken into account. The SURE model allows testing cross-equation restrictions when the errors are contemporaneously correlated. In the present study, we cannot exclude a-priori that the innovations of different equations are independent.

In a SURE model, GLS estimation is required to take into account the covariance structure of the innovations. Feasible GLS estimation is normally performed, by first applying OLS on a univariate basis, then recovering the covariance of innovations from the univariate regression residuals and, finally, applying GLS with the estimated residuals covariance.

Nevertheless, the use of FGLS requires a diagnostic check on the first stage residuals, given the presence of lagged dependent variables (the HAR terms) on the explanatory variables X_j, and because the true unrestricted model is of a $VAR-$type (the general model is a $VARX(22)$ whose parameters are highly restricted in the VAR part). In fact, if the first stage residuals would be serially correlated, this could lead to biases in the first stage OLS. Those biases might be associated with a correlation between regressors and innovations. Differently, if first stage residuals show evidences of serial cross-correlations, those would signal, on the hand, the need of a less restricted HAR component, for instance allowing for interactions across equations, and on the other hand would lead to biases due to the omitted variable problem.

In our empirical analyses, diagnostic checks on the first stage residuals show those problems are not a real concern: in very few cases diagnostic tests lead to a rejection of the null hypotheses, and the first lags in the autocorrelation and cross correlation functions were showing values at maximum equal to 0.1. Therefore, we believe those results support the use of FGLS. However, diagnostic tests show evidences of heteroskedasticity. The latter is taken into account in the estimation of Feasible GLS standard errors by adopting a White-type correction. We replace the traditional FGLS estimator of the parameter covariance matrix with the following expression

$$\mathbb{V}\left[\beta_{FGLS}\right] = \left(X'\left(\hat{\Omega}^{-1} \otimes I_T\right)X\right)^{-1} S \left(X'\left(\hat{\Omega}^{-1} \otimes I_T\right)X\right)^{-1}, \tag{47}$$

where

$$S = \sum_{t=1}^{T} X_t \hat{\Omega}^{-1} \varepsilon_t \varepsilon_t' \hat{\Omega}^{-1} X_t'. \quad (48)$$

and Ω is estimated on the first stage regression residuals. Note that the matrix X_t is block-diagonal with diagonal elements equal to the column-vector $X_{j,t}$; it contains the time t explanatory variables for equation j.