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Dipartimento di Scienze Economiche "Marco Fanno"

OPTIMAL REDISTRIBUTION WITH PRODUCTIVE SOCIAL SERVICES

LUCIANO G. GRECO
Università di Padova

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Luciano G. GRECO*

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Abstract

We analyze the optimality of alternative mechanisms of public provision of private goods affecting the productive capacity of households (e.g. education, health-care) rather than directly their welfare. Opting out mechanisms - often considered a tool to focus social expenditure - are proven to be welfare improving under the assumption that the provided good is not a substitute of households' exogenous productive capacity (say, inherited wealth). Conversely, when publicly provided goods are substitute of inherited productive capacity, topping up mechanisms prove more efficient.

Keywords: In-kind transfers; public provision of private goods; opting out; topping up
JEL classification: H42, H21

*Dipartimento di Scienze economiche, Università degli Studi di Padova, via del Santo 33 - 35123 Padova (Italy), luciano.greco@unipd.it. I thank for their useful comments and suggestions Claude d'Aspremont, Massimo Bordignon, Micael Castanheira, Jean Hindriks, Maurice Marchand, Enrico Minelli, and participants at seminars in PET03 Conference in Peking and 62nd IIPF Conference in Cyprus.

1 Introduction

In many countries, public sector is often responsible for the provision of a wide range of *social services* (e.g. education, health-care, child-, elderly-care, etc.). The specific pattern of social programs is rather articulated across countries: some countries provide a wide range (possibly with relatively high quantity and quality) of social services (e.g. Scandinavian countries), while others provide a relatively narrow set of such services (e.g. Chile, US). The patterns shaping the structure, diversification, and functioning of the welfare states are related to political and economic history of considered countries and, to some extent, to their social and cultural heritage, e.g. Alesina and Glaeser (2004) and Lindbeck (2006). However, in the perspective of reforming welfare states - in order to improve their insurance and redistribution capacity and to reduce distortions related to the second best constraints, it is worth to explore the efficiency features of social programs, taking into account the economic nature of social services.

Social services are, broadly speaking, *private* goods[1]. What normative and positive arguments explain public sector involvement in the provision of such services? Which goods have to be provided? What can be said of their "redistributive" power? And what is the optimal provision framework? These interrelated questions have been addressed by the economic literature on the basis of an interpretation of social services as private *consumption* goods. The alternative view that social services are more related with income production and productive capacity of households is, of course, present in the economic literature - e.g. Aghion and Howitt (1998), Balestrino (1999, p. 346) and López-Casanovas *et al.* (2005), but its implication in terms of specific normative and positive answers to the above questions is still poor.

[1] In this paper, the concept of social services embrace a wider cluster of publicly provided goods, including health-care and education, with respect to statistical definitions that are internationally consolidated (Adema and Ladaique, 2005, p. 7). Moreover, we will abstract from any external effect of considered services. However, in cases such as health and education, services are not pure private goods: some positive externality is relevant. Externalities has been often incorporated in endogenous growth analyses, e.g. Aghion and Howitt (1998), and provide another normative explanation of public policies.

The normative analysis of public provision of private goods has addressed these questions in the framework of the theoretical debate about optimal redistribution tools[2]. The traditional theory, relying on a *first best* approach, claimed the superiority of cash transfers with respect to public provision of any private good. This view has been challenged by several contributions in the last twenty-five years[3]. The seminal paper of Nichols and Zeckhauser (1982) first showed that, in a *second best* economy, the efficiency of redistribution from rich to poor can be improved by in-kind transfers (or equivalently by commodity-specific subsidies or work requirements). The crucial result stressed by Nichols and Zechauser (and confirmed by subsequent contributions) is that in-kind transfers are useful when exogenous features of individuals (say, individual preferences) affect the demand of publicly provided goods: e.g. complementarity between child-care and labor implies that public provision of child-care can improve social welfare in second best economies.

Guesnerie and Roberts (1984) provided a generalization of these results: quotas (i.e. constraints to individuals' consumption of some goods) improve welfare, by relaxing individuals' incentive constraints to taxation. Moreover, Guesnerie and Roberts pointed out that anonymous quantity controls (say, the public provision of a good that households can accept - by opting in the public scheme - or refuse - by opting out) are more powerful than optimal (anonymous) taxation, given that the self-selection mechanism involved in public provision elicits more information about individuals.

The redistribution power of in-kind transfers implemented via self-selection mechanisms (in the following, *opting-out schemes*) was subsequently investigated by other authors. Blackorby and Donaldson (1988) and Besley and Coate (1991) found that pub-

[2]Other research strands that we do not consider in our analysis have explored the political economy arguments explaining the role of the public sector in providing private goods and its working. Epple and Romano (1996a,b) showed that median voter assumption may be very restrictive in a model with public and private alternatives. Moreover, a conflict between the middle class, on the one side, and the low- and high-income classes, on the other, may arise about the optimal level of public provision. However, Blomquist and Christiansen (1999) point out that positive analysis may be nevertheless compatible with efficiency.

[3]See Balestrino (1999, 2000) for a comprehensive review of this literature.

lic provision schemes outperform taxing and subsidizing as redistribution tools. Munro (1992) and Blomquist and Christiansen (1995) focussed on taxation-efficiency: opting out schemes slacken second best (self-selection) constraints limiting optimal income and commodity taxation, and - whenever taxation is constrained (e.g. linear as in our paper) - they enhance redistribution policy, replicating the functioning of non-linear taxation mechanisms.

The efficiency-enhancing role of public provision mechanisms is not a specific feature of opting-out schemes. Boadway and Marchand (1995) and Cremer and Gahvari (1997) considered an alternative mechanism based on uniform public provision and allowing households to privately supplement it (in the following, *topping up schemes*). They found that also these schemes are welfare-improving, in the framework of full-fledged optimal (non-linear) taxation schemes[4], because of the relaxation of taxation constraints they involve.

Blomquist and Christiansen (1998a,b) investigated the interesting issue of the optimal public provision mechanism, depending on the structure of the economy (i.e. of households' heterogeneity in preferences), in the framework of optimal labor-income taxation. Namely, Blomquist and Christiansen (1998a) find that opting-out schemes are welfare-improving when private demand for publicly provided good is increasing in leisure, while topping up schemes are optimal when it is decreasing.

The crucial assumptions underlying these findings are that publicly provided goods are *normal* (say, their demand increases with income) and that a secondary (black or formal) market of provided good is excluded. The latter assumption is relatively mild, as far as goods involved in social services cannot be technologically resold (say, education or health-care). Normality of publicly provided goods is commonly considered a fair approximation of reality[5], though - as it will be clear in the following - it is related to the conventional

[4]Cremer and Gahvari (1997) consider both income and commodity taxation, while Boadway and Marchand (1995) analyze just optimal income taxation.

[5]This seems to contradict Nichols and Zeckhauser (1982, p. 375), stressing that publicly provided goods ought to be inferior. However, the example that Nichols and Zechauser adopt to analyze this issue

wisdom featuring this literature that social services can be treated as *consumption* goods.

As stressed by endogenous growth literature, the economic nature of goods that are publicly provided in many countries is probably more related to production than to consumption: education and health-care primarily affect household's production capacity, namely its human capital[6]; while their features as consumption goods are perhaps relatively less relevant in terms of households' choices. A drawback of such *consumption view* is that the complementarity or substitutability between publicly provided and other private consumption goods might be overestimated and their relationship with other private investment goods underestimated (Balestrino, 1999, 2000)[7]. If this argument were sufficiently convincing, one could suggest to publicly provide - through the appropriate scheme - any good that is, for example, complement of leisure (say, MP3-players or t-shirts). This identification problem has been taken seriously also by the main contributors to this strand of economic literature, e.g. Cremer and Gahvari (1997, p. 114), Blomquist and Christiansen (1998a, p. 407), and Balestrino (1999, p. 349).

The main contribution of this paper is to point out how the analysis of redistribution power and optimal provision mechanisms are affected once we assume that, in a framework of second-best taxation, the publicly provided good affects just households' capacity to produce income, abstracting from any consumption effect. The total productive capacity of individual households is a "composite" asset summing up all physical, financial, and human capital that can be invested to produce income. Households' composite capital is determined by their exogenous *wealth* (again, physical, financial and human exogenous

is somewhat misleading; they consider the case of low-quality housing, as inferior good, though one can - more straightforwardly - think of *quality of housing* to be a normal good, thus implying that low-income households demand lower quality than high-income.

[6]For example, Aghion and Howitt (1998, ch. 10) consider the case of education, while López-Casanovas *et al.* (2005) investigates the relationship between health and growth. It is worth to remark that this literature takes as given the provision mechanisms, very often represented as universal public provision schemes financed through proportional taxes.

[7]As argued, non-separability of households' objective function with respect to publicly provided good plays a fundamental role in this literature. For example, in Blomquist and Christiansen (1995), when consumption good and publicly provided goods are perfectly substitutable, public provision cannot improve social welfare.

endowments) and by the level of investment in capital-enhancing activities, say education or health-care. These activities can be provided by government and/or by private sector.

A focal point of our analysis is complementarity or substitutability of publicly provided goods with respect to households' wealth. Such technological feature can be also read in terms of the intrinsic redistributive power of publicly provided goods (high for substitutes and low for complements). In our simple setting, households' welfare depends just on consumption, hence productive goods that are complement of exogenous household capacity in income production are also normal, while substitute goods are inferior. Our main findings are that opting-out schemes, often considered tools to focus social expenditure, limit their redistributive capacity to normal goods (that in our setting have low redistributive capacity, being complement of households' wealth in terms of income production), while under the assumption that provided goods are inferior, topping-up scheme proves to be superior in terms of social welfare. Quite intuitively, as far as the capacity of government to tax produced incomes shrinks and taxation collapses to lump sum uniform instruments, topping up mechanisms loose any redistributive power, and redistribution is possible only when provided goods are normal, through opting-out mechanisms.

The paper is organized as follows: Section 2 introduces the theoretical setting, and analyzes households' optimization problem; Section 3 analyzes the working of public provision schemes assuming different degrees of taxation efficiency; and Section 4 concludes.

2 The Model

The economy is populated by an infinite number of households, identical up to their exogenous productive capacity (say, *wealth*), $\theta \in [0, 1]$. θ is individual's private information and it is identically and independently distributed across individuals following the cumulative function $F(\theta)$. Household's gross income, $y(\theta, q)$, is strictly increasing, and concave in individual wealth and in *investment*, q, for household's productivity enhancement (e.g. in health-care or education). Each household pays a tax that can be generically defined as

$T = \tau + t \cdot y$, where τ is a lump sum tax (or subsidy, if $\tau < 0$), and t is the rate of taxation on households' produced income.

Individual's utility function, u, is continuously differentiable, strictly increasing and concave in private consumption, c. Household's net income, $y(\theta, q) \cdot (1 - t) - \tau$, finances private expenditure for consumption (also the numerary), c, and possibly for private investment in productivity enhancement, q^m: $y(\theta, q) \cdot (1 - t) - \tau \geq c + p \cdot q^m$, where p is marginal cost of investment. q cannot be resold.

Government is benevolent and maximizes the sum of households' utilities[8]. Whenever government affords all relevant information about households (namely on θ), it could implement first best (lump sum) taxation such that the marginal utility of income would be equalized across households. In the following, we assume that - because of informational (and/or institutional) constraints - government is unable to discriminate households by wealth or consumption and only a linear income tax is available. However, government is allowed to design a scheme of public provision of investment for households' productivity enhancement, q^p[9]. Assuming that public and private provisions are equally efficient, the government faces the budget constraint $t \cdot E(y) + \tau \geq p \cdot q^p \cdot \mathcal{I}$, where $E(y) = \int_0^1 y(\theta, q) \cdot dF(\theta)$.

$\mathcal{I}$ is the share of households covered by the public program, depending on the provision scheme (namely, on the level of the publicly provided investment, q^p). The mechanisms for the public supply of social and economic services can be sorted out on the basis of rules shaping the access of citizens to the program, and of the availability of private substitutes to the public provision (Poterba, 1994). Whenever private supplementing of the public provision is technologically feasible, the government framing the access to the public program may afford a minimum universal provision of public service to all households (hence, $\mathcal{I} = 1$), while (legally) allowing them to privately *top up* the public

[8]In the considered setting, the assumed objective function of government can also be interpreted as a weighted sum of individual consumption levels; where the weights are decreasing in individual consumption, by the concavity of the utility function.

[9]The considered informational and institutional constraints limiting taxation also hinder means-testing in the distribution of public investment.

provision.

In some cases, private supplementing of public provision may be legally unfeasible (e.g. education schemes)[10]. In the considered setting, mandatory enrollment to the public program does never Pareto-dominate leaving households free to choose to opt in or out of the public scheme (Besley and Coate, 1991, p. 981)[11]. Thus, the provision rules of the public and private schemes impose households an *opting in/out* choice. Generally speaking, in this case, only a part of the population is covered by the public program ($\mathcal{I} \in [0,1]$). Finally, public provision programs may mix topping up and opting out schemes[12]. The analysis of a multi-pillar scheme (mixing topping-up and opting-out) proves useful to assess the relative efficiency of these mechanisms as redistribution tools.

The timing of the model is as follows. As first, government chooses the regime for public provision, and credibly fixes its policy parameters (income tax and public provision). Then, households decide whether to opt in the public scheme and/or the level of their private investment (if possible).

2.1 Households' optimization without public provision

Let us first analyze households' investment in absence of any public provision, but assuming that households are taxed with a generic linear income tax. Households are identical up to exogenous wealth, θ. The generic household may choose how much to invest

$$\max_{q^m \geq 0} y(\theta, q^m) \cdot (1-t) - \tau - p \cdot q^m \tag{1}$$

[10]Conditional grants (say, vouchers) can always afford the implementation of topping up functioning also in sectors where using different services at the same time (e.g. for education) is not *technologically* feasible (Blomquist and Christiansen, 1995, pp. 564-5).

[11]A priori, public and private provisions are equivalent on efficiency ground (e.g. assuming that no group-externality features the public program). Moreover, positive analysis, that is not considered here, could introduce "political" differences between public and private programs, e.g. Epple and Romano (1996a,b).

[12]For example, in the experience of some countries, education is based on minimum public financing (say, a first topping-up pillar), often through a system of conditional grants (vouchers or tax allowances) to households or directly through financial support to education institutions; moreover, public sector bears the cost of some (public) institutions providing education to households that opt for them, while other households opt out and supplement public financing to cover private institution education fees (Mitch, 2004).

By the first order condition, private investment is

$$q^m(\theta) = \begin{cases} 0 & if \quad \partial_q y \cdot (1-t) < p \\ q(\theta,t,p) & if \quad \partial_q y \cdot (1-t) = p \end{cases}$$

when the marginal productivity of investment ($\partial_q y \cdot (1-t)$) is lower than its price, the household does not invest - $q^m(\theta) = 0$ (and, as a consequence, private investment is unaffected by tax policy and other exogenous parameters). In the following, we assume that the marginal productivity of investment tends to infinity as q decreases to zero, for any exogenous wealth level; thus, whenever tax rate is below 1, $q^m(\theta) = q(\theta,t,p) > 0$.

By usual comparative statics, household's demand for q is decreasing in taxation (because of reduction in marginal productivity) and in its price[13]. But, the effect of wealth on investment

$$d_\theta q(\theta,t,p) = -\frac{\partial^2_{\theta q} y}{\partial^2_{qq} y}$$

depends on the complementarity or substitutability of wealth and investment in terms of household's income production function (i.e. $\partial^2_{\theta q} y$), and it is independent of taxation. It is useful to discriminate two cases: if investment is (at least weakly) substitute of exogenous wealth in terms of household's income, $\partial^2_{\theta q} y \leq 0$, then the private demand for investment is non-increasing in wealth, $d_\theta q(\theta,t,p) \leq 0$; conversely, when investment is technologically complementary to wealth, $\partial^2_{\theta q} y > 0$, then private investment increases with wealth, $d_\theta q(\theta,t,p) > 0$.

Technological substitutability of investment and wealth could be sufficiently strong to determine a reversal in the relative wealth-ranking of households[14]. Therefore, we

[13]

$$d_t q(\theta,t,p) = \frac{\partial_q y}{\partial^2_{qq} y \cdot (1-t)} < 0 \qquad d_p q(\theta,t,p) = \frac{1}{\partial^2_{qq} y \cdot (1-t)} < 0$$

[14]In such a case, households with higher wealth levels would also be the ones with lower income and consumption levels. A similar behavior would question the concept of (exogenous) wealth and also of

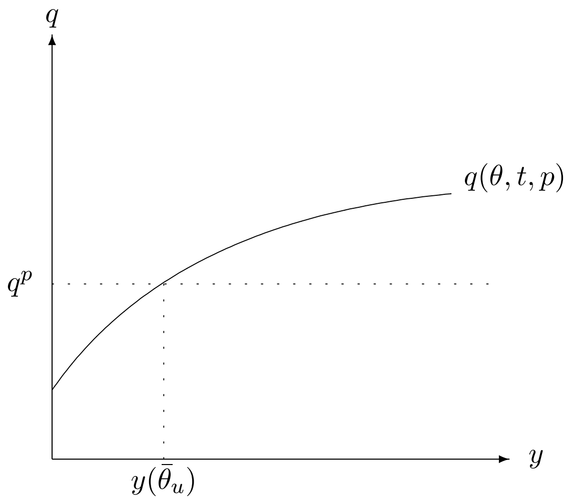

Figure 1: *Wealth-investment complementarity*

restrict the shape of households' income function by imposing the following *lower bound to substitutability between wealth and investment*

$$\partial^2_{\theta q} y > \partial^2_{qq} y \cdot \frac{\partial_\theta y}{\partial_q y} \tag{2}$$

that is necessary and sufficient for increasing monotonicity of household's income in exogenous wealth: $d_\theta y = \partial_\theta y + \partial_q y \cdot d_\theta q(\theta, t, p) > 0$. As a consequence: investment is a normal good, whenever θ and q are complements (Figure 1); conversely, it is an inferior good, whenever θ and q are substitutes (Figure 2).

2.2 Households behavior under topping up schemes

The effect of topping up programs is straightforward: public provision determines a perfect crowding out of private demand, and does not affect optimal allocation of resources unless households are constrained to overinvest (i.e. public provision is strictly higher than their preferred level of q). Moreover, this simple mechanism is redistributive only if income tax is, at least, proportional to (some indicator of) households' wealth.

redistribution in this model.

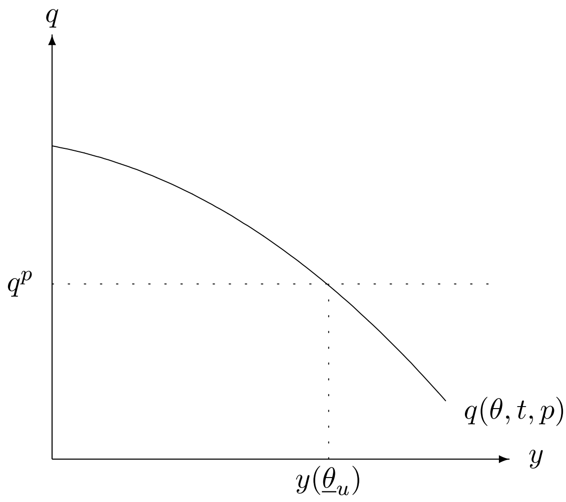

Figure 2: *Wealth-investment substitutability*

When a topping up scheme is introduced, household's private investment, q^m, *supplementing* the minimum universal public provision, q^p, derives by a program similar to (1)

$$\max_{q^m \geq 0} y(\theta, q^m + q^p) \cdot (1-t) - \tau - p \cdot q^m$$

By private investment function

$$q^m(\theta) = \begin{cases} 0 & if \quad \partial_q y \cdot (1-t) < p \\ q(\theta, t, p) - q^p & if \quad \partial_q y \cdot (1-t) = p \end{cases}$$

we see that q^p completely crowds out private demand, unless public provision constrains household to overinvest (hence, marginal productivity of investment is below its price: $\partial_q y \cdot (1-t) < p$), in which case household's investment is identically equal to zero - and it is unaffected by policy and structural parameters[15]. Moreover, we have

Proposition 1 *If investment is a normal (or inferior) good, only households with wealth*

[15]Under topping-up schemes, a lump sum subsidy (i.e. $\tau < 0$) perfectly substitutes minimum public public provision (q^p) for topping up households - given that the price of q is the same for public and private sector.

above $\bar{\theta}_u$ *(or below* $\underline{\theta}_u$*), non-decreasing (or non-increasing) in* t*,* q^p*, and* p*, supplement public provision.*

Proof. See the Appendix. ■

2.3 Households behavior under opting-out schemes

Opting out schemes involve a participation choice of each households: public provision is universally available to individuals willing to opt in the public scheme. But, the decision to privately demand the service involves quitting the public provision program. At individual level, opting out is costly: households deciding for private provision cannot benefit from the public scheme (for which they nevertheless pay the income tax). At collective level, provided that - by decreasing marginal utility of income - some redistribution is socially desirable, opting out is *potentially* able to focus public expenditure on a part of total population also when income tax is very ineffective as redistribution tool (e.g. a poll tax).

For the generic household with wealth θ that opts out of the public program, the private investment, $q^m(\theta)$, derives by program (1) with $q^p = 0$, and yields the maximum utility $v_o(\theta, t, \tau, p) = u(c_o^*(\theta))$, where $c_o^*(\theta) \equiv y(\theta, q^m(\theta)) \cdot (1-t) - \tau - p \cdot q^m(\theta))$ is the consumption corresponding to the optimal investment level. For households opting in the public provision scheme, the maximum utility is $v_i(\theta, t, \tau, q^p) = u(c_i(\theta, q^p))$, where $c_i(\theta, q^p) \equiv y(\theta, q^p) \cdot (1-t) - \tau$ is the consumption corresponding to the public provision.

An household of type θ chooses the public provision scheme if and only if it provides an utility level at least equal to opting out: $v_i(\theta, t, q^p) \geq v_o(\theta, t, p)$; *opting-in condition* is equivalently written as

$$\Delta(\theta) = y(\theta, q^p) \cdot (1-t) - y(\theta, q^m(\theta)) \cdot (1-t) + p \cdot q^m(\theta) \geq 0 \qquad (3)$$

By opting-in condition, households opt out if and only if $\Delta(\theta) < 0$, therefore the optimal

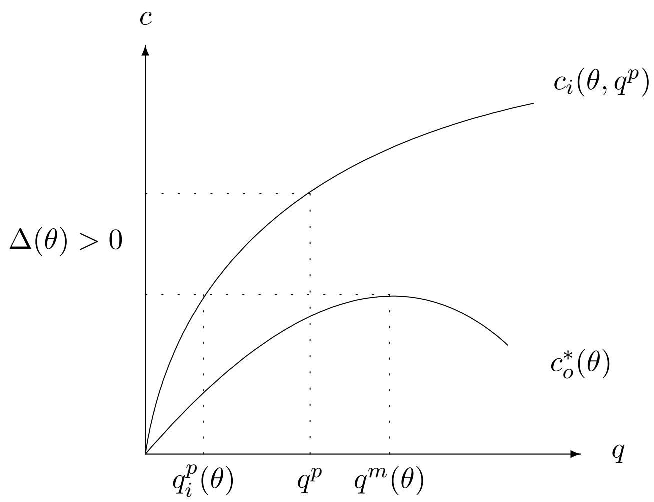

Figure 3: *Opting-in condition*

investment of opting out households is strictly higher than publicly provided investment (Besley and Coate, 1991; Blomquist and Christiansen, 1995).

Households choosing to opt out have to pay for investment: the negative income effect of opting out implies that only a significative increase in income productivity would convince them to exit the public scheme. As a consequence, marginal households opting in the public scheme are rationed (Figure 3). Of course, public-scheme rationing is compensated by the positive income effect related to the fact that public provision is free. The balance of these two effects determines the choice of each household to profit of publicly provided service or not.

Thus, we can compute the minimum public provision required to convince households with wealth θ to opt in: $q_i^p(\theta) \equiv \{q^p \mid \Delta(\theta) = 0\}$[16]; hence, for provision levels below this threshold the household with wealth θ will opt out, and for levels equal or above the considered threshold it opts in. A result similar to Proposition 1 identifies households

[16] Epple and Romano (1996a, pp. 302-3) determine in a similar way the threshold of public provision (p. 302) and the result of Proposition 2 (Corollary 1 at p. 303). Let also remark that

$$d_t q_i^p(\theta) = -\frac{y(\theta, q^m(\theta)) - y(\theta, q_i^p(\theta))}{\partial_q y(\theta, q_i^p(\theta)) \cdot (1-t)} < 0 \quad d_{q^p} q_i^p(\theta) = -\frac{q^m(\theta)}{\partial_q y(\theta, q_i^p(\theta)) \cdot (1-t)} < 0$$

opting in:

Proposition 2 *If investment is a normal (or inferior) good, only households with wealth above $\bar{\theta}_o$ (or below $\underline{\theta}_o$), non-decreasing (or non-increasing) in t, q^p, and p, opt out of the public scheme.*

Proof. See the Appendix. ■

By Proposition 2, for some level of public provision, relatively rich households (with high θ) opt out of the public scheme when relatively poor ones (low-θ) opt in *only if investment is a normal good* (i.e. when it is complementary to households' wealth in terms of income production). In such a case, richer households prefer private provision when public is too limited (in quantity or quality). Conversely, when q is an inferior good, poor households require more investment in productivity enhancement than rich ones; thus, they are eagerer to quit public programs while public level q^p drops. In other terms, to convince poor households to opt in, also rich households have to be covered by public provision.

This observation shed some lights on the limits affecting opting out schemes as redistribution mechanisms. The existing literature has often stressed that opting-out schemes "*[...] allows the government to use its revenues in a targeted fashion, since only the poor participate*" (Besley and Coate, 1991, p. 984). This feature of self-selective schemes heavily relies on the assumption that distributed goods are normal. Conversely, when publicly provided goods are inferior (say, satisfying some basic needs and substituting household's wealth in income production), opting-out schemes lose the capacity to focus expenditure.

Proposition 2 also affords an handful characterization of households' mass covered by the public program, as related to relevant policy (i.e. t, q^p) and technological (p) parameters: $\mathcal{I} = F(\bar{\theta}_o(q^p, t, p))$ in case q is a normal good; and $\mathcal{I} = 1 - F(\underline{\theta}_o(q^p, t, p))$ when q is an inferior good. Of course, when taxation, cost or public provision increase, the balance between income productivity of (privately) investing in q worsens and more households are willing to benefit of public provision.

2.4 Households behavior under two-pillar schemes

As argued, households behavior introduces some limits to possible structures of such public program. Without loss of generality, let q^p be the minimum universal public provision that is afforded to households independently of their behavior (*first pillar* of public provision scheme); and let $q^p + \delta$ be the public provision that households obtain by choosing to enter an optional public scheme forbidding any private supplementing (*second pillar*).

The maximum utility afforded to households that opt out of the second pillar has to take into account the existence of the first pillar provision, q^p, and becomes $v_o(\theta, t, \tau, q^p, p) = u(c_o^*(\theta, q^p))$, where $c_o^*(\theta, q^p)$, the optimal consumption of household, is equal to: $y(\theta, q^m(\theta) + q^p) \cdot (1 - t) - \tau - p \cdot q^m(\theta)$, when household is not constrained to overinvest by q^p; $y(\theta, q^p) \cdot (1 - t) - \tau$, when household is constrained to overinvest. For households opting in the second pillar of the public scheme, the maximum utility is exactly as before $v_i(\theta, t, \tau, q^p + \delta) = u(c_i(\theta, q^p + \delta))$, where $c_i(\theta, q^p + \delta)) = y(\theta, q^p + \delta) \cdot (1 - t) - \tau$.

By inspection of relevant indirect utility functions, we observe that households that are constrained to overinvest by the minimum universal provision (q^p), opt in the second pillar if and only if they obtain a provision level that is not below q^p, hence if and only if $\delta \geq 0$. Moreover, an increase of first-pillar provision (q^p) reduces the interest of households to opt in the second pillar. By the same argument represented in Figure 3, households that opt out of the second pillar (and are not constrained to overinvest by first-pillar provision) implement a total investment such that $q^m(\theta) + q^p > q^p + \delta$, hence $q^m(\theta) > \delta$.

By opting-in condition (3), for any level of public provision within the first pillar, q^p, the minimum additional second-pillar provision (δ) to convince households with wealth θ to opt in is $\delta_i(\theta) \equiv \{\delta \mid \Delta(\theta) = 0\}$. Three features of the two-pillar scheme arise: first, the difference between first- and second-pillar provision decreases in the former[17]; moreover, Proposition 2 still holds, hence a trade-off between first and second pillar arises: the mass of households opting-out of the second pillar grows as the public provision of the first pillar (that allows for private supplementing) increases; finally, all households opting out of the second pillar choose to top up first-pillar minimum provision ($q^m(\theta) > \delta > 0$)[18]. As a consequence, when first-pillar provision q^p is sufficiently high to force households with wealth θ to privately demand $q^m(\theta) = 0$, they opt in as well.

3 The Optimal Design of Public Provision

To keep the analysis as simple as possible, let the population be made by two classes, featured by households' exogenous wealth ($\theta \in \{0, 1\}$), and the mass of poor households be $F(0) = \lambda$.

3.1 Opting out schemes and uniform lump sum tax

As first, we assume that government relies on very rough taxation instruments, being unable to observe or measure exogenous wealth and endogenous incomes of different households: the only feasible taxation takes the form of head tax ($\tau \geq 0$). In such economy, lump sum uniform taxation does not entail second-best distortions. Therefore, public provision does not play any role to correct taxation inefficiency, but opting-out schemes supplement taxation as redistribution tool (Blackorby and Donaldson, 1988; Besley and Coate, 1991). Conversely, any public program based on uniform and universal public pro-

[17]Namely

$$d_{q^p}\delta_i(\theta) = -1 + \frac{\partial_q y(\theta, q^m + q^p)}{\partial_q y(\theta, q^p + \delta_i(\theta))} < 0$$

by concavity of income production function in q. Moreover,

$$d_t\delta_i(\theta) = -\frac{y(\theta, q^m + q^p) - y(\theta, q^p + \delta_i(\theta))}{\partial_q y(\theta, q^p + \delta_i(\theta)) \cdot (1-t)} < 0 \quad d_\theta\delta_i(\theta) = \frac{\partial_\theta y(\theta, q^m + q^p) - \partial_\theta y(\theta, q^p + \delta_i(\theta))}{\partial_q y(\theta, q^p + \delta_i(\theta))}$$

[18]Any household opts out whenever $y(\theta, q^p + \delta) \cdot (1-t) < y(\theta, q^m(\theta) + q^p) \cdot (1-t) - p \cdot q^m(\theta)$, that is always true for $\delta \leq 0$.

vision (topping up scheme) is redundant since it would drain and return to each household the same amount of money.

Under opting out scheme, when investment is a normal (or inferior) good, the thresholds of public provision inducing poor, $q_i^p(0)$, and rich, $q_i^p(1)$, households to enter the public scheme are such that: $q_i^p(1) > q_i^p(0)$ (or $q_i^p(1) < q_i^p(0)$). Thus, the simple distribution of households' exogenous wealth implies three policy regimes: *laissez-faire*, when the public provision is very low ($q^p < \min\{q_i^p(0), q_i^p(1)\}$), no household opts in ($\mathcal{I} = 0$); *discriminating*, an intermediate level of public provision, $q^p \in [q_i^p(0), q_i^p(1))$ (or $q^p \in [q_i^p(1), q_i^p(0))$), induces only poor (or rich) households to opt in, hence $\mathcal{I} = \lambda$ (or $\mathcal{I} = 1 - \lambda$); *inclusive*, for sufficiently high provision ($q^p \geq \max\{q_i^p(0), q_i^p(1)\}$), all households opt in ($\mathcal{I} = 1$).

In the laissez-faire regime, government optimally chooses $\tau = 0$ and the social welfare is $W_0 \equiv \lambda \cdot v_o(0, 0, 0, p) + (1 - \lambda) \cdot v_o(1, 0, 0, p)$. Under inclusive policy, government has to cover involved expenditure: $\tau = p \cdot q^p$; thus, the social welfare is: $W_1(q^p) \equiv \lambda \cdot v_i(0, 0, p \cdot q^p, q^p) + (1 - \lambda) \cdot v_i(1, 0, p \cdot q^p, q^p)$; and $W_1^* \equiv \max_{q^p} W_1(q^p)$. The inclusive regime under lump sum taxation does not involve redistribution, given that every household pays and receives the same amount of resources. Moreover, unless $y(\theta, q)$ is separable in its two arguments (in which case the marginal utility of investment is independent of exogenous wealth), at least one of the two classes in the economy is constrained to overinvest by the inclusive regime[19]. Therefore, the inclusive regime does never dominate laissez-faire in terms of social welfare ($W_0 \geq W_1^*$); in particular, it is dominated when the marginal productivity of investment is affected by wealth.

The analysis of discriminating policies, separating low-demand households (opting in) and high-demand ones (opting out), requires to specify the assumption about the nature of investment. If investment is a normal good, then poor (low-demand) households opt in and rich households opt out. The government has to balance the public budget accordingly,

[19]By the first order condition of the maximization of $W_1(q^p)$ it can be easily checked that the publicly provided investment under inclusive regime is higher or equal to the $\max\{q_i^p(0), q_i^p(1)\}$, thus only the low-demand class of households is constrained to overinvest at the optimum.

$\tau = p \cdot q^p \cdot \lambda$. The maximum social welfare, W^*_λ, is given by the maximization of the social welfare function, $W_\lambda(q^p) \equiv \lambda \cdot v_i(0, 0, p \cdot q^p \cdot \lambda, q^p) + (1 - \lambda) \cdot v_o(1, 0, p \cdot q^p \cdot \lambda, p)$, under the constraints involved by discriminating regime

$$\max_{q^p} W_\lambda(q^p) \quad s.t. \quad q^p \geq q_i^p(0) \quad q^p \leq q_i^p(1) \tag{4}$$

When investment is an inferior good ($q_i^p(1) < q_i^p(0)$), it is impossible to attract poor households in the public (self-selective) scheme unless public provision is so high that also rich are interested to opt in. Intuitively, the discriminating policy regime, $q^p \in [q_i^p(1), q_i^p(0))$ - that determines a uniform lump sum tax $\tau = p \cdot q^p \cdot (1 - \lambda)$ - cannot be welfare-improving whenever marginal utility is decreasing in consumption. Again, maximizing the social welfare function, $W_{1-\lambda}(q^p) \equiv \lambda \cdot v_o(0, 0, p \cdot q^p \cdot (1-\lambda), p) + (1-\lambda) \cdot v_i(1, 0, p \cdot q^p \cdot (1-\lambda), q^p)$, under the constraints involved by discriminating regime

$$\max_{q^p} W_{1-\lambda}(q^p) \quad s.t. \quad q^p \geq q_i^p(1) \quad q^p \leq q_i^p(0)$$

the maximum social welfare, $W^*_{1-\lambda}$, is obtained.

Is any opting-out (discriminating) scheme optimal? When investment is normal, let $\bar{q} \equiv \{\min q^p \mid W_\lambda(q^p) = W_0\}$[20], then

Proposition 3 *An opting-out scheme improves social welfare, provided that utility is strictly concave and investment is normal, if and only if* $q_i^p(1) > \bar{q}$.

Proof. See the Appendix. ■

The capacity of opting-out schemes to improve social welfare (by redistributing incomes) relies on two necessary conditions: as first, redistribution itself has to be socially

[20] This value always exist, when investment is normal. To check this, consider that $W_\lambda(q^m(0)) \geq W_0$, given that poor households obtain their optimal private investment ($q^m(0)$) and pay $\tau = p \cdot q^m(0) \cdot \lambda$, so their utility level is higher than in the laissez faire regime, thanks to the lump sum transfer obtained by the rich households.

relevant (say, the utility function strictly concave), otherwise we simply would not need social service programs; moreover, social services should be normal goods, in order poor households to demand less investment than rich (hence, wealth and investment need to be complement in terms of households' income production), which is a pre-condition for implementing social programs that redistribute in the right direction. Conversely, inferior goods can never be supplied in the framework of such programs, given that they involve a wrong direction in the redistribution of resources. However, to optimally operate an opting-out mechanism (with head taxation), the necessary and sufficient condition in Proposition 3 ($q_i^p(1) > \bar{q}$) requires that the difference in private demands for capital enhancement between poor and rich households has also to be sufficiently wide to implement a discriminating policy regime and afford welfare-improving redistribution.

3.2 Public provision schemes with linear income tax

In this Section, we assume that government has sufficient information and institutional capacity to design and implement a linear income tax: $T = \tau + t \cdot y$. In the considered model, government is able to tax proportionally households' income and redistribute tax revenues through an uniform subsidy ($\tau < 0$). As pointed out by literature on public provision of private goods (see Section 1), an utilitarian government enhances its capacity to redistribute and reduces tax distortions by supplementing taxation with public provision.

But, what is the optimal structure of public programs contingent to different assumptions about the economic nature of investment (normal or inferior good) in productivity enhancement? To provide a thorough exploration of this issue, we consider a two-pillar scheme, that can replicate a single-pillar one, by a suitable choice of policy variables (Blomquist and Christiansen, 1998a). By the analysis of households' response to the two-pillar scheme, we know that public provision of topping-up pillar has to be strictly lower than the one of opting-out pillar, and that there is a threshold of such difference, $\delta_i(\theta)$. Thus, for any first-pillar provision q^p, if investment is a normal (or inferior) good, the following schemes emerge: single topping-up pillar, if $\delta < \delta_i(0)$ (or $\delta < \delta_i(1)$); two-pillar with discriminating opting-out pillar - with poor (or rich) households opting-in - if $\delta \in [\delta_i(0), \delta_i(1))$ (or $\delta \in [\delta_i(1), \delta_i(0))$); inclusive scheme if $\delta \geq \delta_i(1)$ (or $\delta \geq \delta_i(0)$). However, the thresholds discriminating households opting in and out of the second pillar are influenced by the level of first-pillar provision, namely: if investment is a normal (or inferior) good: for $q^p < q^m(0)$ (or for $q^p < q^m(1)$), then $0 < \delta_i(0) < \delta_i(1)$ (or $0 < \delta_i(1) < \delta_i(0)$); for $q^p \in [q^m(0), q^m(1))$ (or for $q^p \in [q^m(1), q^m(0))$), then $0 = \delta_i(0) < \delta_i(1)$ (or $0 = \delta_i(1) < \delta_i(0)$); and for $q^p \geq q^m(1)$ (or for $q^p \geq q^m(0)$), then $0 = \delta_i(0) = \delta_i(1)$.

Therefore, in case both topping-up and opting-out public provision levels are below the minimum thresholds, respectively $\min\{q^m(0), q^m(1)\}$ and $\min\{\delta_i(0), \delta_i(1)\}$, then all households opt out of the second pillar and the public program degenerates to a *pure tax policy regime* (given that topping up provision is not binding for any household, say it is redundant with respect to lump sum transfer). When either topping-up or opting-out public provision is above the maximum threshold, respectively $\max\{q^m(0), q^m(1)\}$ and $\max\{\delta_i(0), \delta_i(1)\}$, then all households opt in and the two-pillar scheme *de facto* becomes a *pure inclusive policy regime.* In the other cases, we have a two-pillar (possibly degenerating in a single pillar) scheme involving a *discriminating policy regime* with just low-investment-demand households opting in the second pillar, or being constrained to overconsume by the first pillar provision (in which case the two-pillar scheme degenerates in a single pillar one).

3.2.1 Pure tax regime

In this case, only the first pillar is possibly active but its provision is redundant with respect to an uniform lump sum subsidy to households. Therefore, we assume without loss of generality that the government does not use any public provision scheme. By government's budget constraint, $\tau = -t \cdot E(y)$ (with $E(y) = \lambda \cdot y(0, q^m(0)) + (1 - \lambda) \cdot y(1, q^m(1))$). The social welfare function is given by $W_0(t) \equiv \lambda \cdot u(y(0, q^m(0)) \cdot (1-t) - p \cdot q^m(0) + t \cdot E(y)) + (1-\lambda) \cdot u(y(1, q^m(1)) \cdot (1-t) - p \cdot q^m(1) + t \cdot E(y))$. By the first order condition of government's optimization (let $W_0^{**} = \max_t W_0(t)$), we have

Proposition 4 *If utility is strictly concave, the optimal tax rate is* $t \in (0, 1)$.

Proof. See the Appendix. ■

This outcome directly derives by the preference for redistribution that decreasing marginal utility involves. The efficiency cost of distorting taxation is here compensated by the redistribution gain (thus, assuming that utility is linear implies that the optimal tax rate is zero). Therefore, under strict concavity (say, strict preference for redistribution) a pure tax regime increases the social welfare with respect to laissez-faire ($W_0^{**} > W_0$).

3.2.2 Inclusive regime

Let us now consider the case every household is covered by public provision. Government's budget constraint implies, without loss of generality, that $\tau = p \cdot q^p - t \cdot E(y)$. The social welfare function is $W_1(t, q^p) \equiv \lambda \cdot u(y(0, q^p) \cdot (1-t) - p \cdot q^p + t \cdot E(y)) + (1-\lambda) \cdot u(y(1, q^p) \cdot (1-t) - p \cdot q^p + t \cdot E(y))$, and the maximum value of it, W_1^{**}, is given by

$$\max_{t, q^p} W_1(t, q^p) \quad s.t. \quad t \geq 0 \quad (\mu) \quad q^p \geq \max\{q_i^p(0), q_i^p(1)\} \quad (\phi)$$

By its first order conditions

Proposition 5 *If utility is strictly concave, optimal inclusive public provision is such that* $E(\partial_q y) = p$*, and optimal tax policy equalizes households' consumption.*

Proof. See the Appendix. ■

What is the ranking between social welfare under respectively pure tax regime, W_0^{**}, and inclusive regime, W_1^{**}? In the case under consideration, the answer is somewhat complicated by the fact that taxation is distorting and redistributive by itself. Under pure tax scheme, redistribution relies just on distorting taxation, but it can be rather flexibly calibrated through a lump sum transfer to households ($\tau < 0$). Conversely, in the case of inclusive policy, taxation is no more distorting - given that households do not choose any more their investment level. However, welfare improvement on the tax collection side entails a welfare loss linked to production inefficiency involved in inclusive policy (all households must invest the same amount of q). What can we say about overall balance?

The welfare cost of inclusive policy is linked to the effect of households' heterogeneity on their investment choices. Let ψ be the cost of inclusive policy in terms of aggregate consumption loss (i.e. loss of income net of investment cost) with respect to laissez-faire

$$\psi = \lambda \cdot [(y(0, q^m(0)) - p \cdot q^m(0)) - (y(0, q^p) - p \cdot q^p)] +$$
$$+(1 - \lambda) \cdot [(y(1, q^m(1)) - p \cdot q^m(1)) - (y(1, q^p) - p \cdot q^p)]$$

The size of ψ is determined by the technological structure of income production: a wider heterogeneity of households' income functions (say, the complementarity or substitutability of wealth and investment) increases this cost[21]. In the extreme case that wealth and investment are separable in terms of income production ($\partial^2_{\theta q} y = 0$) , ψ drops to zero, and inclusive policy always outperforms pure tax one. Contrary to what has been established by literature on publicly provided goods, in this case whenever households heterogeneity does not affect private demand for the publicly provided goods, the latter becomes a powerful redistribution tool and an optimal policy is to provide it equally to all citizens. But, this conclusion is not likely to be robust to extensions of our setting including other productive efforts (say, labor or saving supply) or multiple consumption choices.

[21]Representing $y(\theta, q^m(\theta)) - p \cdot q^m(\theta)$ with second order Taylor approximation around $y(0, q^p) - p \cdot q^p$, it is possible to show that ψ is an increasing function of $|\partial^2_{\theta q} y(0, q^p)|$.

The cost of inclusive policy has to be contrasted to the *inequality premium*, π: the welfare cost of inequality of households that is involved by laissez faire

$$u(\lambda \cdot (y(0, q^m(0)) - p \cdot q^m(0)) + (1-\lambda) \cdot (y(1, q^m(1)) - p \cdot q^m(1)) - \pi) = W_0 \tag{5}$$

π increases in the indicator of *inequality aversion*, $r = |\frac{\partial^2_{cc} u}{\partial_c u}|$[22].

Now, given the cost of inclusive policy, ψ, there is a degree of inequality aversion, $r_0 > 0$, such that for utility functions involving lower inequality aversion laissez-faire policy regime is not dominated, in social welfare terms, by inclusive one ($W_0 \geq W_1^{**}$)[23]. Thus, increasing households' heterogeneity (say, complementarity or substitutability between wealth and investment), the cost of inclusive policies grows and even more pro-redistributive utility functions are compatible with a preference for laissez-faire. On these grounds, we have

Proposition 6 *Pure tax regime always dominates in welfare terms inclusive policy provided that inequality aversion is $r \in (0, \bar{r})$ (with $\bar{r} > r_0$).*

Proof. Let us remark that whenever $r \in (0, r_0]$, $W_0^{**} > W_0 \geq W_1^{**}$. Then, by utility continuous differentiability there is $\bar{r} > r_0$ such that $W_0^{**} > W_1^{**}$. ■

3.2.3 Optimal discriminating regime

Assume, first, that investment is a normal good. The discriminating two-pillar scheme requires that public provision in the first pillar is $q^p \in [0, q(1, t, p)]$, and the second pillar

[22]We borrow the concepts of inequality premium, corresponding to risk premium (Kreps, 1988, p. 74), and inequality aversion, corresponding to absolute risk aversion, from from the literature on decision under uncertainty. Taking the first order Taylor approximation of the right-hand of (5) around the utility of low income households, π can be defined as an implicit function of r, and in particular: $\partial_r \pi_0 > 0$ for any $r > 0$. Quite intuitively: when $r = 0$, the utility function becomes linear, and $\pi = 0$.

[23]By (5), π can be represented as an increasing (implicit) function of r. Moreover, if investment is sensitive to wealth, $q^m(0) \neq q^m(1)$, and for any value of $q^p \in [q^m(0), q^m(1)]$, $\psi > 0$. Thus, either $\psi \geq \pi(r)$ for any $r > 0$ or there is r_0 such that $\pi(r_0) = \psi$ and $W_0^{**} \geq W_1^{**}$ for any $r \leq r_0$.

supplements such provision with a difference that has to satisfy the discrimination constraint ($\delta \in [\delta_i(0), \delta_i(1))$, where $\delta_i(0) = 0$ for any $q^p \geq q(0, t, p)$) only for households that opt in (the poor ones, with a mass λ). Government's budget constraint, somewhat more elaborated than before, can be written as: $\tau = p \cdot (q^p + \delta \cdot \lambda) - t \cdot E(y)$. The social welfare function is now, $W_\lambda(t, q^p, \delta) \equiv \lambda \cdot u(y(0, \delta + q^p) \cdot (1 - t) - p \cdot (q^p + \delta \cdot \lambda) + t \cdot E(y)) + (1 - \lambda) \cdot u(y(1, q^m(1) + q^p) \cdot (1 - t) - p \cdot q^m(1) - p \cdot (q^p + \delta \cdot \lambda) + t \cdot E(y))$. Thus, the maximum social welfare, W_λ^{**}, is obtained by the program

$$\max_{t, q^p, \delta} W_\lambda(t, q^p, \delta) \quad s.t. \tag{6}$$

$$t \geq 0 \quad (\mu) \quad q^p \geq 0 \quad (\eta_0) \quad q(1, t, p) \geq q^p \quad (\eta_1) \quad \delta \geq \delta_i(0) \quad (\phi_0) \quad \delta_i(1) \geq \delta \quad (\phi_1)$$

By the first order conditions of program (6), we have

Proposition 7 *If investment is a normal good, utility is strictly concave, and the economic effect of households' heterogeneity is sufficiently strong to exclude that the upper bound on δ is binding, then the optimal two-pillar scheme degenerates in a pure opting-out scheme: $q^{p*} = 0$, $\delta^* \in (\delta_i(0), \delta_i(1))$, and $t^* \in (0, 1)$.*

Proof. See in the Appendix. ■

Proposition 7 points out a crucial result: the two-pillar scheme slims down to a single opting-out mechanism; though it relies on the assumption that the upper bound on δ is slack (that is related to the relevance of households' heterogeneity on their investment behaviors). Thus, providing a clear-cut indication about the optimality of opting-out schemes when investment is a normal good, as compared to alternative models of public provision. As a consequence, the analysis of Section 3.2.2 applies, and - in particular - by Proposition 3 opting-out discriminating policy outperform in welfare terms pure tax policy (given that we consider the case in which the upper constraint on public provision is not binding).

The reason why the same result may not hold when $\delta = \delta_i(1)$ is that, provided that households' heterogeneity in wealth is not very relevant, improvements of income distribution rely on topping-up mechanism that, though with reduced potential, warrants a certain degree of redistribution when it is financed through proportional taxation.

Let us now turn to the case of inferior investment. In this setting, the discriminating policy within the second pillar implies that only low-demand (rich) households (with mass $1-\lambda$) opt in. Thus, government's budget constraint can be written as: $\tau = p \cdot (q^p + \delta \cdot (1-\lambda)) - t \cdot E(y)$. The social welfare function is $W_{1-\lambda}(t, q^p, \delta) \equiv \lambda \cdot u(y(0, q^m(0)+q^p) \cdot (1-t) - p \cdot q^m(0) - p \cdot (q^p + \delta \cdot (1-\lambda)) + t \cdot E(y)) + (1-\lambda) \cdot u(y(1, \delta + q^p) \cdot (1-t) - p \cdot (q^p + \delta \cdot (1-\lambda)) + t \cdot E(y))$, and the maximum value, $W^{**}_{1-\lambda}$ is obtained by the program

$$\max_{t, q^p, \delta} W_{1-\lambda}(t, q^p, \delta) \quad s.t. \qquad (7)$$

$$t \geq 0 \quad (\mu); \quad q^p \geq 0 \quad (\eta_0) \quad q(0,t,p) \geq q^p \quad (\eta_1) \quad \delta \geq \delta_i(1) \quad (\phi_1) \quad \delta_i(0) \geq \delta \quad (\phi_0)$$

By first order conditions,

Proposition 8 *If investment is an inferior good, and utility is strictly concave, then the optimal two-pillar scheme degenerates in a pure topping-up scheme:* $q^{p*} > q(1,t,p)$, $\delta^* = 0$, *and* $t^* \in [0,1)$.

Proof. See in the Appendix. ■

As regards welfare analysis, considering the optimal program of the government under a pure topping up regime (when investment is inferior), it is possible to show that low-demand (rich) households would be constrained to overinvest ($q^p > q(1,t,p)$). This implies that social welfare under pure topping up regime is higher than under pure tax regime, given that the policy set of the first one is wider than the policy set of the second and that a policy that it is not implementable under pure tax regime is chosen.

By Propositions 7 and 8, we reach the main results of the paper: when inequality aversion is relatively limited (so that inclusive policy is not optimal) and households' heterogeneity is sufficiently large, the optimal public provision scheme is pure opting-out (or pure topping-up) if publicly provided good is normal (or inferior). As argued, limiting inequality aversion below a given threshold implies that inclusive policy regime is always dominated by other regimes. Moreover, households' heterogeneity is required to insure that, when investment is a normal good, a discriminating policy is actually welfare improving.

4 Conclusion

We investigated of the public provision of private goods focussing on the optimal structure of social programs taking into account that social services improve productive capacity of households, and thus directly influence incomes distribution.

The literature on the optimal design of public provision mechanisms has basically considered social services as consumption goods. However, the literature on endogenous growth with human capital highlighted (both theoretically and empirically) that typical publicly provided goods, education and health care, affect households' human capital and productive capacity. Though this latter literature does not consider the implication of this assumption on the optimal design of provision mechanisms. Our contribution addresses this point.

The idea that publicly provided goods improve households' productive capacity delivers a simple characterization of these goods in terms of (technological) complementarity-substitutability between exogenous households' productive capacity (we called it wealth) and investment in productivity enhancement (that can be distributed in kind by government). Thus, publicly provided investment can be a substitute of exogenous productive capacity, involving an high redistributive power, or a complement of it, with a lower intrinsic redistributive power.

An interesting innovation with respect to existing literature on in-kind transfers is that normality of publicly provided good is no more warranted: when publicly provided investment is complement of exogenous productive capacity of households, then it is also a normal good; while it is inferior when it substitutes exogenous capacity. The economic literature on publicly provided goods performs a widespread confidence in normality of social services, however some evidence about health-care in developing countries (López-Casanovas *et al.*, 2005, p. 306) suggest caution on this point, requiring some further empirical investigation.

Our analysis is carried out assuming different degrees of taxation efficiency. When government can finance its expenditure just by head taxation, public provision of investment through a self-selective (opting-out) scheme improves social welfare only if investment is a normal good such that only poor households opt in, while rich opt out. Assuming that taxation is sufficiently efficient to tax households' produced incomes (e.g. linear income tax) public provision is always welfare improving, be investment an inferior or a normal good. But, when investment is normal and, hence, featured by a low intrinsic redistribution power, the optimal redistribution scheme is a pure opting-out one (with some requirement in terms of relevance of households' heterogeneity on their private demands); conversely, when publicly provided goods are *per se* very redistributive, then the optimal public provision scheme is a pure topping up mechanism constraining rich to overinvest.

One of the main implications of our analysis, similar to Blomquist and Christiansen (1998a), is that *a full-fledged two-pillar scheme is never optimal.* Such a conclusion contrasts with reality: some public provision schemes (say, education or health-care schemes) rely on multi-pillar mechanisms. Of course, the two-class setting, that this paper shares with the bulk of the literature on publicly provided private goods, is likely to be responsible of this result. An intuition of the role of that classes and households' heterogeneity play in shaping the optimal structure of (potentially) two-pillar schemes is in the analysis of optimal provision mechanism when investment is a normal good (Proposition 7): when the effect of households' heterogeneity on their private demand of investment is weak, first-pillar provision may be positive, thus determining a non-trivial two-pillar scheme. Conversely, households' heterogeneity plays no direct role in the assessment of the optimality of a pure topping-up scheme when investment is inferior. The intuition that can be drawn by these considerations is that a scope for two-pillar schemes is likely to be there when investment is normal. Further research is required in this direction, through a model with multiple exogenous wealth.

With respect to the existing literature, this model does not account for the interplay between labor supply choices (that play a crucial role in many contributions in the literature) and publicly provided good. Along this intuition, further research will investigate the role of mixing consumption and production dimensions of social services to the design of optimal provision schemes.

Appendix

Proof of Proposition 1. When investment is a normal good, $d_\theta q(\theta, t, p) > 0$, let $\bar{\theta}_u \equiv \{\theta' \in [0,1] \mid q(\theta', t, p) = q^p\}$, $\bar{\theta}_u \equiv \{0 \mid q(\theta', t, p) > q^p\}$ and $\bar{\theta}_u \equiv \{1 \mid q(\theta', t, p) < q^p\}$, then $q^m(\theta) > 0$ for any $\theta > \bar{\theta}_u$. Conversely, when investment is an inferior good, $q^m(\theta) > 0$ for any $\theta < \underline{\theta}_u$, where $\underline{\theta}_u \equiv \{\theta' \in [0,1] \mid q(\theta', t, p) = q^p\}$, $\underline{\theta}_u \equiv \{0 \mid q(\theta', t, p) < q^p\}$ and $\underline{\theta}_u \equiv \{1 \mid q(\theta', t, p) > q^p\}$. By the Implicit Function Theorem[24]: $d_{q^p}\bar{\theta}_u \geq 0$ (or $d_{q^p}\underline{\theta}_u \leq 0$), $d_t\bar{\theta}_u \geq 0$ (or $d_t\underline{\theta}_u \leq 0$), and $d_p\bar{\theta}_u \geq 0$ (or $d_p\underline{\theta}_u \leq 0$). ■

Proof of Proposition 2. Households opt out if and only if $q_i^p(\theta) > q^p$. When investment is a normal good, $d_\theta q_i^p(\theta) > 0$; let $\bar{\theta}_o \equiv \{\theta' \in [0,1] \mid q_i^p(\theta') = q^p\}$, $\bar{\theta}_o \equiv \{0 \mid q_i^p(\theta') > q^p\}$, and $\bar{\theta}_o \equiv \{1 \mid q_i^p(\theta') < q^p\}$, then for any $\theta'' > \bar{\theta}_o$ (or $\theta'' < \bar{\theta}_o$), households opt out (or opt in). When investment is an inferior good, let $\underline{\theta}_o \equiv \{\theta' \in [0,1] \mid q_i^p(\theta') = q^p\}$, $\bar{\theta}_o \equiv \{0 \mid q_i^p(\theta') < q^p\}$, and $\bar{\theta}_o \equiv \{1 \mid q_i^p(\theta') > q^p\}$, then households with $\theta'' > \underline{\theta}_o$ (or $\theta'' < \underline{\theta}_o$) opt in (or opt out). Moreover, by Implicit Function Theorem[25]: $d_{q^p}\bar{\theta}_o \geq 0$ (or $d_{q^p}\underline{\theta}_o \leq 0$), $d_t\bar{\theta}_o \geq 0$ (or $d_t\underline{\theta}_o \leq 0$), and $d_p\bar{\theta}_o \geq 0$ (or $d_p\underline{\theta}_o \leq 0$). ■

Proof of Proposition 3. *Step 1.* $W_0 \geq W_1^*$: By the fact that an inclusive policy cannot be redistributive with lump sum taxation.

Step 2: Assuming inferior investment, $W_{1-\lambda}^* < W_0$: Let $W_{1-\lambda}(q^p, \epsilon) \equiv \lambda \cdot u(y(0, q^m(0)) - p \cdot q^m(0) - p \cdot q^p \cdot (1-\lambda) \cdot \epsilon) + (1-\lambda) \cdot u(y(1, q^p) - p \cdot q^p + p \cdot q^p \cdot \lambda \cdot \epsilon)$, with q^p such

[24] Remark that total differential of $\bar{\theta}_u$ is

$$d\bar{\theta}_u = \begin{cases} 0 & if \quad \bar{\theta}_u \equiv 0 \quad or \quad \bar{\theta}_u \equiv 1 \\ \frac{\partial_q y \cdot dt + dp - \partial^2_{qq} y \cdot (1-t) \cdot dq^p}{\partial^2_{\theta q} y \cdot (1-t)} & if \quad \bar{\theta}_u \in [0,1] \end{cases}$$

and the same expression can be written for $\underline{\theta}_u$.

[25] Remark that total differential of $\bar{\theta}_o$ is

$$d\bar{\theta}_o = \begin{cases} 0 & if \quad \bar{\theta}_o \equiv 0 \quad or \quad \bar{\theta}_o \equiv 1 \\ \frac{\partial_q y(q^p) \cdot (1-t) \cdot dq^p + (y(q^m) - y(q^p)) \cdot dt + q^m \cdot dp}{(\partial_\theta y(q^m) - \partial_\theta y(q^p)) \cdot (1-t)} & if \quad \bar{\theta}_o \in [0,1] \end{cases}$$

and the same expression can be written for $\underline{\theta}_o$.

that $W_{1-\lambda}(q^p, 1) = W^*_{1-\lambda}$. When $\epsilon = 0$, $W_{1-\lambda}(q^p, 0) \leq W_0$. Moreover, $d_\epsilon W_{1-\lambda}(q^p, \epsilon) = -\lambda \cdot (1-\lambda) \cdot p \cdot q^p \cdot (\partial_c u(0) - \partial_c u(1)) < 0$, for any $\epsilon \in [0, 1]$, by strict concavity of utility. Thus, $W_{1-\lambda}(q^p, \epsilon) < W_0$, for any $\epsilon \in [0, 1]$.

Step 3: Assuming normal investment and interior solution of program (4), $W^*_\lambda > W_0$: By the optimization condition $\partial_q y(0, q^p) = p \cdot \left[\lambda + (1-\lambda) \cdot \frac{\partial_c u(1)}{\partial_c u(0)}\right] \geq p$, the optimal investment is $q^p > q^m(0)$. Remarking that $W_\lambda(q^m(0)) > W_0$, when utility is strictly concave (poor households receive their optimal private investment paying only a fraction λ of its cost, $1-\lambda$ being paid by rich households with a lump sum transfer), the argument follows.

Step 4: Assuming normal investment and a corner solution of program (4), $W^*_\lambda > W_0$: By *Step 3*, the non-corner solution would be $q^p > q^m(0)$, therefore $d_{q^p} W_\lambda(q^p) > 0$ $\forall q^p < q^m(0)$. Hence, the corner solution is $q^p = q^p_i(1)$, and the improvement of the social welfare is warranted if and only if $q^p_i(1) > \bar{q}$, by the definition of $\bar{q}$. ■

Proof of Proposition 4. By the first order condition of government's program

$$-E(\partial_c u \cdot y) + E(\partial_c u) \cdot (E(y) + t \cdot E(\partial_q y \cdot d_t q^m)) \leq 0$$

but, strict concavity of utility implies $E(\partial_c u \cdot y) < E(\partial_c u) \cdot E(y)$ for any $t \in [0, 1)$, and by comparative statics on q^m, $E(\partial_q y \cdot d_t q^m)) < 0$, and tends to $-\infty$ as t approaches to 1. ■

Proof of Proposition 5. Whenever utility is strictly concave, the first order condition of (5) with respect to t is such that

$$-E(\partial_c u \cdot y) + E(\partial_c u) \cdot E(y) + \mu - \phi \cdot d_t q^p_i = 0$$

Let us remark that $\mu \geq 0$, $\phi \cdot d_t q^p_i \leq 0$, and $E(\partial_c u \cdot y) \leq E(\partial_c u) \cdot E(y)$; thus, the optimal policy has to be such that: $\mu = 0$, $\phi \cdot d_t q^p_i = 0$, and $E(\partial_c u \cdot y) = E(\partial_c u) \cdot E(y)$, implying in turn that $t^* = 1$ (so that households' consumption is equalized independently of their exogenous wealth); thus, the first order condition with respect to q^p becomes $E(\partial_q y) = p$[26]. ■

Proof of Proposition 7. Assume, by contradiction, $\delta = \delta_i(0)$ (so $\phi_0 > 0$), thus by the first order condition of (6) with respect to δ, $\left(\frac{\partial_c u(0)}{E(\partial_c u)} \cdot (1-t) + t\right) \cdot \partial_q y(0) < p$, but also, by definition of $\delta_i(0)$: $\partial_q y(0) \cdot (1-t) > p$; that implies a contradiction given that $\partial_c u(0) > E(\partial_c u)$. Now, by the first order conditions of program (6) with respect to q^p and δ it follows: $0 < \lambda \cdot (\partial_c u(0) - E(\partial_c u)) \cdot p = \eta_0 - \eta_1 + \phi_1 \cdot (1 + d_{q^p}\delta_i(1))$. Let us remark that $1 + d_{q^p}\delta_i(1) = \frac{\partial_q y(1, q^m(1) + q^p)}{\partial_q y(1, q^p + \delta_i(1))} > 0$; thus a necessary condition for the optimum is that either $\phi_1 > 0$ or that $\eta_0 > 0$. Assuming that heterogeneity of households in wealth (say, the difference between $\delta_i(0)$ and $\delta_i(1)$) is sufficient to exclude $\delta = \delta_i(1)$, then necessarily $q^{p*} = 0$. Finally, we remark that the first order condition of program (6) with respect to t tends to infinity as t approaches to 1, and it is strictly positive when $t = 0$, hence: $t^* \in (0, 1)$. ■

Proof of Proposition 8. By the first order conditions of program (7) with respect to q^p and δ it follows: $0 > \lambda \cdot (\partial_c u(1) - E(\partial_c u)) \cdot p = \eta_0 - \eta_1 - \phi_1 \cdot (1 + d_{q^p}\delta_i(1)) + \phi_0 \cdot (1 + d_{q^p}\delta_i(0))$. Let us remark that $1 + d_{q^p}\delta_i(\theta) = \frac{\partial_q y(\theta, q^m(\theta) + q^p)}{\partial_q y(\theta, q^p + \delta_i(\theta))} > 0$; thus a necessary condition for the optimum is that either $\phi_1 > 0$ or that $\eta_1 > 0$. If $\eta_1 > 0$, then $q^{p*} = q(0, t, p) > q(1, t, p)$, and $\delta^* = \delta_i(0) = \delta_i(1) = 0$. If $\eta_1 = 0$ ($q^{p*} < q(0, t, p)$), then $\phi_1 > 0$, $\delta^* = \delta_i(0) \geq 0$. Let us remark that, if $q^{p*} \geq q(1, t, p)$, then $\delta_i(0) = 0$, and also $q^{p*} + \delta_i(0) \geq q(1, t, p)$. Thus, assume - by contradiction - that $q^{p*} < q(1, t, p)$, hence $q^{p*} + \delta_i(0) < q(1, t, p)$; in the considered case, the first order condition with respect to q^p would be

$$(1 - \lambda) \cdot \Big[\partial_c u(1) \cdot \big(\partial_q y(1) \cdot (1 - t) - p\big) + E(\partial_c u) \cdot t \cdot \partial_q y(1)\Big] - \phi_1 \cdot d_{q^p}\delta_i(1) = 0$$

the second term is positive, hence it would imply that the first term is negative, namely

[26]Moreover, by $t^* = 1$, $\max\{q_i^p(0), q_i^p(1)\} = 0$, and $\phi = 0$ for any $q^{p*} > 0$.

that $\partial_q y(1) \cdot (1-t) < p$, that contradicts the assumption that $q^{p*} + \delta_i(0) < q(1,t,p)$. Finally, we remark that the first order condition of program (7) with respect to t tends to infinity as t approaches to 1, hence: $t^* \in [0,1)$. ■

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