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EFFICIENT PORTFOLIOS WHEN HOUSING NEEDS  
CHANGE OVER THE LIFE-CYCLE

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# *Efficient Portfolios when Housing Needs Change over the Life-Cycle<sup>#</sup>.*

by

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## **Abstract:**

We address the issue of the efficiency of household portfolios in the presence of housing risk. We treat housing stock as an asset and rents as a stochastic liability stream: over the life-cycle, households can be short or long in their net housing position. Efficient financial portfolios are the sum of a standard Markowitz portfolio and a housing risk hedge term that multiplies net housing wealth. Our empirical results show that net housing plays a key role in determining which household portfolios are inefficient. The largest proportion of inefficient portfolios obtains among those with positive net housing, who should invest more in stocks.

***JEL Classification:*** D91, G11

***Keywords:*** Housing and portfolio choice, Portfolio efficiency, Rental risk, Life-cycle.

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## **Introduction**

The role of real estate in personal portfolio management is controversial, and often neglected in the finance literature. Allowing housing wealth into total wealth in the analysis of portfolio choice raises conceptual issues: the main residence provides essential housing services, and it is less than obvious that it should be considered as wealth. Also, housing needs change with age, particularly because of demographics. Demographics drive housing needs up in the first half of the life cycle, down in the second half, very much the same way they also affect non-durable consumption (Attanasio et al, 1999). Consumers can meet their housing need in two different ways: they can rent housing services, or they can purchase housing stock. When the price of renting positively correlates with house prices, home-ownership is a way to reduce the risk related to the consumption of housing services (as argued in Sinai and Souleles, 2005).

In this paper we provide a conceptual framework in which households treat their main residence differently according to their age for the purposes of portfolio management. Elderly households should count most of their main residence as wealth, as they could liquidate it to buy different goods (medical care, long term care, and holidays), while easily meeting their likely housing needs over their remaining years by renting. Single, young households, on the other hand, would be unwise to consider their main residence as wealth, given that they are likely to trade up in the future.

The way we address the issue of optimal portfolio choice in the presence of housing risk is to explicitly treat housing stock as an asset, and rents as a stochastic liability stream (in the sense of Elton and Gruber, 1992). We derive conditions under which standard mean variance analysis holds once wealth includes the value of real estate net of the rent liability. Our wealth definition allows us to distinguish between investors who are long on housing, or “over-housed” (the value of the housing stock they own exceeds the present value of future housing services, that is they have a long net housing position) and short on housing, or “under-housed” (vice-versa, short net housing

position). The former group is more exposed to house price risk, the latter (that includes tenants and young homeowners whose housing needs are increasing) to rent risk.

In this context we show when and how financial portfolio allocations are affected by the presence of a non-zero net housing position. We also show how one can assess the efficiency of financial portfolios when the optimal allocation is indeed affected by the presence of this term.

To do this, we develop a simple dynamic model and derive conditions under which this allows the analysis to be carried out in a static mean-variance framework. Under these conditions, we see that households should allocate financial assets with two objectives in mind: to maximize the expected return of their portfolio, given a certain risk (standard Markowitz portfolio), and to hedge the risk in their net housing position. We can test whether household portfolios are efficient conditionally on housing by computing a statistic that is based on the financial portfolio Sharpe Ratio (the ratio of the mean excess return to the standard deviation) after allowance has been made for the hedge term.

In the empirical application, we derive the optimal financial portfolios for any given net housing position and ask whether household portfolios are in line with these optimal portfolios, that is whether they are efficient, given the presence of housing risk. In our model, we allow housing consumption needs to change with age, but assume that they are given to the household. To satisfy them, households can rent or own housing stock, but in both cases they bear risks, because the price of the house and the rental rate are driven by a single stochastic process that correlates with financial assets returns. This allows us to distinguish between households that have long housing positions (“over-housed”) – or short positions (“under-housed”).

The aim of our application is to evaluate the empirical relevance of housing risk in household portfolios. In particular, we compare the efficiency of asset allocations for households who are over-housed, under-housed or have net housing positions close to zero.

In our application, we analyze household portfolio data from the Italian 2002 Survey on Household Income and Wealth (2002). This survey is run by Bank of Italy and contains detailed information on a number of financial variables, such as self-reported values for household portfolio positions, as

well as on the market and rental value of the main residence. It also contains records on earnings, expected or actual retirement age, occupation and pension income of each individual in the household. For each household we impute a value for human capital and for the present value of future rents, by exploiting information available in previous waves of the survey (SHIW waves from 1989 to 2000). We also use data on financial assets returns and on housing returns from other sources.

This paper is organized as follows. In the next section we present the intuition of our empirical strategy and relate it to the literature on the role of housing investment in portfolio choice (the formal dynamic model is derived in the Appendix). Section 2 describes the data and section 3 presents the empirical results. Section 4 presents robustness analysis and section 5 concludes.

## 1. Analysis conditional on housing

Standard mean-variance analysis (Tobin, 1958, Markowitz, 1952) implies that the vector of asset holdings should satisfy:

$$(1) \quad \underline{X}_0^{*T} = \begin{bmatrix} -\frac{\partial U}{\partial FW} \\ \frac{\partial^2 U}{\partial FW^2} \end{bmatrix} \Sigma^{-1} \underline{\mu}^T$$

where FW is financial wealth,  $\Sigma$  is the variance-covariance matrix of returns on  $n$  risky assets, and  $\underline{\mu}$  is the row vector of expected excess returns (that is, returns in excess of the risk-free rate). The sum of the  $X$ 's is the wealth invested in risky assets.  $U(FW)$  is the utility function – in the simplest case, investors are assumed to maximize the expected utility of end-of-period wealth and returns are normally distributed.

Equation (1) can be derived in a dynamic model where all wealth is invested in liquid financial assets whose returns follow Brownian motions with time-invariant parameters (Merton, 1969).

However, things may change when some assets are illiquid, and therefore are traded infrequently, such as housing. There are circumstances where the standard analysis applies even in the presence

of illiquid real assets over those periods when these are not traded (Grossman and Laroque, 1990, Flavin and Nakagawa, 2004). But in the more general case where housing returns correlate with financial assets returns, the standard analysis fails to capture the presence of a hedge term in the optimal portfolio (Damgaard, Fuglsbjerg and Munk, 2003).

In the vast literature on efficient portfolios, only a few papers incorporate real estate as an asset. Goetzmann and Ibbotson (1990) and Goetzmann (1993) use regression estimates of real estate price appreciation, and Ross and Zisler (1991) calculate returns from real estate investment trust funds, to characterize the risk and return to the real estate investment. Flavin and Yamashita (2002) use data from the 1968-1992 waves of the Panel Study of Income Dynamics that contain records on the owner's estimated value of the house and compute rates of return from regional real estate price data. Flavin and Yamashita characterize the efficiency frontier for house owners, when the house cannot be changed in the short run and there are non-negativity constraints on all assets. They consider the case where financial returns are not correlated with housing returns, and therefore the main effect of housing is to change the background risk faced by investors. Pelizzon and Weber (2006) extend Flavin and Yamashita to cover the case of non-zero correlation between housing and financial asset returns, but concentrate on home-owners and do not take rent-risk into account. Finally, in a model with short-selling constraints, fixed costs of stock market participation and where the mortgage cannot exceed a fraction of the housing stock owned, Cocco (2005) finds that investment in housing can explain the variability of stockholdings in household portfolios, and in particular limited participation among younger and poorer households.

In this paper, we analyze a model where consumers maximize a value function that is the sum of life-time utility of non-durable and durable consumption and a bequest function that depends on total bequeathed wealth, subject to the standard intertemporal constraints. We treat consumption of durable goods services as exogenously determined ("housing needs"), and allow households to either purchase or rent the necessary housing stock (as Sinai and Souleles, 2005, and Yao and Zhang, 2005). We also allow for age-specific survival probabilities, and assume that there is unitary

(or anyhow constant) correlation between the housing stock return and the rental rate of housing services (this assumption is also made by Yao and Zhang, 2005). All asset returns (including the housing return) follow Brownian motions, and they correlate with each other. Human capital is instead assumed to be a risk-free investment (or it has non-systematic risk – see Bodie, Merton and Samuelson, 1992).

In this context, we argue that the relevant notion of housing wealth is the difference between housing stock owned and the present value of current and future rents. Typically, this position is negative for young households who are likely to trade up in the housing market, it is positive for old households, whose housing needs are decreasing and are instead interested in the liquidation value of the house because they wish to trade down (as stressed in Banks et al., 2004) or have a bequest motive. In this model, home-ownership provides insurance against rent risk (see Sinai and Souleles, 2005). Rents are a stochastic liability stream, similarly to pension payments for pension funds (Elton and Gruber, 1992, and Campbell and Viceira, 2005)

In this model (that is presented and developed more formally in the Appendix), if households have non-zero positions in housing (that is, if their home is worth more or less than the present value of their future housing needs), and if financial returns have non-negative correlations with housing returns, the standard analysis is no longer valid.

In fact, we can show that in our model, when owning is a perfect hedge against rent risk, efficient portfolios satisfy the following relation:

$$(2) \quad \underline{X}_0^{*T} = \begin{bmatrix} -\frac{\partial J}{\partial TW} \\ \frac{\partial^2 J}{\partial TW^2} \end{bmatrix} \Sigma^{-1} \underline{\mu}^T - P_0 D_0 \Sigma^{-1} \Gamma_{bP}^T$$

where:

- $TW$  denotes total wealth (the sum of financial wealth,  $FW$ , human capital,  $HC$ , and the value of the home,  $H$ , net of debt and of the present value of future rents).

- $P_0D_0$  is defined as the difference between the value of the home,  $H$ , and the present value of future rents (housing needs). Households are long on housing (over-housed) if  $P_0D_0$  is positive (they have more housing stock than is implied by their consumption requirements), are short on housing (under-housed) otherwise (this includes people who rent their home, or whose current home is inadequate considering their future needs).
- $\Sigma$  is the variance-covariance matrix of returns on risky financial assets, and  $\mu$  is the vector of expected excess returns (that is, returns in excess of the risk-free rate).
- $\Gamma_{bp}$  denotes the (row) vector of covariances between the return on housing and on risky financial assets.
- $J$  is the value function of the intertemporal optimization problem.

Equation (2) reveals that the optimal portfolio is the sum of a standard Markowitz portfolio and a hedge term (see also Mayers, 1973, and Anderson and Danthine, 1981). The former is multiplied by the inverse of absolute risk aversion, whereas the latter is not. This implies that risk-averse investors should hedge housing return risk in exactly the same way, for a given net housing position.

In the sequel of the paper we shall show the importance of the net housing position as defined here. Gross housing wealth apparently accounts for the largest portion of the sum of financial and real wealth, but this does not take into account the future rents liability, that is overall of comparable magnitude as housing wealth, even though it differs greatly across households.

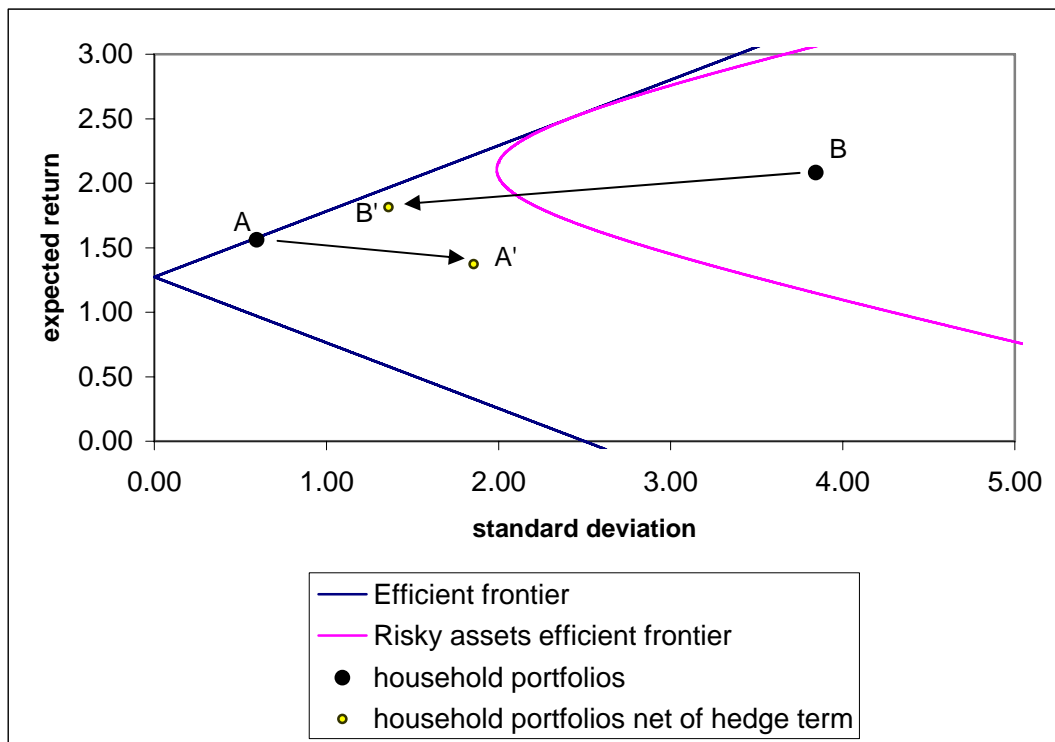
Mean-variance efficiency is usually assessed on the basis of a graphical comparison. The efficient frontier is drawn in the expected return-standard deviation space, and single portfolios are also represented in this space.

In Figure 1 we show how our analysis can be interpreted graphically. The broken line is the mean-variance frontier for financial assets (housing is not included). The dark dots represent two household portfolios, one of which, **A**, clearly efficient according to equation (1) (it lies on the efficient frontier), the other, **B**, probably inefficient (it lies well below). However, these conclusions



are incorrect if the households have non-zero net housing wealth. In fact, equation (2) implies that efficiency should be evaluated once the hedge term is taken out of the portfolio.

**Figure 1. The effects of net housing wealth on portfolio efficiency**



In our graphical example this is shown by the arrows. The resulting portfolios (indicated by circles) look quite different now: the first household portfolio, which apparently lied on the efficient frontier, now appears inefficient, **A'**, whereas the reverse is true for the second household portfolio, **B'**. The economic interpretation of these two cases is simple: the first household closely follows the standard Markowitz rules, but fails to hedge housing risk. The second household instead correctly hedges housing risk, but the resulting portfolio appears inefficient when the standard approach is taken.

Graphical comparisons are extremely useful, but do not take into account that the efficiency frontier is not known, rather it is estimated (and the same applies to the mean-variance performance of any given portfolio). To assess whether a portfolio is indeed efficient we need to take into account this source of variability, that is we need to use a formal statistical test. Jobson and Korkie (1982, 1989)

and Gibbons, Ross and Shanken (1989) have proposed a test of the significance of the difference between the actual portfolio held by an investor and a corresponding efficient portfolio. This test is based on the difference between the slopes of arrays from the origin through the two portfolios in the expected return-standard deviation space. If the actual portfolio is an efficient portfolio, the two slopes will be the same; if the actual portfolio is inefficient, the slope of the efficient portfolio will be significantly greater. These slopes are of course the Sharpe ratios, which relate directly to expected utility, as shown in Gouriéroux and Monfort (2003).

We can test whether household portfolios are efficient conditionally on housing by computing a statistic that is based on the financial portfolio Sharpe Ratio after allowance has been made for the hedge term. In fact, Gouriéroux and Jouneau (1999) derive an efficiency test for the conditional or constrained case, i.e. for the case where a subset of asset holdings is potentially constrained (housing in our case). They define the Sharpe ratio of the unconstrained risky financial assets portfolio as:

$$(3) \quad S_1 = \underline{\mu} \underline{\Sigma}^{-1} \underline{\mu}^T$$

The Sharpe ratio for the observed (constrained) portfolio made of the first  $n$  (financial) assets is defined in this notation as:

$$(4) \quad S_1(Z) = \frac{[\underline{\mu} v_1^T]^2}{v_1 \underline{\Sigma} v_1^T}$$

where  $v_1^T = \underline{X}_0^T + P_0 D_0 \underline{\Sigma}^{-1} \Gamma_{bp}^T$  (see equation 2), that is the actual risky financial asset portfolio after

eliminating the hedge term and  $Z^T \equiv \begin{pmatrix} X_0^T \\ P_0 D_0 \end{pmatrix}$ .

When all asset returns are normally distributed, Gouriéroux and Jouneau show that the Wald statistic:

$$(5) \quad \xi_1 = T \frac{\hat{S}_1 - \hat{S}_1(Z)}{1 + \hat{S}_1(Z) \frac{Z \Omega Z^T}{v_1 \underline{\Sigma} v_1^T}}$$

is distributed as a  $\chi^2(n-1)$  under the null hypothesis that the risky financial assets portfolio (after eliminating the hedge term) lies on the financial efficient frontier<sup>1</sup> where:

$$(6) \quad \Omega = \begin{pmatrix} \Sigma & \Gamma_{bP}^T \\ \Gamma_{bP} & \sigma_p^2 \end{pmatrix}$$

where  $\sigma_p^2$  is the variance of excess returns on housing.

Gourieroux and Jouneau also show that a test for the efficiency of the whole portfolio can be derived as a special case by setting  $v_1 = Z$ . In this special case, this test is asymptotically equivalent to the test derived by Jobson and Korkie (1982, 1989) and Gibbons, Ross and Shanken (1989).

The intuition behind the test is the following. The standard test for portfolio efficiency is based on (the square of) the Sharpe ratio. The Sharpe ratio is in fact the same along the whole efficient frontier (with the exception of the intercept), that is along the capital market line. This test breaks down when one asset is taken as given, because the efficient frontier in the mean-variance space corresponding to all assets is no longer a line, rather a curve. However, equation (2) implies that we can go back to the standard case when the analysis is conducted conditioning on a particular asset, once the hedge term component is subtracted from the observed portfolio. That is, a Sharpe ratio can be used to test for efficiency in the mean variance space corresponding to the “unconstrained” assets, after allowance has been made for the presence of the same hedge term in all efficient portfolios.

## 2. Application.

To show the implications of our theoretical analysis we use data on Italian asset returns and household portfolios. Italy provides a good test case to study the effect of housing on portfolios because home ownership is wide spread and household stock market participation is relatively low

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<sup>1</sup> For the sake of simplicity we do not stress in our notation that the test statistic is defined as a function of sample estimates of the first two moments of the rates of return distribution and takes observed portfolio shares as given.

but has much increased in recent years. As we shall see, in Italy housing returns unambiguously correlate with financial returns, thus providing the need for a hedge term in house owners portfolios. Also, an attractive feature of Italy for our purposes is that pension wealth, whose amount is typically not recorded in survey data, is still almost entirely provided by the public pay-as-you-go social security system and is therefore both out of individual investors' control and not directly related to the financial markets performance. Finally, mortgages are rare compared to countries like the US or the UK, and particularly reverse mortgages (equity lines) are not yet available.

Italian households traditionally have held poorly diversified financial portfolios (Guiso and Jappelli, 2002). In the 1980s and even more in the 1990s, though, the stock exchange has grown considerably and mutual funds have become a commonly held financial instrument. Household financial accounts reveal that the aggregate financial portfolio share in stocks and funds amounted to 16.15% in 1985, 20.69% in 1995 and rose to an unprecedented 46.95% in 1998. It then fell sharply to 35.31% in 2002. This growth in the equity market paralleled the sharp decrease in importance of bank accounts and short-term government debt in household portfolios. These aggregate statistics are uninformative on the participation issue, though. To this end, an analysis of survey data is required. The most widely used Italian survey data, the Bank of Italy-run Survey on Household Income and Wealth (SHIW), shows direct or indirect participation in equity markets (broadly defined to include life insurance and private pensions) to have increased from 26.43% in 1989 to 33.18% in 1995 and to 37.25% in 1998; this was followed by a relatively small decrease to 34.89% in 2002. For comparison, the percentage of house-owners in the same sample hovered around 63-69% over the period.

These summary statistics clearly show that household financial portfolios have changed a great deal over the years, and that a key role in total household wealth is played by real estate. It makes sense to consider the interaction of housing and financial wealth holdings when assessing the efficiency of household portfolios, as stressed by Flavin and Yamashita (2002). A financial portfolio may deviate

from the mean variance frontier for financial assets simply as a result of its covariance properties with the return on housing equity.

In our application we use household portfolio data for 2002 and asset return data for the period 1989-2002.

The 2002 SHIW wave contains detailed information on asset holdings of 8011 households as of 31.12.2002, as well as self assessed value of their housing stock (both principal residence and other real estate) and actual or imputed rent for each dwelling. For each household we also know the region of residence and a number of demographic characteristics. The survey does not over sample the very rich, and it therefore captures about a third of total household financial wealth. It does cover a relatively large number of assets, including individual pension funds: these are still remarkably unimportant in Italy, though, partly because of inadequate tax incentives. Occupation pension schemes are also relatively minor, even though recent reforms of the Italian Social Security system (particularly the Dini reform of 1995) imply that they should become wide-spread.<sup>2</sup>

Asset return data cover four major assets: short term government bonds (6 month BOT), corporate bonds, government bonds, and equity (the MSCI Italy stock index). We treat the short-term bond as risk free, and assume that this is the relevant return on bank deposits, once account is taken of non-pecuniary benefits. For bonds we derive the holding period returns (HPR) as follows. For government bonds we take the MSCI Italian Government bond index after 1993. Prior to December 1993 this index is not available, and we use our own estimates of the term structure based on quoted prices of Italian government bonds to determine the holding period return by assuming a duration of five years<sup>3</sup>. For corporate bonds we compute the HPR by deriving the prices consistent with the RENDIOBB index (the index of Italian corporate bonds yields) and assuming a duration of three years. We express all returns net of withholding tax, on the assumption that for most investors other tax distortions are relatively minor (financial asset income is currently subject to a 12.5%

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<sup>2</sup> Further information on the survey is provided in Guiso and Jappelli (2002) and Biancotti et al. (2004). Information on the Italian pension system and its recent reforms is presented in Brugiavini and Fornero (2001).

<sup>3</sup> We checked the quality of this procedure by regressing our monthly holding period returns on those of the MSCI Italian bond index over the period December 1993 to December 2002 and found an almost perfect fit.

withholding tax. Housing is taxed on the basis of its ratable value, while actual rental income is taxed at the marginal income tax rate).

To evaluate the efficiency of households' portfolio we need to determine the expected return and the expected variance covariance matrix of the assets. Given long, stationary series we could simply compute the corresponding sample moments of the assets excess returns. However, this approach is unlikely to work in our case: our sample period is 1989-2002 (and cannot be extended because some assets did not exist prior to the mid 1980's), and in the 1990s we observe a long convergence process of Italian interest rates to German interest rates that accelerated dramatically in the few years before the introduction of the Euro on January 1999.

Estimation error is of particular concern for first moments and calls for use of prior information in estimation (see for instance Merton, 1980, and Jorion, 1985). In our case, we should estimate the first moments by a Bayesian method that exploits prior information on convergence of particularly long-term government bond rates to its German equivalent, and possibly a multivariate GARCH for the second moments. Unfortunately, we do not have enough data points to perform sophisticated estimation exercises. In fact, housing returns are available at a biannual frequency, and we are therefore forced to use at most twenty-nine data points. However, we can exploit prior information on convergence by using a simple Weighted Least Squares procedure, where the raw return series data are down weighted more the farther away they are from December 1998 (they have a unitary weight from 1999 on). More precisely, we construct the weights to be a geometrically declining function of the lag operator multiplied by  $\alpha$  (where  $\alpha$  is set to 0.9). The weighted series are used to compute sample first and second moments<sup>4</sup>.

In Table 1 we show the first and second moments of the excess returns data we use (1989-2002). These are expressed as percentage annual rates of return net of the time-varying risk-free rate.

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<sup>4</sup> A similar procedure for second-order moments is often used in the financial industry (see RiskMetrics, 1999) and can be shown to be equivalent to particular GARCH models (Phelan, 1995).

**Table 1: Sample first and second moments of annual excess returns**

	<b>Government Bonds</b>	<b>Corporate Bonds</b>	<b>Stocks</b>
<b>Expected return %</b>	4.0981	2.2845	4.9011
<b>Standard Deviation %</b>	5.2383	3.2169	28.9950

<i><b>CORRELATION</b></i>	<b>Government Bonds</b>	<b>Corporate Bonds</b>	<b>Stocks</b>
<b>Government Bonds</b>	<b>1</b>	<b>0.8404</b>	<b>0.0215</b>
<b>Corporate Bonds</b>		<b>1</b>	<b>0.1726</b>
<b>Stocks</b>			<b>1</b>

We see that stocks have higher expected excess return (4.9%) and higher variance than all other risky financial assets. Government bonds also have high expected excess return (4.1%), due to their long maturity. Corporate bonds rank last both in terms of expected excess return (2.3%) and variance.

It is comforting to see that our estimated excess return on equity (4.9%) is not far from what is normally found using much longer sample periods: Dimson et al. (2006) report a 5.7% average equity premium over the 1950-2000 period, in line with secular evidence provided by Panetta and Violi (1999). Our expected excess return for corporate bonds is also similar to the one reported by Dimson et al (2006) for medium term bonds (2.5%) over the 1950-2000 period. Long term government bonds unfortunately did not exist prior to our estimation period, so no comparison with other sources is possible.

Correlation coefficients between bonds are quite high (.84) – correlation coefficients of stocks and bonds are positive, but much smaller. Not surprisingly, stock returns correlate more with corporate bonds (.17) than with government bonds (.02).

This picture is however largely incomplete. We know that two households out of three own real estate, and we argued that this type of investment is highly illiquid. Even those who do not own housing stock consume housing services and should hedge the risk of future purchases of either stocks or services. It is therefore of great interest for us to compute first and second moments of the housing stock. To this end we use province-level biannual price data (source: Consulente

Immobiliare) covering the whole 1989-2002 period. We compute the return on housing according to the formula:

$$(7) \quad r_{H,t} = \frac{P_t - P_{t-1}}{P_{t-1}} + \frac{D_t - COM_t}{P_{t-1}} = \frac{P_t - P_{t-1}}{P_{t-1}} + \kappa$$

where  $D$  denotes rent and  $COM$  maintenance costs. Given that we lack time series information on these, we set  $\kappa=0.025$  (5% on an annual basis), as in Flavin and Yamashita (2002). It is worth stressing that the choice of  $\kappa$  is immaterial in our analysis, as long as  $\kappa$  is a fixed number (see equation 2, where  $\mu_H$  does not appear). We in fact take housing positions as given, and the expected excess return on housing plays no role in evaluating whether financial portfolios are conditionally efficient.

Finally, we aggregate housing returns to the macro-region level (we use provincial resident population numbers to generate weights). The first and second moments are then determined using (prior to 1999) the time-varying weights described above.

Our regional classification splits the country in North West (that includes the three large industrial cities of Milan, Turin and Genoa), the North East (that includes many middle-sized cities and towns, such as Bologna, Venice, Verona, Trieste), the Centre (that includes the capital city, Rome, and many medium-sized town such as Florence, Perugia and Ancona) and the South (largely rural, but including Naples and Bari). The two large islands, Sicily and Sardinia, are also counted as South here.

Table 2 reveals that expected excess returns on housing are highest in the North East and in the South and lowest in Central Italy (they range between 3.3% and 4.2% on an annual basis). They are close to returns on bonds, but are much lower than returns on stocks. Housing excess return standard deviations range between 5.1% and 6.5%, and are therefore much lower than on stocks, but comparable to bonds. Of interest to us is the negative correlation between housing returns and most financial asset returns.



**Table 2: Expected excess returns and correlation matrix of housing (1989-2002)**

	NW	NE	Centre	South
<b>Expected excess return %</b>	3.2922	4.1883	3.2791	3.3036
<b>Standard deviation %</b>	5.5774	5.0755	6.5381	5.0715

	NW	NE	Centre	South
<b>Government bonds</b>	-0.0164	-0.1169	-0.1161	-0.2036
<b>Corporate bonds</b>	-0.0843	-0.1691	-0.2177	-0.1998
<b>Stocks</b>	-0.5057	-0.2790	-0.4172	-0.1506

The issue arises of whether these correlations are negligible. The simplest way to assess this is to estimate the coefficients of the hedge term in equation (2), that is to estimate the beta hedge ratio  $\Sigma^{-1}\Gamma_{bp}$ . This can be done by running the regression of housing returns on financial asset returns, as suggested by de Roon, Eichholtz and Koedijk (2002). Parameter estimates and their standard errors are summarized in Table 3.

**Table 3: Regression of excess return on housing on financial assets excess returns**

Variable	North West	North East	Centre	South
<b>Constant</b>	2.6378 (0.556)	2.8218 (0.591)	2.7910 (0.737)	2.8088 (0.565)
<b>r<sub>GOV.</sub></b>	-0.0128 (0.280)	0.0392 (0.297)	0.1190 (0.371)	-0.1461 (0.284)
<b>r<sub>CORP</sub></b>	-0.2757 (0.477)	-0.5013 (0.507)	-0.7619 (0.632)	-0.3794 (0.484)
<b>r<sub>STOCKS</sub></b>	-0.0968 (0.028)	-0.0427 (0.030)	-0.0844 (0.0374)	-0.0232 (0.0287)
p-value	0.001	0.030	0.015	0.012
<b>R<sup>2</sup></b>	.523	.350	.390	.405

Notes: Standard errors in parentheses. Number of observations = 28

We see that in two regions (NW and CE) there is at least one non-zero parameter at the 95% significance level and in all regions the slope coefficients are jointly significantly different from zero at the 95% level (the p-value of the F-test is reported at the bottom of the table, together with the R<sup>2</sup>). The regions where this test is least significant are the North East (with a p-value of 3%).

On the basis of this evidence, we conclude that housing returns present significant correlations with financial asset returns in Italy, and this provides the basis for introducing a hedge term in household portfolios of house-owners. De Roon, Eichholtz and Koedijk (2002) find that a similar result is also true for some areas in the U.S., but do not analyze the efficiency of U.S. household portfolios. We also find evidence, available upon request, of significant correlations with excess returns on at least some financial assets in other European countries (France, Germany, Spain and the UK).

In Table 4 we report the percentage participation for each asset and liability recorded in SHIW2002. For instance, we see that over 74% of the sampled households have a bank current (i.e. checking) account, and almost 17% have the post-office equivalent.

We also show in the last column of Table 4 where each asset is classified, given that we use asset returns data at a much coarser aggregation level. So, for instance, five of the first seven assets (cash, various deposits) are classified as risk-free.

Of particular interest is the relatively low direct stock market participation (9% hold listed shares; less than 1% shares in unlisted companies). However, 10.76% of all households have mutual funds: in particular, 4.17% have stock funds, 3.16% bond funds, 5.12% mixed funds and 1.13% have monetary funds. These holdings we classify as stocks, government bonds, corporate bonds or risk-free asset on the basis of recorded information on the type of mutual fund held and industry wide information on average investments by type (Assogestioni). We similarly split holdings in managed savings into government bonds, corporate bonds and stocks. Of great relevance for our analysis is also the high proportion of households who own some housing stock (almost 70%). Liabilities are relatively little wide-spread (10% households report mortgage; 11% other forms of consumer debt). We split mortgages in three groups: fix-interest mortgages (3%), floating-interest mortgages (4%) and mortgages intended for home-improvement (3%). We treat fix-interest mortgages as negative positions on government bonds (the longest maturity bonds available in Italy) and floating-interest mortgages as negative positions on the risk-free asset.

**Table 4: Participation decision - individual financial and real assets**

<b>Asset</b>	<b>Participation</b>	<b>Broad Asset</b>
Cash	100%	Risk-free
Bank Current Account Deposits	74.07%	Risk-free
Bank Savings Deposits (Registered)	11.00%	Risk-free
Bank Savings Deposits (Bearer)	2.01%	Risk-free
Certificates of deposit	118%	Gov. Bonds
Repos	0.50%	Gov. Bonds
Post Office Current Accounts and Deposit Books	16.86%	Risk-free
Post Office Savings Certificates	4.76%	Gov. Bonds
BOT (Italian T-bills)	7.13%	Gov. Bonds
CCT (Italian T-certificates)	2.05%	Gov. Bonds
BTP (Italian T-bonds)	1.96%	Gov. Bonds
CTZ (Italian zero-coupon)	0.38%	Gov. Bonds
Other Italian Government Debt (CTE, CTO, etc.)	0.22%	Gov. Bonds
Corporate Bonds	6.00%	Corp. Bonds
Mutual Funds	10.76%	Bonds & Stocks
Shares of listed companies	9.05%	Stocks
<i>of which:</i> of privatized companies	4.80%	Stocks
Shares of unlisted companies	0.89%	Stocks
Shares of limited liability companies	0.18%	Stocks
Managed Savings	1.98%	Bonds & Stocks
Foreign bonds and government securities	0.72%	Bonds
Foreign Stocks and Shares	0.40%	Stocks
Other foreign assets	0.06%	Bonds & Stocks
Loans to co-operatives	1.21%	Stocks
House (main residence and other)	69.01%	Housing wealth
Fix-rate mortgages	3.32%	Gov. Bonds (-)
Floating-rate mortgages	3.78%	Risk-free (-)
Home-improvement mortgages	3.10%	Corp. Bonds (-)
Debt	11.13%	Corp. Bonds (-)

**Notes:** Number of observations: 8011. Population weights have been used.

We treat mortgages for home-improvement and other debt as negative positions on corporate bonds because these debt contracts are normally fix-interest and medium-term maturity. The analysis of Section 1 and the Appendix highlights that the relevant wealth concept is the sum of financial wealth, human capital, and housing wealth net of the present value of future rents (PVR) and total debt. Two key variables are not directly observable and have to be constructed: human capital and the present value of housing needs.

In principle, for each individual in SHIW 2002 we would like to know current and future earnings, current and future pension income, as well as retirement and survival probabilities. This would produce the best possible estimate of human capital wealth. Similarly, for each household in the sample, we would like to know current rent (actual or imputed) and its likely changes in the future that relate to changes in family size and composition, to retirement or death of either spouse, or indeed to changes in economic circumstances of the household. These data, combined with survival probabilities, could be used to calculate a household-specific measure of the PVR, the present value of current and future housing needs.

Only a small part of these data are available in SHIW 2002, but further relevant information can be found in previous waves of SHIW (that refer to 1989, 1991, 1993, 1995, 1998 and 2000).

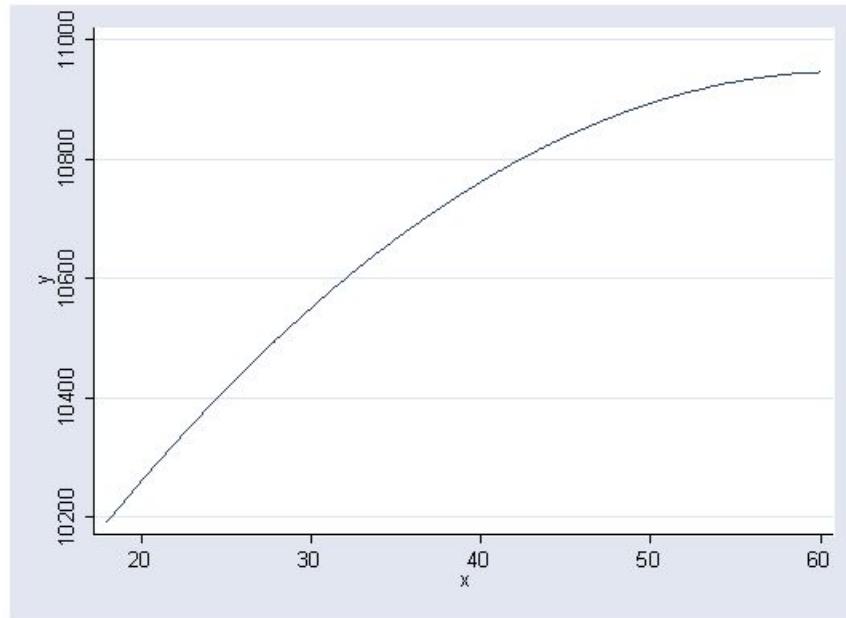
The method we adopt is to use the pooled SHIW data to estimate some relations (for earnings and rent), controlling for age, year of birth and a few characteristics, and use the estimated profiles to project forward the current values reported by SHIW 2002 respondents. These projections are then multiplied by the relevant (age and gender specific) survival probabilities and discounted to get a household-specific estimate of Human Capital and PVR.

Let us consider human capital first. We take individual real earnings for all working individuals in SHIW 1989-2002 (58835 observations in all, spread over 7 different sampling periods), and estimate a multiple regression equation as a function of a second order polynomial in age, education and gender dummies, plus twenty cohort dummies (based on three year intervals for the year of birth). Macroeconomic effects are assumed to be in the error term. The estimated age profile is shown in Figure 2. This is used to impute future earnings to SHIW 2002 observations as follows: an individual reports her current earnings, expected retirement age and replacement rate.

Future earnings up to the expected retirement age are obtained by assuming their growth to be as estimated in the SHIW sample. After retirement, the expected replacement rate determines the first pension payment. Further payments are assumed to be constant in real terms (in agreement with current legislation for public pensions). In the case of couples, we take into account that survivor

pensions are 60% of the original pension. Age, gender and cohort-specific survival probabilities are used to compute expected earnings and pensions; all future terms are discounted at the risk-free rate (net of inflation).

**Figure 2: Estimated age profile for earnings**



The Present Value of Rent was computed along similar lines, using data on actual or imputed rent for the main residence. A real rent equation was estimated at the household level on pooled SHIW data, conditioning on a second-order polynomial in head's age, a tenant dummy plus a set of cohort dummies as described above. The estimation sample has 53367 observations in all.

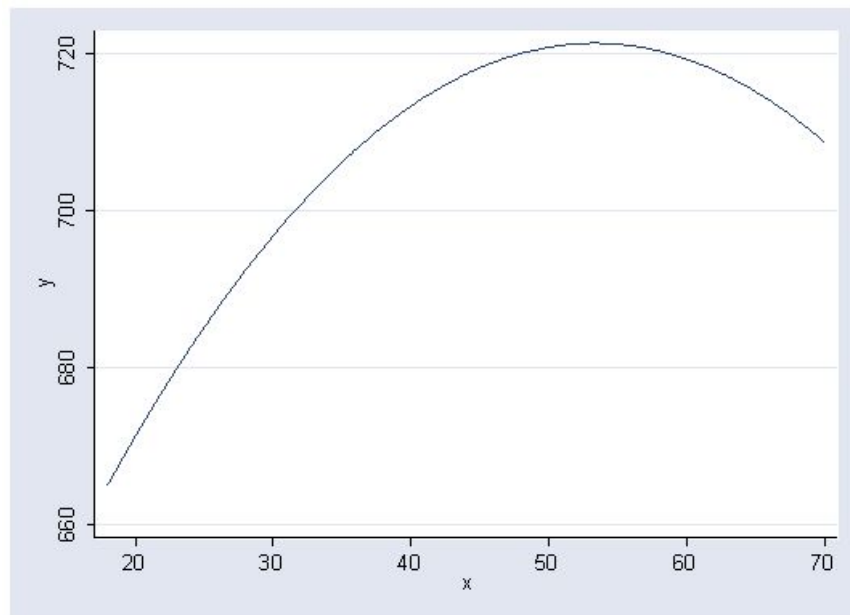
Figure 3 shows the estimated age profile. The most remarkable feature of this profile is that there is a pronounced hump. The initial rise with age is likely due to trading up and/or major home improvements typical of household formation and birth of children. The fall with age estimated to occur after age 58 may be related to actual trading down (purchase of smaller homes once the children have left) or, more likely in the Italian context, to a failure to carry out maintenance and repairs.

The PVR was calculated for each SHIW 2002 household starting from their reported rent, letting future rents evolve according to the estimated profile plus the expected return on housing, taking

into account each spouse's survival probability, and discounting at the expected return on housing in agreement with our theoretical model.

It is worth stressing that our empirical strategy allows for heterogeneity in preferences and/or opportunity sets by taking the current home as an individual-specific basis for the calculation of future needs. We then use (for lack of better, individual-level information) average growth rates in housing consumption (assumed proportional to the real home value) to project forward future needs, and allow for survival probabilities to compute the present value of future housing consumption.

**Figure 3: Estimated age profile for rent**



In the rest of the paper, we focus on those observations with valid records of financial assets and housing stock values and for which we have been able to derive an estimate of both human capital and the present value of rent. This occurs in 7457 cases out of 8011.

Table 5 shows average and median amounts for the broad assets and liabilities we consider: four financial assets, three types of debt, housing, the present value of rent and human capital. We see that financial assets are a relatively small component of total wealth: their average is in the € 23,000 region, whilst average total wealth is close to € 500,000. By far the largest component of total wealth is human capital, which is computed as the present value of future earnings and pension payments and is treated as a risk-free asset. This is a constructed variable, and therefore sensitive to

the particular assumptions made on discount factors, earnings and pensions age profiles, survival probabilities and so forth. For this reason, we have carried out robustness checks of our efficiency analysis with respect to the value of the risk-free position.

**Table 5: Amounts held in financial and real assets**

<b>Asset</b>	<b>(1) Average</b>	<b>(2) Median</b>	<b>(3) Conditional Average</b>
Risk-free Financial Assets	12,728	5,200	15,410
Government Bonds	4,885	0	14,136
Corporate Bonds	2,638	0	7,632
Stocks	3,232	0	9,531
<b>Total Financial Assets</b>	<b>23,482</b>	<b>7,250</b>	<b>46,709</b>
Fix-rate mortgages	1,048	0	3,033
Floating-rate mortgages	1,299		1,334
Other debt and mortgages	949	0	2,745
Housing	132,853	100,000	204,110
Present Value of Rents	141,988	99,985	186,417
Human Capital	485,872	366,224	651,173
<b>Total Wealth</b>	<b>496,924</b>	<b>368,242</b>	<b>708,282</b>

Note: number of observations in columns (1) and (2) = 7457; in column (3): 2577

The second largest components of household wealth are the housing stock and the present value of rents. Their average value is of similar magnitude, but for individual households these two differ considerably over the life-cycle. Younger households are typically under-housed, in the sense that their future needs are worth more than their current housing stock. Older households instead tend to be over-housed (with the notable exception of renters), because the present value of their future rents is reduced by the shortening time horizon, other things being equal.

Even within financial assets, Table 5 reveals that the risk-free position accounts for the largest fraction; with an average of almost € 13,000 (the same is true if we look at the medians). The three risky assets account for an average of € 10,000 overall (their median holdings are instead zero, because participation is not sufficiently wide-spread). Debt positions are relatively small; even though mortgages are sometimes quite large (they exceed € 77,000 for 1% of the sample).

Column (3) of Table 5 presents average holdings for those households who have some financial risky assets or liabilities. The number of observations falls to just 2577 – this is the relevant sample for most of our analysis. This sample is overall richer: average financial wealth is almost twice as

high as in the full sample, with much larger values for risky financial assets (accounting for € 31,000 overall). Total wealth is also higher, but by a more modest 42%.

It is worth recalling that in our efficiency analysis, we treat fix-rate mortgages as negative holdings of government bonds (the only long term bonds available are on government debt), floating-rate mortgages as negative positions on the risk-free asset and all other debt (including home-improvement mortgages) as negative holdings of corporate bonds (other debt typically has medium term maturity like corporate bonds). Thus a household with risk-free assets and a fix-rate mortgage or other debt belongs to this “well diversified” group.

The distinction between households with at least some risky financial assets or liabilities and the remaining households is of particular relevance for us, because for the latter group the test statistic takes the same value for all households in the same broad region. In Table 6 we show how this classification changes according to the broad regions introduced earlier (see Tables 2 and 3).

**Table 6 – Classification by Region.**

	<b>Total</b>		<b>NW</b>		<b>NE</b>		<b>Centre</b>		<b>South</b>	
	<b>n°</b>	<b>%</b>	<b>N°</b>	<b>%</b>	<b>N°</b>	<b>%</b>	<b>n°</b>	<b>%</b>	<b>N°</b>	<b>%</b>
<b>Risk-free asset + housing</b>	4880	65.44	1120	56.22	802	53.18	922	60.22	2036	83.92
<b>Risk-free + risky assets/ liabilities + housing</b>	2577	34.56	872	43.78	706	46.82	609	39.78	390	16.08
<b>Total assets</b>	<b>7457</b>		<b>1992</b>		<b>1508</b>		<b>1531</b>		<b>2426</b>	

We see that the highest proportion of risk-free financial asset portfolios (83.92%) is found in the South, the lowest in the North East (53.18%). This implies that the sample size for our efficiency test differs a lot from the total sample in its regional composition, with a much smaller fraction of households resident in the Southern regions (15.13% as opposed to 32.53%). However, the relative proportions of households resident in the three other macro regions is roughly in line with the full sample.



### 3. Optimal portfolio allocation and efficiency test results

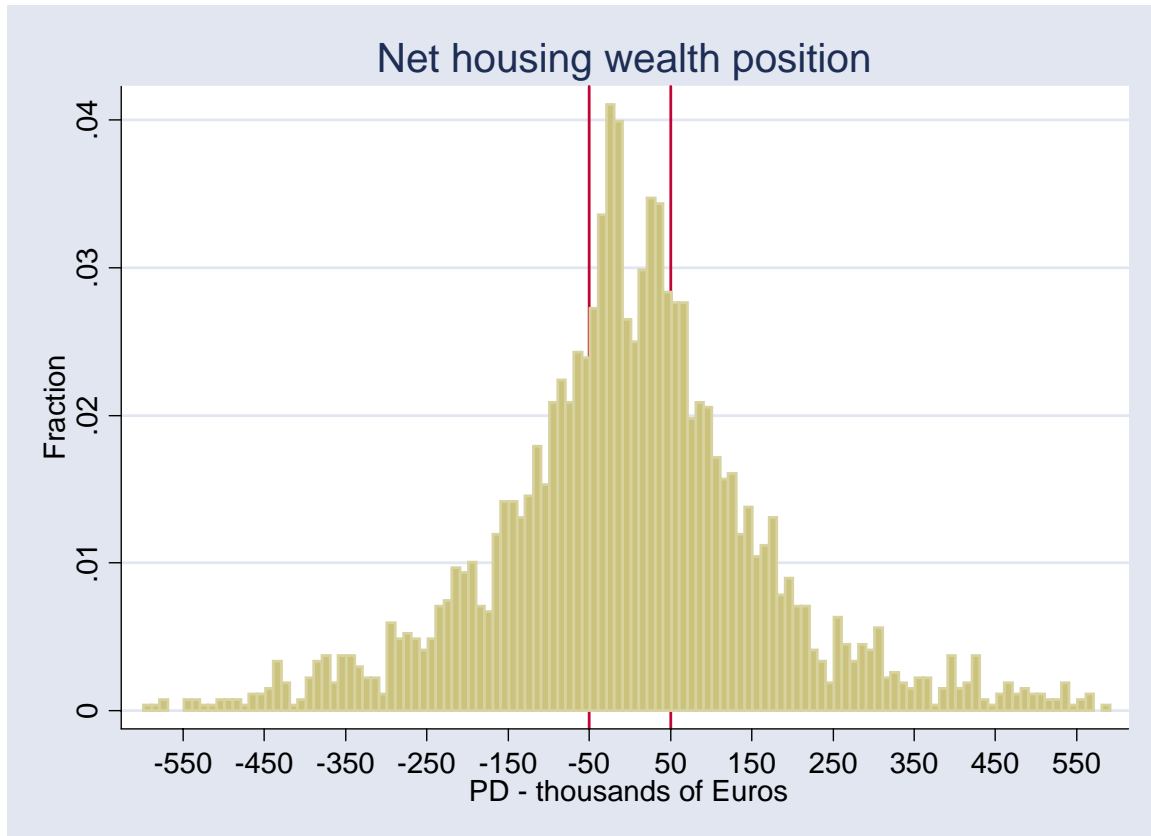
The return data we have described in the previous section have clear-cut implications for optimal portfolio weights, as implied by equation (2). In particular, we find that the Markowitz portfolio has 63% of financial wealth invested in government bonds, 35% in corporate bonds, and a meager 2% in stocks. In each region there is a different set of coefficients multiplying net housing wealth, as shown in Table 3. For instance, in the NW and CE “over-housed” households should invest substantially more in stocks. This is much less true for over-housed households in the NE and SO. For the “under-housed” the pattern is exactly the opposite: households resident in the NW and CE should have even smaller portfolio shares in stocks than the market portfolio.

A first, useful piece of information is the proportion of financial assets portfolios for which the share in stocks is indeed higher than the Markowitz portfolio. We find that 47% portfolios in the NW have more than 2% in stocks. This fraction is 54% in the NE, 37% in the CE and just 25% in the SO. It is useful to see how these proportions vary with net housing wealth.

We therefore split the sample in three groups, according to net housing wealth ( $P_0D_0$  in equation 2). Figure 4 presents a histogram for this variable. We can check that roughly a third of the density lies to the left of € -50,000, roughly a third to the right of € 50,000, and the remaining third in between (see the vertical lines in the graph). These proportions are stable across regions, with the only exception of the SO where relatively more observations fall in the middle, and fewer lie to the left of € -50,000. Our analysis implies that portfolios of the over-housed should have more in stocks than the Markowitz portfolio, and particularly so in the NW and CE. We find that the proportion of portfolios exceeding 2% in stocks is indeed higher for the over-housed in the NE (63%) and SO (36%), but close to the average in the NW and CE. On the basis of this qualitative analysis, we could conclude that many of the over-housed who invest more than 2% in stocks could have efficient portfolios, but we need a formal test to support this conjecture. It is in fact worth stressing that the larger the positive net housing position the larger the share in stocks should be – given that

financial wealth is a relatively small component of total wealth, households with a large, positive net housing position may well require large investment in stocks to be efficient.

**Figure 4 – Distribution of net housing among households with risky financial assets**



We compute the efficiency test statistic described in Section 1 (see equation 5) for all the 7457 household portfolios observed in our data. However, a distinction must be made between the 4880 households who report not having any risky financial assets or liabilities, and the 2577 who instead have at least one such asset or liability. For the former group, by construction the test statistic takes the same value for all households within the same macro-region, irrespective of the amount held in either asset. For the latter group, instead, the test statistic varies across observations, depending on their risky asset shares.

For instance, for all 1049 households who live in the NW and have no risky assets, the test statistic takes a value of 6.34. Under the null of efficiency, this is distributed as a chi-squared random variable with 2 degrees of freedom. The corresponding critical values are 4.60 (test size: 10%), 5.99 (test size: 5%) and 9.21 (test size: 1%). Thus the test always rejects when we choose a 90%

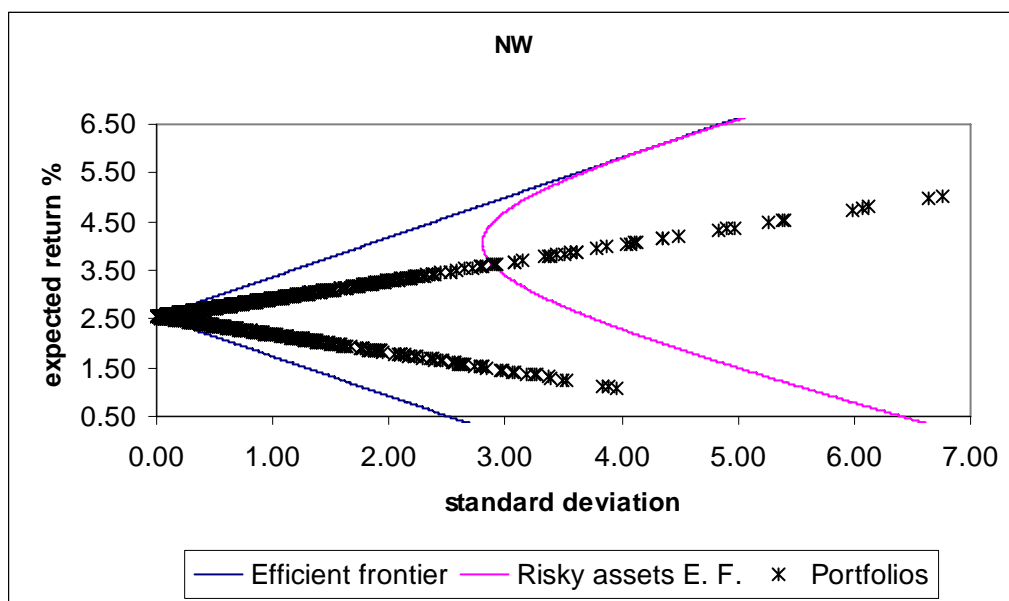
significance level, it also rejects at the 95% level but not at the 99% level. For the NE, the calculated statistics is 2.39, for Centre it is 4.30, 0.41 for households who live in the South, and therefore all these portfolios are efficient for any sensible test size.

It is worth stressing that the test statistic is based on the squared of the Sharpe ratio, thus portfolios with Sharpe ratios of the same magnitude but opposite sign are treated in the same way. In our discussion so far we have ignored this feature.

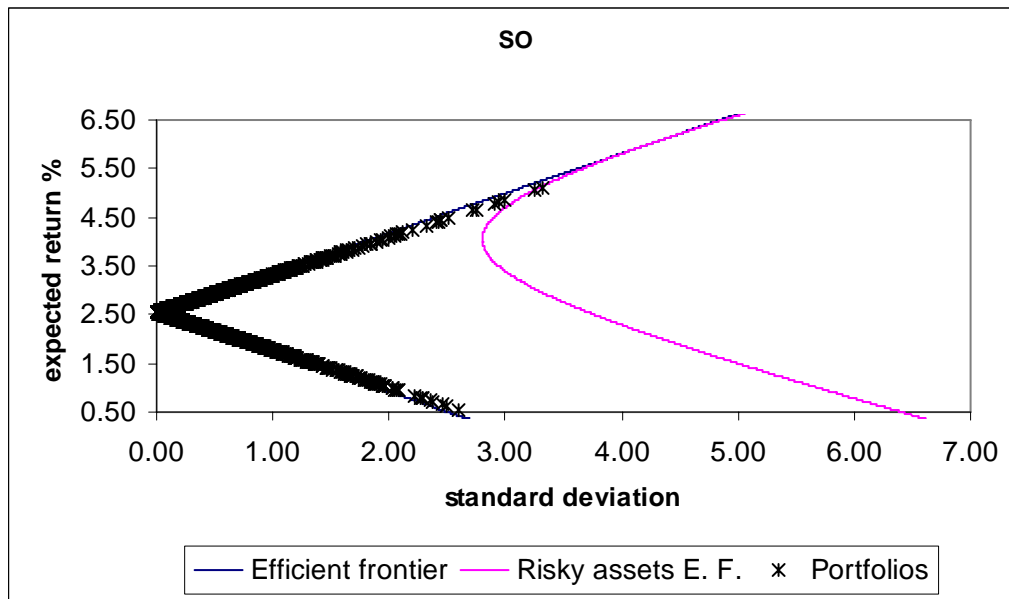
To better understand this point, and the working of conditional efficiency analysis in general, we can use the apparently conventional mean - standard deviation representation, along the lines already shown in Figure 1. In Figures 5 and 6, we represent all portfolios of residents in two macro-regions (NW and SO) who do not own any financial risky assets. We use the December 2002 six month Italian Treasury bill interest rate (2.5%) as the risk-free rate.

The portfolios are represented net of the hedge term corresponding to Table 3 estimates and the specific net housing position (see equation 2). Hence in this simple case positive housing portfolios, net of the hedge term, are short on stocks (and bonds); negative housing portfolios are long on stocks (and bonds).

**Figure 5. Efficiency analysis for portfolios with no risky financial assets in the North-West.**



**Figure 6. Efficiency analysis for portfolios with no risky financial assets in the South.**



In both Figures 5 and 6, portfolios corresponding to negative net housing positions (the “under-housed”) lie on an ascending line that is quite close to the upper part of the broken frontier in the SO and farther away in the NW, but always above the risk-free intercept. Similarly for the over-housed, whose portfolios lie instead on the corresponding descending lines.

This graphical analysis clarifies why the test statistic is very close to zero in the SO, and much higher in the NW, but also suggests the need to consider as inefficient those portfolios that lie below the risk-free intercept even though their test statistic is low.

Hence, in our analysis of diversified portfolios, we shall consider efficiency to be rejected when the test statistic either takes a value higher than the threshold or when the expected return of the hedge-adjusted portfolio is below the risk-free rate. Our graphical analysis of risky portfolios (shown below) shows conclusively that negative excess returns cannot be efficient, given that portfolios with identical standard deviations and higher expected returns are classified as inefficient.

Table 7 reports test results for the sample of well-diversified portfolios, that is for households with some risky financial assets or liabilities. It does so for two different test sizes: In the upper portion of the Table, the chosen test size is 5%; in the lower part, we have set it at 10%.

**Table 7. Test results for all households with risky financial assets**

	Whole country		NW		NE		Centre		South	
	N°	%	N°	%	N°	%	N°	%	N°	%
test size = 5%										
<b>Inefficient</b>	1623	62.98	636	72.94	405	57.37	344	56.49	238	61.03
<b>Efficient</b>	954	<b>37.02</b>	236	<b>27.06</b>	301	<b>42.63</b>	265	<b>43.51</b>	152	<b>38.97</b>
test size = 10%										
<b>Inefficient</b>	1820	70.62	714	81.88	448	63.46	412	67.65	246	63.08
<b>Efficient</b>	757	<b>29.38</b>	158	<b>18.12</b>	258	<b>36.54</b>	197	<b>32.35</b>	144	<b>36.92</b>

Depending on the chosen test size, we find that a fraction of 29 % to 37% of observed portfolios are conditionally efficient overall. There is much regional variability, though, with the lowest proportion of portfolios in the NW that are considered efficient (18% to 27%).

This is partly due to differences in the partial correlations between housing returns and stocks highlighted in Table 3: as we have already pointed out, for both NW and CE there are large, negative and significant coefficients on stocks. But it also reflects differences in financial investments across regions: as we know, households in the NW (and NE) have the highest investments in stocks. Depending on the housing position, these two factors together play a key role in explaining the efficiency or otherwise of NW and CE household portfolios.

In Table 8 we present the number and fraction of efficient portfolios in each of these three groups (with positive, negligible and negative housing wealth) by broad region. For the sake of simplicity, we report them for just one test size (5%).

**Table 8. Proportion of efficient portfolios (test size = 5%) by net housing position**

	Whole country		NW		NE		Centre		South	
	N°	%	N°	%	N°	%	N°	%	N°	%
<b>Over-housed</b>	67	<b>7.20</b>	30	<b>10.00</b>	16	<b>5.97</b>	13	<b>5.78</b>	8	<b>5.80</b>
<b>Negligible</b>	388	<b>47.67</b>	107	<b>41.80</b>	114	<b>52.78</b>	93	<b>53.14</b>	74	<b>44.31</b>
<b>Under-housed</b>	499	<b>59.98</b>	99	<b>31.33</b>	171	<b>77.03</b>	159	<b>76.08</b>	70	<b>82.35</b>
<b>All</b>	954	<b>37.02</b>	236	<b>27.06</b>	301	<b>42.63</b>	265	<b>43.51</b>	152	<b>38.97</b>

We see from the last row of Table 8 that 954 well-diversified households have efficient portfolios, that is 37% of the total. As we have also seen in Table 7, this proportion is highest in the NE and CE (43-44%), lowest in the NW (27%). We also see that the highest proportion of efficient portfolios (60%) obtains among households who are “under-housed”, that is whose net housing

position is below -50,000. This group includes tenants as well as young home-owners. The lowest proportion (7%) is found among those who are “over-housed” (net housing is larger than 50,000). Among those with intermediate positions (“negligible”) the proportion of efficient portfolios are intermediate, but higher than the overall group average. Thus most of the interesting deviations from the overall average are to be found among those with positive net housing wealth, and in the NW broad region.

When we focus on NW households by housing wealth, we find a different pattern: the overall average of 27% masks a low proportion of efficient portfolios (10%) among the positive housing wealth positions, and a much higher one (42%) among those whose housing position is close to zero. In comparison to other regions, NW over-housed households are more often efficient, the under-housed are less often efficient.

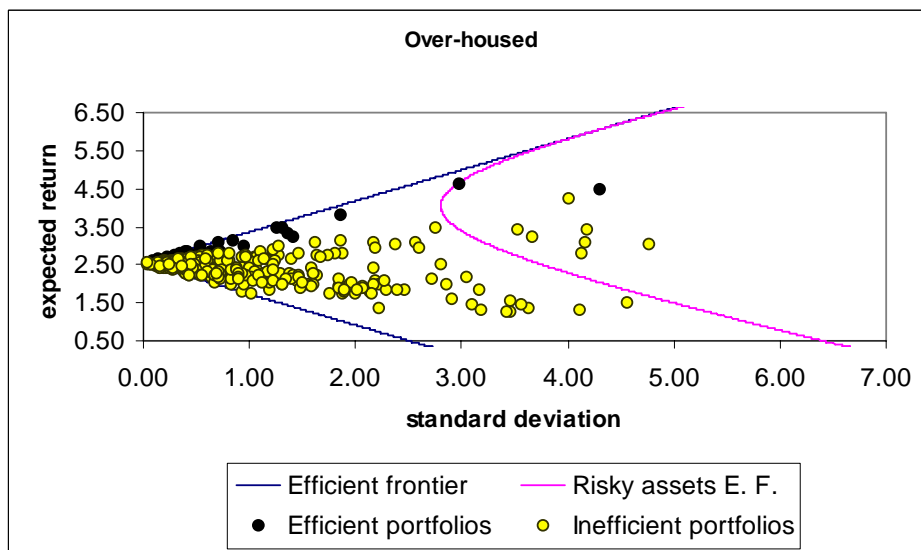
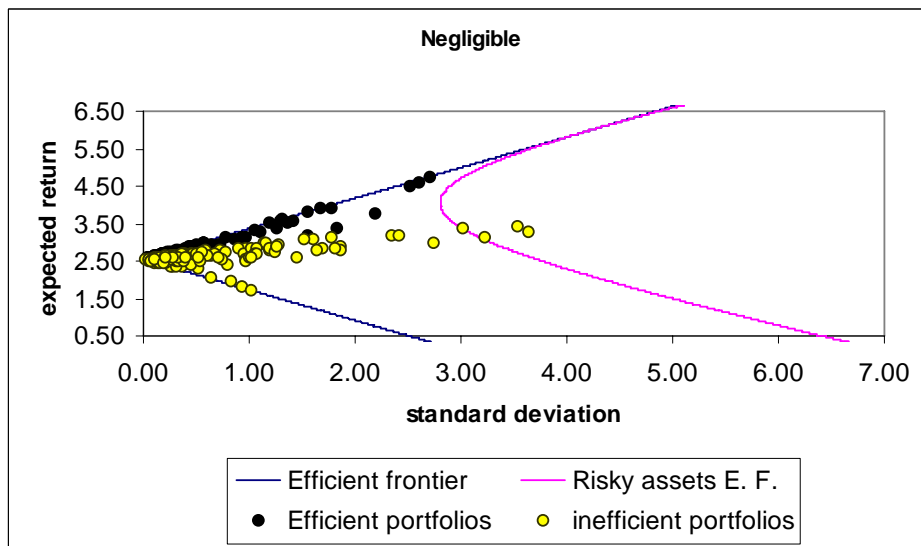
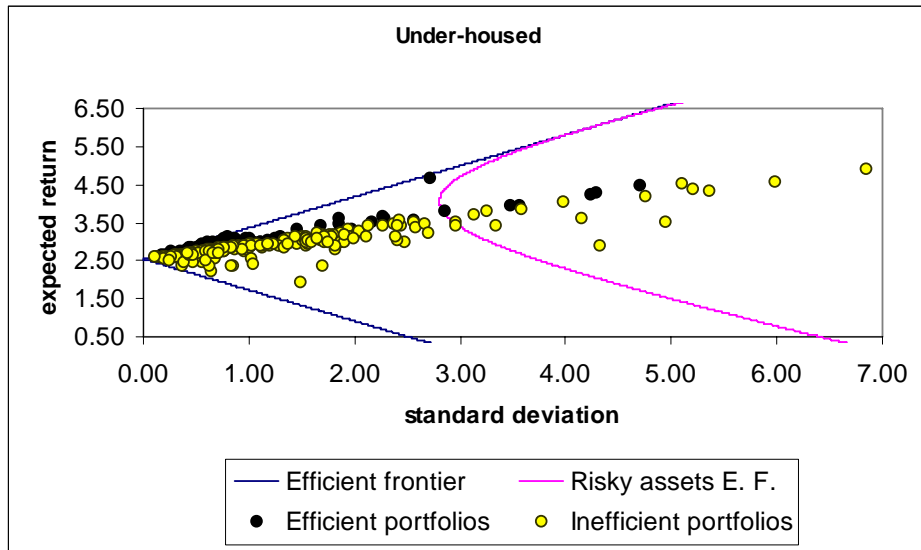
The efficiency pattern is similar to the national average for NE and CE, albeit with fewer over-housed portfolios classified as efficient. In the SO we find a very large fraction of under-housed portfolios efficient. This can be explained by the combination of two factors: the test statistic is very low in the SO for no risky financial assets portfolios (see Figure 6) and even those SO households who invest in risky financial assets have relatively small positions in these assets.<sup>5</sup>

Figures 7 to 10 provide a graphical representation of our efficiency analysis in the mean-standard deviation space, where portfolios are expressed net of the hedge term. We split portfolios according to the net-housing position and mark with a darker dot those that we consider efficient.

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<sup>5</sup> We have checked whether these differences are statistically significant by running a probit regression of the efficiency test outcome on the interactions between housing wealth dummies and broad region dummies, taking as the control group the negligible wealth group in CE. We find strong negative effects for most terms, particularly those involving the NW and the South, with markedly different coefficients across NW variables. We can therefore conclude that the evidence shown in Table 8 is strong despite the relatively small cell sizes.

Figure 7 – Efficiency analysis for the North-West



**Figure 8 – Efficiency analysis for the North-East**

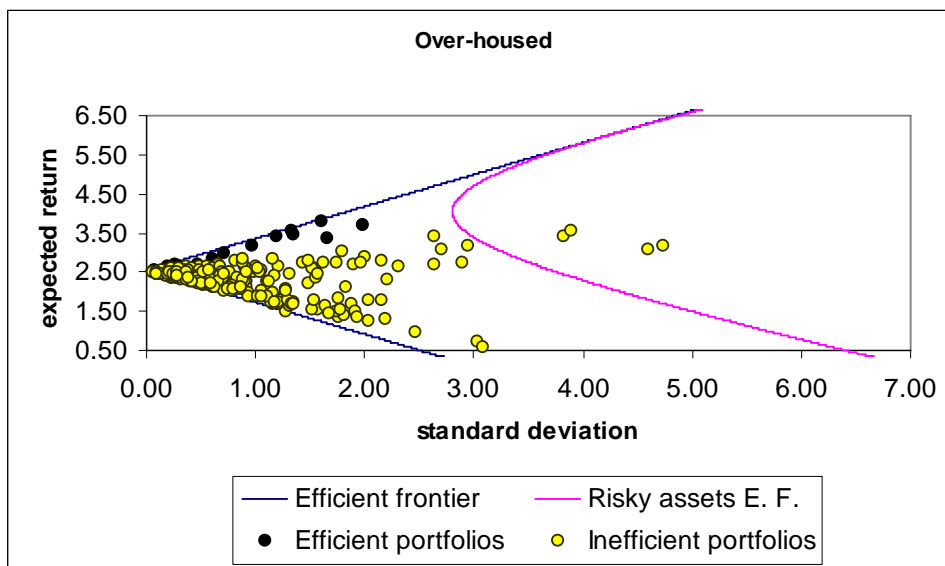
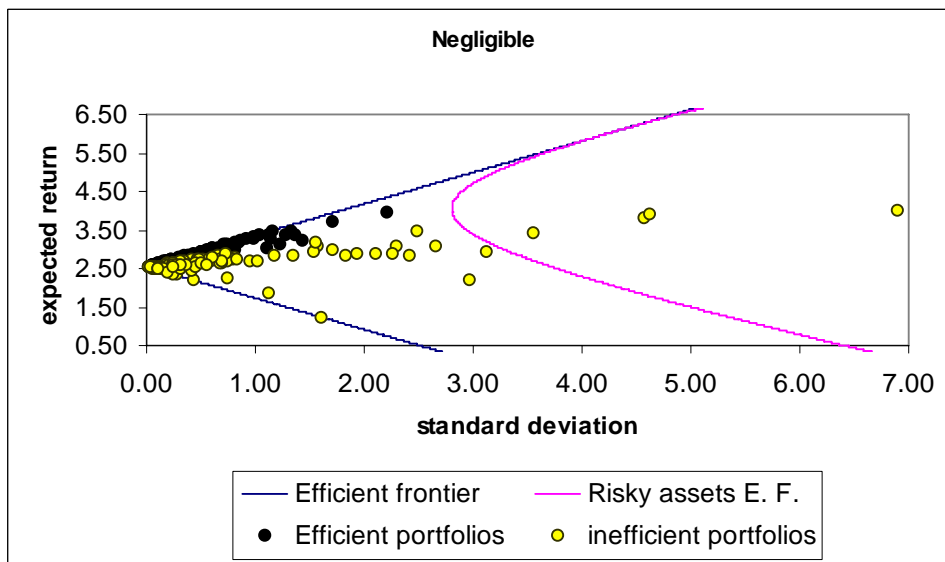
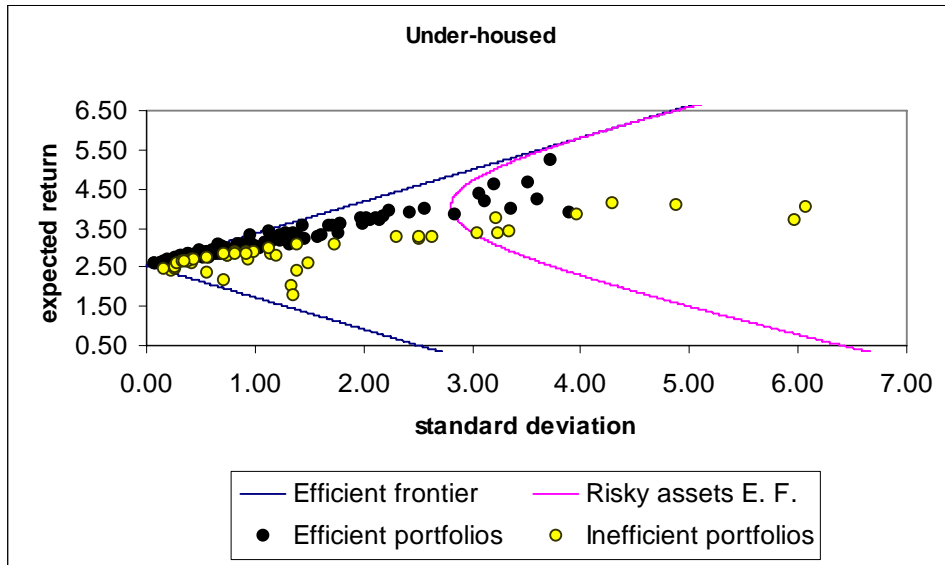
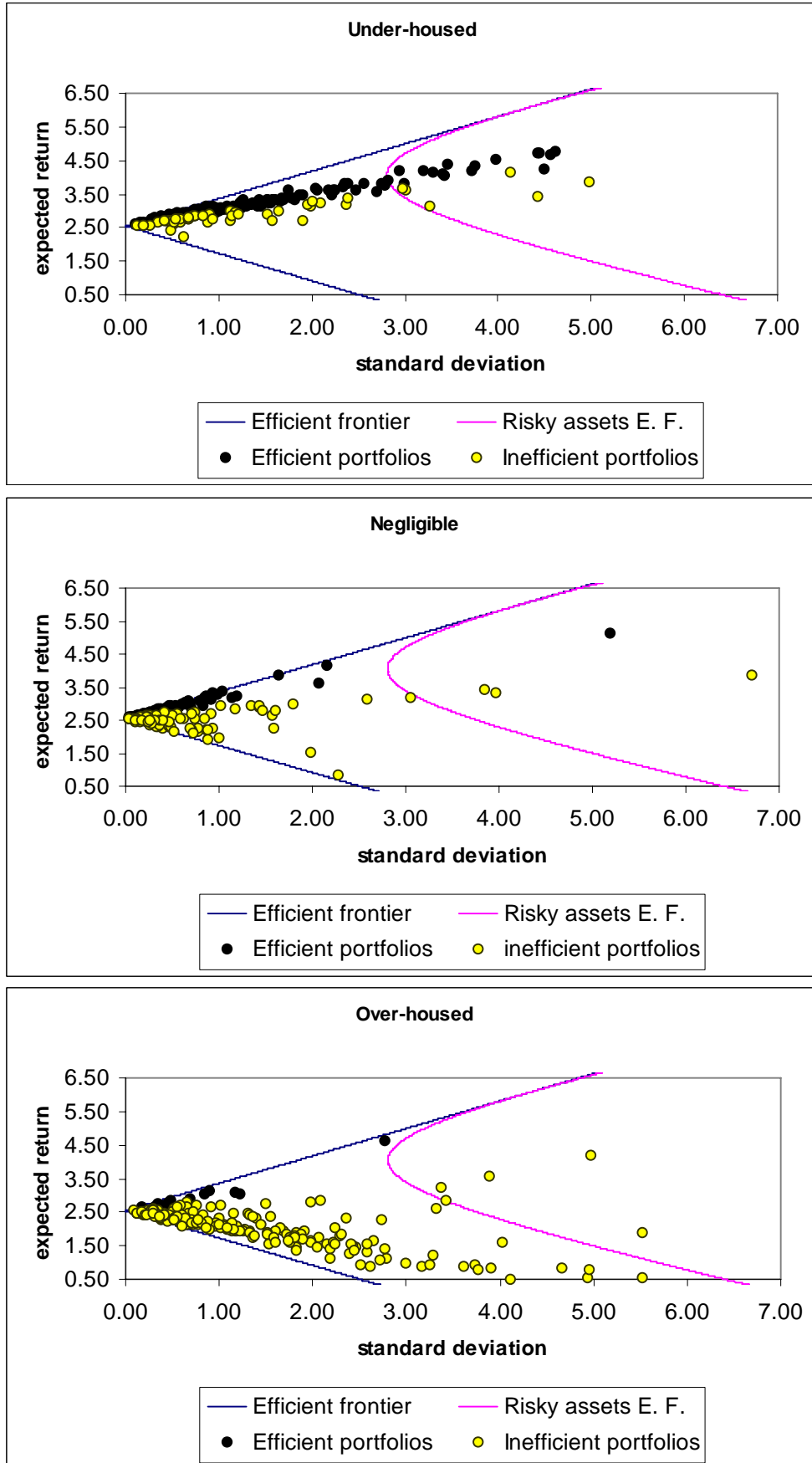
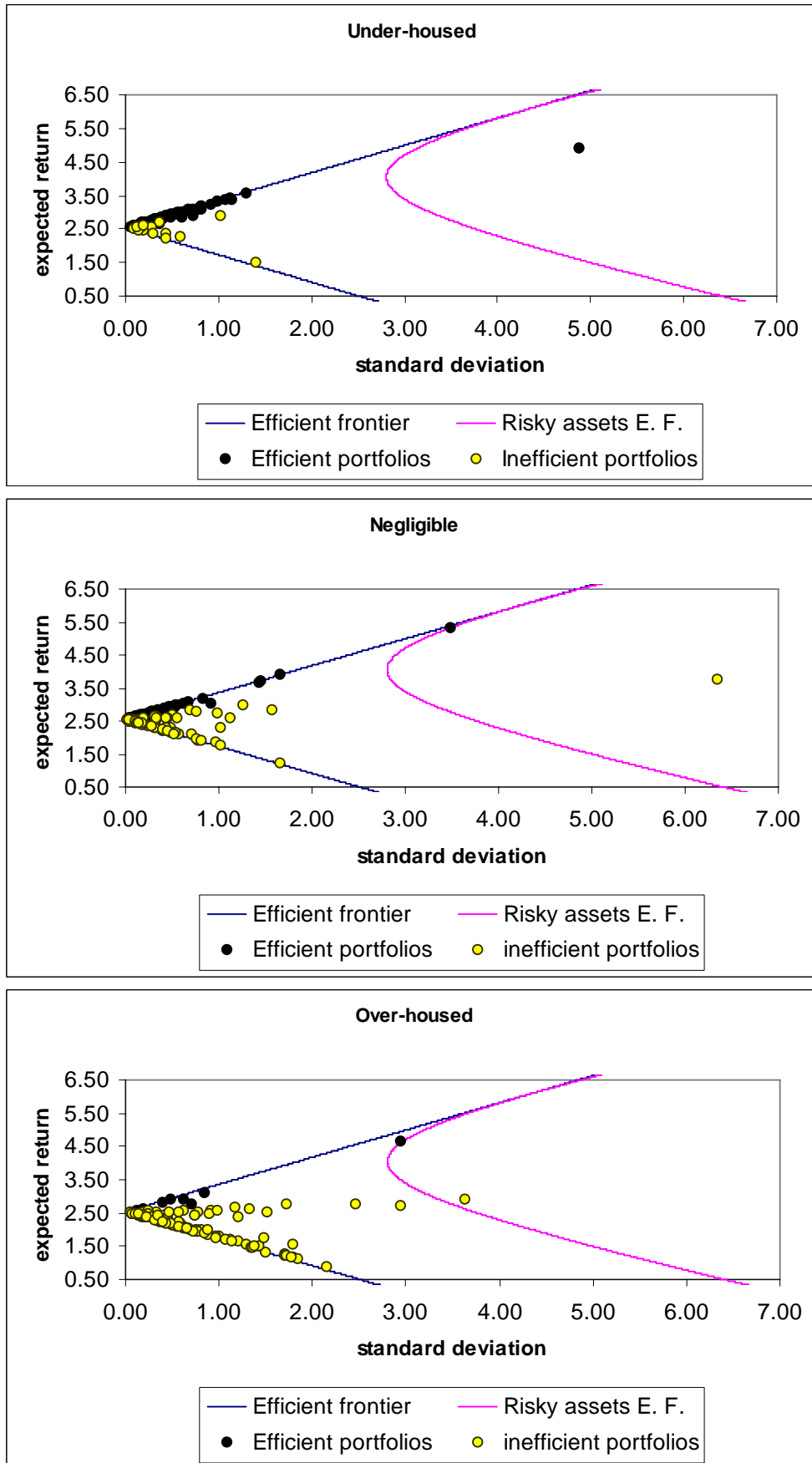




Figure 9 – Efficiency analysis for the Centre



**Figure 10 – Efficiency analysis for the South**



For the under-housed, in all regions many portfolios appear to lie on straight lines, some efficient (see the SO), some inefficient (see the NW). These lines are quite similar to those shown in Figures 5 and 6 (no risky assets), and reflect the importance of the hedge term in determining efficiency when risky assets are a relatively small component of wealth. This feature is much less visible for negligible housing positions, because the hedge term plays a much smaller role (it is multiplied by a small number), and for the over-housed, because these households are typically older and richer with larger fractions of wealth invested in risky assets.

For the over-housed, there is a large fraction of portfolios close to the lower branch of the frontier, even though slightly smaller in the NW than in the other regions. This occurs because they are not hedging housing risk: They should hold more stocks (and corporate bonds) than the Markowitz portfolio and do not.

In all regions (but the SO) there are non-negligible fractions of portfolios that attain a higher expected return than the risk-free rate but are nonetheless considered inefficient. These are the ones for which it is useful to rely on a formal test statistic. Despite important sampling variability in all moments (first moments of financial assets, second moments of both financial assets and housing), we have seen that the test is able to detect departures from efficiency that are due to incorrect assets allocation.

How important is inefficiency in terms of risk-adjusted returns? After taking out the hedge term, we compute Sharpe ratios for the efficient and for the inefficient. The overall average Sharpe ratio for the efficient is 0.61, it is -0.15 for the inefficient (who are mostly below the risk-free rate). The Sharpe ratios difference is highest for the over-housed, lowest for the under-housed. A way to evaluate the importance of our findings is to do back-of-the-envelope calculations on the long-term consequences of inefficiency for the over-housed. For this group, inefficiency brings about a loss of 90 basis points for 1% standard deviation. Over a twenty years time horizon, for every percentage point of risk taken, on average this group loses 20% of final wealth by failing to hedge housing.

We also investigate whether and how household characteristics affect portfolio efficiency, after controlling for region and net housing position. In Table 9 we report the results of a probit regression that efficiency is rejected (at 5%) on the same set of dummy variables described above (corresponding to the different cells in Table 8) and on a few observable characteristics, including head's age, gender and years of education, number of household members (famsize), as well as the total financial wealth to be allocated.

We see from Table 9 that the effects of region interactions with housing position dummies retain their significance, even though the probability of inefficiency is found to be negatively related to education and family size, and positively related to financial wealth.

**Table 9. Probit regression of 5% inefficiency on a number of characteristics (marginal effects)**

Probit estimates Number of obs = 2577  
LR chi2(17) = 917.39  
Prob > chi2 = 0.0000  
Log likelihood = -1239.7061 Pseudo R2 = 0.2701

t95	dF/dx	Std. Err.	z	P> z	x-bar	[	95% C.I.	]
NW_over*	-.343318	.0179386	-10.25	0.000	.116414	-.378477	-.308159	
NW_negl*	-.1052623	.0379265	-2.55	0.011	.09934	-.179597	-.030928	
NW_under*	-.1685377	.0343185	-4.21	0.000	.122623	-.235801	-.101275	
NE_over*	-.3564242	.0158153	-10.55	0.000	.103997	-.387422	-.325427	
NE_negl*	-.0045194	.0454503	-0.10	0.921	.083818	-.0936	.084562	
NE_under*	.2720772	.0554489	4.97	0.000	.086147	.163399	.380755	
CE_over*	-.3495589	.0152234	-10.02	0.000	.087311	-.379396	-.319722	
CE_under*	.2843184	.0550923	5.20	0.000	.081102	.176339	.392297	
SO_over*	-.3262791	.0158361	-7.97	0.000	.053551	-.357317	-.295241	
SO_negl*	-.0488057	.0459757	-1.03	0.305	.064804	-.138916	.041305	
SO_under*	.3833048	.0684051	5.17	0.000	.032984	.249233	.517376	
age	.0011872	.0019525	0.61	0.543	55.5165	-.00264	.005014	
(age-40) <sup>2</sup>	-.0000635	.0000521	-1.22	0.222	428.321	-.000166	.000039	
male*	-.0250597	.0234574	-1.08	0.282	.712068	-.071035	.020916	
famsize	-.0363759	.0101903	-3.57	0.000	2.81102	-.056348	-.016403	
educ	-.0093951	.0029254	-3.21	0.001	10.104	-.015129	-.003661	
f_wealth	.0006659	.0001211	5.47	0.000	46.5295	.000428	.000903	

The effect of the head's age is non-linear: the probability of inefficiency increases with age until age 70, it decreases thereafter.

We can check whether our results depend on the ad-hoc simplifying assumptions made on human capital, particularly on the rate used to discount future earnings and pension benefits, and on its

risk-free nature. When we increase the discount rate for human capital by a third, the efficiency test results are only marginally affected. When we take the self-employed out of the analysis, the patterns highlighted above remain valid.

#### 4. Robustness analysis

In Section 3 we presented results based on a number of assumptions, some of which we can relax.

In this section, we show how our results are affected when we:

- a) allow human capital to be risky;
- b) assume less than unit correlation between the housing service price (rent) changes and house price changes, i.e. owning is less than a perfect hedge against rent risk;
- c) allow households to invest in foreign as well domestic assets.

##### 4.1 Risky human capital

Our theoretical and empirical analysis has so far treated human capital returns as risk-free. This assumption is of course very strong, but it can be relaxed if we are prepared to treat human capital like housing wealth, that is given in the short run. In this case our conditional analysis requires conditioning on two assets, and equation (2) becomes:

$$(8) \quad \underline{X}_0^{*T} = \begin{bmatrix} -\frac{\partial V}{\partial TW} \\ \frac{\partial TW}{\partial^2 V} \\ \frac{\partial TW^2}{\partial TW^2} \end{bmatrix} \Sigma^{-1} \underline{\mu}^T - P_0 D_0 \Sigma^{-1} \Gamma_{bP}^T - HC_0 \Sigma^{-1} \Gamma_{bHC}^T$$

Where  $HC_0$  is the current value of human capital and  $\Gamma_{bHC}^T$  denotes the vector of covariances between human capital return and financial assets. The intuition behind equation (8) is straightforward: optimal portfolios are made of the Markowitz portfolio, net of two hedge terms, one for housing, the other one for human capital.

The test statistic is computed as in equation (5), once  $\Omega$  and  $Z$  are redefined to include the extra given asset, human capital.

The more difficult task is to find good estimates of human capital returns. In principle, one should use panel data spanning a long period and recording hourly wages at a suitable frequency (semi-annual or higher). To our knowledge, such data do not exist for Italy (SHIW has a small panel component, but the survey takes places at 2-3 years intervals). Given the exploratory nature of this robustness exercise, we decided to use aggregate data on earnings per employee, and to remove the effects of work-force aging by taking the residuals of semi-annual changes in the logarithm of earnings per employee on a deterministic trend.

The resulting annual excess return has a negative sample mean (-0.6%) and a relatively low standard deviation (2.43%). It exhibits negative correlations with all three financial assets: -.21 with Government Bonds, -.17 with stocks and -.01 with corporate bonds. Its correlation with housing is positive (.61), even though this is irrelevant for our analysis. The regression of human capital return on a constant and the three financial asset returns produces the following estimates:

$$(9) \quad r_t^{HC} = 0.614 - 0.425 r_t^G + 0.450 r_t^C - 0.025 r_t^S$$

(0.29) (0.14) (0.25) (0.01)

There is a strong negative relation with Government Bonds, positive with Corporate Bonds, and a smaller, but marginally significant, negative relation with stocks.

Equation (9) provides both the weights of the second hedge portfolio in (3), and the discount rate to be used to compute, for each household, an estimate of HC. Based on sample means of the three financial asset returns, and the observed risk free rate at the end of 2002, we can compute the nominal return implied by the model. Once we subtract 1.5% inflation, we obtain a real annual discount rate of 1.39%. We apply this discount rate to all labor income, including pensions and self-employment income, but note that this may bias HC upwards for the self-employed (whose income may be riskier than earnings). It may also be inappropriate to treat HC as risky for the retired, given that pension income is price inflation-index in Italy. This adds a cautionary note to our interpretation of efficiency results for the over-housed, who tend to be older on average.

**Table 10: Amounts held in financial and real assets when human capital is risky**

Asset	(1) Average	(2) Median	(3) Conditional Average
<b>Total Financial Assets</b>	<b>23,482</b>	<b>7,250</b>	<b>46,709</b>
Housing	132,853	100,000	204,110
Present Value of Rents	141,988	99,985	186,417
Human Capital	436,413	333,611	579,850
<b>Total Wealth</b>	<b>447,465</b>	<b>337,278</b>	<b>663,960</b>

Note: number of observations in columns (1) and (2) = 7457; in column (3): 2577

In Table 10 – that compares directly to Table 5 - we show the average, median and conditional average of the new human capital variable, as well as of total financial assets and total wealth. Even though human capital has been reduced by some 10% on average, it remains by far the largest component of total wealth. In the more interesting case of households owning some financial risky asset (column 3), we find that human capital is smaller than housing in 349 cases (one in seven), it is smaller than net housing wealth in just 187 cases (one in thirteen): these households tend to be older than the rest of the sample (on average by 10 and 15 years, respectively).

In Table 11 we show how many portfolios are efficient by region and net-housing position – the table compares directly to Table 8 in Section 2. When we look at the country as a whole, we notice a much lower number of efficient portfolios (460 instead of 954). This is dramatically true for the under-housed (142 instead of 499) and to a lesser extent for those with negligible housing position (131 instead of 388). The over-housed, instead, appear more often efficient (187 instead of 67).

To understand these patterns, we note that the presence of the human capital hedge shown in equation (9) implies that households with negligible housing positions should increase their shares of government bonds and (to a lesser extent) stocks, and decrease their share of corporate bonds. Given that the tangency portfolio already has a very large share of government bonds, investing even more in such bonds may be hard. Also, the relatively small coefficient on shares has to be compared the similarly small share of stocks in the tangency portfolio (2%). Given that few households have large stock positions, the overall fall in efficiency comes as no surprise.

The reason why efficiency decreases most for the under-housed, least for the over-housed, is because the under-housed are young and therefore have more human capital: they should increase

Government Bond holdings most, and avoid (fixed-interest) mortgages. The over-housed, instead, have lower human capital, little or no mortgages, and have higher holdings of Government Bonds and Stocks. To the extent that our measure of human capital risk is appropriate for them, they tend to do the right thing more often than the under-housed.

**Table 11. Proportion of efficient portfolios by net housing position when human capital is risky (test size = 5%)**

	Whole country		NW		NE		Centre		South	
	N°	%	N°	%	N°	%	N°	%	N°	%
<b>Over-housed</b>	187	<b>20.06</b>	44	<b>14.57</b>	50	<b>18.66</b>	60	<b>26.79</b>	33	<b>23.91</b>
<b>Negligible</b>	131	<b>16.11</b>	32	<b>12.50</b>	21	<b>9.81</b>	25	<b>14.29</b>	53	<b>31.55</b>
<b>Under-housed</b>	142	<b>16.90</b>	37	<b>11.60</b>	50	<b>22.32</b>	27	<b>12.74</b>	28	<b>32.94</b>
<b>All</b>	460	<b>17.79</b>	113	<b>12.88</b>	121	<b>17.14</b>	112	<b>18.33</b>	152	<b>38.87</b>

Figures 11 and 12 provide a graphical representation of our efficiency analysis for two macro regions, the NW and the South. These correspond to Figures 7 and 10 of our previous analysis, where human capital was treated as risk-free. In both cases we notice many more portfolios close to the lower branch of the minimum variance frontier, a clear indication of portfolio inefficiency.

We should stress that these results are not driven by the low return on human capital implied by equation (9): even if we assume human capital return to be much higher (4%), and discount future incomes accordingly, we find that the number of efficient portfolios is largely the same as shown in Table 11 (478 instead of 460), with a slight reduction for the under-housed (121 instead of 131) and a slight increase for the other two groups (211 versus 187 for the over-housed; 146 instead of 131 for those with negligible net housing positions).

We conclude from this analysis that the risky nature of human capital may have important consequences for portfolio efficiency.



**Figure 11 – Efficiency analysis for the North-West**

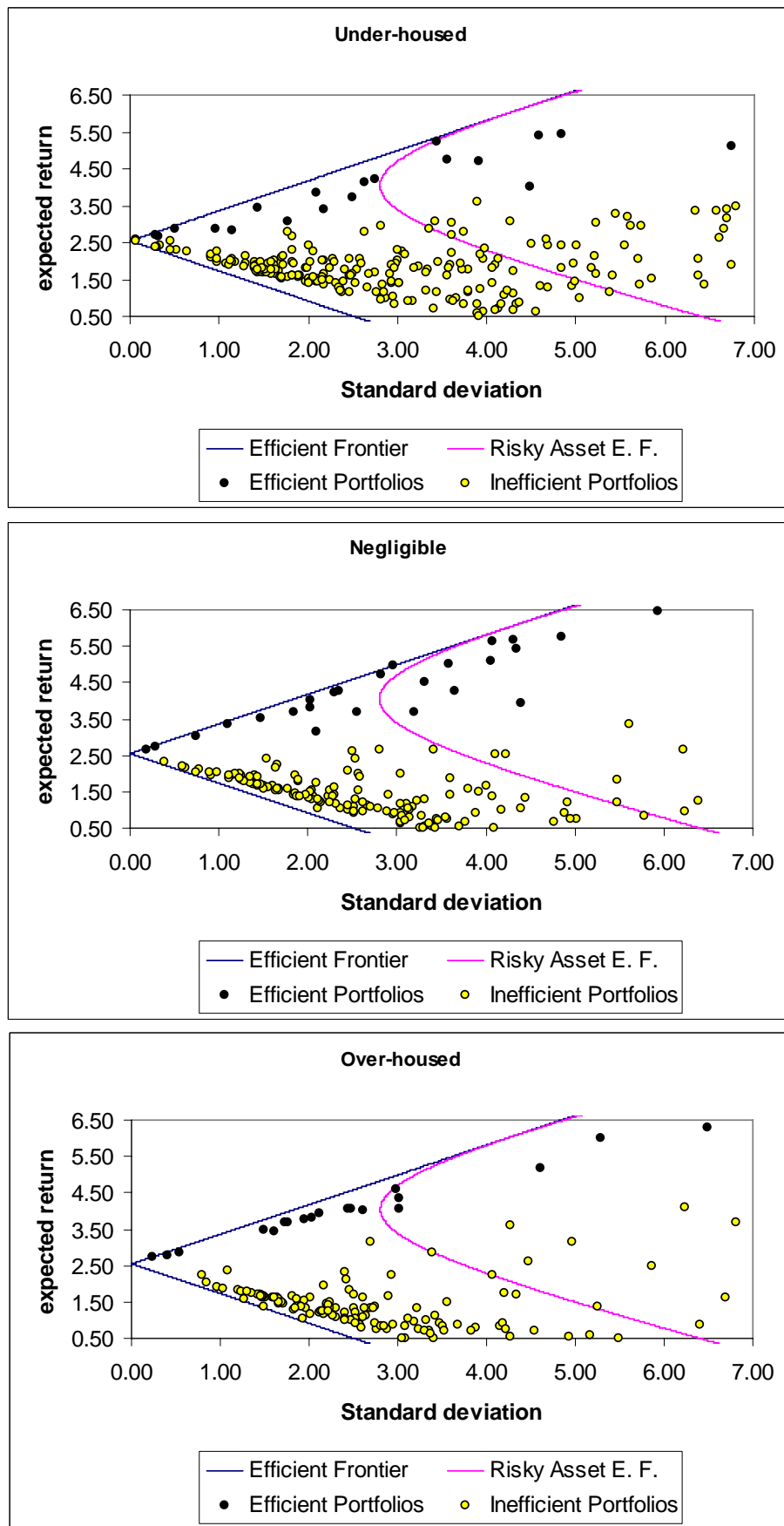
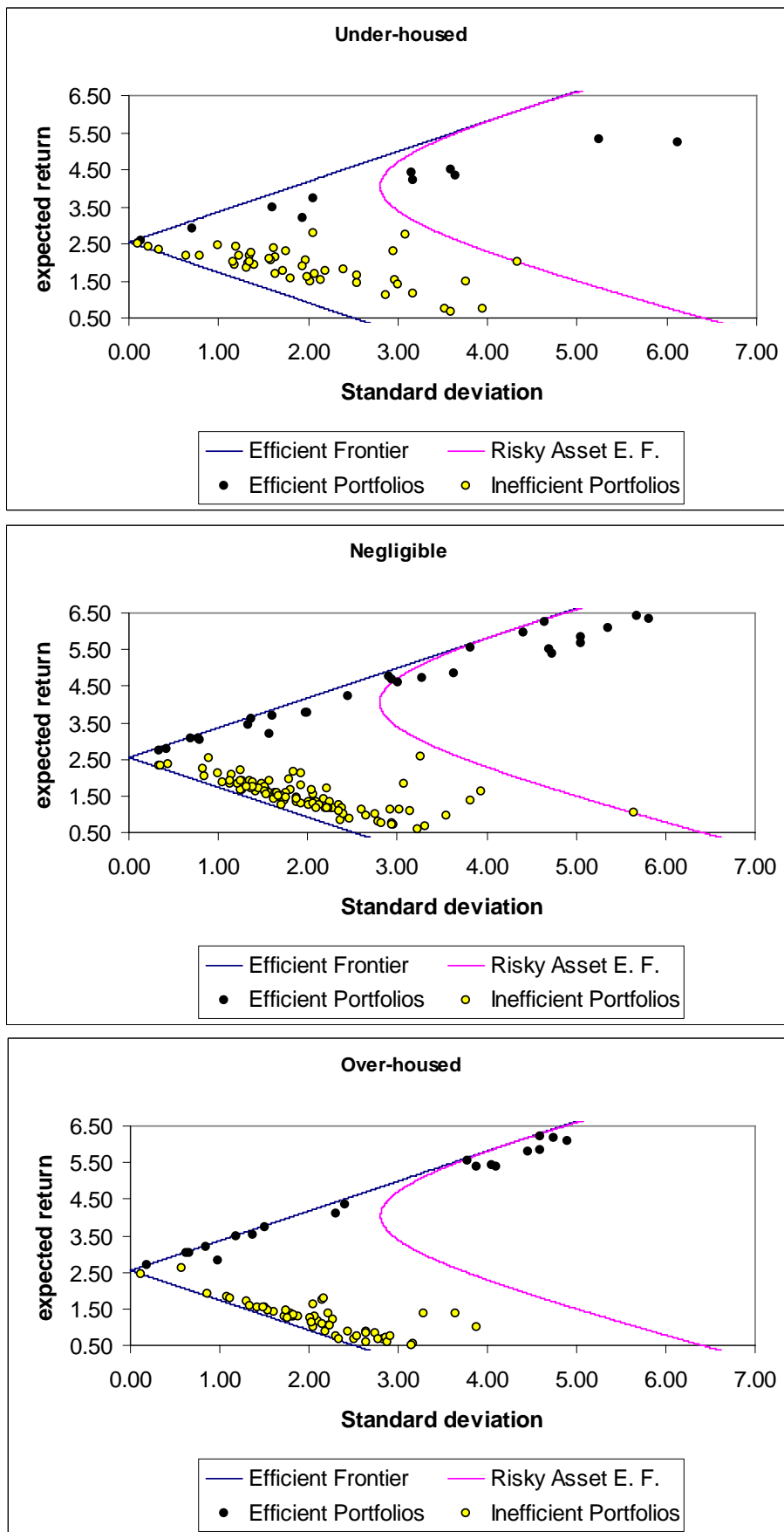


Figure 12 – Efficiency analysis for the South



## 4.2 Owning is a less than perfect hedge against rent risk

In Section 3 we have considered the case where owning is a perfect hedge against rent risk, because the price of housing services has unit correlation with house prices. This case is appealing in the long run (when rents are roughly proportional to house values), but obviously not necessarily true in the short or even medium run. Here we consider what happens to our efficiency analysis when the correlation between house price changes and rental price changes is not perfect, but varies between one (as in Section 3) and zero. This last case is analyzed in some recent papers that ignore the liability side of housing (Flavin and Yamashita, 2002, Cocco, 2005 and Pelizzon and Weber, 2006).

Figure 13 shows the proportions of efficient portfolios for the three net-housing groups considered so far as a function of  $\beta_{p\pi}$ , the hedge ratio between house returns and rents (see the Appendix). We should stress that we keep the groups' composition constant in this comparison, even though their net housing value (defined as value of the house minus  $\beta_{p\pi}$  times the present value of rents) changes – in other terms, net housing positions are defined once and for all with reference to the case where  $\beta_{p\pi}=1$ .

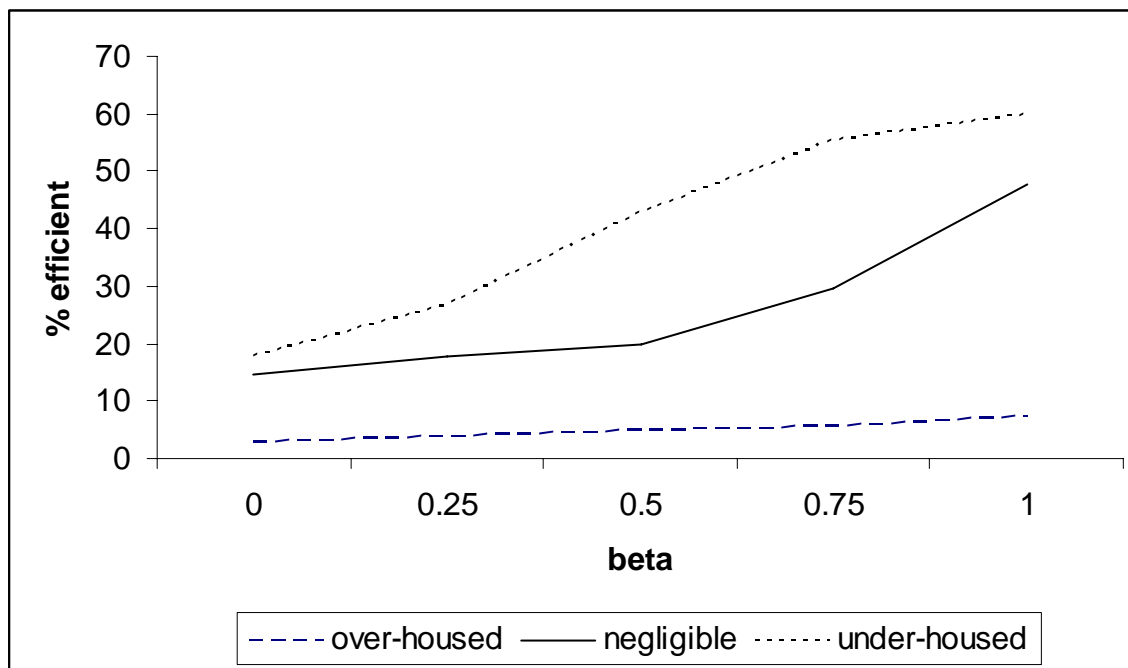
We see from Figure 13 that the over-housed appear mostly inefficient for any beta. This is not surprising: the reason why they are inefficient for  $\beta_{p\pi}=1$  is that they do not hedge their positive housing position. Taking lower values of beta implies even larger positive housing positions, and efficiency is even less often achieved.

Efficiency results also deteriorate going from right to left for the other two groups, whose net housing position becomes less negative and then positive when beta approaches zero, as long as they own at all. For renters instead net housing never becomes positive, and when  $\beta_{p\pi}=0$  the analysis collapses to the standard efficiency analysis of financial portfolios.

Housing wealth plays a role for all three groups and this explains why the lines do not appear to converge as  $\beta_{p\pi}$  approaches zero.

Overall the analysis shows that housing needs plays an important role for portfolio efficiency and ignoring this aspect induces an over-estimation of portfolio inefficiency.

**Figure 13: Proportions of efficient portfolios and the hedge ratio between house price and rent**



### 4.3 International portfolio diversification

In Section 3 we have assumed that Italian households invest in domestic stocks and bonds. This is particularly problematic for indirect holdings of stocks: we know from financial industry sources (Assogestioni) that in 2002 Italian mutual funds stock investment were three quarters in foreign stocks, one quarter in domestic stocks. According to a different source, the Bank of Italy financial statistics, roughly 50% of stocks were from Euro-area markets. Finally, we know that direct and indirect stock market participation were similar in 2002 (see Table 4), and this is also true for average amounts in our sample. Direct holdings are almost exclusively concentrated in domestic stocks.

Based on this information, we checked whether our analysis is robust to assuming that stock holdings were split among domestic stocks (62%) and foreign stocks (32%), and used MSCI Italy and MSCI world returns to compute their returns. We denote this asset as “international stocks”.

In Tables 11 and 12 we report descriptive statistics of the resulting returns. Compared to Table 1, we see from Table 11 that international stocks have lower expected returns (3.96% versus 4.90%) and lower standard deviation (27.65% instead of 28.99%). Remarkably, they have a negative correlation with government bonds return: this insurance property implies that optimal stock holdings are higher than in the case previously considered.

**Table 11 – Sample first and second moments of annual excess returns**

	<b>Government Bonds</b>	<b>Corporate Bonds</b>	<b>International Stocks</b>
<b>Expected return %</b>	4.0981	2.2845	3.9649
<b>Standard Deviation %</b>	5.2383	3.2169	27.6519

<i><b>CORRELATION</b></i>	<b>Government Bonds</b>	<b>Corporate Bonds</b>	<b>Stocks</b>
<b>Government Bonds</b>	1	0.8404	-0.0981
<b>Corporate Bonds</b>		1	0.0565
<b>International Stocks</b>			1

**Table 12 – Correlation matrix of housing and financial returns**

	<b>NW</b>	<b>NE</b>	<b>Centre</b>	<b>South</b>
<b>Government bonds</b>	-0.0164	-0.1169	-0.1161	-0.2036
<b>Corporate bonds</b>	-0.0843	-0.1691	-0.2177	-0.1998
<b>International Stocks</b>	-0.5432	-0.3660	-0.4484	-0.1959

**Table 13 – Regression of excess return on housing on financial assets excess returns**

<b>Variable</b>	<b>North West</b>	<b>North East</b>	<b>Centre</b>	<b>South</b>
<b>Constant</b>	2.7403 (0.5259)	2.894 (0.5618)	2.8884 (0.7110)	2.8548 (0.5534)
<b>r<sub>Gov.</sub></b>	-0.0739 (0.2666)	-0.0289 (0.284)	0.0536 (0.3595)	-0.1929 (0.2798)
<b>r<sub>CORP</sub></b>	-0.2896 (0.4454)	-0.4414 (0.4758)	-0.7547 (0.6021)	-0.3313 (0.4686)
<b>r<sub>STOCKS</sub></b>	-0.1146 (0.0284)	-0.0678 (0.0304)	-0.1049 (0.0384)	-0.0409 (0.0299)
<b>p-value</b>	0.000	0.009	0.006	0.007
<b>R<sup>2</sup></b>	0.576	0.416	0.436	0.433

*Notes: Standard errors in parentheses. Number of observations = 28*

The efficiency portfolio has weights of .67 on government bonds, .29 on corporate bonds and .04 on stocks – this compare to .63, .35 and .02 of the domestic stock return case. Table 12 however shows that correlation coefficients with housing are quite similar, and this results in minor changes in the regression coefficients (see Table 13).

In Table 14 we present the number and fraction of efficient portfolios in each net-housing group by broad region. A comparison with Table 8 shows that there are remarkably little differences.

**Table 14 – Proportion of efficient portfolios (test size 5%) by net housing position**

	Whole country		NW		NE		Centre		South	
	N°	%	N°	%	N°	%	N°	%	N°	%
<b>Over-housed</b>	56	<b>6.01</b>	28	<b>9.27</b>	14	<b>5.22</b>	8	<b>3.57</b>	6	<b>4.35</b>
<b>Negligible</b>	372	<b>45.76</b>	102	<b>39.84</b>	104	<b>48.60</b>	96	<b>54.86</b>	70	<b>41.67</b>
<b>Under-housed</b>	525	<b>62.50</b>	126	<b>39.50</b>	166	<b>74.11</b>	164	<b>77.36</b>	69	<b>81.18</b>
<b>All</b>	953	<b>36.87</b>	256	<b>29.19</b>	284	<b>40.23</b>	268	<b>43.86</b>	152	<b>38.87</b>

## 5. Conclusions

In this paper we have investigated how portfolio choices should be taken when housing represents a perfect hedge for rent risk. We have argued that in this case future rents are a stochastic liability. In this asset-liability framework the relevant housing wealth concept is the difference between the market value of housing stock owned and the present discounted value of current and future housing needs. Under the assumption that main residence housing consumption equals housing needs, the present value of housing needs can be calculated from micro data on rents (actual for tenants, imputed or self-assessed for owner-occupiers).

According to our model, households are short on housing (“under-housed”) when they either rent or own dwellings that are small compared to their future needs. Households who are owner occupiers may be long on housing (“over-housed”) at a late stage in the life cycle, when their future housing needs are declining and their death probability is increasing. However, long positions can also be obtained by purchasing secondary or investment homes. The under-housed are more exposed to rent risk, the over-housed (who are interested in the liquidation value of their home) to house price risk.

In order to evaluate the empirical relevance of housing risk in household portfolios, we have compared the efficiency of financial asset allocations for Italian households who are over-housed, under-housed or who have negligible housing positions. This comparison is of particular interest for financial intermediaries who design and sell securities to the general public, but is also of interest for its economic and policy implications.

Our key result is that many households do not appear to hedge housing risk in a satisfactory way. We have shown that the largest fraction of efficient financial portfolios is found among households who are “under-housed”, and should have less in stocks than the standard Markowitz portfolio. The smallest fraction of efficient portfolios obtains among households who are “over-housed”: even though in this group there is the highest proportion of stock-owners, their investment in stocks is often not sufficient to hedge all the housing risk.

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## ***APPENDIX - A formal dynamic model***

In this appendix we present a formal model that can be used to justify equation (2). We take the view that consumers know how their housing needs evolve over time, and optimally choose whether to purchase or rent housing stock. Consumption of housing services may be lower or higher than the service flow of the housing stock owned: if it is lower, some services are rented, if higher, part of the housing stock is let to other consumers. The existence of a rental market allows consumption and investment motives to be separated, but the presence of housing needs implies that investment decisions are affected by current and future consumption of housing. In this context, we show how the optimal financial portfolio changes when the difference between housing needs and housing owned is non-zero, and how to assess whether observed household portfolios are indeed efficient.

We assume that consumers enjoy utility from non-durable consumption and from housing, and housing services can be obtained by either renting or owning a certain housing stock. We also assume that rent rates correlate with housing stock returns – we first consider the special case where there is unit correlation, then we explore the more general case with a positive, fixed correlation coefficient lower than 1. The former case corresponds to a situation where owning is a perfect hedge (as in Yao and Zhang, 2005 and in Section 3 of this paper). The latter to the more general situation where house owning is not a perfect hedge against rent risk (as in Section 4.2 of this paper).

In our model, consumers do not live forever - the maximum length of life is  $T$  - but they can die in each period with a given, age-specific probability. Consumers care about their children, i.e. there is a bequest motive in their life-time utility function, but they wish to bequeath wealth, not housing. Housing can be bequeathed, but it is only valued for its monetary value, nothing else. Finally, housing needs evolve with age in a deterministic manner (that could be driven by demographic factors, like in Banks et al., 2004, or by an exogenously given income process).

In the model, we make the strong assumption that housing consumption equals housing needs, that is housing consumption is an exogenously given function of age. Consumers can invest their wealth in a risk-free asset, that is an asset whose return is known in terms of the non-durable good,  $n$  risky financial assets and housing stock. Housing services are provided by the housing stock that can be either rented or owned.

Formally, consumers maximize the following life-time expected utility index:

$$(1A) \quad E \left\{ \int_0^{\tau} e^{-\delta t} u(C_t, h_t) dt + e^{-\delta \tau} BB(W_{\tau}) | G_0 \right\}$$

where  $\tau$  is the stochastic end of life,  $C_t$  is non-durable consumption,  $h_t$  is consumption of housing services that relates to the housing stock owned ( $H_t$ ) or rented ( $H_t^R$ ) as follows:

$$(2A) \quad h_t = \rho_1 H_t + \rho_2 H_t^R$$

In the expressions above,  $\delta$  is a time preference parameter,  $u(\cdot)$  is a well-behaved utility function and  $BB$  is a bequest function depending on end-of-life wealth,  $W_{\tau}$ . Crucial to our analysis is the assumption that consumers do not wish to specifically bequeath housing stock – they only care about the money value of their bequests (if at all).

We assume that  $h_t$  is not a choice variable for the consumer, but it changes deterministically with age (housing needs are hump-shaped). Consumers can choose their non-durable consumption, financial asset holdings and housing stock they own.  $H_t^R$  is used to make up for the difference between necessary housing services and the housing services provided by owner-occupied housing<sup>6</sup>.

In this model, wealth includes financial asset holdings, the present value of future earnings (human capital, that we shall assume to be risk-free), as well as the value of the housing stock owned net of

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<sup>6</sup> Housing stock transactions may be subject to transaction costs, as in Grossman and Laroque (1990), Damgaard et al. (2003) and Cauley et al. (2005). We do not take them into account, nor do we make a distinction between deterministic housing needs and housing consumption, because needs are not observed.

the present value of future housing needs. The way human capital (HC) and the present value of housing needs (V) are computed is explained in the sequel.

Formally, wealth at time  $t$  is defined as:

$$(3A) \quad W_t = P_t H_t + B_t + X_t \cdot 1 - V_t + HC_t$$

Human capital enters wealth for reasons explained in Bodie, Merton and Samuelson (1992). A similar argument applies to  $V_t$ , the present value of the future stochastic rents. In equation (3A),  $B_t$  is the value of the risk-free asset position,  $X_t$  is the row vector of the values held in the  $n$  risky securities.

The prices of the housing stock ( $P_t$ ), of the  $n$  risky financial securities ( $S_t$ ) and of the risk-free asset ( $S_{0t}$ ) have the following dynamics:

$$(4A) \quad dP_t = (\mu_H + r_f)P_t dt + P_t dw_{Ht} \quad \text{with} \quad dw_{Ht} = \sigma_{P_1}^T dw_{1t} + \sigma_{P_2} dw_{2t}$$

$$(5A) \quad dS_t = \text{diag}\{S_t\}[\mu dt + \sigma dw_{1t}] \quad \text{with} \quad S_t \in \mathfrak{R}^n$$

$$(6A) \quad dS_{0t} = r_f S_{0t} dt$$

where  $w_{1t}$  and  $w_{2t}$  are two independent Wiener processes.

The housing stock is assumed to have zero depreciation rate, and this implies that expected housing return  $\mu_H$  must be defined as net of maintenance and repairs costs (as in Flavin and Yamashita, 2002).

Let us start from the present value of housing needs. We have to consider two stochastic processes: one for the consumer's death, another for the market price of housing needs. Thus we model a continuous time stochastic rent flow with stochastic expiration date.

We assume that the death process has a deterministic intensity rate (or survival rate). Let  $\tau$  be an exponential random variable with intensity rate  $\lambda_t$  and  $N_t = I_{\tau \leq t}$  be the consumer death process.

Let  $\mathfrak{F}_t = \sigma(\pi_s, s \leq t)$ ,  $H_t = \sigma(N_s, s \leq t)$  and finally  $G_t = H_t \vee \mathfrak{F}_t$  with  $H_t$  independent of  $\mathfrak{F}_t$ .

Assume that the intensity is a deterministic function of time  $\lambda_t = \lambda(t)$  and that at each instant the consumer pays a rent flow  $\pi_t h_t$ , where  $\pi_t$  represents the rental value (per square meter) and  $h_t$  housing consumption (in square meters). In view of the evidence that rents and house prices are strongly related, we follow Yao and Zhang (2005) and further assume  $\pi_t$  to be proportional to the house price:  $\pi_t = \alpha P_t$ , thus:

$$(7A) \quad d\pi_t = \pi_t (\mu_H + r_f) dt + \pi_t dw_{Ht}, \quad \text{with } \pi_0 = \alpha P_0^7$$

We write the following dynamics for the intensity rate and housing needs:

$$(8A) \quad d\lambda_t = -\lambda_t g(a(t)) dt, \quad \lambda_0 \text{ given}$$

$$(9A) \quad dh_t = -h_t f(a(t)) dt, \quad h_0 \text{ given}$$

where  $a(t)$  is a known age function (for instance, a polynomial in age).

The expected present value of the future rent stochastic flow is for the replicability argument (see Bodie, Merton and Samuelson, 1992, for a similar argument on human capital):

$$(10A) \quad V_t = E \left\{ \int_t^{\tau} e^{-(\mu_H + r_f)(s-t)} \pi_s h_s ds \mid G_t \right\} = I_{\tau > t} \alpha P_t h_t Q_t$$

where:

$$(11A) \quad Q_t = \int_t^T e^{-\int_t^s (g(a(u)) + f(a(u))) du} ds$$

This present value can be subtracted from the housing value in the definition of wealth because of unit correlation between housing prices and rent. (We discuss in the Appendix the case where the correlation is lower than one).

Note that the dynamics of the present value is a martingale process obtained as localization of a function of the house price process. In particular the dynamics is:

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<sup>7</sup> An extension to the case where rents and house prices have less than unit correlation is presented in the Appendix.

$$(12A) \quad dV_t = V_t \left( \mu_H + r_f - f(a(t)) + \frac{\partial Q_t}{\partial t} \frac{1}{Q_t} \right) dt + V_t dw_{Ht}$$

which is a geometrical Brownian motion with a non-constant drift.

For human capital the computations are similar in principle, under the (simplifying) assumption that labor income risk can be fully insured. Human capital is defined as the present value of future earnings prior to retirement and of future pension payments after retirement, where the discount rate is the risk-free rate. We assume that future earnings follow a deterministic age profile, and that retirement age and the replacement rate are known to the individual. Death is instead not known, but follows the same stochastic process as above. Under these assumptions, the relevant discount rate is the risk-free rate.

Let  $X_t^T = \theta_t^T \text{diag}\{S_t\}$  be the vector of total amounts invested in each financial asset and assume the self-financing portfolio hypothesis holds:

$$(13A) \quad dX_t = \sum_{i=0}^n \theta_{it} dS_{it}$$

then the wealth process has the following dynamic:

$$(14A) \quad dW_t = \left( r_f W_t - V_t \left( \mu_H - f(a(t)) + \frac{\partial Q_t}{\partial t} \frac{1}{Q_t} \right) + X_t \cdot (\mu^T - r_f \cdot 1) - C_t + P_t H_t \mu_H \right) dt \\ + X_t \cdot \sigma dw_{1t} + P_t H_t dw_{Ht} - V_t dw_{Ht}$$

The value function of the problem is:

$$(15A) \quad J = \sup_{\{\theta, H, C\}} E \left\{ \int_0^{\tau} e^{-\delta t} u(C_t, h_t) dt + e^{-\delta \tau} u(W_\tau) \mid G_0 \right\}$$

and the Hamilton-Jacobi-Bellman equation associated to the value function is:

$$(16A) \quad (\delta + \lambda)J = \sup_{\{\theta, H, C\}} E \left\{ u(h, C) + \frac{\partial J}{\partial W} dW_t + \frac{1}{2} \frac{\partial^2 J}{\partial W^2} dW_t^2 + \frac{\partial J}{\partial P} dP_t + \frac{1}{2} \frac{\partial^2 J}{\partial P^2} dP_t^2 + \frac{\partial^2 J}{\partial W \partial P} dP_t dW_t \right\}$$

which becomes, under the previous assumptions on the wealth process dynamics in (14A):

$$(17A) \quad (\delta + \lambda)J = \sup_{\{\theta, h, c\}} \left\{ \frac{\partial J}{\partial W} \left( r_f W_t - Q_t h_t \alpha P_t \left( \mu_H + f(a(t)) - \frac{\partial Q_t}{\partial t} \frac{1}{Q_t} \right) + X_t \cdot (\mu^T - r_f \cdot 1) - C_t + P_t H_t \mu_H \right) \right. \\ \left. + \frac{1}{2} \frac{\partial^2 J}{\partial W^2} \left( X_t \sigma \sigma^T X_t^T + (P_t (H_t - \alpha h_t Q_t))^2 [\sigma_{P_2}^2 + \sigma_{R_1} \sigma_{R_1}^T] + 2P_t (H_t - \alpha h_t Q_t) X_t \cdot \sigma \sigma_{R_1}^T \right) + \right. \\ \left. + \frac{\partial J}{\partial P} P_t (\mu_H + r_f) + \frac{1}{2} \frac{\partial^2 J}{\partial P^2} P_t^2 [\sigma_{P_2}^2 + \sigma_{R_1} \sigma_{R_1}^T] + \frac{\partial^2 J}{\partial W \partial P} (P_t^2 (H_t - \alpha h_t Q_t) [\sigma_{P_2}^2 + \sigma_{R_1} \sigma_{R_1}^T] + X_t P_t \cdot \sigma \sigma_{R_1}^T) \right\}$$

and taking first order conditions with respect to  $\{C, \theta, H\}$  respectively, we obtain:

$$(18A) \quad \frac{\partial u}{\partial C} - \frac{\partial J}{\partial W} = 0$$

$$(19A) \quad \frac{\partial J}{\partial W} (\mu^T - r_f \cdot 1) + \frac{\partial^2 J}{\partial W^2} (\sigma \sigma^T X_t^T + P_t (H_t - \alpha h_t Q_t) \sigma \sigma_{R_1}^T) + \frac{\partial^2 J}{\partial W \partial P} (P_t \cdot \sigma \sigma_{R_1}^T) = 0$$

$$(20A) \quad \frac{\partial J}{\partial W} P_t \mu_H + \frac{\partial^2 J}{\partial W^2} (P_t^2 (H_t - \alpha h_t Q_t) [\sigma_{P_2}^2 + \sigma_{R_1} \sigma_{R_1}^T] + P_t X_t \cdot \sigma \sigma_{R_1}^T) + \frac{\partial^2 J}{\partial W \partial P} P_t^2 [\sigma_{P_2}^2 + \sigma_{R_1} \sigma_{R_1}^T] = 0$$

Then focusing on the optimal portfolio choice, using the second block of first order conditions

(19A) we derive:

$$(21A) \quad X_t^T = -(\sigma \sigma^T)^{-1} \frac{\frac{\partial J}{\partial W} (\mu^T - r_f \cdot 1)}{\frac{\partial^2 J}{\partial W^2}} - P_t (H_t - \alpha h_t Q_t) (\sigma^T)^{-1} \sigma_{R_1}^T - \frac{\frac{\partial^2 J}{\partial W \partial P} (P_t \cdot (\sigma^T)^{-1} \sigma_{R_1}^T)}{\frac{\partial^2 J}{\partial W^2}}$$

where the first term is the standard Markowitz portfolio, the second term is a hedge term for the net position in housing (housing stock owned net of the present value of future rents) and the third term captures the income and substitution effects of changes in the relative price of housing. We can show that the state variable  $P$  does not affect the marginal value function of wealth (that

is:  $\frac{\partial^2 J}{\partial W \partial P} = 0$ ) when the utility function is additive separable in  $C$  and  $H$  (this extends Damgaard et

al, 2004). Under these conditions, the standard equation (2) holds.

The third block of first order conditions (equation (20A)) can be simplified if we recall that the derivative of housing consumption with respect to the housing stock owned is zero. Re-arranging terms, and substituting for  $X$  from (21A), we derive the following relation:

$$(22A) \quad P_t H_t = -\frac{\frac{\partial J}{\partial W_t}}{\frac{\partial^2 J}{\partial W^2} \sigma_{P_2}^2} [\mu_H + (\mu - r_f \cdot 1^T)(\sigma\sigma^T)^{-1} \sigma\sigma_{P_1}^T] + P_t \alpha h_t Q$$

for the case where  $\frac{\partial^2 J}{\partial W \partial P} = 0$ .

Equation (21A) has been derived without using the first order condition for the housing, and is therefore useful to analyze portfolio holdings even if we believe short run housing stock adjustments to be costly. Equation (22A) instead explicitly uses the optimality condition for  $H_t$ , and is best seen as suggestive of what consumers should try and do in the case where transaction costs are important. Equation (22A) shows that the optimal housing stock is the larger, the higher the present value of future rents: house-owning does indeed play the role of a hedge against rent risk, as suggested by Sinai and Souleles (2005). Our model can be used to justify why home-ownership is so wide-spread, and housing wealth is so important in household portfolios.

How does the analysis change when house prices are more volatile than rents, and the two processes have positive, but less than unitary correlation? To answer this question, we extend the model presented above to the general case of an arbitrary, fixed correlation coefficient between rent rates and housing stock returns.

Let equation (4A) be replaced by:

$$(23A) \quad dP_t = (\mu_H + r_f)P_t dt + P_t dw_{Ht} \quad \text{with} \quad dw_{Ht} = \sigma_{P_1}^T dw_{1t} + \sigma_{P_2} dw_{2t} + \sigma_{P_3} dw_{3t}$$

and assume the equation for the rental value dynamics is instead given by:

$$(24A) \quad d\pi_t = (\mu_\pi + r_f)\pi_t dt + \pi_t dw_{\pi} \quad \text{with} \quad dw_{\pi} = \sigma_{P_1}^T dw_{1t} + \sigma_{P_2} dw_{2t}$$

where  $\pi_t = \alpha P_t$ , but proportionality does not hold in any other period and  $w_{1t}$ ,  $w_{2t}$  and  $w_{3t}$  are independent Wiener processes.

The standard deviation of the rental price process is:

$$(25A) \quad \sigma_\pi = \sqrt{[\sigma_{P_1}; \sigma_{P_2}]^T * [\sigma_{P_1}; \sigma_{P_2}]}$$



the standard deviation of the house price process is instead given by:

$$(26A) \quad \sigma_H = (\sigma_\pi^2 + \sigma_{P3}^2)^{1/2}$$

The covariance of these two processes is given by  $\sigma_\pi^2$  and the slope coefficient of the regression of rental price changes on house price changes is:

$$(27A) \quad \beta_{P\pi} = \frac{\sigma_\pi^2}{\sigma_H^2} \quad \text{and} \quad 0 \leq \beta_{P\pi} \leq 1.$$

It is easily checked that  $\beta_{P\pi}$  is the square of the correlation coefficient.

Let us assume for simplicity that  $\mu_\pi = \beta_{P\pi} \mu_H$ . In this context, owning housing stock provides a hedge against rent risk, but not a perfect hedge. Standard results on optimization by risk averse investors imply that households will reduce this type of large, undiversifiable risk by purchasing housing stock, even though not for the full amount of their present value of housing needs.

We assume that the  $n$  risky financial securities ( $S_t$ ) and of the risk-free asset ( $S_{0t}$ ) have the same dynamics as in (5A) and (6A) and that the intensity rate and housing needs have the same dynamics as in (8A) and (9A).

In line with Bodie et al (1992), by a replicability argument, in order to hedge the risk in  $\pi_s h_s$  (for any  $s > 0$ ) the household should have  $\pi_t h_s \beta_{P\pi}$  in housing. This should hold for all future periods.

The present value of the future rent stochastic flow that should be hedged is:

$$(28A) \quad V_t^\beta = E \left\{ \int_t^\tau e^{-(r_f + \beta_{P\pi} \mu_H)(s-t)} \pi_s h_s \beta_{P\pi} ds \mid G_t \right\} = I_{\tau > t} \alpha P_t h_t \beta_{P\pi} Q_t$$

Note that the dynamics of the present value is a martingale process obtained as localization of a function of the house price process. In particular the dynamics is:

$$(29A) \quad dV_t^\beta = V_t^\beta \left( r_f + \beta_{P\pi} \mu_H - f(a(t)) + \frac{\partial Q_t}{\partial t} \frac{1}{Q_t} \right) dt + V_t^\beta dw_{Ht}$$

which is a geometric Brownian motion with a non-constant drift.

In this context, wealth at time  $t$  is defined as:

$$(30A) \quad W_t = P_t H_t + B_t + X_t \cdot 1 - V_t^\beta + HC_t$$

where  $V_t^\beta$  represents the expected present value of current and future stochastic rents that are hedged, and  $B_t$  is the value of the risk-free asset position,  $X_t$  is the row vector of the values held in the  $n$  risky securities as before.

Then the wealth process has the following dynamics:

$$(31A) \quad dW_t = \left( r_f W_t - V_t \left( r_f + \beta_{P\pi} \mu_H - f(a(t)) + \frac{\partial Q_t}{\partial t} \frac{1}{Q_t} \right) + X_t^T \cdot (\mu - r_f \cdot 1) - C_t + P_t H_t \mu_H \right) dt \\ + X_t \cdot \sigma dw_{lt} + P_t H_t dw_{Ht} - V_t dw_{Ht} - \pi_t h_t dw_{\pi} + \alpha P_t \beta_{P\pi} h_t dw_{Ht}$$

The value function of the problem is:

$$(32A) \quad J = \sup_{\{\theta, H, C\}} E \left\{ \int_0^\tau e^{-\delta t} u(C_t, h_t) dt + e^{-\delta \tau} u(W_\tau) \mid G_0 \right\}$$

and the Hamilton-Jacobi-Bellman equation associated to the value function is:

$$(33A) \quad (\delta + \lambda)J = \sup_{\{\theta, H, C\}} E \left\{ u(C, h) + \frac{\partial J}{\partial W} dW_t + \frac{1}{2} \frac{\partial^2 J}{\partial W^2} dW_t^2 + \frac{\partial J}{\partial P} dP_t + \frac{1}{2} \frac{\partial^2 J}{\partial P^2} dP_t^2 - \frac{\partial J}{\partial \pi} d\pi_t^2 \right. \\ \left. + \frac{1}{2} \frac{\partial^2 J}{\partial \pi^2} d\pi + \frac{\partial^2 J}{\partial W \partial P} dP_t dW_t - \frac{\partial^2 J}{\partial W \partial \pi} d\pi_t dW_t - \frac{\partial^2 J}{\partial \pi \partial P} dP_t d\pi_t \right\}$$

which becomes, under the previous assumptions on the wealth process dynamics in (31A):

$$(34A) \quad 0 = -(\delta + \lambda)J + \sup_{\{\theta, H, C\}} \left\{ \frac{\partial J}{\partial W} \left( r_f W_t - Q_t \beta_{P\pi} h_t \alpha P_t \left( \mu_H + f(a(t)) - \frac{\partial Q_t}{\partial t} \frac{1}{Q_t} \right) \right) \right. \\ \left. + X_t \cdot (\mu^T - r_f \cdot 1) - C_t + P_t H_t \mu_H - \mu_H h_t (\pi_t - \alpha P_t \beta_{P\pi}) \right. \\ \left. + \frac{1}{2} \frac{\partial^2 J}{\partial W^2} \left( X_t \sigma \sigma^T X_t^T + (P_t (H_t - \alpha h_t \beta_{P\pi} (Q_t - 1)))^2 [\sigma_{P_3}^2 + \sigma_{P_2}^2 + \sigma_{P_1} \sigma_{P_1}^T] \right) \right. \\ \left. + (\pi_t)^2 [\sigma_{P_2}^2 + \sigma_{P_1} \sigma_{P_1}^T] + 2P_t (H_t - \alpha h_t \beta_{P\pi} (Q_t - 1)) X_t \cdot \sigma \sigma_{P_1}^T - 2\pi_t h_t X_t \cdot \sigma \sigma_{P_1}^T \right) \\ \left. + \frac{\partial J}{\partial P} P_t (\mu_H + r_f) + \frac{1}{2} \frac{\partial^2 J}{\partial P^2} P_t^2 [\sigma_{P_2}^2 + \sigma_{P_1} \sigma_{P_1}^T] \right. \\ \left. + \frac{\partial^2 J}{\partial W \partial P} \left( P_t^2 (H_t - \alpha h_t \beta_{P\pi} (Q_t - 1)) [\sigma_{P_3}^2 + \sigma_{P_2}^2 + \sigma_{P_1} \sigma_{P_1}^T] + X_t P_t \cdot \sigma \sigma_{P_1}^T \right) \right\}$$

$$\begin{aligned}
& -\frac{\partial J}{\partial \pi} \pi_t (\mu_\pi + r_f) + \frac{1}{2} \frac{\partial^2 J}{\partial \pi^2} \pi_t^2 [\sigma_{P_2}^2 + \sigma_{P_1} \sigma_{P_1}^T] \\
& - \frac{\partial^2 J}{\partial W \partial \pi} ((\pi h_t)^2 [\sigma_{P_2}^2 + \sigma_{P_1} \sigma_{P_1}^T] + X_t \pi_t \cdot \sigma \sigma_{P_1}^T + (P_t H_t - V_t) \pi_t \sigma_\pi^2) - \frac{\partial^2 J}{\partial P \partial \pi} (\pi_t P_t [\sigma_\pi^2])
\end{aligned}$$

and taking first order conditions with respect to  $\{C, \theta, H\}$  respectively, we obtain:

$$(35A) \quad \frac{\partial u}{\partial C} - \frac{\partial J}{\partial W} = 0$$

$$\begin{aligned}
(36A) \quad & \frac{\partial J}{\partial W} (\mu^T - r_f \cdot 1) + \frac{\partial^2 J}{\partial W^2} (\sigma \sigma^T X_t^T + (P_t (H_t - \alpha h_t \beta_{P\pi} (Q_t - 1) - \alpha h_t)) \sigma \sigma_{P_1}^T) \\
& + \frac{\partial^2 J}{\partial W \partial P} (P_t \cdot \sigma \sigma_{P_1}^T) - \frac{\partial^2 J}{\partial W \partial \pi} \pi_t [\sigma \sigma_{P_1}^T] = 0
\end{aligned}$$

$$\begin{aligned}
(37A) \quad & \frac{\partial J}{\partial W} P_t \mu_H + \frac{\partial^2 J}{\partial W^2} (P_t^2 (H_t - \alpha h_t \beta_{P\pi} (Q_t - 1)) [\sigma_{P_3}^2 + \sigma_{P_2}^2 + \sigma_{P_1} \sigma_{P_1}^T] + P_t X_t \cdot \sigma \sigma_{P_1}^T) + \\
& \frac{\partial^2 J}{\partial W \partial P} P_t [\sigma_{P_3}^2 + \sigma_{P_2}^2 + \sigma_{P_1} \sigma_{P_1}^T] - \frac{\partial^2 J}{\partial W \partial \pi} P_t \pi_t [\sigma_\pi^2] = 0
\end{aligned}$$

Then focusing on the optimal portfolio choice, using the second block of first order conditions

(36A) we derive:

$$\begin{aligned}
(38A) \quad X_t^T = & -(\sigma \sigma^T)^{-1} \frac{\frac{\partial J}{\partial W} (\mu^T - r_f \cdot 1)}{\frac{\partial^2 J}{\partial W^2}} - (P_t (H_t - \alpha h_t \beta_{P\pi} (Q_t - 1) - \alpha h_t)) (\sigma^T)^{-1} \sigma_{P_1}^T \\
& - \left[ \frac{\frac{\partial^2 J}{\partial W \partial P} (P_t \cdot (\sigma^T)^{-1} \sigma_{P_1}^T)}{\frac{\partial^2 J}{\partial W^2}} - \frac{\frac{\partial^2 J}{\partial W \partial \pi} (\pi_t \cdot (\sigma^T)^{-1} \sigma_{P_1}^T)}{\frac{\partial^2 J}{\partial W^2}} \right]
\end{aligned}$$

where the first term is the standard Markowitz portfolio, the second term is a hedge term for the net position in housing (housing stock owned net of the present value of future rents that are hedged by purchasing housing stock) and the third term captures the income and substitution effects of changes in the relative price of housing. We can show that the state variables  $P$  and  $\pi$  do not affect

the marginal value function of wealth (that is:  $\frac{\partial^2 J}{\partial W \partial P} = 0$  and  $\frac{\partial^2 J}{\partial W \partial \pi} = 0$ ) when the utility function is additive separable in C and H (this extends Damgaard et al., 2004). With this last assumption equation (38A) collapses to equation (21A) when  $\sigma_\pi^2 = \sigma_H^2$  (i.e.  $\beta_{P\pi} = 1$ ). The robustness analysis performed in Section 4.2 is based on this general case, where  $\beta_{P\pi}$  is allowed to vary from zero to one.