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PARTNERSHIPS VS. FIRMS ENTRY STRATEGIES

MICHELE MORETTO
Università di Padova

GIANPAOLO ROSSINI
Università di Bologna

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Partnerships vs. Firms Entry Strategies*

Michele Moretto† and Gianpaolo Rossini‡

Abstract

From 1997 to 2001 we observe a faster growth in the number of *Nonemployer* businesses (mostly *Partnerships) vis-à-vis Firms* in the USA, a country with the mildest asymmetries between the two types of enterprise with respect to taxation, administrative entry barriers and other institutional aspects.

The different speed of net entry may be due to the internal organisation of the two types of enterprise and its relation to some market features.

In a continuous time stochastic environment, with sunk costs, we model entry as a growth option. *Partnerships* and *Firms* display specific entry patterns in terms of output price and size since they react in diverse fashions to market uncertainty. In most cases, the *Partnership* is less risky and better suited to enter under conditions of high volatility, as during the years between 1997 and 2001.

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†Dipartimento di Scienze Economiche, University of Padova, via del Santo, 33, Padova Italy. michele.moretto@unipd.it

‡Corresponding author. Dipartimento di Scienze Economiche, University of Bologna, Strada Maggiore, 45, Bologna, Italy. rossini@spbo.unibo.it

1 Prologue

From 1997 to 2001 in the USA we observe an overwhelming expansion of *Nonemployer vis-à-vis Employer* business.

Net entry of *Nonemployer*, proxied by the number of establishments (EST, in Table 1),[1] is more than twice that of *Employer*. Between the Censuses of 1997 and 2001 the number of *Nonemployer* grew by 10%, compared to 3% of *Employer*. *Nonemployer* business is smaller (average receipt (RE) is 43,000 dollars in 2001) than *Employer* (average payroll (PA) is 442,000 dollars).[2]

TABLE 1
Employer, Nonemployer businesses in all US industries

		Nonempl.			*Empl.*	
			change			change
1997	EST	15,438,609	01/97: 10%	EST	6,894,869	3%
1998		15,708,727	98/97: 1.7%		6,941,822	0.7%
1999		16,152,604	99/98: 2.8%		7,008,444	0.9%
2000		16,529,955	00/99: 2.3%		7,070,048	0.9%
2001		16,979,498	01/00: 2.7%		7,095,302	0.3%
1997	RE[3]	586,315,757	01/97 : 24%	PA	3,047,907,469	30%
1998		643,720,460	98/97: 9.7%		3,309,405,533	7.4%
1999		667,219,733	99/98: 3.7%		3,554,692,909	9.1%
2000		709,378,836	00/99: 6.3%		3,879,430,052	9.1%
2001		729,922,063	01/00: 2.8%		3,989,086,323	2.8%

Evidence presented in Table 1 is confined to the US, where better data are coupled with the mildest asymmetries between the two kinds of enterprise with respect to fiscal, financial, administrative entry barriers and other institutional aspects (OECD, 2000; OECD, 2006).

The *Employer* category is made up by enterprises which maximize profit and display separation between workers and owners. We call them simply *Firms*.

The *Nonemployer* category contains enterprises of three distinct legal and/or organizational forms: *Individual Proprietorship,*[4] *Partnership,*[5]

[1] *Nonemployers* do not live, on average, longer than *Employers,* (Parker, 2004; Taylor, 1999).

[2] Receipt and payroll are heterogeneous magnitudes: we use them to approximate enterprise size for broad comparisons.

[3] Payrolls (PA) and receipts (RE) are in thousands of dollars.

[4] These enterprises are close to the *self-employed* category of the European nomenclature. See for instance Parker, Barmby and Belghitar (2004).

[5] Over the same period covered in Table 1 ESTs of *Partnerships* increased by 26%.

Corporation, all without employees.[6] Among them, the most common are the first two. The most dynamic and fast growing is the *Partnership*, on which we concentrate our comparative analysis.[7]

The internal organization of a *Partnership* closely replicates that of a Labor Managed Enterprise (*LME*). In an *LME,* owners and employees coincide and share the governing power of the enterprise on an equal foot, maximizing individual dividend.[8]

What are the implications for entry strategies and size of this odd similarity between *Partnership*, one of the most dynamic form of start-up, and *LME*, often regarded as a sort of bulky legacy of socialism?[9]

The answer to this question is the main object of this study, whose aim is the comparison of entry strategies and size of *Partnerships* (N) and *Firms* (F).

In a dynamic setting, where a new venture project is carried out at distinct times and at distinct entry-trigger market prices, most differences between the N and the F come from uncertainty coupled with sunk costs. We shall see that N enters at less favorable conditions than F since the trigger price increases in peculiar fashions for the two enterprises as uncertainty unfolds. Higher risk makes the investment return more volatile and the value of the entry option goes up like the incentive to wait.[10] In N each member shares the enterprise risk with colleagues and bears only a fraction of the corresponding cost. The consequence is a higher value of the investment option without any increase in the incentive to delay entry.

A related question concerns the size at entry. By theoretically ex-

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This is the largest rate of growth among all categories. Average RE of *Partnership* in 2001 was 123,000 dollars, larger than the overall figure for *Nonemployers,* but still smaller than that of *Employers.* RE of *Partnerships* increased over the same time span by 39%.

[6]We follow the US Bureau of Census nomenclature (US Census Bureau, 2003a).

[7]Here is the US Bureau of Census definition: "*Individual proprietorship*....is an unincorporated business owned by an individual". Self-employed persons are included in this category. "*Partnership* ......is an unincorporated business owned by two or more persons having a shared financial interest in the business", i.e. sharing profits and losses and responsibilities having a general or limited liability. " A *Nonemployer Corporation* is a legally incorporated business under state laws", without employees. See: http://www.census.gov/epcd/nonemployer/view/define.html

[8]Individual *Proprietorship* is close to an *LME* with a single member.

[9]There are long run affinities between a competitive *LME* and the corresponding *Firm,* despite heterogeneities mostly due to perverse response of the short run *LME* supply function (Ward (1958), Vanek (1970), Pestieau and Thisse (1979), Pencavel and Craig (1994), Delbono and Rossini (1992)). These oddities vanish with tradeability of memberships i.e. in *workers' enterprises* (Sertel, 1993, 1997).

[10]This effect follows from the "bad news principle of irreversible investment" (Bernanke, 1983).

ploring entry and size of N and F we provide a basic interpretation of the smaller dimension and of the recent growth of N, during a period of intense financial volatility in the US.

In the next section we analyze the entry option of the two enterprises. In the third and fourth sections we define the values of the two enterprises. In the fifth section we investigate the distinct entry strategies. In the sixth section we assess the effect of uncertainty. In the seventh part we provide a numerical example. The sum up is contained in the epilogue.

2 A start-up option

Let's begin by comparing entry options. Each enterprise is supposed to own an infinitely-lived investment project. We model entry with a set of common assumptions plus some specific hypotheses referring to each enterprise.

Assumption 1 *The project, corresponding to the start-up decision, is of finite size and requires an investment I.*

Assumption 2 *The investment is irreversibly sunk. It can neither be changed, nor temporarily stopped, nor shut down.*[11].

Assumption 3 *The instantaneous short run revenue of the project is:*

$$R(p_t; L_t) \equiv p_t Q(L_t) \tag{1}$$

where p_t is the market output price, L_t is labor, $Q(L_t)$ is the short run Marshallian production function, with $Q(0) = 0$, $Q'(L_t) > 0$, $Q''(L_t) < 0$ and $L \in \left[\underline{L}, \bar{L}\right]$.

Assumption 4 *The uncertain market price evolves according to the following trendless stochastic differential equation:*

$$dp_t = \sigma p_t dB_t \qquad \text{with } \sigma > 0 \text{ and } p_0 = p, \tag{2}$$

where dB_t is the standard increment of a Wiener process (or Brownian motion), uncorrelated over time and satisfying the conditions that $E(dB_t) = 0$ and $E(dB_t^2) = dt$ (Dixit, 1993). Therefore $E(dp_t) = 0$ and $E(dp_t^2) = (\sigma \pi_t)^2 dt$, i.e., starting from the initial value p_0,

[11] This avoids the analysis of operating options differing across the two kinds of enterprise. The most relevant comes from the ability to reduce output and to shut down. Operating options increase the value of the enterprise. See McDonald and Siegel (1986) and, for a thorough discussion, Dixit and Pindyck (1994, chs. 6 and 7).

the random position of the price p_t at time $t > 0$ has a normal distribution with mean p_0 and variance $p_0^2(e^{\sigma^2 t} - 1)$ which increases as we look further and further into the future. The process "has no memory" (i.e. it is Markovian), and hence i) at any time t, the observed p_t is the best predictor of future profits, ii) p_t moves at any $t+1$ upwards or downwards with equal probability.[12]

Assumption 5 The market unitary wage w is constant.

Assumption 6 The investment is financed either by the founder members[13] (N), or by shareholders (F).

Assumption 7 Employees-members of N are homogeneous. They invest in the project and maximize the discounted value of expected individual net dividends. They receive a "supplemented wage", equal to dividends plus the market wage w.

Assumption 8 N and F maximize respectively the individual and the aggregate discounted value of expected cash flows.

Assumption 9 The size (L) of both enterprises is held fixed after entry.[14]

3 The value of a Partnership

Only if the price is high enough, N enters and sets the optimal size (L). The decision process requires a backwards procedure. First, we compute, for any L, the value of the individual option to enter. Subsequently, we choose L that maximizes the individual option value at entry. The discounted value of expected net individual dividend is:

$$
\begin{aligned}
y(p; L) - \frac{w}{\rho} &= \frac{E_0\left\{\int_0^\infty e^{-\rho t} R(p_t; L)dt \mid p_0 = p\right\} - I}{L} - \frac{w}{\rho} \qquad (3)\\
&= \frac{\frac{pQ(L)}{\rho} - I}{L} - \frac{w}{\rho}
\end{aligned}
$$

[12] By the Markov property of the process p_t, the results do not change qualitatively assuming a positive (or negative) trend of price.

[13] This is consistent with the assumption of the existence of a market for memberships, operating according to standard financial canons (Sertel, 1982, 1993, 1997).

[14] New enterprises are usually small. It sounds plausible to assume that at their entrance they choose the size of the labor force to hire and shun from adjusting it to variation of demand, preferring alternative ways which do not damage fresh internal organization.

where $E_0(.)$ is the expectation operator, with the information available at time zero, ρ is the riskless interest rate.[15] $\frac{w}{\rho}$ is the discounted flow of the market wage, i.e. the minimum that N grants its members and corresponds to a participation constraint: below it, members are better off supplying their labor in the market rather than founding a new company.

The employee-member of an N of size L decides whether and when to start the new project by solving an optimal stopping time problem and choosing the investment timing which maximizes:

$$f_N(p;L) = \max_T E_0\left[\left(y(p_T;L) - \frac{w}{\rho}\right)e^{-\rho T} \mid p_0 = p\right] \tag{4}$$

By Assumption 7 the employees-members of N are homogeneous. Each one holds an option to invest corresponding to (4) and has an interest in exercising it cooperatively at the same time.[16]

The employees-members wait up to time T, when p_t, starting from p_0, reaches an upper value, say p_N, and then invest. T is a random variable whose distribution can be obtained from that of (2). Assuming that p_N exists, taking expectation of (4) and using the distribution of T, we are able to write the member's value function, before investing, as (Dixit and Pindyck, 1994; Dixit et al., 1999):[17]

[15] Introducing risk aversion does not change the results since the analysis can be developed under a risk neutral probability measure (Cox and Ross, 1976; Harrison and Kreps, 1979).

[16] Members have just founded the firm of the optimal size and they have no incentive to behave non-cooperatively from the beginning.

[17] The solution to $E_0\left[e^{-\rho T}\right]$ can be obtained via the usual dynamic programming decomposition (Dixit et al., 1999 p.184). Since the process p_t is continuous, the expected discount factor is increasing in p and decreasing in p_N ; then it can defined by a function $D(p;p_N)$. Over the infinitesimal time interval dt, p will change by the small value dp, hence we get the following Bellman equation:

$$\rho D(p;p_N)dt = E(dD(p;p_N)),$$

By applying Ito's Lemma to dD we obtain the following differential equation:

$$\frac{1}{2}\sigma^2 p^2 D'' - \rho D = 0\,,$$

We solve it subject to the two boundary conditions:

$$\lim_{p\to\infty} D(p;p_N) = 0$$

$$\lim_{p\to p_{NF}} D(p;p_N) = 1$$

$$f_N(p;L) = \left(y(p_N;L) - \frac{w}{\rho} \right) E_0 \left[e^{-\rho T} \mid p_0 = p \right] \tag{5}$$
$$= \left(y(p_N;L) - \frac{w}{\rho} \right) \left(\frac{p}{p_N} \right)^{\beta} \quad \text{for } p < p_N.$$

The member option value (5) represents the expected net individual dividend of the project, i.e., $y(p_N;L) - \frac{w}{\rho}$, multiplied by the expected discount factor, i.e., $\left(\frac{p}{p_N}\right)^{\beta}$. Then, the optimal investing rule implies that $f_N(p;L) > y(p;L) - \frac{w}{\rho}$ for all $p < p_N$. By some algebra (5) can be written as:

$$f_N(p;L) = \frac{Q(L)}{L} \left[\frac{p_N}{\rho} - AC(L) \right] \left(\frac{p}{p_N} \right)^{\beta} \quad \text{for } p < p_N \text{ and } p_N \geq \rho AC(L) \tag{6}$$

where $AC(L) \equiv \frac{\frac{wL}{\rho}+I}{Q(L)}$ is the long-run average total cost. $AC(L)$ stands for the (deterministic) Marshallian entry trigger. Entry occurs if the discounted cash flow generated by the project is weakly larger than the long-run average cost.

Furthermore, from (6), the option value to invest of each member goes to zero in two extreme cases: 1) when the optimal price threshold p_N is exactly equal to $\rho AC(L)$; 2) when the optimal trigger p_N goes to infinity. In the latter case the option vanishes since it is optimal to delay the investment indefinitely. In the former case, the option value evaporates because of lack of flexibility: each member carries out the project if and only if p is larger than $\rho AC(L)$.

4 The value of a Firm

As before, we first derive the entrepreneur's value of the option to invest for any given L, and subsequently we choose L at the optimal entry time. By Assumption 9, we know whether and when to ignite the project from the solutions of the following optimal stopping time problem:

$$F_F(p;L) = \max_T E_0 \left[\left(V(p_T;L) - I \right) e^{-\rho T} \mid p_0 = p \right] \tag{7}$$

where the market value of a project of dimension L is:[18]

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and we get $D(p;p_N) = \left(\frac{p}{p_N}\right)^{\beta}$, where $1 < \beta < \infty$ is the positive root of the auxiliary quadratic equation $\Psi(\beta) = \frac{1}{2}\sigma^2\beta(\beta-1) - \rho = 0$.

[18] By Assumption 9, F selects its project from a set of ventures with total cost, $\frac{w}{\rho}L + K$.

$$V(p;L) = E\left\{\int_0^\infty e^{-\rho t}\left(R(p_t,L) - wL\right)dt\right\} \equiv \left(\frac{pQ(L)}{\rho} - \frac{wL}{\rho}\right).$$

Owing to the homogeneity of (3) and the properties of the stopping time T, it can be shown that:

$$F_F(p;L) = f_N(p;L)L \tag{8}$$

where $f_N(p;L)$ is the value of the project for the L-th member of N, given by (4). Then, by (6) we get:

$$F_F(p;L) = Q(L)\left[\frac{p_F}{\rho} - AC(L)\right]\left(\frac{p}{p_F}\right)^\beta \quad \text{for } p < p_F \text{ and } p_F \geq \rho AC(L) \tag{9}$$

where p_F is the optimal threshold that triggers the investment by F. The remarks made for (6) extend to (9): also for F the option value to invest goes to zero when p_F equals the Marshallian trigger $\rho AC(L)$ or when p_F goes to infinity.

5 Entry strategies

Maximizing (6) and (9) with respect to both p_N and p_F, we obtain the optimal entry policies. The optimal investment strategy for both firms requires investing as soon as the market price exceeds the break-even threshold:

$$p_i(L) \equiv \frac{\beta}{\beta-1}\rho AC(L) \quad \text{for } i = N, F \tag{10}$$

These limits are the Marshall triggers $\rho AC(L)$ multiplied by $\frac{\beta}{\beta-1} > 1$ due to the irreversibility of entry.[19] Substituting (10) back into (6) and (9) and maximizing with respect to L we have:

Lemma 1 *The optimal entry size of N can be obtained from:*

$$\frac{L_N Q'(L_N)}{Q(L_N)} = 1 - \frac{(\beta-1)}{\beta}\frac{I}{(\frac{w}{\rho}L_N + I)} \tag{11}$$

while for F it comes from:

$$\frac{L_F Q'(L_F)}{Q(L_F)} = \frac{(\beta-1)}{\beta}\left(1 - \frac{I}{(\frac{w}{\rho}L_F + I)}\right) \tag{12}$$

[19] With new observations on market profitability obtained by waiting, the enterprise reduces downside risk (Dixit and Pindyck, 1994, p. 142).

Proof. See *Appendix* and subsequent arguments. ■

Although the optimal triggers (10) look alike, they are not since, at entry, the two enterprises have different size. As a proof, consider first N. Substituting (10) into (6) and rearranging we write the L-th employee-member's value of the project prior to investing:

$$f_N(p; L) = A(L)p^{\beta} \qquad \text{for } p < p_N(L), \tag{13}$$

where the constant $A(L)$ is given by:

$$A(L) \equiv \frac{(\beta-1)^{\beta-1}}{(\rho\beta)^{\beta}} AC(L)^{-\beta} \frac{(\frac{w}{\rho}L + I)}{L} > 0 \tag{14}$$

By (13) the optimal dimension of N requires choosing L for which $A(L)$ is the largest. This is equivalent to maximizing

$$a(L) \equiv AC(L)^{-\beta} \frac{(\frac{w}{\rho}L + I)}{L},$$

which gives the first order condition (FOC) described in *Lemma 1.* Since the r.h.s. of (11) is less than one, a necessary condition for an optimal solution is an output elasticity $\varepsilon_{QL} \equiv \frac{LQ'(L)}{Q(L)} < 1$, i.e., the average productivity $\frac{Q(L)}{L}$ must be a decreasing function of labor, as from Assumption 3.

A necessary, yet non sufficient, requirement for the second order condition (SOC) entails the output elasticity to decrease in L (see *Appendix*). If this is not the case, the optimum comes from a binary comparison between the smallest dimension $\underline{L}$ and the largest one $\bar{L}$.

For F, substituting (10) into (9) and rearranging we get the shareholders value as:

$$F_F(p; L) = B(L)p^{\beta}, \qquad \text{for } p < p_F(L) \tag{15}$$

where the constant $B(L) = LA(L)$.

By (15), optimality requires finding L that maximizes $B(L)$, which, by (14), is equivalent to maximize $La(L)$, yielding the FOC contained in *Lemma 1.* As we have seen for N, since the r.h.s. of (12) is less than one, a necessary condition[20] is a production elasticity $\varepsilon_{QL} < 1$, while $\frac{d\varepsilon_{QL}}{dL} < 0$ is necessary but not sufficient to get the optimal dimension within the range $[\underline{L}, \bar{L}]$ (see *Appendix*).

[20] If entry costs are nul, condition (12) reduces to: $\frac{LQ'(L)}{Q(L)} = \frac{(\beta_1 - 1)}{\beta_1} < 1$, equivalent to the condition proposed by Dixit (1993) for a F choosing among investment projects of different dimensions. In Moretto (2003) there is an analogous condition for a F that incrementally reduces capacity.

If an interior solution exists we can compare the entry strategies of F and N, setting first the optimal dimension at entry and then the trigger price. On the basis of *Lemma 1* we can show that:

Proposition 1 *a) Over the range where the SOC holds, F is operating with a larger dimension than N, i.e.:*

$$L_N < \hat{L} < L_F,$$

where $\hat{L} = \arg\min AC(L)$ is the minimum efficient scale.

b) The entry trigger prices react in distinct ways for N and F, i.e.:

$$\frac{\partial p_F}{\partial L_F} > 0 \qquad \frac{\partial p_N}{\partial L_N} < 0.$$

Proof. See *Appendix.* ■

To appreciate the intuition behind this result we go back to *Lemma 1* rewriting the *FOCs* for the optimal dimension (11) and (12) at entry. By multiplying both sides of (12) by $p_F(L_F)$ and by simplifying we get:

$$p_F(L_F)Q'(L_F) = w. \tag{16}$$

Then, F, at entry, decides the optimal dimension equating the value marginal product to the market wage w. Similarly, we obtain:

$$p_N(L_N)Q'(L_N) = w + \frac{1}{\beta - 1}(w + \rho\frac{I}{L_N}) > w. \tag{17}$$

Unlike F, N chooses the optimal size equating the value marginal product to the "supplemented wage", which exceeds the market wage w. The Marshallian full cost of the investment imputed to each employee-member is $w + \rho\frac{I}{L_N}$, larger than w since the members of N possess an option (to delay entry), not owned by employees of F. Would-be employees-members are workers endowed with an option to build a *Partnership* making for a compensation larger than w. By the decreasing marginal product of labor, N will have a smaller size at entry than its twin mate F, i.e. $L_N < L_F$. This is consistent with the empirical finding that N is on average smaller than corresponding F.[21]

The conclusion that N and F have different dimensions opens the way to questions about the entry price as size changes.

[21] This is also the case of *LMEs*. "... smaller than their capitalist counterparts in the short-run when profits are positive" (Bonin and Putterman, 1987, p.15). The same applies to the long run if profits are strictly positive (ibidem, p.57).

6 The effects of uncertainty

N and F enter when the market price is larger than the average total cost $AC(L) \equiv \frac{\frac{wL}{\rho}+I}{Q(L)}$ multiplied by a coefficient $\frac{\beta}{\beta-1}\rho$. However, we do not know the reactions to uncertainty of the two enterprises. We fill this gap by going through some comparative statics. First of all we see whether *Proposition 1* holds when uncertainty disappears. We can show that:

Proposition 2 *If $\sigma = 0$, F and N operate at the minimum efficient scale, i.e.:*

$$L_N = \hat{L} = L_F$$

with coincident entry strategies:

$$p_F(\hat{L}) = p_N(\hat{L}).$$

Proof. Straightforward ■

By referring to (6) and (9), we may better understand this result in the Marshallian context. Under certainty the competitive pressure dissipates rents. Free entry leads to zero expected profit (the option value goes to zero) and both enterprises produce at the minimum of the U-shaped average cost curve. The equilibrium "supplemented wage" in N is equal to the competitive wage paid by F and both enterprises enter at the Marshallian trigger $\rho AC(\hat{L})$.

Uncertainty destroys this symmetry. Both enterprises require positive expected profits before committing to an irreversible investment. If, at the time of entry, $V(p;L)-I$ is positive, the discounted value of expected net individual dividend $y(p;L)$ exceeds w because employees-members pocket the rents. Since the dimension of the project is fixed, N will be more "capital-intensive" than F (i.e. $L_N < L_F$), whose cost of labor is w.[22]

Consider now the effect of uncertainty via a larger σ:

Proposition 3 *As market price volatility increases, the entry price increases for both enterprises:*

$$\frac{\partial p_N}{\partial \sigma} > 0 \text{ and } \frac{\partial p_F}{\partial \sigma} > 0$$

and the size gap widens, i.e.

$$\frac{\partial (L_F - L_N)}{\partial \sigma} > 0.$$

[22] A similar conclusion emerges in the comparison of $LMEs$ with Fs (Bonin and Putterman, 1987, p. 57; Delbono and Rossini, 1992).

Proof. See *Appendix.* ■

As the *Real Option Theory* predicts, increasing risk puts off investment timing, i.e. the entry price increases with uncertainty, due to the "bad news principle of irreversible investment". Higher market risk drives up the investment return volatility with positive effects on the option to invest. However, the net marginal benefit of waiting, arising from shunning investment in the bad state, increases with uncertainty. This induces an entry delay (Bernanke, 1983).

As uncertainty grows, F gets larger, N smaller. The higher entry price makes F react by increasing the optimal size so as to keep the value marginal product in line with the market wage. On the contrary, for N, the "supplemented wage" imputed to each employee-member goes up with σ, (down with β) and the enterprise downsizes to adjust the value marginal product.

According to *Proposition 3* uncertainty makes the two enterprises delay entry. Unfortunately, there is no global ranking in terms of entry prices.

When $\sigma \to \infty$, both p_N and p_F tend to infinity: N and F look alike because it is optimal to delay investment indefinitely. However, by investigating entry prices for low price volatility we see that an enterprise invests before the other showing distinct "riskiness", since the N set of entry prices is "less convex" than that of F. That is:

Proposition 4 *For low price volatility, N is "less risky", since the N entry price turns out to be lower than that of F*

Proof. See *Appendix.* ■

The entry boundary increases in distinct manners for N and F. Since the members of N equally share the option to invest, they may demand a higher reward and require a smaller price to compensate for the increased risk. This lowers the net marginal benefit of waiting of each individual member, reducing the incentive to delay entry.

7 A numerical example

A numerical example may better illustrate the relationship between entry triggers, optimal dimension and price volatility.

We assume a standard Cobb-Douglas technology: $Q(L) = \lambda L^{\alpha}$ with $\alpha \in (0, 1]$ and $\lambda \in (0, \infty)$.

We adopt parameter values, fairly common in numerical examples (Dixit and Pindyck, 1994; Smit and Trigeorgis, 2004; Pastor and Veronesi, 2004): $\rho = 0.08; \lambda = 1; \alpha = 0.5;\ w = 0.2$. We analyze two groups of cases selected according to the level of the investment ($I = 50; I = 100$) and we see how the trigger price and the size[23] of the two enterprises change as uncertainty unfolds. For each group we deal with 3 levels of uncertainty.

First group: $I = 50$.

I) Low uncertainty ($\sigma = 0.01; \beta = 40.50$): N enters at price 1.78 and size 20, F at price 1.78 and size 22.

II) Medium uncertainty ($\sigma = 0.08; \beta = 5.52$): N enters at price 2.17 and size 13, F at price 2.18 and size 33.

III) High uncertainty ($\sigma = 0.25; \beta = 2.18$): N enters at price 5.60 and size 2, F at price 6.68 and size 326.

Second group: $I = 100$.

I) Low uncertainty ($\sigma = 0.01; \beta = 40.50$): N enters at price 2.50 and size 40, F at price 2.50 and size 44.

II) Medium uncertainty ($\sigma = 0.08; \beta = 5.52$): N enters at price 3.05 and size 27, F at price 3.06 and size 66.

III) High uncertainty ($\sigma = 0.25; \beta = 2.18$): N enters at price 7.87 and size 4, F at price 9.39 and size 652.

From the numerical example it appears that N enters always at a trigger which is weakly smaller than that of F, i.e. at less favorable market conditions and at smaller dimension.

8 Epilogue

From the four Propositions the *Partnership* turns out to be a more suitable entrepreneurial organization in times of high volatility, such as the 1997 - 2001 period. It enters at a lower market price and smaller size. This is consistent with the statement that volatility boosts the value of an enterprise even if there is no bubble, as shown in Pastor and Veronesi (2004; 2005) who explain the stock exchange growth between 1997 and 2001 with the increasing uncertainty due to IT revolution.

Our results explain:

1. why there were so many entries of *Partnerships* during a period of high volatility such as the years between 1997 and 2001 in the USA, a country in which administrative, financial and fiscal conditions are the most symmetric for *Firms* and *Partnerships*;

2. the smaller operation scale of *Partnerships*.

The divergence between the two entry policies is due to the irreversible commitment under uncertainty and the distinct internal organization of the two types of enterprise.

Employees-members hold an option to enter based on their ability to set up a new enterprise. The option value increases with market

[23] The figures shown in the example are natural numbers, since size refers respectively to the number of members of N and employees of F. Figures are approximations since we do not use integer programming.

volatility and the size of the required irreversible commitment. The value of this option adds to the market wage making the total “salary” paid to employees-members higher with respect to the *Firm*, even in the long run.

The employees-members equally share the option to invest. By demanding a higher reward and requiring a smaller size to compensate for the increased risk, they lower the net marginal benefit of waiting, reducing the incentive to delay entry. Then, the *Partnership* turns out to be “less risky” and more suitable than the *Firm* for periods of high volatility.

Possible avenues for future research should consider the opportunity to vary the size of the investment in a two-factor technology and the possibility of exit.

9 Appendix

9.1 Proof of Lemma 1

To prove the first part of *Lemma 1* take *logs* of $a(L)$. The *FOC* (11) follows by deriving $\ln a(L)$ with respect to $L:$

$$\frac{Q'(L)}{Q(L)} - \frac{(\beta-1)}{\beta}\frac{\frac{w}{\rho}}{(\frac{w}{\rho}L+I)} - \frac{1}{\beta L} = 0.$$

Multiplying both sides by L and recalling that $\frac{\frac{w}{\rho}L}{(\frac{w}{\rho}L+I)} \equiv 1-\eta_{QL}$, where $\eta_{QL} \equiv \frac{I}{(\frac{w}{\rho}L+I)}$, we get:

$$\frac{LQ'(L)}{Q(L)} - \left[1 - \frac{(\beta-1)}{\beta}\eta_{QL}\right] = 0. \tag{18}$$

Since $\frac{(\beta-1)}{\beta} < 1$ and $\eta_{QL} < 1$ a necessary condition for an interior solution requires that $\varepsilon_{QL} \equiv \frac{LQ'(L)}{Q(L)} < 1$. The *SOC* for an interior solution is:

$$\frac{Q''(L)Q(L) - Q'(L)^2}{Q(L)^2} + \frac{(\beta-1)}{\beta}\frac{(\frac{w}{\rho})^2}{(\frac{w}{\rho}L+I)^2} + \frac{1}{\beta L^2} < 0.$$

Rearranging and making use of (18) we get the following local condition:

$$\frac{Q''(L)L^2}{Q(L)} + (1-\varepsilon_{QL})(\varepsilon_{QL}+\eta_{QL}) \equiv \varepsilon_{QL}(\frac{d\varepsilon_{QL}}{dL}\frac{L}{\varepsilon_{QL}} - \eta_{QL}) + \eta_{QL} < 0,$$

where:

$$\frac{d\varepsilon_{QL}}{dL} = \frac{1}{L}\left[\frac{Q''(L)L^2}{Q(L)} + (1-\varepsilon_{QL})\varepsilon_{QL}\right].$$

To prove the second part of *Lemma 1* we take *logs* of $La(L)$. The *FOC* (12) follows by deriving $\ln La(L)$ with respect to $L:$

$$\frac{Q'(L)}{Q(L)} - \frac{(\beta-1)}{\beta}\frac{\frac{w}{\rho}}{(\frac{w}{\rho}L+I)} = 0.$$

Multiplying both sides by L and recalling η_{QL} we get:

$$\frac{LQ'(L)}{Q(L)} - \frac{(\beta-1)}{\beta}\eta_{QL} = 0. \tag{19}$$

Again, a necessary condition for an interior solution requires that $\varepsilon_{QL} \equiv \frac{LQ'(L)}{Q(L)} < 1$, while the *SOC* is:

$$\frac{Q''(L)Q(L) - Q'(L)^2}{Q(L)^2} + \frac{(\beta-1)}{\beta}\frac{(\frac{w}{\rho})^2}{(\frac{w}{\rho}L+I)^2} < 0.$$

Rearranging and making use of (19) we get the following local condition:

$$\frac{d\varepsilon_{QL}}{dL}L+\varepsilon_{QL}(\frac{2\beta-1}{\beta-1}\varepsilon_{QL}-3)+\frac{(\beta-1)}{\beta}<0.$$

9.2 Proof of Proposition 1

First part of the *Proposition.*

Defining $b(L)\equiv La(L)$, we know from *Lemma 1* that F optimal size is given by:

$$b'(L)=a(L)+La'(L)=0,$$

while the SOC is:

$$b''(L)=2a'(L)+La''(L)<0.$$

In general $a''(L)<0$ does not imply that $b''(L)<0$: the two regions, where the SOC holds, overlap only partially. Over the range where the SOC holds $a'(L_N)=0$. Therefore, $b'(L_N)=a(L_N)>0$. If an L_F exists such that $b'(L_F)=0$, this will necessarily be

$$L_N<L_F.$$

Second part of the *Proposition.*

Define the average cost function $AC(L)\equiv\frac{\frac{w}{\rho}L+I}{Q(L)}$. By the concavity of $Q(L)$ it is easy to show that $\lim_{L\to0}AC(L)=+\infty$ and $\lim_{L\to+\infty}AC(L)=+\infty$. By taking the derivative with respect to L, we get:

$$\frac{\partial AC}{\partial L}=\frac{\frac{w}{\rho}Q(L)-(\frac{w}{\rho}L+I)Q'(L)}{Q(L)^2}=\begin{cases}<0 & \text{if} \quad \varepsilon_{QL}=\frac{LQ'(L)}{Q(L)}>1-\frac{I}{(\frac{w}{\rho}L+I)}\\ >0 & \text{if} \quad \varepsilon_{QL}=\frac{LQ'(L)}{Q(L)}<1-\frac{I}{(\frac{w}{\rho}L+I)}\end{cases} \tag{20}$$

Then, a value $\hat{L}>0$ exists such that $\frac{\partial AC}{\partial L}=0$ and it is given by:

$$\frac{\hat{L}Q'(\hat{L})}{Q(\hat{L})}=\left(1-\frac{I}{(\frac{w}{\rho}\hat{L}+I)}\right). \tag{21}$$

The second order condition confirms that $AC(L)$ is a convex function with a minimum represented by $\hat{L}$.

Since $\frac{(\beta-1)}{\beta}<1$, by comparing (21) and (12), we have:

$$\frac{(\beta-1)}{\beta}\left(1-\frac{I}{(\frac{w}{\rho}L+I)}\right)<1-\frac{I}{(\frac{w}{\rho}L+I)},$$

which, in the range where the SOC holds, implies that $\hat{L}<L_F$. On the contrary, by comparing (21) and (11), we notice that N operates

only in the descending branch of the average cost curve to the left of the minimum. That is:

$$1 - \frac{(\beta - 1)}{\beta} \frac{I}{(\frac{w}{\rho}L + I)} > 1 - \frac{I}{(\frac{w}{\rho}L + I)}$$

or:

$$\frac{1}{\beta} \frac{I}{(\frac{w}{\rho}L + I)} > 0,$$

which implies that $\hat{L} > L_N$. QED

9.3 Proof of proposition 3

Applying the implicit function theorem to (12) and (11), it can be shown that $\partial L_F/\partial\beta \leq 0 \leq \partial L_N/\partial\beta$. Then, since $\frac{\partial\beta}{\partial\sigma} < 0$, $\frac{\beta-1}{\beta}$ decreases and the opposite effect on optimal dimension follows. Moreover, totally differentiating (10) for the two enterprises yields:

$$\frac{\partial p_F}{\partial\sigma} = \frac{\partial(\frac{\beta}{\beta-1})}{\partial\sigma} AC + \frac{\beta}{\beta - 1} \frac{\partial AC}{\partial L_F} \frac{\partial L_F}{\partial\sigma} > 0 \qquad \text{for } L_F > \hat{L}, \tag{22}$$

$$\frac{\partial p_N}{\partial\sigma} = \frac{\partial(\frac{\beta}{\beta-1})}{\partial\sigma} AC + \frac{\beta}{\beta - 1} \frac{\partial AC}{\partial L_N} \frac{\partial L_N}{\partial\sigma} > 0 \qquad \text{for } L_N < \hat{L}. \tag{23}$$

By the above result and (20) it is easy to ascertain the positivity of both. In particular, if $\sigma \to \infty$, we have $\beta \to 1$ and $\frac{\beta-1}{\beta} \to 0$: neither type of enterprise enters. QED

9.4 Proof of Proposition 4

The slope of the entry price at $\sigma = 0$ can be found by evaluating (22) and (23) at $L_F = L_N = \hat{L}$. Since $AC'(\hat{L}) = 0$ we get:

$$\frac{\partial p_F}{\partial\sigma} |_{\sigma=0} = \frac{\partial(\frac{\beta}{\beta-1})}{\partial\sigma} |_{\sigma=0} AC(\hat{L}) > 0,$$

$$\frac{\partial p_N}{\partial\sigma} |_{\sigma=0} = \frac{\partial(\frac{\beta}{\beta-1})}{\partial\sigma} |_{\sigma=0} AC(\hat{L}) > 0.$$

Then, both enterprises have the same slope of the entry price at $\sigma = 0$. Differentiating (22) and (23) once more with respect to σ and evaluating the result at zero yields:

$$\frac{\partial^2 p_F}{\partial\sigma^2} |_{\sigma=0} = \frac{\partial^2(\frac{\beta}{\beta-1})}{\partial\sigma^2} |_{\sigma=0} AC(\hat{L}) + \frac{\beta}{\beta - 1} AC''(\hat{L}) \frac{\partial L_F}{\partial\sigma} |_{\sigma=0}$$

$$\frac{\partial^2 p_N}{\partial \sigma^2}\mid_{\sigma=0} = \frac{\partial^2(\frac{\beta}{\beta-1})}{\partial \sigma^2}\mid_{\sigma=0} AC(\hat{L}) + \frac{\beta}{\beta-1} AC''(\hat{L}) \frac{\partial L_N}{\partial \sigma}\mid_{\sigma=0}$$

Since $\frac{\partial L_F}{\partial \sigma}\mid_{\sigma=0} > 0$ and $\frac{\partial L_N}{\partial \sigma}\mid_{\sigma=0} < 0$ we conclude that $\frac{\partial^2 p_F}{\partial \sigma^2}\mid_{\sigma=0} > \frac{\partial^2 p_N}{\partial \sigma^2}\mid_{\sigma=0}$. QED

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