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OPTIMAL PENALTY FOR INVESTMENT DELAY  
IN PUBLIC PROCUREMENT CONTRACTS

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# Optimal penalty for investment delay in public procurement contracts

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## Abstract

Our aim in this paper is to provide a general framework in which to determine the optimal penalty fee inducing the contractor to respect the contracted delivery date in public procurement contracts (PPCs). We do this by developing a real option model that enables us to investigate the contractor's value of investment timing flexibility which the penalty rule - *de facto* - introduces. We then apply this setting in order to evaluate the range of penalty fees in the Italian legislation on PPC. According to our calibration analysis, there is no evidence that the substantial delays recorded in the execution times of Italian investments are due to incorrectly set penalty fees. This result opens the way for other explanations of delays in PPCs: we thus extend our model to include the probability that the penalty is ineffectively enforced and study how calibration results are accordingly affected. We finally show how our model can be used to investigate both the case of a penalty/premium rule and that of an optimal penalty fee in a concession contract.

**Keywords:** public procurement contracts, penalty fee, investment timing flexibility, investment irreversibility, contract incompleteness, enforceability of rules.

**JEL:** L33; H57; D81

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# 1 Introduction

The deterioration of public finance and the increase in global competition have forced governments and public institutions to obtain “the best value for money” through the purchase of goods, works and services in the form of procurement contracts. Efficient public procurement contracts (henceforth PPCs) are thus emerging as a “core necessity for ... the public’s sector effectiveness in obtaining resources for social spending and/or lower taxes” (Dimitri et al., 2006). These contracts have recently recorded a rapid increase both in number and in value, reaching 16% of GDP in the EU, and around 20% in the United States.<sup>1</sup> However, PPCs have both costs and benefits: their benefits (i.e. allocative and productive efficiency) can be quickly erased by the costs (i.e. inefficiency) which often arise from contractual incompleteness and all the issues that ensue therefrom.<sup>2</sup>

In this paper we specifically address the source of inefficiency which pertains to delays in PPC execution times<sup>3</sup> by investigating the optimal penalty design which should provide the *right* incentive to prevent delays. Indeed, delays in delivery dates in PPCs may negatively affect all the actors involved, i.e. they may determine direct costs for the procurer, lower firms’ profits (i.e. firms other than the contractor) and reduce consumers’ utility. The typical illustrative example in this regard is provided by a PPC for roadway resurfacing, rehabilitation and restoration: if these activities are undertaken in heavily urbanized areas, they may cause extreme traffic congestion and severe inconvenience to the travelling public and the business community. Thus, delays in the completion of these works prolong the negative impact on users (i.e. a social cost), and also cause overruns in the planned execution costs.<sup>4</sup>

There is evidence that delays in delivery dates have been particularly large and harmful in the recent Italian experience of PPCs. The data-base compiled by the Italian Authority in charge of controlling PPCs (*Autorità per la Vigilanza sui Contratti Pubblici di Lavori, Servizi e Forniture - AVLP*) records all contracts of a value between 150,000 and 15,000,000 euros awarded by munic-

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<sup>1</sup>Note that between 1995 and 2002 PPCs in the EU underwent a 31% increase in value (Dimitri, *et al.*, 2006: Ch. 1). See also: [http://europa.eu.int/comm/internal\\_market/publicprocurement/index\\_en.htm](http://europa.eu.int/comm/internal_market/publicprocurement/index_en.htm)

<sup>2</sup>The economic and engineering literatures give different explanations for the main issues arising in PPCs. Most of the economic analysis on this topic focusses on the information asymmetry concerning production costs between the supplier and the procurer (Laffont and Tirole, 1993), while engineering and construction management analysis concentrates on the uncertainty which affects the contract after it has been signed and its effects on both the supplier and the procurer (Bartholomew, 1998). For an economic methodological discussion on contract incompleteness and unforeseen contingencies see Maskin and Tirole (1999).

<sup>3</sup>In the economic literature on PPCs, delivery delays in contract execution are often considered along with the issue of the supplier’s performance regarding contracted aims (i.e. quality). See on this: Engel *et al.* (2006b).

<sup>4</sup>Cost overruns in different procurement contracts have been investigated in the seminal paper by Bajari and Tadelis (2001): they showed that cost plus contracts are better than fixed price contracts when the project carried out through the procurement is more complex. Focusing on fixed price contracts, Ganuza (2007) found that when the procurement market is more competitive, cost overruns are lower and decreasing with the design specification level.

ipalities, local/regional public authorities and public firms. Our examination of this data-base highlighted that out of 43,863 fully exploited contracts in the period 2000-2006, about 32,731 had been completed with delays. For about 27,826 contracts, the delay contracted days ratio<sup>5</sup> was always larger than 1, and it was higher, the greater the contract value (with a small slowdown in the trend for the more expensive contracts, see Table 1 below).<sup>6</sup>

		N° of contracts	Delay contracted days ratio
<b>By range value (euro)</b>			
	≥150,000 € < 500,000 €	23,297	1.12
	≥ 500,000 € < 1,000,000 €	545	1.30
	≥ 1,000,000 € < 5,000,000 €	3,678	1.51
	≥ 5,000,000 € < 15,000,000 €	241	1.89
	≥ 15,000,000 €	65	1.82
<b>By awarding procedure</b>			
	"Open Procedure"	21,019	1.18
	"Negotiated Procedure"	10,210	1.27

Table 1: Delays on Italian PPCs, 2000-2006

Similar findings on the substantial body of evidence on delays in the Italian PPCs have been recorded by other empirical analyses<sup>7</sup> and call - primarily - for investigation into the effectiveness of the penalty rules currently adopted in such contracts. According to Italian law,<sup>8</sup> the penalty fee in PPCs should be defined as a percentage - ranging between 0.03% and 0.1 % - of the total contract price for each day of delay in completion of the contracted works.<sup>9</sup>

Is there something wrong with the definition of these ranges? Answering this question should inform the decision on whether to insert a penalty rule in

<sup>5</sup>This ratio has the following interpretation: for example, if 200 days are the ex-ante contracted time for the infrastructure's execution and then its delivering occurs with 220 days of delay, the delay contracted days ratio results equal to 1.1.

<sup>6</sup>These estimations and further analysis on the data-set are available from the authors on request.

<sup>7</sup>See Bentivogli *et al.* (2007) and ANCE (2002). The former study reports interviews conducted with 32 local Contracting Authorities managing about 280 PPCs: only one third of these contracting authorities declared that the contract had been executed within the contracted time, or with very small delays. In the latter study, a detailed investigation conducted at regional level by ANCE Emilia Romagna highlights that of the 776 PPCs concluded in the period 2001-2002, more than 68% recorded delays and/or increases in cost execution. Similar results have been found at national level (see ANCE, 2005 and AVL, 2005).

<sup>8</sup>See Government Decree n° 163/2006 and D.P.R. 554/1999, art.117.

<sup>9</sup>This penalty rule for Italian PPCs is determined in similar fashion in other countries; in regard to the US, Herbsaman *et al.* (1995, Table 6, p. 276) show that for PPCs in highway construction, the contracting authority usually sets penalties ranging from 0.03% to 0.3% of the contract value for each day of delay.

the PPC and what its optimal design should be. Thus, the aim of this paper is twofold: first, to provide a theoretical and general framework in which to investigate how the inclusion of a penalty fee affects the contract value; second, to verify whether the penalty range currently adopted in the Italian legislation on PPCs correctly induces the contractor to avoid delays.

Our starting point is that the inclusion of a penalty clause in a PPC gives the contractor - to some extent - the option of deciding the investment timing for the contract's execution. Thus - in order to be effective - the penalty fee should consider the investment timing flexibility which, *de facto*, increases the supplier's contract value. To correctly approach the issue, we propose a simple Real Option model which allows us to ascertain the value of investment timing flexibility induced by the inclusion of the penalty clause in the contract.<sup>10</sup> Then, calibrating the model, we verify the range of penalty fees defined by the Italian legislation on PPCs: quite surprisingly, we find that this range indeed seems able to induce the contractor to respect the contractual execution time.

This result sheds new light on the determinants of investment delays in PPCs which include explicit penalty rules, and it adds a new aim to our study because it leads to other cause-and-effect explanations specifically related to the enforcement of the penalty clause itself. Following results generally acknowledged in the incomplete contract literature, failures by the Contracting Authority (henceforth, CA) to enforce the penalty clause can arise when: i) the contractor's work execution time is non-verifiable<sup>11</sup>; ii) default by the contractor triggers costly and time-consuming litigation because the "quality" of the judicial system is unsatisfactory; iii) the court of law<sup>12</sup> - to which the parties refer for settlement of the dispute on the penalty payment - reduces (or even does not enforce) the committed fee. The last two issues seem to play a major role in the Italian experience: indeed, Albano *et al.* (2007) found that, in the data-set of Italian PPCs that they investigated, only in less than 5% of verifiable delays was the penalty enforced by the CAs. Moreover, with specific regard to elements determining the low enforcement of penalties in Italy, it is a matter of fact that in the period 2004-2005 the average duration of an Italian civil trial was around 876 working days<sup>13</sup>: this, if anticipated by the contractor, may well override the incentive provided by the penalty design.

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<sup>10</sup>As Brennan and Schwartz (1985) and McDonald and Siegel (1985; 1986) highlighted in their seminal works, there is a close analogy between security options and investment timing flexibility.

<sup>11</sup>"To ensure the contract enforceability, the court must first be able to verify that an agent has disobeyed the agreed clauses of the contract" (Laffont and Martimort, 2002, p.348). To our knowledge, since the seminal paper by Manelli and Vincent (1995), the analysis of the non-verifiability of quality aspects in procurement (and concession) contracts has been carried out - with differing emphases - by Dalen *et al.* (2004), Calzolari and Spagnolo (2006) and Moretto and Valbonesi (2007).

<sup>12</sup>See Eggleston *et al.* (2000) for discussion of the role of courts in enforcing contract clauses.

<sup>13</sup>See the Corte Suprema di Cassazione (2007). Moreover, an old but impressive Report by the Commissione per la garanzia dell'informazione statistica of the Italian Government (2000, pp. 27-28) found that in the period between 1988 and 1997, the average time taken by the administrative (regional) courts to make a final ruling increased from 2617 to 4261 working days.

We thus characterize our model to investigate how the CA's ineffective enforcement of a penalty rule would affect the extent of the optimal penalty. Accordingly, we assume in the model that the probability of enforcement is correlated to a parameter identifying the "quality" of the judicial system (i.e. the average time taken to resolve disputes) and to the level of the penalty itself.<sup>14</sup>

Our calibration results show that the probability of penalty enforcement decreases both when i) the quality of the judicial system is low and ii) the civil court has high discretionality in reducing the penalty imposed. Specifically, the greater the level of the penalty that the CA would like to introduce in the contract, the lower the probability that it will be enforced by the court. This, in turn, calls for higher penalties - well beyond the range prescribed by the Italian legislation - to make the firm comply with the contracted execution time. However, if the degree of the court's responsiveness to the penalty imposed by the CA is high, the latter will find it convenient to set the fee considered *reasonable* by the court.

Finally, a caveat concerning our analysis should be mentioned. In principle, delays in the PPCs' completion time weight differently on the contract value according to the procedure adopted in awarding it. Indeed, in the "negotiated" procedure, the contract value is directly agreed between the parties and includes an explicit trade-off between the contract value and the investment's delivery time. Differently, in the "open" procedure, the investment's execution time can itself be part of the successful bid, thus representing a strategic variable in the competition among bidders.<sup>15</sup> Although our simple model refers in principle to PPCs awarded through the negotiated procedure because it assumes an exogenous and fixed price contracted between the CA and the contractor, it is interesting that the Italian delays contracted days ratio seem not to be correlated with the nature of the awarding procedure - see second part of Table 1 above. This suggests that our results could be reasonably extended to the Italian PPCs awarded through the open procedure as well.

The rest of the paper is organized as follows. Section 2 presents a PPC basic model where a penalty fee is included. Section 3 presents an extension of the model which considers a simple and symmetric penalty/premium scheme where the contractor is punished/rewarded if it decides to delay/anticipate the delivery date. Section 4 concludes by providing a brief summary of our findings and policy implications. Finally, the Appendix contains all the proofs and shows how our framework can be used to define the optimal penalty in a concession

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<sup>14</sup>The enforcement of a penalty clause may also be ineffective (i.e. costly) in the case of "multiple contacts" between the CA and the contractor. Indeed, when the latter has (or expects to have) other ongoing (or future) procurements with the same CA and perceives the committed fee as highly punitive sanction, it may take revenge on the other ongoing (or future) PPCs. Our model does not specifically address this case.

<sup>15</sup>Indeed, of the four EU procedures for procurement ("open procedure", "restricted procedure", "negotiated procedure", and "competitive dialogue"), only the "open procedure" and - partially - the "restricted procedure" make the trade-off between the contract's price and the delivery date explicit: the contractor makes a lower bid (i.e. asks for a lower price to execute the contract) if it can delay the construction time. For a survey on the current EU legislation on PPCs, see: <http://europa.eu/scadplus/leg/en/lvb/l22009.htm>

setting.

## 2 Optimal penalty fee for delays in delivery date

We consider the simple case where a CA awards a contract - a PPC - to an economic operator (i.e. a contractor firm) to build a public infrastructure with exogenous and ex-ante defined technical characteristics.<sup>16</sup> We assume that the up-front investment does not depreciate and that the contractor is selected according to a “negotiated” procedure whereby the CA first consults some economic operators of its choice and then agrees the terms of the contract only with one of them.

According to the PPC, the contractor commits itself to constructing the infrastructure immediately (i.e. at time  $t$ ) in return for a fixed payment  $p$ , which is agreed by both the parties. Furthermore, the contract includes the contractor’s liability for completion on time: i.e. if the contractor delays the contracted delivery date it will pay a constant penalty  $c$  for each period (usually for each day) of verifiable delay.

Under these assumptions, the net benefit for a risk-neutral contractor (i.e. the project’s NPV) that complies with the contract delivery time is simply given by:

$$F_t = p - C_t , \quad (1)$$

where  $C_t \leq p$  is the estimated cost of building the infrastructure at time  $t$ , when the contract is signed.

However, the introduction of the penalty clause gives - *de facto* - the contractor some flexibility in deciding its optimal time-to-completion. This investment timing flexibility has a value that should be added to the project’s NPV as expressed in (1). In particular if, for simplicity, we assume that the project’s cost  $C_t$  evolves according to a geometric Brownian motion where  $\alpha$  and  $\sigma$  are constants reflecting the drift and the volatility of the cost process respectively, we get:<sup>17</sup>

$$dC_t = \alpha C_t dt + \sigma C_t dz_t ,$$

and the contractor’s possibility of deferring the infrastructure’s completion date becomes analogous to a Perpetual Put Option whose value is equal to:

$$P_t \equiv \Phi_t - \pi \Lambda_t . \quad (2)$$

where  $\Phi_t \equiv E_t(e^{-r(\tau-t)})F_\tau$  and  $\Lambda_t \equiv E_t \left[ \int_t^\tau c e^{-r(s-t)} ds \right]$ . Specifically,  $\Phi_t$  is the expected and discounted net benefit from investing at a general cost  $C_\tau < C_t$ ,

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<sup>16</sup>This setting does not fit the issue of delay caused by an erroneous original project: to investigate this issue, one should add i) a preliminary stage where the CA evaluates the contractor’s proposal and ii) a further stage where the CA controls ex-post the infrastructure’s execution.

<sup>17</sup>In the following equation,  $dz_t$  is the increment of a standard Brownian process with mean zero and variance  $dt$  (Dixit, 1993; Dixit and Pindyck, 1994).

$\Lambda_t$  is the expected value of the penalty at time  $t$ ,  $\pi \in [0, 1]$  is the probability that a third party - i.e. a court of law - is able to enforce the penalty,  $r$  is the risk-adjusted expected rate of return that investors would require to own the project.<sup>18</sup> and  $\tau$  is the exercise time of the option. According to (2), since  $\Lambda_t = [1 - E_t(e^{-r(\tau-t)})] \frac{c}{r}$ , the ex-ante value of the procurement contract for the contractor turns out to be (Dixit and Pindyck, 1994):<sup>19</sup>

$$P_t = E_t(e^{-r(\tau-t)}) \left( F_\tau + \pi \frac{c}{r} \right) - \pi \frac{c}{r}. \quad (3)$$

Further, since  $F_t$  is also driven by a geometric Brownian motion, i.e.  $dF_t = \alpha(F_t - p)dt + \sigma(F_t - p)dz_t$ , the discount rate can be expressed as  $E_t(e^{-r(\tau-t)}) = \left( \frac{F_t - p}{F_\tau - p} \right)^\beta$ , where  $\beta < 0$  is the negative root of the quadratic equation  $\frac{1}{2}\sigma^2x(x - 1) + \alpha x - r = 0$ .<sup>20</sup> By substituting the expression for the discount rate into (3) we obtain the final expression for  $P_t$  as:

$$P_t = \left( \frac{F_t - p}{F_\tau - p} \right)^\beta \left( F_\tau + \pi \frac{c}{r} \right) - \pi \frac{c}{r}. \quad (4)$$

Equation (4) states that for any fixed  $p$ , whenever  $P_t > F_t$ , it will be profitable for the contractor to infringe the contract's provision on the investment's delivery date. In particular, the firm will be better off by maximizing (4) with respect to  $F_\tau$  and thus determine its optimal delay. The net benefit that will trigger the firm's investment is:<sup>21</sup>

$$F_\tau = \frac{1}{1 - \beta} \left( p + \beta \pi \frac{c}{r} \right). \quad (5)$$

Equation (5) yields the following investment rule: if  $F_\tau \leq F_t$ , it is optimal for the firm to invest immediately, while if  $F_\tau > F_t$ , it is optimal to wait until the net benefit is equal to  $F_\tau$ . Finally, if the CA wishes to incentivate the firm to

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<sup>18</sup>The discount rate  $r$  can either be adjusted for risk or the expectation for the discount factor can be taken with respect to a risk-adjusted probability measure with  $r$  as the risk-free discount rate (Cox and Ross, 1976; Harrison and Kreps, 1979).

<sup>19</sup>When it is established in the PPC that the CA can revoke the contract if the total penalty reaches an upper bound  $G$ , the previous Perpetual Put Option in (2) turns into an American Put Option, with maturity time  $T$ , as follows:

$$\int_0^T ce^{-rs} ds \equiv \frac{c}{r} (1 - e^{-rT}) = Gp$$

Modelling this option is more complicated than the previous (2) but the results do not substantially differ.

<sup>20</sup>See Dixit and Pindyck (1994), p. 315-316.

<sup>21</sup>The first order condition is:

$$\begin{aligned} \frac{\partial P}{\partial F_\tau} &= \beta \left( \frac{F_t - p}{F_\tau - p} \right)^{\beta-1} \left( -\frac{F_t - p}{(F_\tau - p)^2} \right) \left( F_\tau + \pi \frac{c}{r} \right) + \left( \frac{F_t - p}{F_\tau - p} \right)^\beta \\ &= \left( \frac{F_t - p}{F_\tau - p} \right)^\beta \left[ \beta \left( -\frac{1}{F_\tau - p} \right) \left( F_\tau + \pi \frac{c}{r} \right) + 1 \right] = 0 \end{aligned}$$



respect the contractual time, it must fix a penalty fee such that  $F_\tau = F_t$ . From (5), the optimal penalty fee is:

$$c^*(\pi) = \frac{r}{\pi} \left( \frac{\beta - 1}{\beta} C_t - p \right) \quad (6)$$

which, *ceteris paribus*, depends on  $\pi, \sigma$  (via  $\beta$ ) and  $C_t$ .

According to (6), if the CA expects to be endorsed with a low probability  $\pi$  of enforcing the penalty clause and/or a high current investment cost  $C_t$  (i.e. for decreasing mark-up  $p - C_t$ ), it should increase the value of the penalty fee to discourage the firm from delays. Yet, since  $d((\beta - 1)/\beta)/d\sigma > 0$ , by (6) the CA must set a higher penalty fee to induce the firm not to infringe the contractual delivery date.

We now investigate the probability of the penalty's enforcement more deeply: specifically, we assume that a low probability of penalty enforcement may arise from at least two different sources.

First, if the court of law - to which the parties refer in case of dispute - considers the penalty imposed to be "excessive", it may decide not to enforce it or to reduce it to an extent estimated as reasonably covering the damages caused by the contractor's breach.<sup>22</sup> In order to include this case in our model, we assume that the probability of enforcement by the court  $\pi$  depends on the value of penalty  $c$  with the properties that  $\pi'(c) < 0$ ,  $\pi(\underline{c}) = 1$  and  $\lim_{c \rightarrow \infty} \pi(c) = 0$ , where  $\underline{c} \geq 0$  represents the minimum value of a time unit (i.e. fee per day) considered *reasonable* by the court of law.<sup>23</sup>

A second element affecting the enforceability of the penalty clause is the "quality" of the judicial system. Following Guasch *et al.* (2003), we thus multiply the probability  $\pi(c)$  by a parameter  $\theta \in [0, 1]$  which refers to the average time that the court of law takes to resolve disputes.<sup>24</sup>

According to these assumptions, the optimal penalty design (6) is now given by the implicit function:

$$\pi(c^*)c^* - \frac{r}{\theta} \left( \frac{\beta - 1}{\beta} C_t - p \right) = 0, \quad \text{for } c^* \geq \underline{c}. \quad (7)$$

<sup>22</sup>In the literature on the firm's breach of the contract, this discretionality by the court of law is commonly referred to as the "liquidated damage principle". Delay in delivering the contracted investment should be referred to as a specific case of the firm's breach of the contract and the court can apply the above principle to cover the reasonable damage caused by delays to society. For a discussion of the application of the "liquidate damage principle" in PPCs, see Dimitri *et al.* (2004, Ch. 4, pp. 85-86); for an analysis of the economic incentives pertaining to it, see Anderlini *et al.* (2007).

<sup>23</sup>In the US experience of PPC in the highway construction industry, the "unit time value" is typically expressed as a cost per day. It is calculated by the State highway agency (the CA in our model) referring to the "daily road-user cost", which include items such as travel time, travel distance, fuel expense, etc.. See Herbsman *et al.* (1995) for an example of the "daily road-user cost" calculation used by the Kansas Department of Transportation.

<sup>24</sup>We are aware that the quality of justice is often discussed in the economic literature with reference to many other dimensions, such as accuracy and costs in pursuing legal actions. Considering only the timing dimension, we would stress here the relevance of the common saying "justice delayed is justice denied".

In order to illustrate the properties of (6) and (7), in what follows we provide some numerical solutions of both and discuss their applications with reference to the Italian case. For the sake of simplicity, the choice of parameters for the calibration has been made following indications from related studies as far as possible (Dixit and Pindyck, 1994; Herbsman *et al.*, 1995). The price of the contracted investment is normalized to one, i.e.  $p = 1$ , and the parameters of the model take the following values:  $r = 0.10$ ;<sup>25</sup>  $\alpha = -0.05$ ;<sup>26</sup>  $C_t = 0.7, 0.8, 0.9$  and  $\sigma = 0.3, 0.4, 0.5$ .

Let us first evaluate (6). As far as the probability  $\pi$  is concerned, we simply consider three cases: perfect enforceability, i.e.  $\pi = 1$ , and two cases of reduced enforceability with  $\pi = 0.5$ , and  $\pi = 0.25$  respectively.

Table 2 shows the optimal penalties expressed in day terms as a percentage of  $p$  for different levels of  $C_t$  and  $\pi = 1$ . In the case of perfect enforceability, the optimal penalty falls within the interval 0.04% to 0.08% : this range is consistent with the one suggested by the Italian legislation on penalty fees in PPCs (i.e., 0.03% – 0.1%). However, since  $c^*(\pi) = \frac{1}{\pi}c^*(\pi = 1)$ , this penalty range rapidly increases to 0.08%– 0.15 % and to 0.16%– 0.30 % when the probability  $\pi$  falls to 0.5 and 0.25 respectively.<sup>27</sup>

$\alpha=-0.05$		$c^*$		
		$r=10\%$		
		$\sigma=30\%$	$\sigma=40\%$	$\sigma=50\%$
Ct	0.7	0.04131	0.05011	0.06045
	0.8	0.04760	0.05766	0.06947
	0.9	0.05390	0.06521	0.07850

**Table 2:** Optimal penalty for different values of  $C_t$  and  $\sigma$ ,  $\pi = 1$ ,  $\alpha = -0.05$ , and  $r = 10\%$  expressed in % and in day terms.

Inspection of Table 2 shows that, according to the Real Option Theory, the higher the cost of the investment  $C_t$  and/or the uncertainty (i.e.  $\sigma$ ), the higher

<sup>25</sup>Although  $r$  should be the return that an investor can earn on other investments with comparable risk characteristics, throughout our analysis we simply refer it to the social rate of discount that the Italian government suggests should be used to evaluate most public projects. For Italy this ranges between 8% and 12%, with the possibility of dropping to 5% for projects undertaken in the southern regions (see: Pennisi and Scandizzo, 2003).

<sup>26</sup>We assume  $\alpha < 0$  in order to stress the value of waiting to invest. The results do not differ qualitatively if  $\alpha \geq 0$  as long as  $\alpha < r$ .

<sup>27</sup>We have performed other simulations which show that this result is robust on changing the value of some parameters, as for  $r = 8\%$ , 15%, and  $\alpha = -0.1$ .

becomes the optimal penalty  $c^*$ . In other words, both the investment cost  $C_t$  and the uncertainty  $\sigma$  incentivate the firm to defer the contracted investment. This, in turn, calls for higher penalties to make the firm comply with the contracted execution time.

For example, for  $C_t = 0.7$ , when  $\sigma$  increases from 30% to 50%, the penalty  $c^*$  increases by about 42%. This increment is larger as  $C_t$  increases, but at a decreasing rate: when  $\sigma$  increases from 30% to 50%,  $c^*$  is subject to an increase of about 46% for  $C_t = 0.8$  and of about 45% for  $C_t = 0.9$ . Finally, when  $\sigma$  increases from 30% to 50% and  $C_t$  rises from 0.7 to 0.9, the total effect on the optimal  $c^*$  is roughly a 90% increase.

Let us now calculate (7). For the sake of simplicity, we assume  $\pi(c) = (\underline{c}/c)^\eta$  for  $c \geq \underline{c}$ , and the elasticity  $\eta$  of the probability  $\pi$  as taking two different values,  $\eta = 0.3$  and  $\eta = 0.7$ , respectively. In other words, when the CA sets a penalty higher than the  $\underline{c}$  - i.e. the lower value considered *reasonable* by the court - an increase in the elasticity  $\eta$  determines a rapid decrease in the probability  $\pi$  that the court will enforce the penalty. If the elasticity is less than one, so that higher values of  $\underline{c}$  are deemed excessive by the court, increasing values of both  $\sigma$  and  $C_t$  lead to higher optimal penalties. In the calibration,  $\underline{c}$  takes the values of 0.03% and 0.1%, which are respectively the lower and upper bound of the penalty fee in PPC as set by the Italian legislation.

Finally, interpreting  $\theta$  as the probability that a court of law is able to resolve a dispute in a year, in order to gauge the effect of the “quality” of the judicial system we set  $1/\theta = 3$  so as to refer to average number of years the Italian courts take to resolve legal disputes.<sup>28</sup>

All the above assumptions allow us to rewrite equation (7) as follows:

$$c^* = \max \left\{ \underline{c} , \frac{\left[ 3r \left( \frac{\beta-1}{\beta} C_t - p \right) \right]^{1/1-\eta}}{(\underline{c})^{\eta/1-\eta}} \right\}. \quad (8)$$

Tables 3 and 4 below show the optimal penalties obtained by simulations of (8) for  $\underline{c} = 0.03\%$  and  $\eta = 0.3$ , and  $\eta = 0.7$ . In both the Tables, one observes that the higher the values of  $C_t$  and  $\sigma$ , the higher is the optimal penalty  $c^*$ . Moreover, one notes that the optimal penalties are highly sensitive to the value of  $\underline{c}$ . Specifically, when  $\underline{c} = 0.03\%$ , for both  $\eta = 0.3$  and  $\eta = 0.7$ , the optimal penalty  $c^*$  always exceeds the range 0.03% – 0.1%.

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<sup>28</sup>We refer here to the average duration of a civil trial in Italy, because the civil court of law is the forum authorized to deal with these disputes. Note that the average duration of a civil trial adopted in the calibration is consistent with the period to which our data-set on the Italian PPCs refers.

$\alpha=-0.05$		$c^*$		
		$r=10\%$		
		$\sigma=30\%$	$\sigma=40\%$	$\sigma=50\%$
$C_t$	0.7	0.13069	0.17219	0.22511
	0.8	0.16003	0.21042	0.27462
	0.9	0.19109	0.25086	0.32698

**Table 3:** Optimal penalty for different values of  $C_t$  and  $\sigma$ ,  $\underline{c} = 0.03\%$ ,  
 $\theta = 1/3$ ,  $\alpha = -0.05$ ,  $r = 10\%$ ,  $\eta = 0.3$   
expressed in % and in day terms.

$\alpha=-0.05$		$c^*$		
		$r=10\%$		
		$\sigma=30\%$	$\sigma=40\%$	$\sigma=50\%$
$C_t$	0.7	0.16545	0.31488	0.58841
	0.8	0.26542	0.50271	0.93577
	0.9	0.40149	0.75761	1.40607

**Table 4:** Optimal penalty for different values of  $C_t$  and  $\sigma$ ,  $\underline{c} = 0.03\%$ ,  
 $\theta = 1/3$ ,  $\alpha = -0.05$ ,  $r = 10\%$ ,  $\eta = 0.7$   
expressed in % and in day terms.

By contrast, when the value of the penalty considered *reasonable* by the court of law is  $\underline{c} = 0.1\%$ , if the elasticity of the probability is  $\eta = 0.3$ , the optimal penalty  $c^*$  is higher than 0.1% only for a high value of  $C_t$  and/or  $\sigma$ . In all the other cases, the CA will find it convenient to set the fee proposed by the court (Table 5).

$\alpha=-0.05$		$c^*$		
		$r=10\%$		
		$\sigma=30\%$	$\sigma=40\%$	$\sigma=50\%$
$C_t$	0.7	0.1	0.10278	0.13437
	0.8	0.1	0.12560	0.16393
	0.9	0.11406	0.14975	0.19518

**Table 5:** Optimal penalty for different values of  $C_t$  and  $\sigma$ ,  $\underline{c} = 0.1\%$ ,  $\theta = 1/3$ ,  $\alpha = -0.05$ ,  $r = 10\%$ ,  $\eta = 0.3$  expressed in % and in day terms.

Finally, when  $\underline{c} = 0.1\%$  and  $\eta = 0.7$ , the optimal penalty is always  $c^* = \underline{c} = 0.1\%$ , whatever the value of  $C_t$  and  $\sigma$ . In other words, when both  $\underline{c}$  and the elasticity  $\eta$  are high, the probability that the court of law will enforce a penalty greater than  $0.1\%$  decreases dramatically to zero.

### 3 A penalty/premium scheme

In the previous section we investigated how a PPC should comprise a penalty clause for delay designed to optimally induce the contractor to invest at the contracted time  $t$ . Many PPCs, however, commit the contractor to invest at a future date  $t' > t$  and also include an incentive/disincentive (I/D) clause stating that, on the one hand, if the contractor is able to complete the project ahead of scheduled  $t'$  it will be entitled to premium fee  $I$  whilst, on the other hand, if the contractor delays completion, a penalty  $D$  will be imposed.

Although the CA may introduce different and alternative I/D designs, we consider here the simplest one where the firm receives a constant premium/penalty fee  $c$  for each period (day, month, year, etc.) with which it anticipates/delays the investment with respect to  $t' > t$ .<sup>29</sup> In other words, the present section investigates how this I/D rule - where the premium and the penalty are identically defined in their amounts, but with opposite signs - should be optimally designed.

Following the approach presented in the previous section, the current NPV, say  $N$ , of the project for the contractor complying with the contractual delivery

<sup>29</sup>Herbsman *et al.*, 1995 underline that in the real world when CAs adopt the I/D rule, the same value for both the incentive and the disincentive fee is generally used.

time now becomes:<sup>30</sup>

$$\begin{aligned} N(F_t, t') &\equiv N_t = e^{-r(t'-t)}p - e^{-\delta(t'-t)}C_t \\ &= e^{-\delta(t'-t)}F_t + \left[ e^{-r(t'-t)} - e^{-\delta(t'-t)} \right] p \end{aligned} \quad (9)$$

where  $\delta = r - \alpha$ .<sup>31</sup>

As in the previous section, the inclusion in the procurement contract of a I/D rule makes the contractor's investment decision equivalent to exercising a Perpetual Put Option whose value is now given by  $P_t \equiv \Phi_t - \pi\Lambda_t$ , where  $\Phi_t$  and  $\pi$  are as in (2) and  $\Lambda_t$  is now equal to (see Appendix):

$$\begin{aligned} \Lambda_t &= \mathcal{E}_t \left[ \int_t^{\min(\tau, t')} 0e^{-r(s-t)} ds + \int_{\min(\tau, t')}^{t'} ce^{-r(s-t)} ds + \right. \\ &\quad \left. - \int_{t'}^{\max(\tau, t')} ce^{-r(s-t)} ds \right] \\ &= \left[ E_t(e^{-r(\tau-t)}) - e^{-r(t'-t)} \right] \frac{c}{r}. \end{aligned} \quad (10)$$

In (10), the expected value  $\mathcal{E}_t$  is calculated with respect to both  $\tau$  and the probability that  $\tau$  is lower (greater) than  $t'$ . According to (2) and (10) the PPC's ex-ante value where an I/D rule is included is now:

$$P_t = \left( \frac{F_t - p}{F_\tau - p} \right)^\beta \left( F_\tau + \pi \frac{c}{r} \right) - \pi \frac{c}{r} e^{-r(t'-t)} \quad (11)$$

which should be maximized with respect to  $F_\tau$ .

From (11), if the contractual time is very long, i.e.  $t' \rightarrow \infty$ , the second term on the l.h.s. disappears and the contractor will get the premium by investing before  $t'$  with probability one. If, conversely, the contractual time  $t'$  is very short, i.e.  $t' \rightarrow t$ , the second term on the l.h.s. of (11) reduces to  $\frac{c}{r}$  as in (4). The contractor will then incur a penalty since, with probability one, it invests when the contractual time is over. Finally, because the term  $\frac{c}{r} e^{-r(t'-t)}$  enters (11) as a constant, the optimal investment trigger  $F_\tau$  is still given by (5) as well as the firm's investment decision rule. That is, the contractor defers the infrastructure delivery date until  $F_t$  reaches the trigger  $F_\tau$  from below for the

<sup>30</sup>Note that if  $t'$  is set by the CA to allow the contractor to maximize the NPV (9), depending on the parameter values,  $t'$  is greater than  $t$  only if  $r < \delta$ . In particular, maximizing (9) we obtain:

$$t' = \max \left[ \frac{1}{r - \delta} \log \left( \frac{\delta C_t}{r p} \right), 0 \right] + t$$

which is an increasing function of the current investment cost  $C_t$  and is always greater than  $t$  if  $\frac{\delta}{r} < \frac{p}{C_t}$ . In the case of  $r = \delta$ , we get  $N_t = e^{-r(t'-t)}F_t$ . Since  $F_t > 0$  it is optimal to invest immediately, i.e.  $t' = t$ . If  $r > \delta$  the solution of the first order condition represents a minimum as  $\frac{\partial^2 N_t}{\partial (t')^2} > 0$  and, then, the optimal value is found on one of the boundaries, i.e. it is given by  $\max [F_t, \lim_{t' \rightarrow \infty} N_t]$ . However, since  $\lim_{t' \rightarrow \infty} N_t = 0$  it is still optimal to invest immediately.

<sup>31</sup>The term  $r - \delta$  can be interpreted as the certainty-equivalent rate of return (see Mc Donald and Siegel, 1984; Dixit and Pindyck, 1994).

first time. In this respect, if the exercise time  $\tau$  is lower than  $t'$ , the contractor gains a premium, otherwise it must pay a fee.

As before, whenever  $P_t > N_t$  it will be profitable for the contractor to infringe the contractual time  $t'$ . Thus, the difference  $P_t - N_t$  represents the contractor's opportunity cost in delivering the investment according to the contracted date  $t'$  instead of taking advantage of the investment timing flexibility which pertains to the I/D clause.

We complete the analysis by calculating the optimal I/D fee which induces the contractor to respect the completion date  $t'$ . In this regard, note that since the exercise time  $\tau$  is stochastic and  $c$  is constant (i.e.  $c$  is not contingent on  $\tau$ ), the CA must set a policy-rule referring to the probability distribution of  $\tau$ . For the sake of simplicity, we follow the simple average-time rule<sup>32</sup>:

$$E(\tau) = t' \quad (12)$$

In this case, as we show in the Appendix, the mean time that  $F_t$ , with starting point  $F_t > F_\tau$ , takes to hit the upper trigger  $F_\tau$  for the first time is given by:

$$E(\tau) = m^{-1} \log \left( \frac{C_t}{C_\tau} \right) + t, \quad (13)$$

with  $m \equiv (\frac{1}{2}\sigma^2 - (r - \delta)) > 0$  and  $C_\tau = p - F_\tau$ .<sup>33</sup> According to (13) and (12), the optimal penalty is then equal to:

$$c^*(\pi) = \frac{r}{\pi} \left( \frac{\beta - 1}{\beta} C_t e^{-m(t'-t)} - p \right) \quad (14)$$

Since when  $t' = t$  and  $t' > t$ , (14) is respectively equal to and smaller than (6), the results obtained in the previous section can be replicated for the I/D scheme as well.

It is worth noting that as  $(t' - t)$  increases, the optimal I/D fee diminishes. That is, when the contractual time  $t'$  is a long way from the current time  $t$ , the incentive for the contractor to delay the investment decreases and the CA is able to minimize premia outpayment by undercutting the I/D fee. When the interval  $t' - t$  is very long, we have the paradox that the contractor must somehow be incentivated, by means of a premium, to respect the contracted delivery date for the investment.

## 4 Final remarks and policy implications

Public procurement contracts account for a sheer volume of economic activity in many countries, and the significant evidence of harmful delays in the delivery

<sup>32</sup>Depending on different assumptions about the CA's risk aversion, this rule can be made more stringent by giving greater weights to different moments in the delivery timing distribution.

<sup>33</sup>Obviously  $m$  should be positive; otherwise  $E(\tau) = \infty$  (see Cox and Miller, 1965, p. 221-222).

of the contracted investment is a very important issue for investigation. Various empirical analyses of these delays in Italian PPCs seem to demonstrate that the penalty rules included in these contracts are generally ineffective.

The main goal of this study has thus been to explain investment delays when a penalty clause is included in the PPC. To this end, we set out a Real Option model enabling us to correctly define the PPC's value when a penalty clause is incorporated. Our starting point has been that when such a penalty fee is present, the contractor has - to some extent - the option of deciding its optimal investment timing, so that its contract value becomes higher. Taking this investment timing flexibility into account, we have modelled the investment decision as a Perpetual Put Option that the contractor can exercise. Then, in order to evaluate the properties of this setting and define the optimal penalty range (i.e. the range which induces the contractor to comply with the contracted delivery time), we have performed some calibrations of the model specifically referring to PPCs in Italy. To our knowledge this is the first analysis on investment delivery delays in PPC which adopts a Real Option approach. Our findings - quite surprisingly - show that in the case of perfect enforceability of the penalty, the optimal fee falls within an interval which is consistent with the one suggested by the Italian legislation on PPCs'.<sup>34</sup> These results thus suggest that the large delays in the Italian PPCs are due not to incorrectly set penalty fees, but - as highlighted by the PPC data-set investigated by Albano *et al.* (2007) - to the low enforceability of the penalty rule itself. We then extended our model to include this issue. Specifically, in case of verifiable delays, a low probability of enforcing the committed penalty may arise from at least two different sources: a) when the court of law - to which the parties refer in the case of litigation - has discretionality in reducing (or even not enforcing) the penalty imposed; b) when the "quality" of the judicial system is unsatisfactory. Indeed, as to the former source, if the court of law deems the penalty excessive, it may decide to reduce it to an extent considered as reasonably covering the damages caused by the contractor's breach. As to the latter source, we have specifically defined the quality of the judicial system by referring to the average time taken to resolve disputes between the parties. Both these elements - if anticipated by the contractor - can play a crucial role in overriding the incentive provided by the penalty, and this seems to be the case in the Italian PPCs.

Thus, while our analysis is motivated by the Italian experience in PPCs, it offers the following suggestions in regard to designing the optimal penalty for delay in procurement contracts:

i) since the volatility of the project costs incentivates the firm to defer the contracted investment time, this, in turn, again calls for higher penalties to reduce delays;

ii) further, low quality of the judicial system coupled with a court of law with high discretionality in reducing the penalty calls for higher penalties to induce the contractor to comply with the contracted execution time.

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<sup>34</sup>Our calibrations also show that the optimal penalty should be higher, the larger the investment cost and the uncertainty; and that the penalty grows in amount, but at a decreasing rate, as the investment cost increases.



These suggestions also open the way to considerations on designing a mechanism alternative to the penalty fee for preventing delays in the PPC investment delivery. This has not been investigated in this paper but is part of our research agenda: indeed, if the CA's contractual payments to the contractor can be made contingent on the contract's execution timing, this would by itself induce the contractor to avoid delays. Specifically, if the contingent payments to the contractor are equal to its opportunity benefits in deferring the investment, there is no reason to resort to a penalty rule in the contract, i.e. the more complete the *ex-ante* contract design, the less likely it becomes that the contractor will delay delivery.<sup>35</sup>

We have then extended the model to an incentive/disincentive rule (I/D), that is, a framework where the contractor is respectively rewarded/fined if it anticipates/delays the investment's execution. Within a very simple symmetric I/D design (i.e the contractor receives a constant premium/penalty fee for each period that it anticipates/delays the contracted timing of delivery), we found that the CA has the incentive to reduce the I/D fee to minimize expected premia outpayment to the contractor.

Finally, we show in the Appendix how our model can be applied to define the optimal penalty for delays in concession contracts, where, differently from procurements, once the contractor has made the contracted investment, it recovers its cost by managing the service, i.e. with the revenues from users' tariffs.

## A Appendix

### A.1 Proof of (11)

Consider (10). After some calculations and arrangements we obtain:

$$\begin{aligned}
\Lambda_t &= E_t \left[ \int_t^{\min(\tau, t')} 0e^{-r(s-t)} ds + \int_{\min(\tau, t')}^{t'} ce^{-r(s-t)} ds - \int_{t'}^{\max(\tau, t')} ce^{-r(s-t)} ds \right] \quad (15) \\
&= E_t \left[ c \left( -\frac{1}{r} e^{-r(t'-t)} + \frac{1}{r} e^{-r(\min(\tau, t')-t)} \right) - c \left( -\frac{1}{r} e^{-r(\max(\tau, t')-t)} + \frac{1}{r} e^{-r(t'-t)} \right) \right] \\
&= \frac{c}{r} E_t \left[ -e^{-r(t'-t)} + e^{-r(\min(\tau, t')-t)} + e^{-r(\max(\tau, t')-t)} - e^{-r(t'-t)} \right] \\
&= \frac{c}{r} E_t \left[ e^{-r(\min(\tau, t')-t)} + e^{-r(\max(\tau, t')-t)} \right] - 2\frac{c}{r} e^{-r(t'-t)}
\end{aligned}$$

where the optimal exercise time  $\tau$  is defined as

$$\tau = \min(t \geq 0 \mid F_\tau = \arg \max P_t) \quad (16)$$

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<sup>35</sup>However, as highlighted by Bajari and Tadelis in their seminal contribution 2001, "a more complete *ex-ante* design... imposes higher *ex-ante* cost on the buyer" (i.e. on the CA). Thus, our results highlight a new trade-off which weights - on the one hand - the *ex-ante* cost for the CA in designing a more complete contract and - on the other - the probability of the penalty's enforcement.

According to (16), at time  $t$ , the probability of having a bonus is the probability of having an optimal exercise time  $\tau$  less than (or equal to) the contractual time  $t'$ . In other words, this is the probability of the geometric Brownian motion  $F_t$  reaching the critical value  $F_\tau^*$  within  $[t, t']$  starting from an initial condition  $F_t < F_\tau^*$ . This can be expressed as (Harrison, 1985)

$$\Pr(\tau \leq t') = N(s_1) + \left(\frac{F_\tau^*}{F_t}\right)^{2(r-\delta)/\sigma^2-1} N(s_2) \quad (17)$$

where:

$$\begin{aligned} s_1(F_t, F_\tau^*) &= \frac{\ln(F_t/F_\tau^*) + (r - \delta - \sigma^2/2)(t' - t)}{\sigma\sqrt{t' - t}} \\ s_2(F_t, F_\tau^*) &= s_1 - \left(\frac{2(r - \delta)}{\sigma^2} - 1\right) \sigma\sqrt{(t' - t)}. \end{aligned}$$

By (17), we rewrite (15) as:

$$\begin{aligned} \Lambda_t &= \frac{c}{r} E_t \left[ \Pr(\tau \leq t') \left( e^{-r(\tau-t)} + e^{-r(t'-t)} \right) + (1 - \Pr(\tau \leq t')) \left( e^{-r(t'-t)} + e^{-r(\tau-t)} \right) \right] + \\ &\quad - 2\frac{c}{r} e^{-r(t'-t)} \\ &= \frac{c}{r} E_t \left[ e^{-r(t'-t)} + e^{-r(\tau-t)} \right] - 2\frac{c}{r} e^{-r(t'-t)} \\ &= \frac{c}{r} E_t \left[ e^{-r(\tau-t)} \right] - \frac{c}{r} e^{-r(t'-t)} \end{aligned}$$

## A.2 Proof of (13)

Consider the process  $C_t$  on an interval  $0 < a < C_t < b < \infty$ , with left boundary  $a$  and right boundary  $b$ . Defining  $t_{a,b}$  as the stochastic variable that describes the time it takes  $C_t$  to hit for the first time either  $a$  or  $b$ , we are able to evaluate the first moment (Saphores, 2002):

$$E(t_{a,b}) = \frac{2}{\kappa^2 \sigma^2} \left\{ \frac{C_t^\kappa - a^\kappa}{b^\kappa - a^\kappa} \left[ \left(\frac{b}{C_t}\right)^\kappa - 1 - \kappa \log\left(\frac{b}{C_t}\right) \right] + \frac{b^\kappa - C_t^\kappa}{b^\kappa - a^\kappa} \left[ \kappa \log\left(\frac{C_t}{a}\right) + \left(\frac{a}{C_t}\right)^\kappa - 1 \right] \right\}$$

where  $\kappa = 1 - \frac{2(r-\delta)}{\sigma^2}$ . Since  $\kappa > 0$ , letting  $b \rightarrow \infty$  and  $a \rightarrow C^* < C_t$  we obtain the expected time that the construction cost will take to reach the lower boundary  $C^*$  starting from  $C_t$ .

$$\begin{aligned} &\lim_{a \rightarrow C^*, b \rightarrow \infty} E(t_{a,b}) = E(t_{C^*}) = \\ &= \frac{2}{\kappa^2 \sigma^2} \left\{ \frac{C_t^\kappa - a^\kappa}{b^\kappa - a^\kappa} \left[ \left(\frac{b}{C_t}\right)^\kappa - 1 - \kappa \log\left(\frac{b}{C_t}\right) \right] + \frac{b^\kappa - C_t^\kappa}{b^\kappa - a^\kappa} \left[ \kappa \log\left(\frac{C_t}{a}\right) + \left(\frac{a}{C_t}\right)^\kappa - 1 \right] \right\} \\ &= \frac{2}{\kappa \sigma^2} \log\left(\frac{C_t}{C^*}\right) \end{aligned}$$

To prove this limit, let us consider the first and second term separately:

$$\begin{aligned}
& \lim_{b \rightarrow \infty} \frac{b^\kappa - C_t^\kappa}{b^\kappa - a^\kappa} \left[ \kappa \log \left( \frac{C_t}{a} \right) + \left( \frac{a}{C_t} \right)^\kappa - 1 \right] = \kappa \log \left( \frac{C_t}{a} \right) + \left( \frac{a}{C_t} \right)^\kappa - 1 \\
& \lim_{b \rightarrow \infty} \frac{C_t^\kappa - a^\kappa}{b^\kappa - a^\kappa} \left[ \left( \frac{b}{C_t} \right)^\kappa - 1 - \kappa \log \left( \frac{b}{C_t} \right) \right] \\
& = \lim_{b \rightarrow \infty} \frac{C_t^\kappa - a^\kappa}{b^\kappa - a^\kappa} \left( \frac{b}{C_t} \right)^\kappa - \lim_{b \rightarrow \infty} \frac{C_t^\kappa - a^\kappa}{b^\kappa - a^\kappa} - \lim_{b \rightarrow \infty} \frac{C_t^\kappa - a^\kappa}{b^\kappa - a^\kappa} \log \left( \frac{b}{C_t} \right)^\kappa \\
& = \lim_{b \rightarrow \infty} \frac{b^\kappa}{b^\kappa - a^\kappa} \frac{C_t^\kappa - a^\kappa}{C_t^\kappa} - 0 - 0 = \frac{C_t^\kappa - a^\kappa}{C_t^\kappa} = 1 - \left( \frac{a}{C_t} \right)^\kappa
\end{aligned}$$

Putting the two limits *together*, we get:

$$\frac{2}{\kappa^2 \sigma^2} \left\{ 1 - \left( \frac{a}{C_t} \right)^\kappa + \kappa \log \left( \frac{C_t}{a} \right) + \left( \frac{a}{C_t} \right)^\kappa - 1 \right\} = \frac{2}{\kappa^2 \sigma^2} \left\{ \kappa \log \left( \frac{C_t}{a} \right) \right\}$$

### A.3 A concession contract

In this Appendix we show how the model presented in Section 2 can be fitted into the framework of a concession contract. Concession contracts mainly differ from procurement contracts in the level of responsibilities transferred to the contractor: in particular, through a concession contract a public authority entrusts a third party (i.e. the concessionaire) with the total or partial management of an economic activity for which the concessionaire assumes the operating risk.<sup>36</sup> As an example, suppose that a firm has signed a contract for the provision of a service to be started immediately (i.e. at time  $t$ ) and which requires, on the part of the firm, a completely irreversible capital outlay  $K$  (e.g. building an infrastructure). The contract also includes a clause requiring the concessionaire to pay a penalty if the provision of the service is delayed. Specifically, the concessionaire pays a constant fee  $d$  for each period (e.g. day, month, year, etc.) of delay in its starting of the service.

Under these assumptions, the value of the concession for a concessionaire that complies with the contracted delivery time is:

$$G_t = V_t - K \tag{18}$$

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<sup>36</sup>The European legislation envisages two main types of concession contract: works concessions and service concessions. Directive 93/37/EC distinguishes a work concession from a public works contract by the fact that the concessionaire is granted the right to exploit a construction as a consideration for having erected it. Directive 2004/18/EC defines service concessions as contracts of the same type as public service contracts except for the fact that the consideration for the provision of services consists in the right to exploit the service. A service concession exists when the concessionaire bears the risks involved in establishing and exploiting the service and obtains revenues from users by charging fees or tariffs (<http://europa.eu/scadplus/leg/en/lvb/122011.htm>).

where  $V_t \geq K$  is the project's value (i.e. the future discounted cash flow generated by the project and the provision of the service).

As before, if we assume that  $V_t$  is driven by the process  $dV_t = \nu V_t dt + \lambda V_t dz_t$  with  $\lambda > 0$  and  $\nu \geq 0$ , the inclusion of the penalty clause gives the concessionaire the option of deferring the investment. The value of this option is analogous to a perpetual American Call option:

$$B_t = \Psi_t - \pi \Delta_t = E_t(e^{-r(\tau-t)})G_\tau - \pi E_t \left[ \int_t^\tau de^{-r(s-t)} ds \right] \quad (19)$$

where  $\Psi_t$  is the expected and discounted profit deriving from exercising the option to invest when the net benefit has increased to  $G_\tau > G_t$  instead of doing it now and obtaining  $G_t$ ,  $\Delta_t$  is the expected value of the penalty,  $r$  is the risk-free interest rate and  $\tau$  is the exercise time. Since  $E_t \left[ \int_t^\tau de^{-r(s-t)} ds \right] = [1 - E_t(e^{-r(\tau-t)})] \frac{d}{r}$  and  $E_t(e^{-r(\tau-t)}) = \left( \frac{G_t + K}{G_\tau + K} \right)^\gamma$ , according to (19), the ex-ante concession value for the firm that takes the opportunity to defer starting the service is:

$$B_t = \left( \frac{G_t + K}{G_\tau + K} \right)^\gamma \left( G_\tau + \pi \frac{d}{r} \right) - \pi \frac{d}{r}$$

where  $\gamma > 1$  is the positive root of the quadratic function  $\frac{1}{2}\lambda^2 x(x-1) + \nu x - r = 0$ . By maximizing  $B_t$  with respect to  $G_\tau$  we obtain  $G_\tau = \frac{1}{\gamma-1}(K - \gamma\pi\frac{d}{r})$  and imposing  $G_\tau = G_t$ , we get the optimal penalty:

$$d^* = \frac{r}{\pi} \left( K - \frac{\gamma-1}{\gamma} V_t \right) \quad (20)$$

## References

- [1] Albano, G.L., F. Dini and G. Spagnolo (2007): "Strumenti a sostegno della qualità negli acquisti pubblici", *Quaderni Consip*, forthcoming.
- [2] Anderlini, L. , L. Felli, and A. Postlewaite, (2007): "Courts of Law and Unforeseen Contingencies," *Journal of Law, Economics and Organization*, 23: 662-684.
- [3] AVLP, Autorità per la Vigilanza sui lavori pubblici (2005), *Relazione al Parlamento Anno 2004*, Volume I, Roma.
- [4] ANCE Emilia Romagna (2002): "Rapporto sui lavori pubblici in Emilia Romagna 2000-2001. Bologna.
- [5] ANCE (2005): "Rapporto sulle infrastrutture in Italia", mimeo presented at ANCE Conference in Turin, March, 11, 2006.

- [6] Bajari, P., and S. Tadelis, (2001), "Incentives versus Transaction Costs: A Theory of Procurement Contracts", *The Rand Journal of Economics*, 32/3: 387-407.
- [7] Bartholomew, S. H. (1998), *Construction contracting: business and legal principles*, Prentice Hall, Inc.
- [8] Bentivogli C., Casadio P., Cullino R. and C. Fabrizi (2007): "Concorrenza e trasparenza nel mercato delle opere pubbliche locali: un'indagine empirica", *Economia e Politica Industriale*, 3:65-98.
- [9] Brennan, M.J., and E.S. Schwartz, (1985), "Evaluating Natural Resource Investments", *The Journal of Business*, 58/2: 137-157.
- [10] Calzolari, G., and G. Spagnolo, (2006): "Reputation and Collusion in Procurement", University of Bologna, mimeo.
- [11] Commissione per la Garanzia dell'informazione statistica, (2000), "Indicatori per l'analisi dell'attività della giustizia e, in particolare, della giustizia amministrativa", Rapporto di Ricerca per la Presidenza del Consiglio dei Ministri.
- [12] Corte Suprema di Cassazione (2007): "Relazione sulla amministrazione della giustizia nell'anno 2006".
- [13] Cox, J.C., and S.A. Ross, (1976), "The Valuation of Options for Alternative Stochastic Processes", *Journal of Financial Economics*, 3: 145-166.
- [14] Cox, D.R., and H.D. Miller, (1965) *The Theory of Stochastic Process*, London, Chapman and Hall.
- [15] Dalen M.G., E.R. Moen, and C. Riis, (2004), "Contract renewal and incentives in public procurement", Working Paper CEPR, 4540.
- [16] Dimitri, N., G. Piga, and G. Spagnolo (2006), *Handbook of Procurement*, Cambridge University Press, Cambridge, MA.
- [17] Dixit, A., (1993), *The art of smooth pasting*, Chur, Harwood Academic.
- [18] Dixit, A., and R.S. Pindyck (1994), *Investment under Uncertainty*, Princeton University Press.
- [19] Eggleston, K., E.A. Posner, and R.J. Zeckhauser (2000), "The design and interpretation of contracts: why complexity matters", *Northwestern University Law Review*, 95/1: 91-132.
- [20] Engel, E., R. Fisher and A. Galetovic (2006a), "Renegotiation Without Holdup: Anticipating Spending and Infrastructure Concessions", NBER Working Papers, 12399, National Bureau of Economic Research, Inc.

- [21] Engel, A. R., J. J. Ganuza, E. Hauk, and A. Wambach, (2006b). "Managing Risky Bids", Ch. 13, in N. Dimitri, G. Piga, and G. Spagnolo (eds.), *Handbook of Procurement*, Cambridge University Press, Cambridge, MA.
- [22] Ganuza, J.J. (2007), "Competition and cost overruns in procurement", *The Journal of Industrial Economics*, 55:4, pp.633-660.
- [23] Guash, J.L., J.J. Laffont and S. Straub (2003), "Renegotiating of Concession Contracts in Latin America", Policy Research Working Paper, 3011, World Bank, Washington.
- [24] Harrison, J. M., (1985), *Brownian motion and stochastic flow systems*, Wiley & Son, Inc., New York.
- [25] Herbsman, Z. J., W. T. Chen., and W.C. Epstein, (1995), "Time Is Money: Innovative Contracting Methods in Highway Construction", *Journal of Construction Engineering and Management*, September: 273-281.
- [26] Laffont, J.J., and D. Martimort (2002), *The Theory of Incentives, The Principal-Agent Model*, Princeton University Press, Princeton.
- [27] Laffont and Tirole, (1993): *A Theory of Incentives in Procurement and Regulation*, MIT Press.
- [28] Manelli, A. and D.R. Vincent, (1995), "Optimal procurement mechanisms", *Econometrica*, 63: 591-620.
- [29] Maskin, E. and J. Tirole, (1999): "Unforeseen contingencies and incomplete contracts", *Review of Economic Studies*, 66: 83-114.
- [30] McDonald, R., and D.R. Siegel, (1984), "Option pricing when the underlying asset earns a below-equilibrium rate of return: A note", *Journal of Finance*, 39/1: 261-265.
- [31] McDonald, R., and D.R. Siegel, (1985), "Investment and the Valuation of Firms When There is an Option to Shut Down", *International Economic Review*, 26: 331-349.
- [32] McDonald, R., and D.R. Siegel, (1986), "The Value of Waiting to Invest", *The Quarterly Journal of Economics*, 101: 707-728.
- [33] Moretto, M. and P. Valbonesi, (2007): "Firm Regulation and Profit Sharing: A Real Option Approach," *The B.E. Journal of Economic Analysis & Policy*: 7/1, (Topics), Article 56.
- [34] Pennisi, G. and P. Scandizzo (2003), "Valutare l'incertezza. L'analisi dei costi benefici nel XXI secolo", Milano: Giappichelli.
- [35] Saphores, J.D., (2002), "Barriers and Optimal Investment Rules", Department of Economics, *Working Paper*, 02-03-06, University of California, Irvine.