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MODELING EXCHANGE RATE AND INDUSTRIAL  
COMMODITY VOLATILITY TRANSMISSIONS

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# **Modeling Exchange Rate and Industrial Commodity Volatility Transmissions**

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## **Abstract**

This paper examines the inclusion of the dollar/euro exchange rate together with important commodities in two different BEKK, or multivariate conditional covariance, models. Such inclusion increases the significant direct and indirect past shock and volatility effects on future volatility between the commodities, as compared with their effects in the all-commodity basic model (Model 1), which includes the highly-traded aluminum, copper, gold and oil. Model 2, which includes copper, gold, oil and exchange rate, displays more direct and indirect transmission than does Model 3, which replaces the business cycle-sensitive copper with the highly energy-intensive aluminum. Optimal portfolios should have more Euro than commodities, and more copper and gold than oil. The multivariate conditional volatility models reveal greater volatility spillovers than their univariate counterparts.

**JEL:** C51, E27, Q43

**Keywords:** Multivariate GARCH; shocks; volatility; transmission; portfolio weights

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## **1. Introduction**

Commodity and other asset markets have been highly volatile in recent years. Commodities like oil have had significantly greater volatility than other commodities such as gold. Volatility brings risk and opportunity to traders and investors, and thereby should be examined. There are many reasons for volatility in commodity markets. Market participants form different expectations of profitable opportunities, process information at different speeds, perform cross-market hedging and build and draw inventories at different levels. These factors contribute volatility to commodities over time and to volatility spillovers across commodity markets

Shocks or news can also create and exacerbate volatility in commodity markets. Shocks to the US dollar, for example, may exacerbate commodity fluctuations in the long-run equilibrium, and hence lead to volatility transmission across markets. Oil and gold are also more sensitive to changes in the dollar than are copper and aluminum. On the other hand, copper seems to be the most sensitive to the business cycle (Hammoudeh, Sari and Ewing, 2008). This heterogeneous sensitivity to news should also spawn different volatilities among commodities.

The tradability and liquidity of futures contracts usually affect commodity fluctuations. The more liquid are contracts, the smoother will be commodity movements. Oil, gold, aluminum and copper are all exchange traded, but it is not known, if they all have the same contract liquidity and similar fluctuations during trading. Even within global oil benchmarks which belong to “one great pool”, liquidity, tradability and volatility vary. For example, the contracts of the light crude benchmarks, WTI and Brent oil, are more liquid at NYMEX and ICE than their own contracts and the contracts of the

medium crude benchmark Dubai/Oman at the Dubai Mercantile Exchange. Moreover, WTI is less volatile than non-exchange traded Maya, the Mexican heavy crude benchmark (Hammoudeh, Ewing and Thompson, 2008). If gold contracts, for example, are more liquid than those of copper or aluminum, then gold should have less volatile fluctuations.

The same argument applies to the LME-traded copper, which is particularly sensitive to economic activity. Copper may be more volatile because its market participants do not significantly stockpile this metal, and do not speculate heavily relative to other metals because it is cheap, heavy and plentiful. On the other hand, the price of copper generally represents an accurate barometer of its demand in the real world, rather than an irrational bet on its future value.

Changes in, and the availability of, commodity inventories may also affect volatility, depending on whether the change will add to or subtract from inventories, and on the size of the build-up compared with their long-run averages. Moreover, owners of oil storage tankers can use their knowledge of the fullness or emptiness of the tanks to spread news to induce traders to act quickly on false information, and may affect the speed and direction of adjustments. Unlike most stock markets, insider trading is usually not illegal in commodity markets. Oil companies, for example, can use their information of future production to trade during positive and negative shocks. Varying inventories and the backwardation/contagion state of commodity markets may also affect volatility.

In this paper, we concentrate on representatives of four types of fuel and industrial commodity classes, namely aluminum, copper, gold and oil. Aluminum represents an energy-intensive commodity class, copper represents base metals, gold represents

precious metals, and oil represents energy commodities. We also include a major macroeconomic variable, the dollar/euro exchange rate, as a link and policy variable. The commodities dominate asset trading, have strong linkages with the macro economy, and/or influence or are influenced by policy decisions.

We use multivariate GARCH models to estimate simultaneously the means and variances of the four commodity price and exchange rate returns to analyze volatility and its transmission. Furthermore, we use the multivariate BEKK model, which does not impose the restriction of constant conditional correlations across the commodity shocks. This procedure allows an examination of covariance spillovers across commodities.

This paper fills the voids in the literature on commodity volatility in three important areas. First, it uses multivariate conditional volatility models to determine volatility progression and transmission among the four commodities across different classes. Second, it examines the bi-directional impacts between the exchange rate and commodities, taking into account flight to safety, asset reallocation and responsiveness to policy decisions. Third, it uses the volatility results to calculate dynamic hedge ratios and risk-minimizing optimal portfolio weights for two commodities, or for one commodity and the exchange rate.

The remainder of the paper is organized as follows. Section 2 provides a review of the literature. Section 3 discusses the data and descriptive statistics. Section 4 presents the empirical model, and Section 5 discusses the empirical results. Section 6 gives some concluding comments.

## **2. Review of the Literature**

The literature on commodities has concentrated on their price co-movements and their roles in transmitting information on returns. The research on commodity volatility has been considerably less than on their counterparts in commodity prices and returns. This research has typically focused on volatility behavior for a single commodity over time, and not on volatility transmission across commodities and over time due to methodology complexities. The single commodity volatility research has used univariate models of conditional volatility (or GARCH) to examine the behavior of volatility over time, with a focus on own shocks and volatility dependencies over time, while ignoring volatility interdependencies across commodity markets and/or classes.

Bracker and Smith (1999) and Smith and Bracker (2003) apply the GARCH and EGARCH models to copper futures prices, and find these specifications to better explain volatility behavior for copper than do other models. McKenzie et al. (2001) explored the applicability of the univariate power ARCH (PARCH) model to precious metals futures contracts traded at the London Metal Exchange (LME), and found that asymmetric effects are not present, and the model did not provide an adequate explanation of the data. Tully and Lucey (2007) used the univariate asymmetric power GARCH (APGARCH) model to examine the asymmetric volatility of gold. They concluded that the exchange rate is the main macroeconomic variable that influences the volatility of gold, with few other macroeconomic variables having an impact.

Batten and Lucey (2007) studied the volatility of gold futures contracts traded on the Chicago Board of Trade (CBOT) using intraday and interday data. They used the univariate GARCH model to examine the volatility properties of the futures returns and

the alternative nonparametric volatility static model of Garman and Klass (1980) to provide further insights into intraday and interday volatility dynamics of gold. The results of both measures provided significant variations within and between consecutive time intervals. They also found a low correlation between volatility and volume. Bhar et al. (2008) used the univariate GARCH model to examine the behavior of the short-run stationary components of four oil benchmarks

In terms of nonlinearity and chaotic structure, Yang and Brorsen (1993) concluded that palladium, platinum, copper and gold futures have chaotic structures. In contrast, Adrangi and Chatrath (2002) found that the nonlinearity in palladium and platinum is inconsistent with chaotic behavior. They concluded that ARCH-type models with controls for seasonality and contractibility explained the nonlinear dependence in their data for palladium and platinum.

In comparison with other studies on commodities, Plourde and Watkins (1998) compared the volatility in the prices of nine non-oil commodities to the volatility in oil prices. On the basis of several non-parametric and parametric tests, they found that oil prices tend to be more volatile than the prices of gold, silver, tin and wheat, and argued that the differences are more evident in the case of precious metals. Hammoudeh and Yuan (2008) used three different univariate GARCH models to investigate the volatility and leverage properties of two precious metals (gold and silver) and one base metal (copper). They found that in the standard univariate GARCH model, gold and silver have almost the same conditional volatility persistence, which is higher than that of the pro-cyclical copper. In the EGARCH model, they found that only copper has an asymmetric effect, and in the CGARCH model the transitory component of volatility converges to



equilibrium faster for copper than for gold and silver. Using a rolling AR(1)-GARCH model, Watkins and McAleer (2008) showed that the conditional volatility for two nonferrous metals, namely aluminum and copper, is time-varying over a long horizon.

Finally, there are few studies that use multivariate GARCH to examine volatility transmissions across commodities. Hammoudeh et al. (2004) use a trivariate BEKK model to examine the volatility between oil prices and oil industry equity indices. Ewing et al. (2002) employ a bivariate BEKK model for the oil and natural gas sectors to examine how volatility changes over time and across the two sectors. Moschini and Myers (2002) develop a different bivariate GARCH parameterization for cash and futures markets, with a flexible functional form for time-varying volatility that is suitable for testing whether the optimal hedge ratio is constant, and whether the time variations in the optimal hedge ratios are solely due to deterministic seasonality and time-to-maturity effects. Statistical tests rejected both null hypotheses.

Thus, these studies, except for the last three, do not examine cross-volatility and shock effects between commodities using multivariate GARCH models. Even these three studies did not use a four variable GARCH model. This could be a major shortcoming when one considers that real world applications such as hedging, portfolio diversification and inter-commodity volatility predictions are conducted in multivariate settings. In this regard, we are interested in ascertaining to what extent commodity volatility interdependencies across markets and over time exists, and what role hedging and optimal portfolio formation play in mitigating their risks. Policy makers, traders and portfolio managers, as well as manufacturers, would be interested in this information because precious and industrial metals are investment assets, feed into inflation and have

important and diversified industrial uses in the jewelry, electronic and autocatalytic industries.

### 3. Data Description

We use daily time series data (five working days per week) for the four commodity (aluminum, copper, gold and oil) closing spot prices and the US dollar/euro exchange rate for the period 4 January 1999 to 5 November 2007. The exchange rate is the value of the US dollar to one euro, suggesting that a rise in the rate implies devaluation of the dollar, and vice-versa. Aluminum, gold and oil are traded at COMEX in New York. Copper is traded at LME. Oil is represented by the benchmark West Texas Intermediate (WTI). The daily US dollar/euro exchange rate series is obtained from the database of the Federal Reserve Bank of Saint Louis. All commodity and exchange rate series are modeled in natural logarithms and depicted in **Figure 1**.

The ADF and PP unit root tests for both the drift and without drift specifications demonstrate that the commodity and exchange rate variables have unit roots with and without drift.<sup>1</sup> Therefore, we will examine and model the returns instead of the levels for the five variables. **Table 1** provides the descriptive statistics for those variables. Among the four commodities, oil followed by copper yielded the highest average return, while gold had the lowest return over the sample period. Oil also has the highest volatility, as defined by standard deviation, while gold has the lowest. It is not surprising that oil has the highest volatility because it is periodically managed by OPEC, and is also sensitive to

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<sup>1</sup> The results are available from the authors upon request.

weather, frequent inventory changes and political tensions and military conflicts in the oil-producing countries.

Some studies have interpreted volatility as a proxy for information flow, in the sense that increases in information should translate into greater volatility (Lin and Chiang, 2005). Moreover, gold has been subdued due to low inflation during much of the sample period. All the series are leptokurtic, that is, have fat tails, which requires testing the individual mean equations for ARCH effects. The results show that there are strong ARCH effects for the four commodities and the exchange rate, thereby warranting estimation of the GARCH model.

### 3. Empirical Model

The commodities and the exchange rate in our empirical systems are indexed by  $i$ , and  $n$  is the total number of commodities and the exchange rate when the latter is included in the various models. Each system, whether all commodities or a combination of commodities and the dollar/euro exchange rate, has four variables, so that  $n = 4$ . The mean equation for the  $i^{\text{th}}$  commodities/exchange rate in this system is AR(1), and is given by:

$$R_{i,t} = a_i + b_i R_{i,t-1} + c_i D03 + \varepsilon_{i,t} \quad (1)$$

$$\varepsilon_{i,t} = H_t^{1/2} \eta_t, \quad \eta_t \sim \text{iid } N(0, I)$$

where  $R_{i,t}$  is the return on the  $i^{\text{th}}$  commodity (or exchange rate) of the  $nx1$  vector  $R_t$ , which is defined as a log difference. The innovation  $\eta_t$  is an  $nx1$  vector of *i.i.d.* random shocks, and  $H_t$  is the conditional covariance matrix of commodities (and exchange rate) at time  $t$ . D03 denotes the dummy variable for the 2003 Iraq War.

We follow Engle and Kroner (1995) to form the evolution of the conditional covariance matrix as the BEKK model, which permits an examination of the cross-commodity effects. It is also more practicable than the VECH specification given in Bollerslev, Engle and Wooldridge (1988), which is highly over-parameterized. The BEKK model restricts the estimated covariance matrix to be positive definite, and is given as:

$$H_{t+1} = C'C + A'\varepsilon_t\varepsilon_t'A + B'H_tB. \quad (2)$$

The coefficient matrices are given as:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \quad C = \begin{bmatrix} c_{11} & 0 & 0 & 0 \\ c_{21} & c_{22} & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix}$$

where C is a  $4 \times 4$  lower triangular matrix with 10 parameters. The  $4 \times 4$  matrices A and B represent the effects of past shocks and past conditional variances and covariances on the current conditional variances and covariances of the various commodities/foreign exchange rate, respectively. The total number of estimated elements for the variance equation (2) in the four-variable system is 42.

The interpretations of the basic estimated elements are not obvious. Ignoring the constant term, the conditional variance equations can be re-expressed as:

$$h_{ii,t+1} = \sum_{j=1}^4 a_{ji}^2 \varepsilon_{j,t}^2 + \sum_{j=1}^3 \sum_{k=j+1}^4 2a_{ji} a_{ki} \varepsilon_{j,t} \varepsilon_{k,t} + \sum_{j=1}^4 b_{ji}^2 h_{jj,t} + \sum_{j=1}^3 \sum_{k=j+1}^4 2b_{ji} b_{ki} h_{jk,t} \quad i=1,2,3,4 \quad (3)$$

Equation (3) shows how shocks and volatilities are transmitted across commodity/foreign exchange markets and over time.

We maximize the following likelihood function, assuming the errors are normally distributed:

$$L(\theta) = -T \ln(2\pi) - \frac{1}{2} \sum_{t=1}^T (\ln |H_t| + \varepsilon_t' H_t^{-1} \varepsilon_t),$$

where  $T$  is the number of observations and  $\theta$  is the estimated parameter vector. Numerical maximization techniques are employed in order to maximize the non-linear log-likelihood function. Initial conditions are obtained by performing several initial iterations using the simplex algorithm, as recommended in Engle and Kroner (1995). The Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm was then used to obtain the final estimate of the covariance matrix, with corresponding standard errors for the commodity/exchange rate models.

#### 4. Empirical Results

We will estimate empirical models for three combinations of the four commodities and the exchange rate because of the well known convergence limitations of

the BEKK model.<sup>2</sup> Model 1 will be considered as the basic model, and will include the four commodities, namely aluminum, copper, gold and oil. Model 2 consists of copper, gold, oil and the dollar/euro exchange rate. Finally, Model 3 is comprised of aluminum, gold, oil and the exchange rate. We included copper with the exchange rate in Model 2, and aluminum and exchange rate in Model 3 because, while we recognize that these commodities are purely industrial commodities, they are different because aluminum is highly energy-intensive compared with copper. However, both commodities are included in Model 1.

*Model 1:*

We will only examine the statistically significant estimates in this model. We start with the conditional variance (volatility),  $h_{11}$ , for aluminum in **Table 2**. This highly energy-intensive and industrial metal is significantly and positively affected by news (unexpected shocks),  $\varepsilon^2_{1t}$ , from its own market without being affected by any news spillovers from the other markets, including copper, gold or oil. In terms of sensitivity to past volatility,  $h_{11}$ , aluminum is also significantly and positively affected by past volatility originating only from its own market. The aluminum ambivalence to both oil and copper news and volatilities is surprising, and may underline the nature of this metal as both an industrial and energy intensive metal, thereby placing it in a separate metal class from the others, even from copper.

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<sup>2</sup> The BEKK model did not converge with five variables. We then estimated the DCC model for all five variables combined. The results show that the conditional correlation coefficients for the shocks are less than 1%, which implies that the DCC matrix converges to a constant matrix in the long run. We also estimated the VARMA-GARCH model of Ling and McAleer (2003), but did not obtain convergence.

Copper volatility,  $h_{22}$ , is significantly and positively affected by news or shocks generated in its own market,  $\varepsilon_2^2$ . In contrast to aluminum, the copper volatility is significantly and positively impacted by news in the gold market,  $\varepsilon_2 \varepsilon_3$ . When it comes to the effects of past volatility, copper is impacted only by its own shocks, as is the case in the aluminum market.

The volatility of gold,  $h_{33}$ , is much more heavily impacted by news from other markets than the other three commodities. Specifically, it is significantly and positively affected by news from its own market,  $\varepsilon_3^2$ . The interaction,  $(\varepsilon_2 \varepsilon_3)$ , of shocks emanating from the copper and gold markets significantly reduces the conditional volatility of the gold market. For example, news about power deficiency in major copper-producing countries, associated with news about explosions in a major gold mine, indirectly affects volatility in the gold market. This indirect impact is due to cross-market hedging, or sharing common information between two markets. The volatility in the gold market is influenced by news because it is a safe haven in times of high risk and rising inflation. During bad times, investors dump copper and aluminum, and buy gold as part of a risk hedging asset reshuffling strategy.

When it comes to sensitivity to past volatilities, gold volatility is indirectly affected by the interaction of volatilities in the aluminum and copper markets, aluminum and own market, and copper and own market. It is also affected directly by its own market. It seems that gold volatility is impacted by other commodity volatilities because traders and investors revert to it as a safe haven during times of high volatility in other markets. It is interesting that gold volatility is not impacted by volatility in the oil market, which is also involved in the flight to safety when the dollar exchange rate is impacted.

Oil is, however, periodically managed by OPEC, and has its own trajectory. It is also possible that oil is overplayed by speculators.

The oil market volatility in this model seems to be independent of the other three metals markets. Its volatility is significantly and directly affected only by its own past shocks and volatility, as is the case with aluminum volatility. Oil has the highest unconditional volatility, as shown in **Table 1**, due to its manipulation by OPEC and speculators and the geopolitics of its supply.

In summary, in a simultaneous four variable setting, gold receives more shock and volatility spillovers than any other commodity, with copper second. Aluminum and oil are explained by their own markets. In the four commodity BEKK model, there are no volatility independencies, with the transmissions being generally significant.

### *Model 2*

This model contains copper, gold, oil and the dollar/euro exchange rate, with the highly energy-intensive industrial metal, aluminum, included in the following model. As mentioned above, the exchange rate is included to account for a feedback mechanism between dollar-denominated metals and oil, and the exchange rate.

The inclusion of the exchange rate in this model increases substantially the direct and indirect effects of past shocks and volatilities on future volatility of the three commodities, as compared with their effects in Model 1 (the basic model), as displayed in **Table 3**.

There are direct effects ( $\varepsilon_i^2$ ) of news from and to own markets for all four commodities. Moreover, the direct news effects from the other markets on own are as



follows: Gold on copper, and vice-versa; oil on exchange rate, and vice-versa; exchange rate on gold, and vice versa; and gold on oil. It is interesting to find that news (shock) impacts are bidirectional between gold and the exchange rate, in lieu of the fact that gold, dollar and euro are used for foreign reserves. Furthermore, gold news unidirectionally affects oil volatility, despite the fact that gold and oil are dollar-denominated assets and are considered safe havens and hedges against inflation and a depreciating dollar.

There are also indirect effects ( $\varepsilon_i, \varepsilon_j$ ) from news interactions between markets on own markets. The most notable of these indirect effects is for the exchange rate and oil. There are not, however, as many indirect effects for copper and gold.

When we focus on the direct and indirect effects of past volatilities on future volatilities, we see more significant relationships than in the shock effects, indicating that commodity volatility is predictable, even in a simultaneous setting. The results show that there are significant volatility effects ( $h_{ii}$ ) on own market volatility for all four markets in this model.

Direct volatility effects from other markets to own are: copper on three markets; exchange rate on three markets; gold on three markets; and oil on the exchange rate and gold markets, which is different from what we have in the case of shocks. These volatilities are simultaneously affected by fundamental forces such as macroeconomic factors and cross-market hedging.

Finally, there are many indirect volatility transmissions, representing interactions of volatilities between markets. There are transmissions of volatility interactions in exchange rate and gold on all four markets; between exchange rate and oil on the foreign exchange, gold and oil markets; and between gold and oil on the foreign exchange, gold

and oil markets. It seems that transmissions of indirect volatility interactions are the strongest among exchange rate, gold and oil, and weakest for the more business cycle sensitive copper.

Some of the simultaneous direct results indicated above are consistent with those of the univariate GARCH model, which had an impact of exchange rate on commodity volatility, particularly that of the exchange rate on gold (Tully and Lucey, 2006). Others are different from the univariate transmissions for oil, gold and copper (Hammoudeh and Yuan, 2006). These arise because of the shortcomings of the univariate GARCH model, in that they block simultaneous feedbacks and spillovers.

### *Model 3*

The composition of this model differs from that of Model 2 as it replaces copper with aluminum, but retains gold, oil and the exchange rate. It examines the simultaneous interactions and transmissions when a business cycle sensitive base industrial metal is replaced by a highly energy intensive industrial metal which does not have the same economic interactions with the overall economy.

The results reveal that the simultaneous relationships are not as significant as in Model 2. Copper is known to have many more linkages with various economic sectors, and it is more directly sensitive to business cycles than aluminum. Some economists call it Dr. Copper because of its ability to predict business cycles (Lahart, 2006). Copper also seems to share a greater sensitivity with gold and oil for common macroeconomic factors than with other commodities, including aluminum.

The empirical findings reveal that the direct shocks and volatility transmissions between the markets are still strong in this model compared with the all commodity model, but the indirect transmissions are much weaker than in Model 2. There are direct effects of news from and to own markets for the four markets in this model, as for Model 2. On the other hand, the direct news effects from other markets on own markets are evident only from the exchange rate to gold. Even in this direct news spillover case, there is no reciprocal news impact from gold to exchange rate as is the case in Model 2. The indirect effects from news interactions  $(\varepsilon_i, \varepsilon_j)$  between markets on own markets are also limited compared with the previous model. There are transmissions of (indirect) news interactions in the exchange rate and gold on the gold market, and between the exchange rate and oil on the oil market, as in Model 2.

The direct volatility transmissions from and to own markets are the same for all four markets, as in Model 2, but the direct volatility transmissions from other markets to own are concentrated primarily on the exchange rate and gold, and to a lesser extent on oil. This is largely due to cross hedging among these asset classes, but these transmissions are irrelevant for the aluminum market. The same analysis applies to indirect volatility transmissions.

## **5. Implications for Portfolio Designs and Hedging Strategies**

We now provide two examples using the estimates of Model 2 for the copper, foreign exchange, gold and oil markets, and for the aluminum market in Model 3, to analyze portfolio design and hedging strategies.

### 5.1. Portfolio weights

The first example follows Kroner and Ng (1998) by considering a portfolio that minimizes risk without lowering expected returns. If we assume the expected returns to be zero, the optimal portfolio weight of one commodity (or asset) relative to the other in a two commodity (asset) portfolio is given by:

$$w_{12,t} = \frac{h_{22,t} - h_{12,t}}{h_{11,t} - 2h_{12,t} + h_{22,t}}$$

and

$$w_{12,t} = \begin{cases} 0, & \text{if } w_{12,t} < 0 \\ w_{12,t}, & \text{if } 0 \leq w_{12,t} \leq 1 \\ 1, & \text{if } w_{12,t} > 1 \end{cases}$$

where  $w_{12,t}$  is the portfolio weight for, say, commodity (asset) 1 relative to commodity (asset) 2 in one dollar portfolio of the two commodities (assets) 1 and 2 at time  $t$ ,  $h_{12,t}$  is the conditional covariance between commodity returns, or assets 1 and 2, and  $h_{22,t}$  is the conditional variance of the commodity, or asset 2. The portfolio weight of the second commodity, or asset, in the one dollar portfolio is  $1-w_{12,t}$ .

The average values of  $w_{12,t}$  for the commodities or assets in Model 2 are reported in **Table 5**. For instance, the average value of  $w_{12,t}$  of a portfolio comprising copper and exchange rate is 0.14.<sup>3</sup> This suggests that the optimal holding of copper in one dollar of copper/euro portfolio in Model 2 is 14 cents, compared with 86 cents for euro. Similar

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<sup>3</sup> Hassan and Malik (2007) used the BEKK model and estimated the average weight between the financial and technology sectors at 0.66, while the average risk-minimizing hedge ratio between these sectors was 0.64.

results are obtained for gold/euro and oil/ euro in Model 2, and for aluminum/euro in Model 3. These optimal portfolio weights suggest that investors should own more euro than commodities in their portfolios. For purely commodity portfolios, investors should hold more copper and gold than oil, and hold more gold than copper and aluminum in their portfolios.

### 5.2. Hedge ratios

As a second illustration, we follow the example given in Kroner and Sultan (1993) regarding risk-minimizing hedge ratios, and apply it to these markets. In order to minimize risk, a long position of one dollar taken in one commodity/asset market should be hedged by a short position of  $\beta_t$  in another market at time  $t$ . The  $\beta_t$  is given by:

$$\beta_t = \frac{h_{12,t}}{h_{22,t}} ,$$

where  $\beta_t$  is the risk-minimizing hedge ratio for two commodities/assets,  $h_{12,t}$  is the conditional covariance between markets 1 and 2, and  $h_{22,t}$  is the conditional variance of the second market.

The second column of Table 5 reports the average values of  $\beta_t$  for the markets. By following this hedging strategy, one dollar long in the copper market, for example, should be shorted by 31 cents in the foreign exchange market, 34 cents in the gold market, and by 9 cents in the oil market. Similarly, one dollar long in the gold market

should be shorted by 4 cents in the oil market. It seems that the most effective hedging is by shorting oil.

## **6. Conclusions**

There has been a significant amount of research that modeled simultaneous transmissions of *returns* among commodity markets using VARs. A growing number of studies have also examined the behavior of shocks and volatility of oil and industrial commodities using univariate versions of the GARCH family of volatility models. These studies did not examine the transmission of shocks and volatility among commodities in a simultaneous setting. Commodity markets employ cross-market hedging and share common information that affects future volatilities simultaneously. Commodity markets are lagging behind stock markets in this regard. With the increasing globalization of the world's economies and commodity markets, analyzing commodity volatility spillovers is important and useful. We have tried to fill the gap for commodities in this paper.

While univariate volatility models examine the impacts arising from markets such as foreign exchange on another market, such as gold, the simultaneous commodity/foreign exchange multivariate volatility models found many direct and indirect shock and volatility transmissions, while confirming the direct impacts estimated in the univariate GARCH model, particularly between gold and the exchange rate.

Including the exchange rate in the commodity model increases the direct and indirect shocks and volatility transmissions, particularly between the exchange rate, gold and oil. Replacing the business cycle sensitive copper with the energy intensive aluminum diminished the transmission, but affected the spillovers between the exchange

rate and gold, and oil to a lesser extent. Traders, investors and the policy market should be aware of the strong transmissions of shocks and volatilities between the exchange rate, gold and oil. They should also be aware that in a two-asset portfolio, optimal portfolios hold a greater weight of the euro than commodities, and more gold than aluminum, copper and oil. It would seem that the most effective way of hedging long positions with a shorting position is to short with oil.

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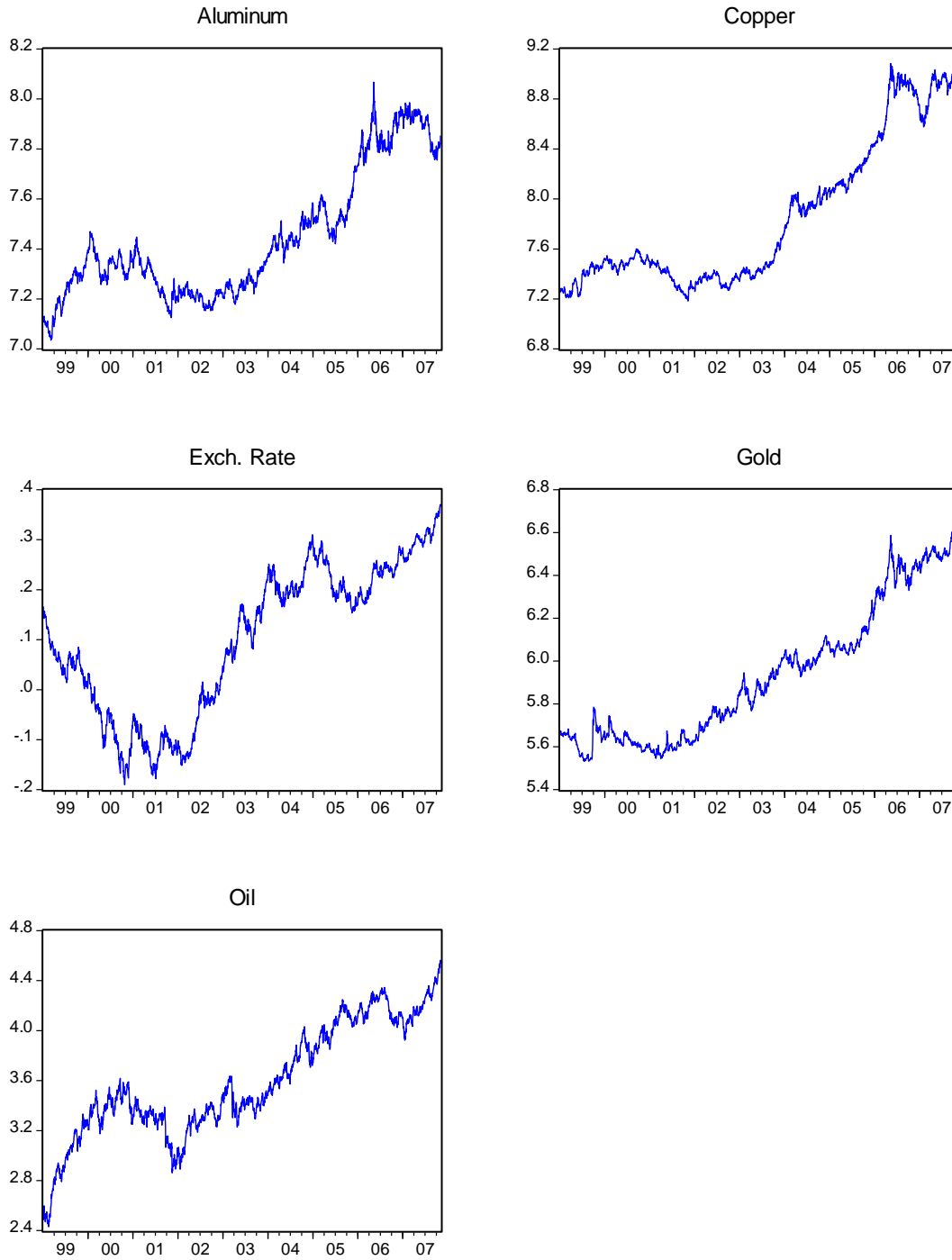
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Table 1: Descriptive Statistics

| <b>Statistics</b>          | <b>Aluminum</b> | <b>Copper</b> | <b>Exch. Rate</b> | <b>Gold</b>   | <b>WTI Oil</b> |
|----------------------------|-----------------|---------------|-------------------|---------------|----------------|
| Mean                       | 0.0003          | 0.0007        | 0.0001            | 0.0004        | 0.0009         |
| Median                     | 0.0000          | 0.0000        | 0.0000            | 0.0001        | 0.0008         |
| Maximum                    | 0.0520          | 0.1155        | 0.0271            | 0.0701        | 0.1244         |
| Minimum                    | -0.0826         | -0.1036       | -0.0247           | -0.0625       | -0.1709        |
| Std. Dev.                  | 0.0123          | 0.0153        | 0.0058            | 0.0098        | 0.0236         |
| Skewness                   | -0.3288         | -0.0957       | 0.0090            | 0.1160        | -0.5517        |
| Kurtosis                   | 6.3808          | 8.1807        | 4.0321            | 8.9191        | 7.0413         |
| Jarque-Bera<br>Probability | 1139.294<br>0   | 2581.253<br>0 | 102.3437<br>0     | 3370.049<br>0 | 1685.484<br>0  |
| ARCH Effect                | 11.75           | 18.09         | 18.4              | 4.05          | 8.38           |
| Sum                        | 0.7422          | 1.6463        | 0.2029            | 1.0305        | 2.0246         |
| Sum Sq. Dev.               | 0.3509          | 0.5389        | 0.0782            | 0.2199        | 1.2802         |
| Observations               | 2305            | 2305          | 2305              | 2305          | 2305           |

Notes: All commodity and dollar/euro exchange rate variables are log differences. The ARCH effect test was conducted on the AR(1) mean equations for up to 12 lags. The 5% critical value for this test is 1.75.

Figure 1



Note: The graphs are for the log of the variables.

Table 2. Model 1 (Basic) for Aluminum, Copper, Gold and Oil

| Independent Variable                    | $h_{11, t+1}$       | $h_{22, t+1}$       | $h_{33, t+1}$         | $h_{44, t+1}$       |
|---|---------------------|---------------------|-----------------------|---------------------|
| $\varepsilon_{1, t}^2$                  | 0.0198 <sup>a</sup> | 1.58E-06            | 8.47E-04              | 5.20E-04            |
| $\varepsilon_{1, t} \varepsilon_{2, t}$ | -5.95E-04           | 1.65E-04            | 0.0020                | -6.63E-04           |
| $\varepsilon_{1, t} \varepsilon_{3, t}$ | 0.0043              | 8.38E-05            | -0.0097               | 2.78E-04            |
| $\varepsilon_{1, t} \varepsilon_{4, t}$ | 0.0011              | -3.24E-06           | -1.11E-04             | 0.0052              |
| $\varepsilon_{2, t}^2$                  | 1.79E-05            | 0.0173 <sup>a</sup> | 0.0047                | 8.45E-04            |
| $\varepsilon_{2, t} \varepsilon_{3, t}$ | -1.28E-04           | 0.0088 <sup>a</sup> | -0.0229 <sup>a</sup>  | -3.54E-04           |
| $\varepsilon_{2, t} \varepsilon_{4, t}$ | -3.25E-05           | -3.39E-04           | -2.64E-04             | -0.0066             |
| $\varepsilon_{3, t}^2$                  | 9.17E-04            | 0.0045 <sup>c</sup> | 0.1108 <sup>a</sup>   | 1.48E-04            |
| $\varepsilon_{3, t} \varepsilon_{4, t}$ | 2.33E-04            | -1.72E-04           | 0.0013                | 0.0028              |
| $\varepsilon_{4, t}^2$                  | 5.93E-05            | 6.67E-06            | 1.47E-05              | 0.0520 <sup>a</sup> |
| $h_{11, t}$                             | 0.9615 <sup>a</sup> | 7.34E-05            | 3.73E-04              | 8.56E-05            |
| $h_{12, t}$                             | 0.0036              | -0.0085             | 3.23E-04 <sup>a</sup> | 6.03E-05            |
| $h_{13, t}$                             | 0.0049              | 1.20E-04            | 0.0174 <sup>b</sup>   | 1.53E-05            |
| $h_{14, t}$                             | -0.0017             | -8.22E-06           | -5.74E-05             | 0.0086              |
| $h_{22, t}$                             | 1.34E-05            | 0.9883 <sup>a</sup> | 2.80E-04              | 4.25E-05            |
| $h_{23, t}$                             | 1.83E-05            | -0.0139             | 0.0151 <sup>b</sup>   | 1.08E-05            |
| $h_{24, t}$                             | -6.52E-06           | 0.0010              | -4.97E-05             | 0.0061              |
| $h_{33, t}$                             | 2.51E-05            | 1.95E-04            | 0.8114 <sup>a</sup>   | 2.73E-06            |
| $h_{34, t}$                             | -8.93E-06           | -1.34E-05           | -0.0027               | 0.0015              |
| $h_{44, t}$                             | 3.18E-06            | 9.21E-07            | 8.83E-06              | 0.8697 <sup>a</sup> |

Notes: Market subscripted 1 is aluminum, market 2 is copper, market 3 is gold, And market 4 is oil.  $h_{ii}$  refers to the variance in market  $i$ , while  $h_{ij}$  is the covariance of market  $i$  in response to past volatility in market  $j$ . Shocks are defined similarly.

Table 3. Model 2 for Copper, Exchange Rate, Gold and Oil

| Variable                             | $h_{11, t+1}$          | $h_{22, t+1}$          | $h_{33, t+1}$         | $h_{44, t+1}$          |
|--------------------------------------|------------------------|------------------------|-----------------------|------------------------|
| $\mathcal{E}_{1,t}^2$                | 0.0218 <sup>a</sup>    | 2.00E-06               | 4.33E-04 <sup>b</sup> | 0.00E+00               |
| $\mathcal{E}_{1,t}\mathcal{E}_{2,t}$ | -0.0032 <sup>a</sup>   | -1.35E-04              | 0.0044 <sup>a</sup>   | 1.08E-04               |
| $\mathcal{E}_{1,t}\mathcal{E}_{3,t}$ | 0.0084 <sup>a</sup>    | -1.70E-05              | -0.0072 <sup>a</sup>  | 3.50E-05               |
| $\mathcal{E}_{1,t}\mathcal{E}_{4,t}$ | 0.0006 <sup>c</sup>    | 4.00E-06               | -3.40E-05             | 6.10E-05               |
| $\mathcal{E}_{2,t}^2$                | 4.72E-04               | 0.0115 <sup>a</sup>    | 0.0439 <sup>a</sup>   | 0.0234 <sup>a</sup>    |
| $\mathcal{E}_{2,t}\mathcal{E}_{3,t}$ | -0.0012 <sup>a</sup>   | 0.0014 <sup>a</sup>    | -0.0723 <sup>a</sup>  | 0.0077 <sup>a</sup>    |
| $\mathcal{E}_{2,t}\mathcal{E}_{4,t}$ | -9.20E-05              | -3.80E-04 <sup>a</sup> | -3.47E-04             | 0.0132 <sup>a</sup>    |
| $\mathcal{E}_{3,t}^2$                | 0.0033 <sup>a</sup>    | 1.73E-04 <sup>a</sup>  | 0.1190 <sup>a</sup>   | 0.0025 <sup>b</sup>    |
| $\mathcal{E}_{3,t}\mathcal{E}_{4,t}$ | 2.41E-04               | -4.70E-05 <sup>a</sup> | 5.71E-04              | 0.0043 <sup>a</sup>    |
| $\mathcal{E}_{4,t}^2$                | 1.80E-05               | 1.30E-05 <sup>b</sup>  | 3.00E-06              | 0.0074 <sup>a</sup>    |
| $h_{11,t}$                           | 0.9750 <sup>a</sup>    | 1.00E-06 <sup>a</sup>  | 1.01E-04 <sup>a</sup> | 2.80E-05 <sup>a</sup>  |
| $h_{12,t}$                           | 0.0064 <sup>a</sup>    | 0.0007 <sup>a</sup>    | 3.46E-04 <sup>a</sup> | -2.33E-04 <sup>a</sup> |
| $h_{13,t}$                           | -0.0130 <sup>a</sup>   | -4.00E-06 <sup>a</sup> | 0.0092 <sup>a</sup>   | -1.37E-04 <sup>a</sup> |
| $h_{14,t}$                           | 1.13E-04               | 2.00E-06 <sup>a</sup>  | 2.50E-05 <sup>a</sup> | 0.0053 <sup>a</sup>    |
| $h_{22,t}$                           | 4.30E-05 <sup>a</sup>  | 0.9892 <sup>a</sup>    | 0.0012 <sup>a</sup>   | 0.0019 <sup>a</sup>    |
| $h_{23,t}$                           | -8.60E-05 <sup>a</sup> | -0.0052 <sup>a</sup>   | 0.0317 <sup>a</sup>   | 0.0011 <sup>a</sup>    |
| $h_{24,t}$                           | 1.00E-06               | 0.0022 <sup>a</sup>    | 8.70E-05 <sup>a</sup> | -0.0435 <sup>a</sup>   |
| $h_{33,t}$                           | 1.74E-04 <sup>a</sup>  | 2.80E-05 <sup>a</sup>  | 0.8451 <sup>a</sup>   | 6.66E-04 <sup>a</sup>  |
| $h_{34,t}$                           | -2.00E-06              | -1.20E-05 <sup>a</sup> | 0.0023 <sup>a</sup>   | -0.0256 <sup>a</sup>   |
| $h_{44,t}$                           | 1.30E-08               | 5.00E-06 <sup>a</sup>  | 6.00E-06 <sup>a</sup> | 0.9880 <sup>a</sup>    |

Notes: Market subscripted 1 is copper, market 2 is dollar/euro foreign exchange, market 3 is gold, and market 4 is oil.  $h_{ii}$  refers to the variance in market  $i$ , while  $h_{ij}$  is the covariance of market  $i$  in response to past volatility in market  $j$ . Shocks are defined similarly.

Table 4: Model 3 for Aluminum, Exchange Rate, Gold and Oil

| Variable                               | $h_{11, t+1}$<br><i>Aluminum</i> | $h_{22, t+1}$ Exchange | $h_{33, t+1}$<br><i>gold</i> | $h_{44, t+1}$<br><i>oil</i> |
|--|----------------------------------|------------------------|------------------------------|-----------------------------|
| $\mathcal{E}_{1, t}^2$                 | 0.0229 <sup>a</sup>              | 6.00E-06               | 3.29E-04                     | 2.49E-04                    |
| $\mathcal{E}_{1, t}\mathcal{E}_{2, t}$ | 2.32E-04                         | 2.76E-04               | 0.0038                       | 0.0029                      |
| $\mathcal{E}_{1, t}\mathcal{E}_{3, t}$ | 0.0053 <sup>c</sup>              | 3.00E-05               | -0.0063                      | 5.70E-04                    |
| $\mathcal{E}_{1, t}\mathcal{E}_{4, t}$ | 0.0014                           | -9.00E-06              | 3.70E-05                     | 0.0015                      |
| $\mathcal{E}_{2, t}^2$                 | 2.00E-06                         | 0.0118 <sup>a</sup>    | 0.0444 <sup>a</sup>          | 0.0347                      |
| $\mathcal{E}_{2, t}\mathcal{E}_{3, t}$ | 5.40E-05                         | 0.0013 <sup>c</sup>    | -0.0733 <sup>a</sup>         | 0.0067                      |
| $\mathcal{E}_{2, t}\mathcal{E}_{4, t}$ | 1.40E-05                         | -3.94E-04              | 4.33E-04                     | 0.0173 <sup>b</sup>         |
| $\mathcal{E}_{3, t}^2$                 | 0.0012                           | 1.44E-04               | 0.1211 <sup>a</sup>          | 0.0013                      |
| $\mathcal{E}_{3, t}\mathcal{E}_{4, t}$ | 3.15E-04                         | -4.40E-05              | -7.15E-04                    | 0.0034                      |
| $\mathcal{E}_{4, t}^2$                 | 8.10E-05                         | 1.30E-05               | 4.00E-06                     | 0.0086 <sup>a</sup>         |
| $h_{11, t}$                            | 0.9655 <sup>a</sup>              | 3.58E-07               | 4.07E-04 <sup>c</sup>        | 4.00E-06                    |
| $h_{12, t}$                            | 0.0022                           | 6.17E-04               | 7.68E-04 <sup>a</sup>        | -9.20E-05                   |
| $h_{13, t}$                            | -0.0028                          | -3.00E-06              | 0.0184 <sup>a</sup>          | -3.60E-05                   |
| $h_{14, t}$                            | -0.0018                          | 1.00E-06               | 7.80E-05                     | 0.0020                      |
| $h_{22, t}$                            | 5.00E-06                         | 0.9895 <sup>a</sup>    | 0.0014 <sup>a</sup>          | 0.0021 <sup>b</sup>         |
| $h_{23, t}$                            | -7.00E-06                        | -0.0053 <sup>b</sup>   | 0.0347 <sup>a</sup>          | 8.10E-04                    |
| $h_{24, t}$                            | -4.00E-06                        | 0.0021 <sup>a</sup>    | 1.47E-04                     | -0.0454 <sup>a</sup>        |
| $h_{33, t}$                            | 8.00E-06                         | 2.90E-05               | 0.8312 <sup>a</sup>          | 3.13E-04                    |
| $h_{34, t}$                            | 5.00E-06                         | -1.10E-05 <sup>c</sup> | 0.0035 <sup>c</sup>          | -0.0176                     |
| $h_{44, t}$                            | 3.00E-06                         | 5.00E-06 <sup>b</sup>  | 1.50E-05                     | 0.9852 <sup>a</sup>         |

Notes: Market subscripted 1 is aluminum, market 2 is dollar/euro foreign exchange, market 3 is gold, and market 4 is oil.  $h_{ii}$  refers to the variance in market  $i$ , while  $h_{ij}$  is the covariance of market  $i$  in response to past volatility in market  $j$ . Shocks are defined similarly.

Table 5: Optimal Portfolio Weights and Hedge Ratios

| Portfolio      | Weight ( $w_{12,t}$ ) of First Commodity/Asset in 1\$ Portfolio (Kroner and Ng, 1998) | Short/Long Beta $\beta_t$ (Kroner and Sultan, 1993) |
|----------------|---|---|
| <u>Model 2</u> |   |   |
| Copper/Euro    | 0.14  | 0.31  |
| Copper/Gold    | 0.27  | 0.32  |
| Copper/Oil     | 0.72  | 0.09  |
| Euro/Gold      | 0.78  | 0.22  |
| Euro/Oil       | 0.95  | 0.01  |
| Gold/Oil       | 0.87  | 0.04  |
| <u>Model 3</u> |   |   |
| Aluminum/Euro  | 0.17  | 0.30  |
| Aluminum/Gold  | 0.35  | 0.24  |
| Aluminum/Oil   | 0.80  | 0.07  |

Notes:  $w_{12,t}$  is the portfolio weight of commodity or asset 1 relative to commodity or asset 2 in a two-commodity/asset holding at time  $t$ , while average  $\beta_t$  is the risk-minimizing hedge ratio for the two commodities/assets