UNIVERSITÀ DEGLI STUDI DI PADOVA

Dipartimento di Scienze Economiche ed Aziendali “Marco Fanno”

MONETARY POLICY NEUTRALITY? SIGN RESTRICTIONS GO TO MONTE CARLO

EFREM CASTELNUOVO
University of Padova

October 2012

“MARCO FANNO” WORKING PAPER N.151

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Efrem Castelnuovo
University of Padova

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Abstract

A new-Keynesian DSGE model in which contractionary monetary policy shocks generate recessions is estimated with U.S. data. It is then used in a Monte Carlo exercise to generate artificial data with which VARs are estimated. VAR monetary policy shocks are identified via sign restrictions. Our VAR impulse responses replicate Uhlig's (2005, *Journal of Monetary Economics*) evidence on unexpected interest rate hikes having ambiguous effects on output. The mismatch between the true (DSGE-consistent) responses and those produced with sign-restriction VARs is shown to be due to the low relative strength of the signal of the monetary policy shock. We conclude that Uhlig's (2005) finding is not inconsistent with monetary policy non-neutrality.

JEL classification: C3, E4, E5.

Keywords: Monetary policy shocks, VARs, sign restrictions, Dynamic Stochastic General Equilibrium models, monetary neutrality.

*We thank Fernando Alvarez, Guido Ascari, Christopher Bowdler, Giovanni Caggiano, Martin Ellison, Renee Fry-McKibbin, Stefano Gnocchi, Nicolas Groshenny, Robert G. King, Alexander Kriwoluzky, Francesco Lippi, Antonio Nicolò, Matthias Paustian, Giorgio Primiceri, Roberto Rigobon, Konstantinos Theodoridis, Harald Uhlig, Tony Yates, and participants to presentations held at the Reserve Bank of New Zealand, Royal Economic Society (University of Cambridge), 1st DEFAP-LASER TEAM Summer School (Pavia), the Universities of Auckland, Dunedin, Münster, the Australian National University-CAMA, and EIEF for useful comments and suggestions. Part of this research was conducted while visiting the Nuffield College/Department of Economics of the University of Oxford and the Reserve Bank of New Zealand, whose hospitality is gratefully acknowledged. All errors are ours. Author's details: Efrem Castelnuovo, Department of Economics, University of Padova, Via del Santo 33, I-35123 Padova (PD). E-mail: efrem.castelnuovo@unipd.it. Phone: +39 049 8274257. Fax: +39 049 8274211.

1 Introduction

The conventional view on the effects of monetary policy shocks is the following. An unexpected policy rate hike increases the real interest rate, depresses aggregate demand, and pushes inflation down in the short-run. An intriguing exercise proposed by Uhlig (2005) casts doubts on this transmission mechanism. Working with a VAR estimated with post-WWII U.S. data, Uhlig (2005) shows that the response of output to a monetary policy shock is surrounded by a large amount of uncertainty. As a matter of fact, output may increase or decrease following a shock that triggers conventional reactions of other macroeconomic indicators.

This result is thought-provoking. As stated by Uhlig (2005, p. 406),

> *" "Contractionary" monetary policy shocks do not necessarily seem to have contractionary effects on real GDP. One should therefore feel less comfortable with the conventional view and the current consensus of the VAR literature that has been the case so far."*

In light of this finding, Uhlig (2005, p. 382) concludes that

> *"Neutrality of monetary policy shocks is not inconsistent with the data."*

This paper shows that Uhlig's (2005) intriguing result is *consistent* with the conventional view on the real effects of monetary policy shocks. In short, we show that ambiguous effects of a monetary policy tightening on output may be found with VARs estimated with artificial data generated by structural models in which monetary policy is *non-neutral*. We do so by setting up a Monte Carlo exercise in which the Data Generating Process (DGP) is a new-Keynesian DSGE model predicting "textbook" effects as for the short-run reaction of output to monetary policy shocks. Such model is estimated with U.S. data and employed to generate artificial data with which we feed our VARs. Monetary policy shocks are identified by imposing a set of widely-accepted, model-consistent sign restrictions on the modeled variables. Following Uhlig (2005), we leave the reaction of output *unconstrained* at all horizons.

Our main result reads as follows. The estimated DSGE model of the business cycle predicts a phase of economic bust and a deflation after an unexpected policy tightening. However, our VARs return quite uncertain indications as for the reaction of output. In particular, about 2/3 of the VAR responses of output conditional on an unexpected policy tightening turn out to be *positive*, a result consistent with Uhlig's (2005) evidence. In other words, our exercise replicates Uhlig's (2005) finding in a controlled environment in which monetary policy shocks *do* exert an influence on the U.S. business cycle. This result proves that an uncertain reaction of output to a monetary policy shock obtained with sign-restriction VARs is *not* inconsistent with the conventional view.

This result is driven by the relatively little role played by policy shocks in influencing the volatility of output. Given the weakness of the signal, the estimated VAR monetary policy "shock" is actually a combination of all shocks hitting the economic system. In particular, supply shocks contaminate the estimated dynamic responses and induce a positive output reaction. Sign restrictions are shown to correctly identify the negative effects on output exerted by policy shocks in alternative environments in which such shocks play a (counterfactually) larger role as for the volatility of output. Therefore, *in principle*, nothing is wrong with the sign restrictions methodology. If the signal associated to the shocks one aims at identifying is strong enough, sign restrictions represent a powerful procedure to recover its macroeconomic effects (see also Paustian (2007) and Canova and Paustian (2011)). Unfortunately, monetary policy shocks are typically found to be of limited quantitative importance as for the U.S. output volatility. Therefore, the uncertain reaction of output to monetary policy shocks found with sign restrictions VARs may be due to an identification issue, more than representing a truly genuine empirical fact.[1]

We verify the robustness of our finding to a number of perturbations of our baseline exercise. These include alternative sets of sign restrictions, which enable us to identify other shocks (non-policy demand shocks, supply shocks); different sample-sizes of artificial data; different sets of constrained horizons as for our restrictions; and a different estimated DGP. In particular, our baseline exercise is conditional on a small-scale model à la King (2000), Woodford (2003), and Carlstrom, Fuerst, and Paustian (2009). As an alternative, we employ the larger-scale framework proposed by Smets and Wouters (2007). Our result turns out to be very robust to these departures from our benchmark exercise.

Previous papers have already investigated the ability of VARs to recover the true effects of structural shocks in DSGE frameworks. Alternative identification schemes featuring short-run zero-restrictions to recover the effects of monetary policy shocks are scrutinized by Canova and Pina (2005) and Carlstrom, Fuerst, and Paustian (2009). Christiano, Eichenbaum, and Vigfusson (2006) and Chari, Kehoe, and McGrattan (2008) assess the ability of truncated VARs to recover the true (DSGE-consistent) effects of a technology shock on hours worked. Fernández-Villaverde, Rubio-Ramírez, Sargent, and Watson (2007) derive a condition to ensure the existence of the VAR representation of a DSGE model. Ravenna (2007) discusses the assumptions needed for a finite order VAR representation of a DSGE model to exist, and shows that truncated VARs may produced severely biased impulse responses when the true DGP is an infinite order VAR. Our paper focuses on the ability of sign restrictions VARs to recover the effects of a monetary policy shock. It alternatively deals with two estimated, state-

[1] We note that, as a matter of fact, Uhlig (2005) never explicitly claims that his result is not consistent with conventional wisdom. What our paper shows is that models in line with conventional wisdom are very likely to produce an outcome like the one proposed by Uhlig (2005). The search for DSGE models *not* implying VAR reactions as those in Uhlig (2005), and which are therefore "rejected" by the data (as processed by such VARs), is left to future research.

of-the-art DSGE framework in the attempt to be as informative as possible as for the issue of interest, i.e., the interpretation of an uncertain output reaction to monetary policy shocks. Kilian and Murphy (2012) demonstrate that sign restrictions alone are insufficient to infer the responses of the real price of oil to demand and supply shocks in the market for crude oil. They show that the imposition of empirically plausible bounds on the magnitude of the short-run oil supply elasticity and on the response of real activity reduces the set of admissible model solutions to a smaller number of qualitatively similar estimates. Caldara and Kamps (2012) study the relevance of information coming from the imposition of boundaries on fiscal elasticities in SVARs dealing with fiscal policy shocks. Differently, we deal with monetary policy shocks whose identification can hardly rely on external information concerning elasticities. Paustian (2007) derives a sufficient condition for sign restrictions VARs to recover the correct sign of the response of a given variable to a shock of interest, and shows that the relative variance of such shock is relevant to correctly identify the effects of such shock. Canova and Paustian (2011) elaborate further on this issue by conductive extensive Monte Carlo simulations. Moreover, they show how sign restrictions may be employed to validate classes of DSGE models. Differently, this paper focuses on the macroeconomic effects of monetary policy shocks as measured by sign restrictions VARs, and shows that an uncertain reaction of output may be obtained even when conditioning on textbook monetary policy models that are suited to fit the U.S. quarterly data.

The paper develops as follows. Section 2 presents and estimates a standard new-Keynesian DSGE model with U.S. data. Such model is employed as a DGP in Section 3, which sets up our Monte Carlo experiment. Then, we contrast the impulse responses generated with our estimated DSGE with those coming from the sign-restriction VARs in a controlled environment. Section 4 collects our robustness checks. Section 5 concludes.

2 A DSGE model as DGP

This Section presents the DSGE model we will use as a DGP in our Monte Carlo simulations and its estimation.

2.1 Model presentation

We work with a standard three-equation DSGE model à la King (2000), Woodford (2003), and Carlstrom, Fuerst, and Paustian (2009). The log-linearized version of the model is presented in Table 1. Eq. (1) is an expectational new-Keynesian Phillips curve (NKPC) in which π_t stands for the inflation rate, β represents the discount factor, y_t identifies the output gap, whose impact on current inflation is influenced by the slope-parameter κ, α identifies indexation to past inflation, and ε_t^{π} represents a supply shock (e.g., a "price mark-up" shock); γ is the weight of the forward-looking component in the intertemporal IS curve (2); σ^{-1} is the households' intertemporal elasticity of substitution; ν is the inverse of the Frisch labor elasticity, and ε_t^{a} identifies a demand shock (e.g., a "technology" shock); τ_{π}, τ_y, and τ_R are policy parameters in the Taylor rule (3); the monetary policy shock ε_t^{R} allows for a stochastic evolution of the policy rate. All shocks $\varepsilon_t^{i}, i = (\pi, a, R)$ are assumed to follow mutually independent AR(1) processes that feature autocorrelation coefficients $(\rho_{\pi}, \rho_a, \rho_R)$, respectively. The standard deviation of the structural innovations $u_t^{i}, i = (\pi, a, R)$ are $(\sigma_{\pi}, \sigma_a, \sigma_R)$.

2.2 Model estimation

We estimate the model (1)-(4) with Bayesian methods. We work with quarterly U.S. data, sample: 1984:I-2008:II. This sample, which identifies a period of great moderation from a macroeconomic standpoint, does not include the bulk of the Volcker disinflation. We do so to avoid dealing with a phase of imperfect credibility by the Federal Reserve (Erceg and Levin (2003), Goodfriend and King (2005)). Our sample choice is also justified by our willingness to control for policy parameters' instability (Clarida, Galí, and Gertler (2000) and subsequent contributions); heteroskedasticity of the structural shocks (Justiniano and Primiceri (2008)); omitted variables as, e.g., real money balances, which may have played an important role in determining output in the 1970s (Castelnuovo (2012)); and instabilities concerning VARs estimated over the post-WWII and possibly due to the appointment of Paul Volcker as Federal Reserve Chairman in 1979 have been detected by Bagliano and Favero (1998), Boivin and Giannoni (2006), and Castelnuovo and Surico (2010). Our sample ends in 2008:II to exclude the acceleration of the financial crises began with the bankruptcy of Lehman Brothers in September 2008, which triggered non-standard policy moves by the Federal Reserve.

We employ three observables to estimate our model. The output gap is computed as log-deviation of the real GDP with respect to the potential output estimated by the Congressional Budget Office. The inflation rate is the quarterly growth rate of the GDP deflator. For the short-term nominal interest rate we consider the effective federal funds rate expressed in quarterly terms (averages of monthly values). The source of the data is the Federal Reserve Bank of St. Louis' website. The vector $\boldsymbol{\theta} = [\beta, \upsilon, \kappa, \alpha, \gamma, \sigma, \tau_\pi, \tau_y, \tau_R, \rho_a, \rho_\pi, \rho_R, \sigma_a, \sigma_\pi, \sigma_R]'$ collects the parameters characterizing the model. We set $\beta = 0.99$ and $\nu = 1$, a very standard calibration in the literature. The remaining priors are collected in Table 2. Details on the Bayesian algorithm employed to estimate our DSGE model are discussed in our Appendix.

Our posterior estimates are reported in Table 2. All the estimated parameters take quite conventional values. The parameters of the policy rule suggest an aggressive conduct to dampen inflation fluctuations, and a high degree of policy gradualism; the estimated degree of price indexation (posterior mean) is 0.09 (90% credible set: $[0.01, 0.17]$); the estimated weight of the forward looking component in the IS curve is 0.78 (90% credible set: $[0.70, 0.86]$). A comparison involving actual series and the

DSGE model's one-step ahead predictions confirms the very good-short term predictive power of our small-scale model (evidence provided in our Appendix).

3 Impulse responses: DSGE vs. SRVARs

This Section presents our Monte Carlo exercise and our main results.

3.1 Monte Carlo exercise

Our Monte Carlo experiments aim at comparing the true (DSGE-consistent) impulse responses with those produced with a VAR whose monetary policy shocks are identified with sign restrictions (SRVAR). We calibrate the vector of our estimated structural parameters $\boldsymbol{\theta}$ of the DSGE framework with our posterior means. Then, we compute the DSGE model-consistent impulse responses conditional on $\boldsymbol{\theta}$ to an unexpected nominal interest rate hike, and store them in the $[3xHxJ]$ $\boldsymbol{DSGE\_IRFs}$ matrix, which accounts for the $[3x1]$ vector of variables we focus on, the $h \in \{1, ..., H\}$ step-ahead responses of interest, and the $j \in \{1, ..., J\}$ draws of such responses. Subsequently, we run the following algorithm.

For $j = 1$ to J, we

1. feed our VARs with the artificial data $\boldsymbol{x}^j_{ps,[3:T]}$ (variables: inflation, output gap, nominal rate) generated with the DSGE model conditional on $\boldsymbol{\theta}$;
2. compute the impulse responses to a monetary policy shock with sign restrictions (as explained below);
3. store them in the $[3xHxJ]$ $\boldsymbol{SRVAR\_IRFs}$ matrix.

We set the number of repetitions $K = 1,000$, the horizon of the impulse response functions $H = 15$, and the length of the pseudo-data sample $T = 98$. This sample numerosity coincides with that of the actual data sample (1984:I-2008:II) that we employed to estimate our DSGE model. Monetary policy shocks are normalized to induce an on-impact equilibrium reaction of the nominal rate equivalent to 25 quarterly basis points. Our VAR include all endogenous state variables of the DSGE model. Moreover, the number of shocks equals the number of observables. Importantly, Ravenna (2007) and Carlstrom, Fuerst, and Paustian (2009) show that, in this case, a model like ours has got a VAR representation of order 2, which is the one we employ in our simulations. This is important, in that it ensures that our results are not affected by any truncation bias, at least in population.[2]

3.2 Sign restrictions

Sign restrictions represent a strategy to identify a structural shock in VAR analysis. In a nutshell, the idea is that of imposing *ex post* sign restrictions on a set of moments generated with the VAR, e.g., a set of impulse responses to a given shock. In our application, we estimate the reduced-form VAR coefficients $\boldsymbol{A}(L)$ and covariance matrix $\boldsymbol{\Lambda}$ from the data via OLS. Then, we orthogonalize the VAR residuals via an eigenvalue-eigenvector decomposition such that $\boldsymbol{\Lambda} = \boldsymbol{PHP}^T$, where $\boldsymbol{P}$ is the matrix of eigenvectors and $\boldsymbol{H}$ is the diagonal matrix of eigenvalues. The non-uniqueness of the MA representation of the VAR is exploited to provide a set of alternative proposals for the shock(s) of interest via the employment of three Givens rotation matrixes $\boldsymbol{Q}_{ij}(\omega)$, where $\omega \in (0, 2\pi)$, and $\boldsymbol{R} = \boldsymbol{Q}_{12}(\omega_1)\boldsymbol{Q}_{13}(\omega_2)\boldsymbol{Q}_{23}(\omega_3), \boldsymbol{RR}^T = \boldsymbol{I}_3$. The "impulse" matrix loading the VAR with candidate "shocks" is therefore given by $\widetilde{\boldsymbol{B}}(\boldsymbol{\omega}) = \boldsymbol{PD}^{1/2}\boldsymbol{R}(\boldsymbol{\omega})$. If the impulse responses to the "candidate" shock satisfy all the required restrictions, then the draw of the orthonormal vector $\boldsymbol{\omega}$ and the corresponding responses are retained. Otherwise, they are discarded. In so doing, we assign equal, strictly positive weight to

[2]Robustness checks dealing with the optimal choice of the VAR lag-length based on the Schwarz criterion delivered virtually identical results.

the draws we retain (those that meet our restrictions), and assign zero prior weight to those that violate our constraints. A non-exhaustive list of recent applications of the sign-restriction strategy to identify structural shocks includes Faust (1998), Canova and de Nicoló (2002), Peersman (2005), and Uhlig (2005). Rubio-Ramírez, Waggoner, and Zha (2010) propose an algorithm to compute rotations of the impulse matrix efficiently. Such algorithm works well also when the number of variables in the vector is large and several restrictions are imposed to identify more than one structural shock. Canova and Paustian (2011) propose an algorithm which derives a set of robust restrictions from a class of structural DSGE models that one may exploit to identify the shock(s) of interest with Vector Autoregressions. Fry and Pagan (2011) critically review the estimation of structural VARs with sign restrictions.

We identify the monetary policy shock by imposing "textbook" constraints on the impulse responses of inflation and the policy rate to a monetary policy shock. The signs to achieve identification are collected in Table 3. Such signs are robust in the sense of Canova and Paustian (2011), because they hold true for a variety of different calibrations of the parameters of interest (see our Appendix). Such constraints are imposed on the first $K = 2$ quarters, i.e., the one in which the shock occurs and the following one. This choice is in line with Uhlig's (2005), which sets $K = 5$ but deals with monthly (as opposed to quarterly) data. Importantly, we leave the reaction of output unconstrained in order to let the (artificial) data free to speak as for the effects of an unexpected interest rate hike (on output itself). Notice that, in this Monte Carlo exercise, the set of restrictions associated to the monetary policy shock only is sufficient to identify such shock. This is because an unexpected contractionary monetary policy move is the only shock able to generate an on-impact negative (conditional) correlation between the short-term policy rate and inflation according to our DGP.[3] Section 4 documents the

[3] There are important differences between this exercise and Uhlig's (2005). Uhlig (2005) also considers total reserves, non-borrowed reserves, and a commodity price index, which he exploits to identify

robustness of our results in presence of additional restrictions that identify two more shocks (price mark-up, technology).

3.3 Results

We recall our research question, which is:

> *"Suppose that the Data Generating Process is a standard DSGE framework in which monetary policy is not neutral. Would a VAR with sign restrictions imposed on the responses of inflation and the policy rate only be capable of uncovering the authentic reaction of output to a monetary policy shock?"*

Figure 1 depicts the impulse responses to a monetary policy shock obtained in our in lab-exercise. It collects ten randomly drawn realizations as well as pointwise 90% response intervals. The reaction of output turns out to be quite uncertain. Realizations suggesting a "boom" after a policy tightening are all but rare. Positive realizations do not only occur on impact, but also for a number of periods after the shock. When looking at this evidence, one could hardly interpret these monetary policy shocks as truly "contractionary". However, this VAR evidence is, *by construction*, consistent with a "textbook" transmission of a monetary policy shock. Interestingly, this VAR evidence occurs in spite of a positive short-run reaction of the policy rate. Notice that a large number of policy rate realizations go negative from the third quarter onward (an evidence in line with Uhlig, 2005). However, one may easily verify that the real interest rate stays positive along all the horizons considered here. Therefore, a negative long-run interest rate is not the explanation for our frequently positive response of output.

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monetary policy shocks with actual data. In contrast, our exercise deals with a world in which inflation, output, and the policy rate are the only relevant variables, and non-borrowed reserves are just left unmodeled. Another important difference regards the frequency of the data, which is monthly in Uhlig's case vs. quarterly in our exercise. Therefore, our exercise should be seen as inspired by Uhlig's (2005) findings, more than else.

As anticipated, the ambiguous reaction of output resembles the main finding in Uhlig (2005). He documents that, two times out of three, an unexpected policy tightening will move real GDP *up* on impact. Figure 2 documents the uncertainty surrounding the on-impact output reaction. Realizations are more in favor of a *positive* reaction of output, which goes against conventional wisdom. The number of positive realizations amounts to 61%, which is very close to the 2/3 figure proposed by Uhlig (2005).[4]

Figure 3 depicts the DSGE-consistent impulse responses and the median reactions computed over our $K = 1,000$ draws. The true reactions of inflation and output (red dashed lines with diamonds) to a monetary policy shocks are negative (the zero value is outside the estimated Bayesian 90% credible set, not shown here). This is not surprising, in light of the fact that the DSGE model (1)-(4) features a standard demand channel that implies a negative correlation between the real ex-ante interest rate and output conditional on monetary policy shocks. However, all the pointwise median reactions suggested by our structural VARs differ substantially from the true responses. In particular, the reaction of output is clearly wrongly signed, and persistently so.

A possible drawback of this exercise is the way in which we compute the median VAR responses. In general, the pointwise location measure one computes across different models do not necessarily deliver a model-(rotation-)consistent response. Fry and Pagan (2011) propose to select the impulse responses conditional on the retained rotation matrix which minimizes the distance with respect to the pointwise median. We document in our Appendix that our results are robust to this alternative way of

[4] Technically, we are plotting a density constructed by considering impulse responses conditional on 1,000 different randomly drawn datasets. In other words, we consider, per each given dataset, a single rotation meeting our sign restrictions. Clearly, a (much more time-consuming) alternative would be to consider, per each given database, a large number of rotations meeting our sign restrictions. A robustness check documented in our Appendix considers 1,000 different rotations (meeting our requirements) per each one of our 1,000 different samples. Our main result, i.e., the large uncertainty surrounding the reaction of output in a context in which the true DGP is a standard textbook model, turns out to be very robust to this perturbation of our benchmark case.

computing our impulse responses to a monetary policy shock.[5]

Importantly, what our simulations show is that, even conditional on a standard demand channel effectively being at work, an agnostic identification procedure like the one based on sign-restrictions *may* produce findings interpretable as support to the monetary neutrality hypothesis. This may happen (and, in our simulations, it does happen) even if shocks are not rare, but actually hit the economic system in each period. Moreover, our exercise shows that the evidence provided in Figures 1 and 2 is fully-consistent with a "textbook" AD-AS new-Keynesian model and the transmission of monetary policy impulses embedded in it. Of course, Uhlig's (2005) evidence is *not necessarily* the outcome of an exercise conducted by employing realizations generated by a DGP like ours. However, in light of our exercise, this is a possibility to consider.

3.4 Understanding the driver

What is the driver of our result? Our interpretation hinges upon the low relative contribution that monetary policy shocks exert on the variance of output. As shown in Paustian (2007) and Canova and Paustian (2011), sign restrictions work well when the "signal" of the shock of interest is strong enough.

Paustian (2007) provides a simple example to understand why the relative strength of a shock matters. Assume the structural model of a given economy to be the following:

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} \underset{+}{d_{11}} & \underset{-}{d_{12}} \\ \underset{+}{d_{21}} & \underset{+}{d_{22}} \end{bmatrix} \begin{bmatrix} u_{yt} \\ u_{xt} \end{bmatrix}, \mathbf{\Sigma} = \begin{bmatrix} \sigma_y^2 & 0 \\ 0 & \sigma_x^2 \end{bmatrix}.$$

The econometrician imposes some sign restrictions to identify the two shocks u_{yt} and u_{xt}, whose variance covariance is $\mathbf{\Sigma}$. In particular, she imposes

[5] We focus on the pointwise median as opposed to the pointwise mean or the trimmed pointwise mean because of its larger precision in this context (see Canova and Paustian (2011)). As proposed by Liu and Theodoridis (2012), an alternative choice (not entertained here) would be to select the unique rotation matrix which minimizes the distance between the on-impact VAR pointwise medians and the on-impact responses predicted by a (possibly misspecified) DSGE model.

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} \underset{+}{m_{11}} & \underset{-}{m_{12}} \\ \underset{+}{m_{21}} & \underset{?}{m_{22}} \end{bmatrix} \begin{bmatrix} \widehat{u}_{yt} \\ \widehat{u}_{xt} \end{bmatrix}.$$

Notice that the sign of the impact of the shock $\widehat{u}_{xt}$ on the variable x_t is not imposed. Can the econometrician recover such sign correctly? One may work out the equations implied by the system $\boldsymbol{D\Sigma D'} = \boldsymbol{MM'}$, where the matrix $\boldsymbol{D}$ contains the structural parameters of the economy, while the matrix $\boldsymbol{M}$ is a rotation matrix satisfying the three signs imposed by the econometrician. Paustian (2007) shows that there exist a critical value $\sigma^2 \equiv \sigma_x^2/\sigma_y^2 = -d_{11}d_{21}(d_{12}d_{22})^{-1}$ such that, for $\sigma_x^2 > \sigma^2$, the sign of the impact of u_{xt} on x_t is uniquely determined by the three restrictions imposed by the econometrician. If, instead, $\sigma_x^2 \leq \sigma^2$, then such sign is not pinned down without further assumptions on the values of the structural parameters of the economy.

This condition tells us that the relative strength of the signal of a shock is important to identify the effects of such shock correctly. According to our estimated DSGE model, the contribution of policy shocks in explaining the variance of the forecast errors of our variables is low, i.e., it amounts to about 10%, 4%, and 21% as for inflation, output, and the policy rate, respectively (figures referring to the variance of one year-ahead forecast errors). In this case, what happens is that the estimated "shock" is actually a combination of all the true, structural disturbances hitting the economic system. Formally, $\widehat{u}_t^R = \phi_R^R u_t^R + \phi_R^\pi u_t^\pi + \phi_R^a u_t^a$, where $\widehat{u}_t^R$ is the monetary policy shock estimated by the sign restrictions VAR, and $\phi_R^i, i = (\pi, a, R)$ are the loadings taken by the true structural shocks. The larger the standard deviation of the shock one is after (the monetary policy shock, in our context) with respect to the remaining ones, the smaller the values of the loadings associated to the remaining shocks (ϕ_R^π and ϕ_R^a in our case).

Two exercises are conducted to give support to this interpretation. The first one re-runs our Monte Carlo exercise by switching some shocks off.[6] Figure 4 depicts the

[6] Technically, we do so by setting the standard deviations of the shocks we aim at switching off to

reaction of output in four alternative scenarios. The first one is our baseline scenario, which is represented by the top-left panel. The second one is a scenario in which the demand shock is suppressed (top-right panel). The outcome of this experiment should be judged by contrasting it to the baseline case (depicted in the top-right panel to ease comparability). It is possible to see that things go even worse than in the baseline scenario, in that the reaction of output as suggested by the VAR is even more distant than the one in the benchmark case. This suggests that i) the demand shock enters the linear combination which determines $\widehat{u}_t^R$, and ii) the loading ϕ_R^a is negative. In other words, $\widehat{u}_t^R$ picks up also the dynamics of output after a negative demand shock. An interesting scenario is that in which the supply shock is muted (bottom-left panel). It is easy to see that i) the reaction of output is largely negative; ii) it suggests in fact a deeper recession than the true one. Therefore, $\widehat{u}_t^R$ picks up also the effects of a negative supply shock on output, and such shock is responsible for the positive reaction of output in our benchmark simulations. Finally, and as expected, when the monetary policy shock is made the sole responsible for the macroeconomic volatilities in our economy (bottom-right panel), our VARs perfectly recover the true effects of a monetary policy shock. This is not surprising, because shutting down the volatilities of the (non-policy) demand and supply shocks is equivalent to imposing $\phi_R^\pi = \phi_R^a = 0$. Consequently, $\widehat{u}_t^R$ perfectly recovers the conditional correlations induced by the true monetary policy shock u_t^R.

We conduct a second exercise to support the role of monetary policy shocks' signal in our VAR context. In this exercise, the standard deviation of the monetary policy shock in our DSGE model (otherwise calibrated with our posterior means) is counterfactually inflated. Figure 5 collects the results of our simulations. Evidently, the stronger the signal, the more precise the estimation of the median effect of the monetary policy shock on output. An increase of 25% of the standard deviation of output leads to

$10^{-3} > 0$ to avoid singularity issues.

a substantial improvement in terms of (reduction of the) distance between the true response and that implied by our VAR estimates. Nevertheless, the sign of output is still wrong, also on impact. An increase of 50% of the standard deviation leads to an appreciable reduction in the "distortion", with the median reaction of output being correctly signed for the first two quarters. A (dramatic) increase of 400% of the standard deviation of the monetary policy shock in our estimated DSGE model leads to a spectacular performance by sign restrictions. Consistently, the percentage of wrongly signed output realizations falls as the signal becomes stronger. The responses of output, on impact, are wrongly signed in 46% of the cases when the standard deviation of the policy shock is scaled up by 25%, 37% of the times when it is scaled up by 50%, and just 2% of the times when the shock's standard deviation is multiplied by a factor of 5. However, this scenario features a counterfactually strong monetary policy shock, which is responsible of about 75%, 52%, and 87% of the forecast error variance of inflation, output, and the policy rate (again, these figures refer to the variance of the four-step ahead forecast errors). According to the estimates available in the literature, this is a different world with respect to the U.S. economy. Uhlig (2005) finds monetary policy shocks to be responsible of about 5-10% for the variations in real GDP at all horizons as for the period 1965-2003, monthly data. Similar estimates are proposed by Smets and Wouters (2007) with their DSGE structural analysis dealing with quarterly data, sample 1957-2004, and by Justiniano, Primiceri, and Tambalotti (2010), who analyze a similar sample. Christiano, Eichenbaum, and Evans (2005) analyze the sample 1965-1995 with a Cholesky-VAR and document a larger contribution of monetary policy shocks on output variation of about 38% (two-year ahead forecast error). However, the authors themselves advise to treat this conclusion with caution, in that the uncertainty surrounding this figure is large - the 5th percentile of the distribution suggests a much smaller contribution of about 15%. With the same methodology, Altig, Christiano, Eichenbaum, and Lindé (2011) find monetary policy shocks to be responsible for about 9% of the movements of the real GDP at business cycle frequencies in the sample period 1982-2008. Justiniano and Primiceri (2008) document the time-dependence of the contribution of such shock to output growth, but the largest realizations, which are estimated to occur in the mid-1970s and early-1980s, never exceed 15% (median values).[7]

Wrapping up, our simulations show that a particular set of sign restrictions imposed on VAR impulse responses may generate a very uncertain and often positive responses of output even in a world in which such responses are in line with conventional wisdom. This may occur due to the weakness of the signal associated to the policy shocks. Therefore, Uhlig's (2005) evidence if consistent with monetary policy shocks having the power of affecting the business cycle.

One should bear in mind that Uhlig's (2005) result is obtained by employing a longer sample (1965-2003), modeling extra variables such as total reserves, non-borrowed reserves, and a commodity price index, and employing monthly observations. Importantly, in our Appendix we show that this VAR evidence can be obtained also by estimating a trivariate VAR modeling GDP deflator inflation, the CBO output gap, and the federal funds rate with U.S. quarterly data, 1984:I-2008:II. Hence, our simulations replicate an empirical result which appears to be quite robust as for the U.S. economy.

4 Robustness checks

We verify the robustness of our results to a number of variations of our baseline exercise. These variations consider the identification of non-policy shocks on top of the monetary policy disturbance, a sample size much larger than the one employed in our

[7]Of course, different modeling assumptions may lead to different results as for the macroeconomic effects of monetary policy shocks. Faust (1998) shows that, if one is willing to search for a prior that places the largest possible mass on the impulse vector that explain the largest share of output variation, some 86% of the variance of output may be attributed to monetary policy shocks.

baseline simulations, a different number of periods during which our sign restrictions are imposed, and a different structural model as our DGP. We analyze these perturbations in turn.

Identification of extra-shocks. Our DSGE model (1)-(4) features three shocks, i.e., a monetary policy shock, a supply (mark-up) shock directly influencing the inflation rate, and a demand (technology) shock that affects the output gap in first place. In our baseline exercise, we appeal to restrictions regarding the monetary policy shock only. Paustian (2007) and Canova and Paustian (2011) suggest to use as many restrictions as possible to identify the effects of a given shock and distinguish it with respect to other disturbances in the economy. We then identify also the non-policy demand shock and the supply shock by imposing "textbook" sign-restrictions (consistent with our DSGE model) as indicated in Table 2.

Figure 6 (top-left panel) shows the density of our on-impact output responses obtained by adding these extra-sign restrictions. The reaction of output remains quite uncertain. The median value of the distribution is 0.32, i.e., the on-impact reaction of output is, once again, wrongly signed. As in our baseline case, the evidence is dramatically different when moving to an alternative world in which the standard deviation of the monetary policy shock is five times larger. In this latter case, which is depicted in the top-right panel of Figure 6, the signal associated to the policy shock is strong enough, and the true effects on output are, as a matter of fact, correctly recovered not only on impact, but also as for the entire span of interest.

Sampling uncertainty. The analysis conducted so far involves two types of uncertainties. One is the so called "identification uncertainty", which regards the ability of sign restrictions *per se* to recover the true effects of monetary policy shocks on output (see our discussion in Section 3.4). Such uncertainty refers to the inability of the sign restrictions VARs to recover the true macroeconomic effects of a monetary policy shock.

The other one is the "sampling uncertainty" which is typically faced by an econometrician endowed with a small sample. The sample-size we used in our analysis is the typical sample size employed in macroeconometric analysis of the U.S. data during the great moderation. Of course, it is of interest to understand if an econometrician endowed with a much larger sample would do a better job in recovering the true effects of a monetary policy shock on output. Figure 6 (central-left panel) plots the results of a simulation in which the sample-size equals 100,000 observations. Evidently, the uncertainty surrounding the reaction of output is much smaller. This implies that part of our baseline result is indeed due to sample uncertainty. However, some 61% of the on-impact reactions of output are positive, and the on-impact median reaction reads 0.09. Again, a quite different picture emerges when the signal is made much stronger (Figure 6, central-right panel), with the dynamics of output being correctly recovered by our VARs, and the share of positive reaction of output dramatically falling down to zero (not shown).

Number of constrained horizons. When working with sign restrictions, one of the key-choices is how many restrictions to place per each given shock/variable. Our baseline choice is $K = 2$, i.e., two periods (including the one in which the shock realizes). It is therefore of interest to check if our results are sensitive to a variation of K. We then set $K = 6$, and re-run our experiments. Figure 6 (bottom panels) shows that our results are robust to this perturbation. As a matter of fact, in the scenario in which the contribution of the monetary policy shock is counterfactually boosted up, the reaction of output replicates the true one just perfectly. This last result may be easily interpret in light of the fact that $K = 6$ is actually consistent with the true dynamics of output in response to a monetary policy shock in the DSGE model.

Smets and Wouters (2007) model as DGP. Our Monte Carlo results are conditional on a set of assumptions, the most important one possibly being that of the DGP in place. Small-scale models like the one we employ in our baseline analysis have proved to be useful also for empirical investigations. However, they miss to consider a number of nominal and real frictions which may be relevant to model the transmission of a monetary policy impulse to the real side of the economy. Hence, we consider an alternative framework that is (one of those) currently used by central banks and research institutes to perform policy analysis, i.e., the Smets and Wouters (2007) model (see, for similar frameworks, Christiano, Eichenbaum, and Evans (2005) and Justiniano and Primiceri (2008)). We estimate this model with U.S. data, sample 1984:I-2008:II. (Details on the structure of the model, the data employed to estimate it, our Bayesian estimation, and our posterior estimates can be found in our Appendix.) Then, we calibrate such model with our posterior means and conduct our Monte Carlo experiment. Notice that, as in the case of the small-scale model, the *short-run* restrictions imposed on the responses of inflation and the policy rate are *theoretically sufficient* to achieve the identification of the monetary policy shock. This is so because, out of the seven shocks in Smets and Wouters' (2007) model, three of them (TFP shock, price mark-up shock, wage mark-up shock) induce a policy trade-off that implies a positive correlation between inflation and the policy rate in the short run. Other three shocks (risk-premium shock, investment shock, and fiscal spending shock) act as "demand" shocks, which also induce a positive correlation between inflation and the nominal interest rate in the first quarters after the shock. Therefore, the only shock leading to a negative conditional correlation between inflation and the federal funds rate in the short run is the monetary policy shock.

Figure 7 (top panels) shows the outcome of our exercise. Again, when sticking to the baseline calibration, the average reaction of output (here expressed in growth rates) is at odds with respect to the conventional view.[8] Some 70% of the realizations of the

[8] We model our artificial data with a VAR(2). Our results are robust to the employment of a variety of alternative VAR(p) models, with p ranging from 3 to 16. Our qualitative message remains unchanged when employing either the log-level of output or the model-consistent output gap in place of the growth rate of output in our VARs.

distribution of the on-impact response of output to a monetary policy shock suggest a positive reaction. But, exactly as in the case of the small-scale model, this is a result due to the low strength of the signal associated to the monetary policy shock. If such strength is counterfactually increased, the picture changes drastically once again. In particular, the average reaction of output as suggested by the VAR analysis nicely lines up with the true reaction as predicted by the Smets-Wouters model.

Unfortunately, as already pointed out when dealing with the small-scale version of the DSGE framework, the magnified standard deviation of the monetary policy shock we deal with is clearly counterfactual. The estimated Smets and Wouters (2007) model conditional on our great moderation sample implies a contribution of the monetary policy shocks to output growth of about 5%, a result in line with Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). To have a sense of the likelihood of a larger contribution of such shock to the volatility of output, we conduct an alternative estimation in which we set the prior mean of the standard deviation of the policy shock to 0.60, a value five times larger than the one estimated in our baseline case. Moreover, we set the standard deviation of the $\mathcal{IG}$ distribution of such standard deviation of the policy shock to 0.25, much smaller than our baseline calibration (that is, 2). Our estimate of the standard deviation of the policy shock turns out to be larger, i.e., 0.16, and the contribution of such shock to the volatility of output is estimated to be twice as large, i.e., almost 10%. However, we also record a drop of the marginal likelihood of about 20 log-points, which indicates a much worse fit of the model, overall. According to our estimates, the scenario that should take place in order to have the VAR able to recover the true effects of a monetary policy shock is just very unlikely to occur.

The reason why we get a pointwise median reaction of output which is very different with respect to the one suggested by the Smets-Wouters mode in the baseline scenario is that, again, the signal associated to the monetary policy shock is weak. Therefore, the reaction of output is due to a combination of shocks, not only to the monetary policy shock. Figure 8 plots the medians obtained by simulating the Smets-Wouters model in different scenarios characterized by the absence of one or more structural shocks. Shocks to the TFP, risk-premium, fiscal condition, and investment appear to exert a quantitatively negligible impact on the reaction of output. Differently, two supply shocks, i.e., those to the price and wage mark-ups, are clearly important drivers of such a reaction. In particular, when shutting the price mark-up shock off, the VAR gets the on impact sign right (but it overstates the impact of the policy shock), and it correctly captures the dynamic response over the horizons of interest. The wage-mark up shock clearly dampens the pointwise median response. In other words, these two shocks importantly affect the estimation of the effects of policy shocks on output one can obtain with sign restrictions VARs. When jointly shutting these two shocks down, we actually obtain a negative reaction of output which overstates the true one (Figure not shown here for the sake of brevity). This implies that the reaction of output is demand shocks also enter the linear combination of structural shocks which is interpreted by our VARs as pure monetary policy shocks, and they do so acting as negative demand shocks (on aggregate). Finally, and not surprisingly, when the true economy features the monetary policy shock only, the VAR is perfectly able to recover the true effects of a monetary policy innovation.

Restrictions on the response of output. An obvious way to fix the distortion affecting the response of output to a monetary policy shock would seem to be that of placing a sign restriction on the reaction of output. Of course, this is somewhat problematic for our study, whose aim is to offer a possible interpretation of the conditional correlation proposed by Uhlig (2005) that hinges upon the choice of *not* imposing such sign restriction on output - in general, if one wants to stay as agnostic as possible as regards the response of output, it would seem natural not to impose any restriction on its reaction to the shock of interest. However, to have a sense of the impact of such a possible restriction, we go back to our small-scale model, produce artificial data, and ask our rotation matrices to return a non-positive reaction of output (on top of a non-negative reaction of the policy rate and a non-positive reaction of inflation) for the first $K = 2$ horizons. Therefore, a negative output reaction to an unexpected monetary policy tightening is now an assumption - and not a result - in the very short run.

Figure 9 documents the outcome of our exercise. Two scenarios are proposed. The first one - top row panels - employs a calibration for our DGP in line with our estimated posterior means. An interesting result emerges. The reaction of output is estimated to be non positive for the first five periods, i.e., a longer horizon than the one involved by our sign restrictions. This is not entirely surprising, in light of the fact that we are dealing with a VAR that well captures the persistence of the series. In other words, the "initial conditions" dictated by our sign restrictions matter for periods over those of the imposition of the signs, and work in favor of reducing the wedge between the true output response and the one estimated by the VAR. Said so, evident discrepancies between the true DSGE-based responses and those estimated with our VARs are still present. As shown by the panels at the bottom of Figure 9, in a counterfactual world in which monetary policy shocks' contribution to output volatility is (substantially) inflated, we are instead able to recover the correct reaction of output.

5 Conclusions

A standard new-Keynesian DSGE model of the business cycle featuring monetary policy non-neutrality is estimated with quarterly U.S. data. It is then used in a Monte Carlo exercise to generate artificial data with which VARs are estimated. Sign restrictions are imposed to identify the effects of a monetary policy shock with such VARs. We replicate Uhlig's (2005) evidence on the ambiguous effects of a contractionary monetary policy shock on output. We show that this result is due to the weak signal associated to the policy shock in this environment (i.e., to the low contribution of such shock to the volatility of output). Sign restrictions are shown to correctly identify the negative effects on output in an alternative world in which the share of output variance explained by monetary policy shocks is counterfactually magnified. Our results reconcile Uhlig's (2005) evidence with the conventional view on the real effects of monetary policy shocks.

After stating what this paper is about, it is worth pointing out what this paper is *not* about. This paper does *not* represent, in any manner, a "rejection" of Uhlig's (2005) empirical findings. If anything, it is quite the opposite. Uhlig's (2005) empirical result is very intriguing because it is obtained with a clean VAR-based econometric investigation. As stressed by Uhlig (2012), the challenge is that of understanding why that result is there and what it implies as for macroeconomic modelling. Our exercise suggests that a researcher who believes in monetary policy non-neutrality should *expect* to get empirical results in line with Uhlig's (2005) when dealing with sign-restriction VARs that do not impose any constraints on the response of output to a monetary policy shock.

Our paper contains a suggestion to practitioners working with sign restrictions. Canova and Paustian (2011) suggest to use robust sign restrictions to identify shocks of interest with VAR estimated with actual data, which can then be exploited to assess the ability of DSGE models to replicate the VAR responses to such identified shocks. In light of our findings, the comparison between DSGE responses and VAR responses may be problematic in presence of shocks whose signals are weak. Our suggestion is to compare the VAR responses computed with actual data to the VAR responses computed with artificial data generated with DSGE models and identified via the same set of sign restrictions. In other words, our suggestion is to use the class of DSGE models one is interested into not only to isolate robust sign restrictions, but also to form a correct a-priori on what a VAR exercise run with actual data may actually deliver in terms of dynamic responses to the shocks of interest.

References

ALTIG, D., L. J. CHRISTIANO, M. EICHENBAUM, AND J. LINDÉ (2011): "Firm-Specific Capital, Nominal Rigidities and the Business Cycle," *Review of Economic Dynamics*, 14(2), 225–247.

BAGLIANO, F. C., AND C. A. FAVERO (1998): "Measuring monetary policy with VAR models: An evaluation," *European Economic Review*, 42, 1069–1112.

BOIVIN, J., AND M. GIANNONI (2006): "Has Monetary Policy Become More Effective?," *Review of Economics and Statistics*, 88(3), 445–462.

CALDARA, D., AND C. KAMPS (2012): "The Analytics of SVARs: A Unified Framework to Measure Fiscal Multipliers," Finance and Economics Discussion Series, Federal Reserve Board, Paper Number 2012-20.

CANOVA, F., AND G. DE NICOLÓ (2002): "Monetary Disturbances Matter for Business Fluctuations in the G-7," *Journal of Monetary Economics*, 49, 1131–1159.

CANOVA, F., AND M. PAUSTIAN (2011): "Business cycle measurement with some theory," *Journal of Monetary Economics*, 58, 345–361.

CANOVA, F., AND J. P. PINA (2005): "What VAR Tell Us About DSGE Models?," in Diebolt. C. and Kyrtsou C. (Eds): New Trends In Macroeconomics, Springer Verlag, New York, NY, 89-124.

CARLSTROM, C., T. FUERST, AND M. PAUSTIAN (2009): "Monetary Policy Shocks, Choleski Identification, and DNK Models," *Journal of Monetary Economics*, 56(7), 1014–1021.

CASTELNUOVO, E. (2012): "Estimating the Evolution of Money's Role in the U.S. Monetary Business Cycle," *Journal of Money, Credit and Banking*, 44(1), 23–52.

CASTELNUOVO, E., AND P. SURICO (2010): "Monetary Policy Shifts, Inflation Expectations and the Price Puzzle," *Economic Journal*, 120(549), 1262–1283.

CHARI, V., P. J. KEHOE, AND E. R. MCGRATTAN (2008): "Are Structural VARs with Long-Run Restrictions Useful in Developing Business Cycle Theory?," *Journal of Monetary Economics*, 55, 1337–1352.

CHRISTIANO, L., M. EICHENBAUM, AND C. EVANS (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

CHRISTIANO, L. J., M. EICHENBAUM, AND R. VIGFUSSON (2006): *Assessing Structural VARs*. Daron Acemoglu, Ken Rogoff, and Michael Woodford (Eds.): NBER Macroeconomics Annual, MIT Press, Cambridge.

Clarida, R., J. Galí, and M. Gertler (2000): "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics*, 115, 147–180.

Erceg, C., and A. Levin (2003): "Imperfect Credibility and Inflation Persistence," *Journal of Monetary Economics*, 50(4), 915–944.

Faust, J. (1998): "The robustness of identified VAR conclusions about money," *Carnegie Rochester Conference Series on Public Policy*, 49, 207–244.

Fernández-Villaverde, J., J. F. Rubio-Ramírez, T. J. Sargent, and M. W. Watson (2007): "ABCs (and Ds) of Understanding VARs," *American Economic Review*, 97(3), 1021–1026.

Fry, R., and A. Pagan (2011): "Sign Restrictions in Structural Vector Autoregressions: A Critical Review," *Journal of Economic Literature*, 49(4), 938–960.

Goodfriend, M., and R. G. King (2005): "The Incredible Volcker Disinflation," *Journal of Monetary Economics*, 52(5), 981–1015.

Justiniano, A., and G. Primiceri (2008): "The Time-Varying Volatility of Macroeconomic Fluctuations," *American Economic Review*, 98(3), 604–641.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2010): "Investment shocks and business cycles," *Journal of Monetary Economics*, 57(2), 132–145.

Kilian, L., and D. Murphy (2012): "Why Agnostic Sign Restrictions Are Not Enough: Understanding the Dynamics of Oil Market VAR Models," *Journal of the European Economic Association*, forthcoming.

King, R. G. (2000): "The New IS-LM Model: Language, Logic and Limits," *Federal Reserve Bank of Richmond Economic Quarterly*, 86(3), 45–103.

Liu, P., and K. Theodoridis (2012): "DSGE Model Restrictions for Structural VAR Identification," *International Journal of Central Banking*, forthcoming.

Paustian, M. (2007): "Assessing Sign Restrictions," *The B.E. Journal of Macroeconomics*, 7:1 (Topics), Article 23, 1–31.

Peersman, G. (2005): "What Caused the Early Millennium Slowdown? Evidence Based on Vector Autoregressions," *Journal of Applied Econometrics*, 20, 185–207.

Ravenna, F. (2007): "Vector Autoregressions and Reduced Form Representations of DSGE Models," *Journal of Monetary Economics*, 54, 2048–2064.

Rubio-Ramírez, J. F., D. F. Waggoner, and T. Zha (2010): "Structural Vector Autoregressions: Theory of Identification and Algorithms for Inference," *Review of Economic Studies*, 77, 665–696.

Smets, F., and R. Wouters (2007): "Shocks and Frictions in US Business Cycle: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

Uhlig, H. (2005): "What are the Effects of Monetary Policy? Results from an Agnostic Identification Procedure," *Journal of Monetary Economics*, 52, 381–419.

——— (2012): "Economics and Reality," *Journal of Macroeconomics*, 34, 29–41.

WOODFORD, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press. Princeton, New Jersey.

Variables
π_t: Inflation, y_t: Output gap, R_t: Nominal interest rate

Log-linearized equations	
$\pi_t = \xi_1 E_t \pi_{t+1} + \xi_2 \pi_{t-1} + \xi_3 y_t + \xi_4 \varepsilon_t^\pi$	(1)
$y_t = \varphi_1 E_t y_{t+1} + \varphi_2 y_{t-1} - \varphi_3 (R_t - E_t \pi_{t+1}) + \varphi_4 \varepsilon_t^a$	(2)
$R_t = (1 - \tau_R)(\tau_\pi \pi_t + \tau_y y_t) + \tau_R R_{t-1} + \varepsilon_t^R$	(3)
$\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + u_t^i, u_t^i \sim N(0, \sigma_i^2), i \in \{\pi, a, R\}$	(4)

Compound parameters
$\xi_1 \equiv \beta \xi_4, \xi_2 \equiv \alpha \xi_4, \xi_3 \equiv \kappa \xi_4, \xi_4 \equiv (1 + \alpha\beta)^{-1}$, $\varphi_1 \equiv \gamma, \varphi_2 \equiv (1 - \gamma), \varphi_3 \equiv \sigma^{-1}, \varphi_4 \equiv \frac{(\rho_a - 1)(1+\nu)}{(\sigma+\nu)}$

Calibrated parameters
Discount factor: $\beta = 0.99$; Inverse of the Frisch labor elasticity: $\nu = 1$

Table 1: **Description of the small-scale DSGE Model - Log-linearized Equations.** *The definitions of the structural parameters are given in this Table and Table 2.*

Param.	*Interpretation*	*Priors*	*Posterior Means* [5h,95th]
β	Discount factor	*Calibrated*	0.99 [−]
υ^{-1}	Frisch elasticity	*Calibrated*	1 [−]
κ	NKPC, slope	$\mathcal{N}(0.1, 0.015)$	0.12 [0.10,0.14]
α	Price indexation	$\mathcal{B}(0.5, 0.2)$	0.09 [0.01,0.17]
γ	IS, forw. look. degree	$\mathcal{B}(0.5, 0.2)$	0.78 [0.70,0.86]
σ	Inverse of the IES	$\mathcal{N}(3, 1)$	5.19 [3.95,6.45]
τ_π	T. Rule, inflation	$\mathcal{N}(1.5, 0.3)$	2.21 [1.85,2.56]
τ_y	T. Rule, output gap	$\mathcal{G}(0.3, 0.2)$	0.16 [0.05,0.25]
τ_R	T. Rule, inertia	$\mathcal{B}(0.5, 0.285)$	0.81 [0.77,0.86]
ρ_a	AR tech. shock	$\mathcal{B}(0.5, 0.285)$	0.89 [0.84,0.94]
ρ_π	AR cost-push shock	$\mathcal{B}(0.5, 0.285)$	0.98 [0.97,0.99]
ρ_R	AR mon. pol. shock	$\mathcal{B}(0.5, 0.285)$	0.43 [0.30,0.56]
σ_a	Std. tech. shock	$\mathcal{IG}(1.5, 0.2)$	1.50 [1.10,1.91]
σ_π	Std. cost-push. shock	$\mathcal{IG}(0.35, 0.2)$	0.09 [0.07,0.11]
σ_R	Std. mon. pol. shock	$\mathcal{IG}(0.35, 0.2)$	0.14 [0.12,0.15]

Table 2: **Bayesian estimates of the DSGE model.** *1984:I-2008:II U.S. data. Prior densities: Figures indicate the (mean,st.dev.) of each prior distribution. Legend: (N, B, G, IG) stand for (Normal, Beta, Gamma, Inverse Gamma) densities. Posterior densities: Figures reported indicate the posterior mean and the [5th,95th] percentile of the estimated densities. Details on the estimation procedure provided in the text.*

Shocks\Imposed signs	π	y	R
MP shock	$\leqslant$		$\geqslant$
Supply shock	$\geqslant$	$\leqslant$	$\geqslant$
Demand shock	$\leqslant$	$\leqslant$	$\leqslant$

Table 3: **Sign restrictions to achieve structural shocks' identification.** *Identification in the baseline case achieved by imposing restrictions for K=2 and as for the monetary policy shock only. Alternative scenarios discussed in the text.*

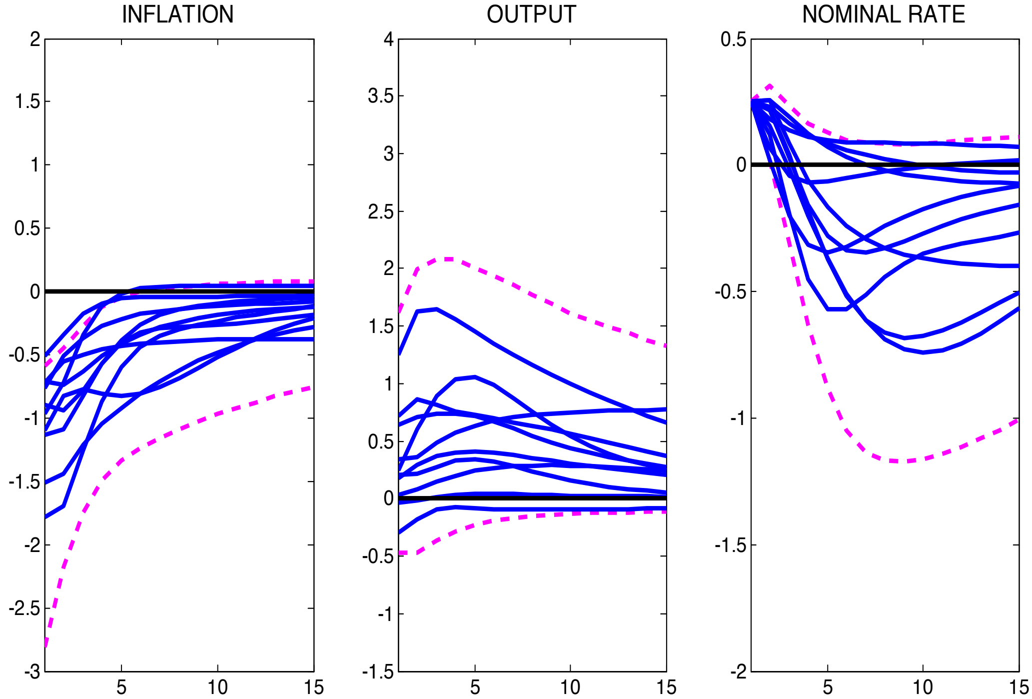

Figure 1: **Impulse response functions to a monetary policy shock identified with sign restrictions.** *Realizations conditional on sign restrictions imposed for K=2 and concerning the monetary policy shock only. Blue solid lines represent 10 randomly selected impulse responses meeting the imposed sign restrictions. Dashed magenta lines identify the 5th and 95th percentiles of the distribution. Figure based on 1,000 draws.*

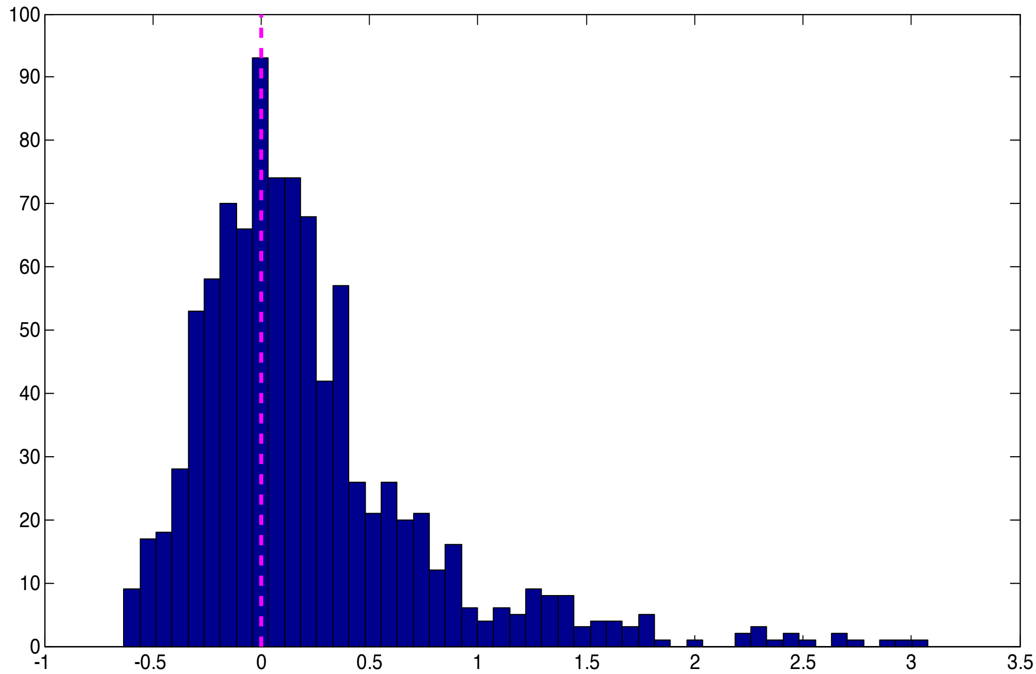

Figure 2: **On impact impulse response function of output to a monetary policy shock.** *Realizations conditional on sign restrictions imposed for K=2 and concerning the monetary policy shock only. On impact realizations (i.e., at horizon 0) only. Outliers excluded by trimming the realizations not belonging to the [2.5th,97.5th] percentiles interval out. Figure based on 1,000 draws.*

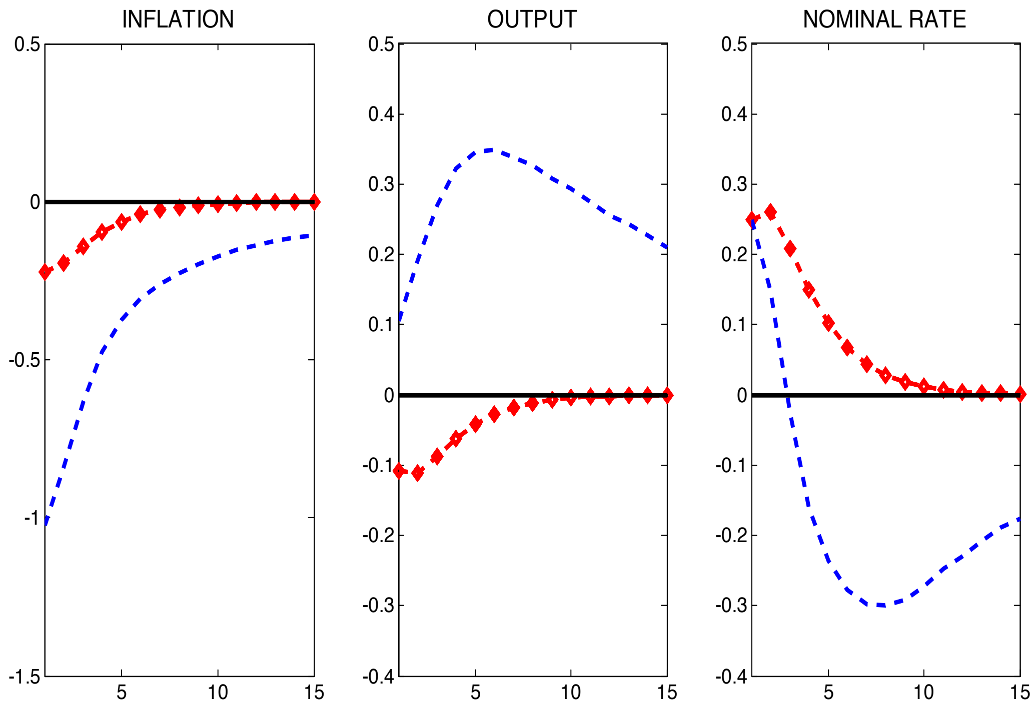

Figure 3: **Impulse responses to a monetary policy shock: DSGE vs. VAR.** *Realizations conditional on sign restrictions imposed for K=2 and concerning the monetary policy shock only. Red dashed lines with diamonds identify the reaction to a monetary policy shock conditional on the DSGE model calibrated with posterior-mean values. Blue dashed lines represent the median response across all the VAR impulse responses meeting the imposed sign restrictions.*

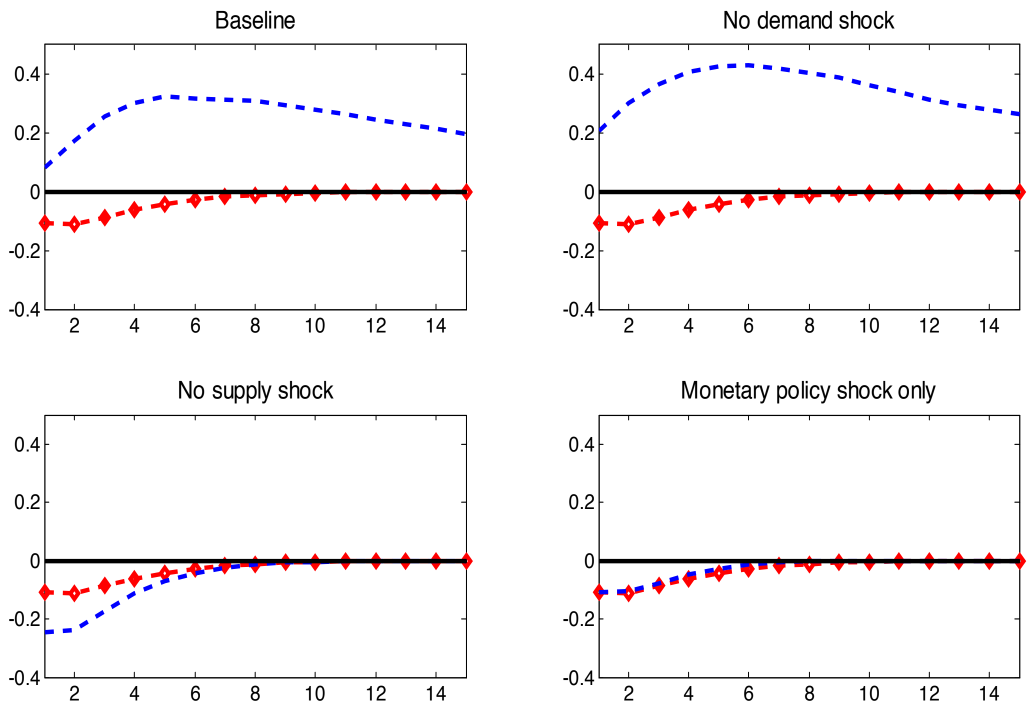

Figure 4: **Output response to a monetary policy shock: Selections of structural shocks.** *Top-left panel: Baseline case. Top-right panel: Simulations conditional on the monetary policy and supply shocks only. Bottom-left panel: Simulatios conditional on the monetary policy and demand shocks only. Bottom-right panel: Simulations conditional on the monetary policy shock only. Realizations conditional on sign restrictions imposed for K=2 and concerning the monetary policy shock only. Blue dashed lines represent the median response across all the VAR impulse responses meeting the imposed sign restrictions. Red dashed lines with diamonds identify the reaction to a monetary policy shock conditional on the DSGE model calibrated with posterior-mean values. Figure based on 1,000 draws.*

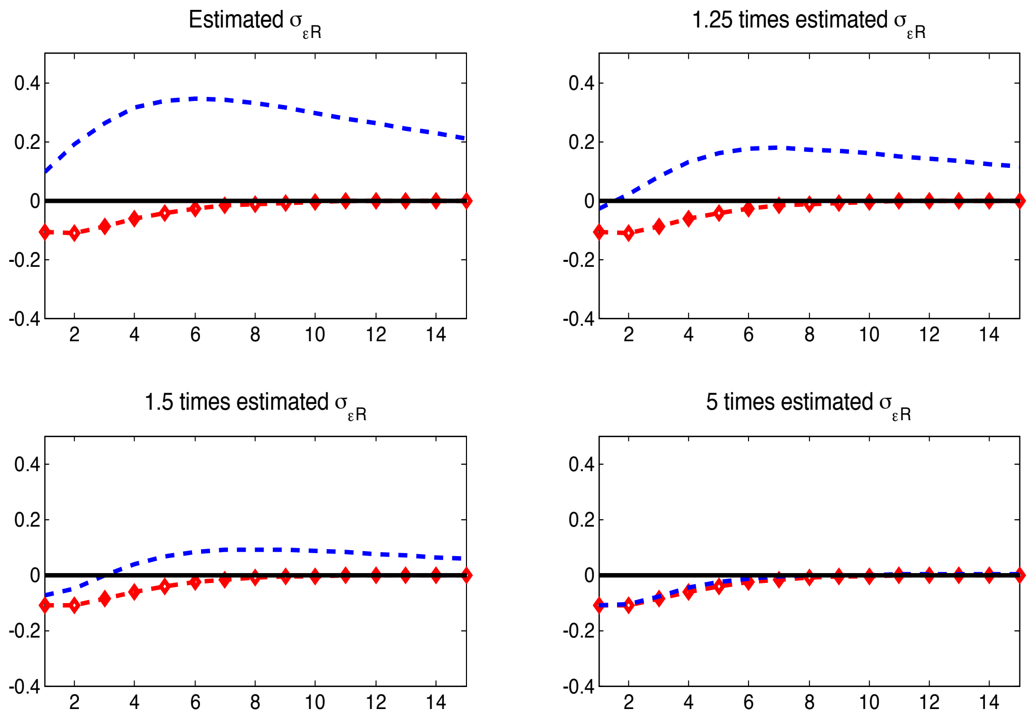

Figure 5: **Output response to a monetary policy shock: Role of the signal.** *Standard deviations of the monetary policy shock in the DGP increased by 25%, 50%, and 400% in panels [1,2], [2,1], and [2,2], respectively. Realizations conditional on sign restrictions imposed for K=2 and concerning the monetary policy shock only. Blue dashed lines represent the median response across all the VAR impulse responses meeting the imposed sign restrictions. Red dashed lines with diamonds identify the reaction to a monetary policy shock conditional on the DSGE model calibrated with posterior-mean values. Figure based on 1,000 draws.*

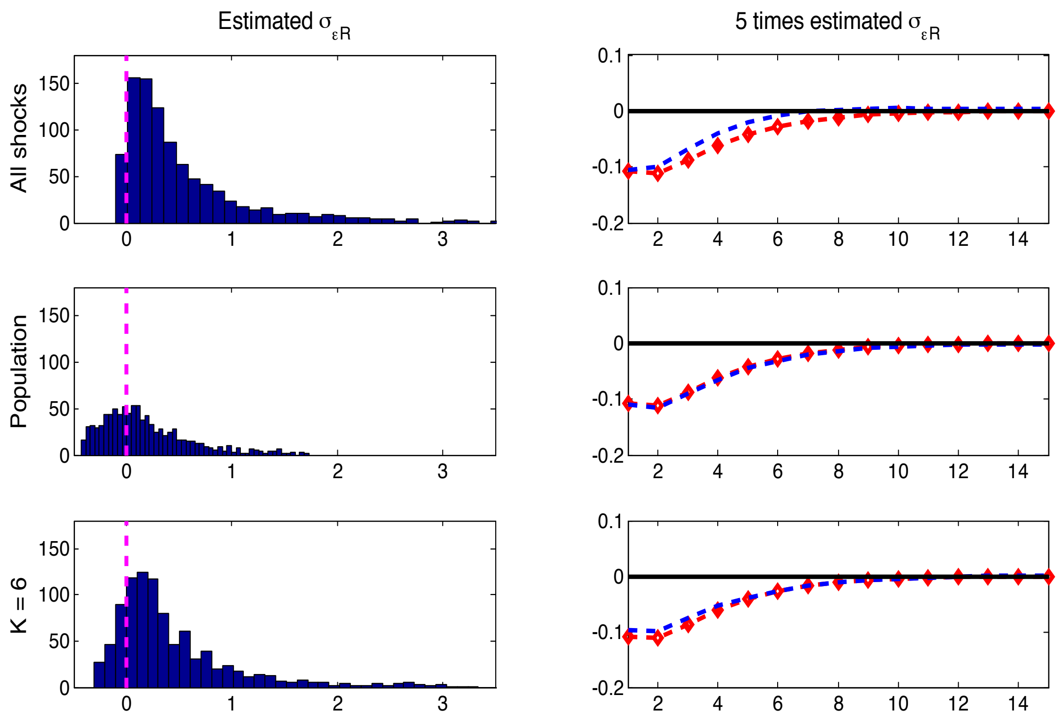

Figure 6: **Output response to a monetary policy shock: Robustness checks.** *Perturbation of the baseline case as follows. "All Shocks": All three shocks (monetary policy shock, mark-up shock, technology shock) are jointly identified. "Population": Simulations conducted conditional on a sample size equal to 10,000 observations. "K = 6": Sign restrictions imposed over the period of the shock and the following five periods. Left column: On impact (i.e., at horizon 0) impulse response function of output to a monetary policy shock. Realizations conditional on different scenarios as indicated by the y-axis labels. Outliers excluded by trimming the realizations not belonging to the [2.5th,97.5th] percentiles interval out. Figure based on 1,000 draws. Right column: Standard deviations of the monetary policy shock in the DGP increased by 400%. Blue dashed lines represent the median response across all the VAR impulse responses meeting the imposed sign restrictions. Red dashed lines with diamonds identify the reaction to a monetary policy shock conditional on the DSGE model calibrated with posterior-median values where not otherwise specified.*

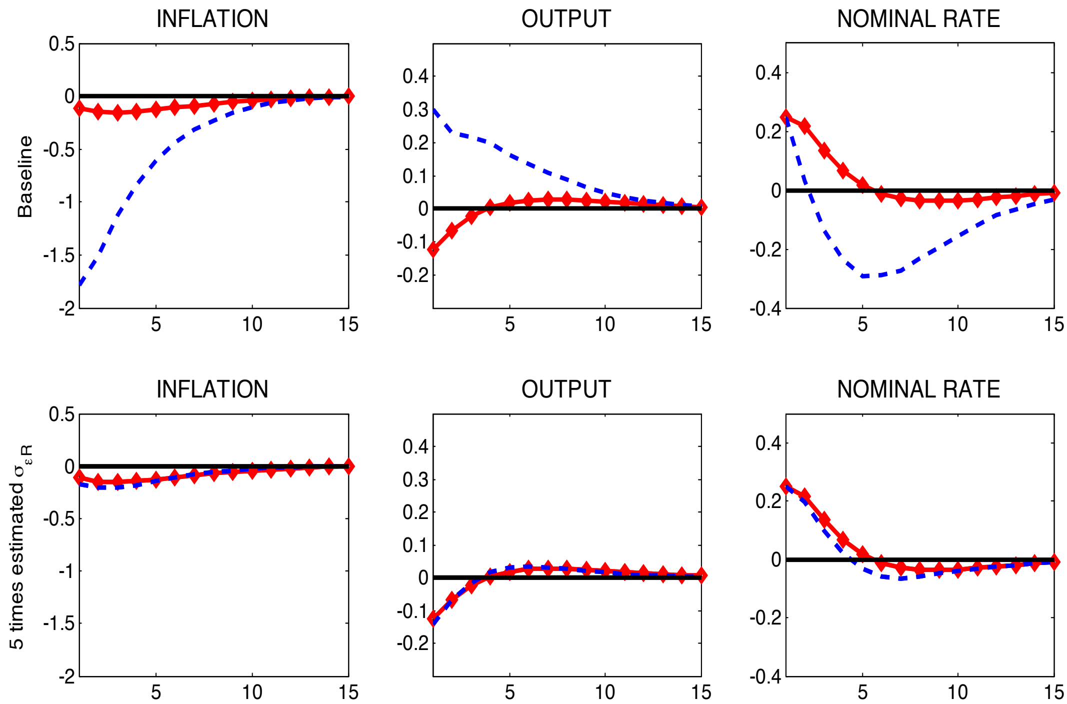

Figure 7: **Relevance of the strength of the policy shock signal in the Smets-Wouters (2007) model.** *Realizations conditional on sign restrictions imposed for K=1 and concerning the monetary policy shock only. Blue dashed lines identify the median response across all the VAR impulse responses meeting the imposed sign restrictions. Dashed red lines with diamonds: Reaction to a monetary policy shock conditional on the Smets-Wouters (2007) DSGE model calibrated with posterior-mean values. Standard deviations of the monetary policy shock in the DGP increased by 400% in panels [2,1], [2,2], and [2,3]. "Output" expressed in growth rates. Figure based on 1,000 draws.*

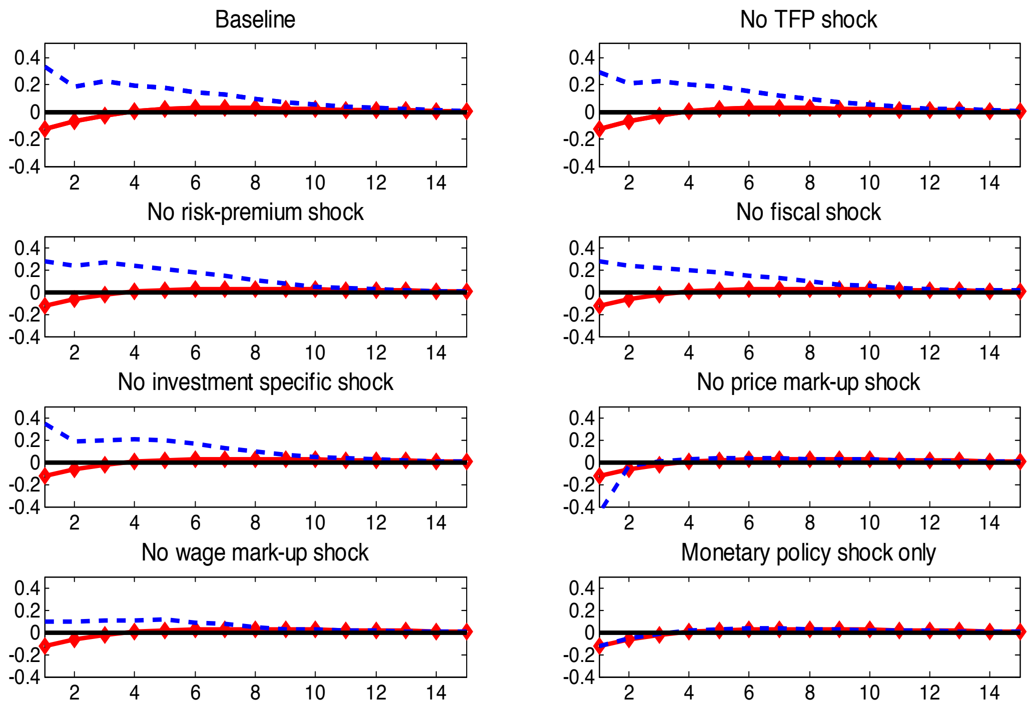

Figure 8: **Output response to a monetary policy shock: Selections of structural shocks in the Smets-Wouters model.** *Top-left panel: Baseline case, all shocks. Other panels: Perturbation of the baseline case implemented by switching off one or more shocks. Blue dashed lines represent the median response across all the VAR impulse responses meeting the imposed sign restrictions. Red dashed lines with diamonds identify the reaction to a monetary policy shock conditional on the DSGE model calibrated with posterior-mean values. Figure based on 1,000 draws.*

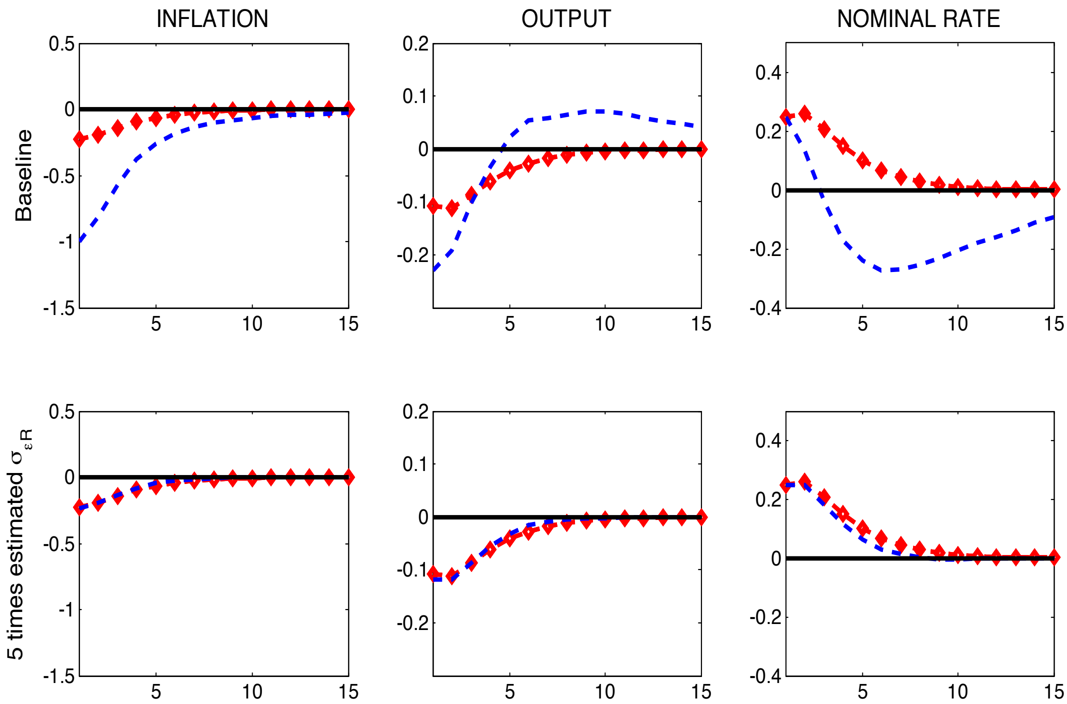

Figure 9: **Sign imposed on output, small-scale model.** *Realizations conditional on sign restrictions imposed for K=2 and concerning the monetary policy shock only. Blue dashed lines identify the median response across all the VAR impulse responses meeting the imposed sign restrictions. Dashed red lines with diamonds: Reaction to a monetary policy shock conditional on the small-scale DSGE model calibrated with posterior-mean values. Standard deviations of the monetary policy shock in the DGP increased by 400% in panels [2,1], [2,2], and [2,3]. Figure based on 1,000 draws.*

Appendix of the paper "Monetary Policy Neutrality? Sign Restrictions Go to Monte Carlo"

Bayesian estimation

To perform our Bayesian estimations we employed `DYNARE`, a set of algorithms developed by Michel Juillard and collaborators (Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto, and Villemot (2011)). `DYNARE` is freely available at the following URL: http://www.dynare.org/.

The simulation of the target distribution is basically based on two steps.

- First, we initialized the variance-covariance matrix of the proposal distribution and employed a standard random-walk Metropolis-Hastings for the first $t \leq t_0 = 20,000$ draws. To do so, we computed the posterior mode by the "csminwel" algorithm developed by Chris Sims. The inverse of the Hessian of the target distribution evaluated at the posterior mode was used to define the variance-covariance matrix C_0 of the proposal distribution. The initial VCV matrix of the forecast errors in the Kalman filter was set to be equal to the unconditional variance of the state variables. We used the steady-state of the model to initialize the state vector in the Kalman filter.

- Second, we implemented the "Adaptive Metropolis" (AM) algorithm developed by Haario, Saksman, and Tamminen (2001) to simulate the target distribution. Haario, Saksman, and Tamminen (2001) show that their AM algorithm is more efficient than the standard Metropolis-Hastings algorithm. In a nutshell, such algorithm employs the history of the states (draws) so to 'tune' the proposal distribution suitably. In particular, the previous draws are employed to regulate the VCV of the proposal density. We then exploited the history of the states sampled up to $t > t_0$ to continuously update the VCV matrix C_t of the proposal distribution. While not being a Markovian process, the AM algorithm is shown to possess the correct ergodic properties. For technicalities, see Haario, Saksman, and Tamminen (2001).

We simulated two chains of 200,000 draws each, and discarded the first 90% as burn-in. To scale the variance-covariance matrix of the chain, we used a factor so to achieve an acceptance rate belonging to the [23%,40%] range. The stationarity of the chains was assessed via the convergence checks proposed by Brooks and Gelman (1998). The region of acceptable parameter realizations was truncated so to obtain equilibrium uniqueness under rational expectations.

Predictive power of the estimated small-scale model

We checked the predictive power of the estimated small-scale model. Figure A1 contrasts the actual series employed in our empirical exercise with the DSGE model's one step-ahead predictions. As shown by the Figure, the model performs well along the one-step ahead forecasting dimension.

Robustness of our sign restrictions in the sense of Canova-Paustian (2011)

The sign-restrictions imposed in our exercise are robust to in the sense of Canova and Paustian (2011). We prove this by sampling 1,000 different calibrations of our small-scale model from uniform densities whose domains are large enough to contain the most common calibrations employed in the literature, computing the impulse responses of interest to a monetary policy shock suggested by each given different calibration, and plotting the 90% sets. Figure A2 shows the robustness of our sign restrictions to such a wide variety of different calibrations.

Fry and Pagan (2011) computation of the model-consistent impulse responses

A possible drawback of our exercise is the way in which we compute the median VAR responses. We do so by appealing to the empirical distribution constructed with all the $\omega^{(j)}$, that induce impulse responses meeting our constraints. Fry and Pagan (2011) identify two possible drawbacks in doing so. Call $C_{ik,h}^{(j)}$ the set of responses of a variable i to a shock k at a horizon h, where j indexes the value of the estimated responses in the set of the theory-consistent models $j \in \{1, ..., J\}$. First, for a given j, $med(C_{ik,h}^{j})$ may very well be none of the selected theory-consistent models. Second, assume that all $med(C_{ik,h}^{(j)})$, $j \in \{1, ..., J\}$ are actually selected models. As a matter of fact, nothing guarantees that, for whatever pair of different (h_1, h_2), $(med(C_{ik,h_1}^{(j)}), med(C_{ik,h_2}^{(j)}))$ is generated by the same model. Fry and Pagan (2011) suggest a way to search for the single model j whose associated responses are as close as possible to the medians shown in Figure 3.[1]

The Fry-Pagan median impulse responses are reported in Figure A3. Interestingly, negligible differences arise as for the reactions of inflation and the policy rate to a monetary policy shock. More importantly for this study, the reaction of output is also robust to the Fry-Pagan way of constructing the median, at least as for the run dynamics. The two medians start differing from the sixth quarter after the shock. The Fry and Pagan response suggests larger positive values and a delayed peak (at the seventh quarter vs. the fifth one in the case of the baseline median reaction). However, the main message is clearly confirmed when checked via the Fry-Pagan lenses, i.e., median measures suggest a positive reaction of output in a context in which, as a

[1] In seeking for such a single model, one has to recognize that the impulses need to be made unit-free by standardizing them. This is done by subtracting from each model-specific impulse response (conditional on a given horizon h) its median and dividing such difference by the standard deviation, where the median and the standard deviation are computed across all the retained models models - see Pagan and Fry (2011, p. 950-951).

matter of fact, such reaction in negative.

SRVAR with actual U.S. data

Uhlig (2005) works with monthly data, a richer VAR including extra variables (total reserves, non-borrowed reserves, a commodity price index) and with a sample (1965-2003) much longer than the one we employed to estimate our DSGE models, i.e., 1984:I-2008:II. Is the VAR evidence on an uncertain reaction of output robust to the employment of a trivariate VAR modeling the U.S. inflation, output gap, and federal funds rate as for the great moderation? To answer this question, we estimate such a trivariate VAR(4) with actual U.S. data, 1984:I-2008:II, and identify a monetary policy shock by imposing the same signs imposed in our baseline Monte Carlo exercise, i.e., a non-positive reaction of inflation and a non-negative reaction of the federal funds rate for $K = 2$. Our observables are defined as in Section 2.2. The choice of including a measure of the output gap is justified by two reasons. First, Giordani (2004) shows that a VAR including a measure of potential output is likely to return less distorted impulse responses to a monetary policy shock. Second, the inclusion of the output gap in our VAR estimated with actual data makes such VAR comparable to the ones employed in our Monte Carlo experiments.

Figure A4 reports the impulse responses over different horizons. One may easily notice the huge uncertainty surrounding the response of output, which clearly resembles the one produced with our Monte Carlo experiment and presented in Figure 1 in the paper. The empirical density of the on-impact response of output to a monetary policy shock is presented in Figure A5. Again, the similarity with our Monte Carlo-based Figure 2 in the main text is striking. The share of on-impact positive realizations of output is even larger than that recorder by Uhlig, in that our density suggests that eight responses out of ten take a positive value.

Our evidence reinforces the empirical result proposed by Uhlig (2005) on the uncertain reaction of output to a monetary policy shock identified with sign restrictions.

Rotation uncertainty

Our baseline model hinges upon the selection of a single model (rotation) meeting our sign constraint per each given database. It is of interest to understand if our findings go through if we retain more rotations satisfying our constraints per each given dataset. Then, we re-run our simulations by retaining, per each one of our $1,000$ artificial datasets, $1,000$ models models meeting our constraints. Then, we compute the median realization of the on-impact output responses to a monetary policy shock over such 1,000 models per each given dataset. We end up having a density based on 1,000 medians (here 1,000 is the number of different datasets), which is depicted in Figure 6. Clearly, all the mass cover positive values, an evidence suggesting that our main result is robust to employing different models per each given artificial dataset. Moreover, the median value of the median realizations reads 0.63, a value very close to the one computed in our baseline case (see our main text). The 5th and 95th percentiles of the distribution are 0.43 and 0.87, respectively. We conclude that our results are robust to the employment of a large number of rotations per each given dataset.

The Smets-Wouters (2007) model

The Smets and Wouters (2007) model is a Dynamic Stochastic General Equilibrium framework extremely popular in academic and institutional circles. The model features a number of shocks and frictions, which offer a quite rich representation of the economic environment and allow for a satisfactory in-sample fit of a set of macroeconomic data (Del Negro, Schorfheide, Smets, and Wouters (2007)). Moreover, Smets and Wouters (2007) show that this model is quite competitive when contrasted with Bayesian-VARs as for forecasting exercises, in particular for the elaboration of medium-term predictions.

The Smets and Wouters (2007) model features sticky nominal price and wage settings that allow for backward-looking inflation indexation; habit formation in consumption; investment adjustment costs; variable capital utilization and fixed costs in production. The stochastic dynamics is driven by seven structural shocks, namely a total factor productivity shock, two shocks affecting the intertemporal margin (risk premium shocks and investment-specific technology shocks), two shocks affecting the intratemporal margin (wage and price mark-up shocks), and two policy shocks (exogenous spending and monetary policy shocks).

In a nutshell, the model features the following main ingredients. Households maximize a nonseparable utility function in consumption and labor over an infinite life horizon. Consumption appears in the utility function in quasi-difference form with respect to a time-varying external habit variable. Labor is differentiated by a union, so there is some monopoly power over wages, which results in explicit wage equation and allows for the introduction of sticky nominal wages à la Calvo (1983). Households rent capital services to firms and decide how much capital to accumulate given the capital adjustment costs they face. The utilization of the capital stock can be adjusted at increasing cost. Firms produce differentiated goods, decide on labor and capital inputs, and set prices conditional on the Calvo model. The Calvo model in both wage and price setting is augmented by the assumption that prices that are not reoptimized are partially indexed to past inflation rates. Prices are therefore set in function of current and expected marginal costs, but are also determined by the past inflation rate. The marginal costs depend on wages and the rental rate of capital. Similarly, wages depend on past and expected future wages and inflation. The model features, in both goods and labor markets, an aggregator that allows for a time-varying demand elasticity depending on the relative price as in Kimball (1995). This is important because the introduction of real rigidity allows us to estimate a more reasonable degree of price and wage stickiness.

The log-linearized version of the DSGE model around its steady-state growth path reads as follows:

$$
\begin{aligned}
y_t &= c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g && (1)\\
c_t &= c_1 c_{t-1} + (1-c_1)E_t c_{t+1} + c_2(l_t - E_t l_{t+1}) - c_3(r_t - E_t \pi_{t+1} + \varepsilon_t^b) && (2)\\
i_t &= i_1 i_{t-1} + (1-i_1)E_t i_{t+1} + i_2 q_t + \varepsilon_t^i && (3)\\
q_t &= q_1 E_t q_t + 1 + (1-q_1)E_t r_{t+1}^k - (r_t - E_t \pi_{t+1} + \varepsilon_t^b) && (4)\\
y_t &= \phi_p(\alpha k_t^s + (1-\alpha)l_t + \varepsilon_t^a) && (5)\\
k_t^s &= k_{t-1} + z_t && (6)\\
z_t &= z_1 r_t^k && (7)\\
k_t &= k_1 k_{t-1} + (1-k_1)i_t + k_2 \varepsilon_t^i && (8)\\
\mu_t^p &= \alpha(k_t^s - l_t) + \varepsilon_t^a - w_t && (9)\\
\pi_t &= \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p + \varepsilon_t^p && (10)\\
r_t^k &= -(k_t - l_t) + w_t && (11)\\
\mu_t^w &= w_t - (\sigma_l l_t + (1-\lambda/\gamma)^{-1}(c_t - \lambda/\gamma c_{t-1})) && (12)\\
w_t &= w_1 w_{t-1} + w_2(E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w && (13)\\
r_t &= \rho r_{t-1} + (1-\rho)(r_\pi + r_Y(y_t - y_t^p)) + r_{\Delta y}[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_t^R && (14)\\
\varepsilon_t^x &= \rho_x \varepsilon_{t-1}^x + \eta_t^x, x = (b, i, a, R) && (15)\\
\varepsilon_t^g &= \rho_g \varepsilon_{t-1}^g + \eta_t^g + \rho_{ga}\eta_t^a && (16)\\
\varepsilon_t^z &= \rho_x \varepsilon_{t-1}^z + \eta_t^z - \chi_z \eta_{t-1}^z, z = (p, w) && (17)\\
\eta_t^j &\sim N(0, \sigma_j^2) && (18)
\end{aligned}
$$

where:

$$c_y = 1 - g_y - i_y \quad (19)$$

and g_y and i_y are the steady-state exogenous spending-output ratio and investment-output ratio, with:

$$i_y = (\gamma - 1 + \delta)k_y \tag{20}$$

where γ is the steady-state growth rate, δ is the depreciation rate of capital, k_y is the steady-state capital-output ratio; $z_y = R_*^y k_y$ is the steady-state rental rate of capital. Notice that eq. (16), the one of the stochastic process of the government spending, allows for the productivity shock to affect it. This is so because exogenous spending, in this model, includes net exports, which may be affected by domestic productivity development.

As for the consumption Euler equation (2):

$$c_1 = \frac{\lambda}{\gamma}\left(1 + \frac{\lambda}{\gamma}\right) \tag{21}$$

$$c_2 = \frac{(\sigma_c - 1)\frac{W_*^h L_*}{C_*}}{\sigma_c(1 + \frac{\lambda}{\gamma})} \tag{22}$$

$$c_3 = \frac{1 - \frac{\lambda}{\gamma}}{\left(1 + \frac{\lambda}{\gamma}\right)\sigma_c} \tag{23}$$

Current consumption is a function of past and expected future consumption, of expected growth in hours worked, of the ex ante real interest rate, and of a disturbance term ε_t^b. Under the assumption of no habits ($\lambda = 0$) and that of log-utility in consumption ($\sigma_c = 1$), $c_1 = c_2 = 0$, then the standard purely forward looking consumption equation is obtained. The disturbance term ε_t^b represents a wedge between the interest rate controlled by the central bank and the return on assets held by the households. A positive shock to this wedge increases the required return on assets held by the households. At the same time, it increases the cost of capital and it decreases the value of capital and investment (see below). This is basically a shock very similar to a net-worth shock. This disturbance is assumed to follow a standard AR(1) process.

The dynamics of investment is captured by the investment Euler equation (3), where:

$$i_1 = \frac{1}{1+\beta\gamma^{1-\sigma_c}} \quad (24)$$

$$i_2 = \frac{1}{1+\beta\gamma^{1-\sigma_c}\gamma^2\varphi} \quad (25)$$

where φ is the steady-state elasticity of the capital adjustment cost function, and β is the discount factor applied by households. Notice that capital adjustment costs are a function of the change in investment, rather than its level. This choice is made to introduce additional dynamics in the investment equation, which is useful to capture the hump-shaped response of investment to various shocks. In this equation, the stochastic disturbance ε_t^i represents a shock to the investment-specific technology process, and is assumed to follow a standard first-order autoregressive process.

The value-of-capital arbitrage equation (4) suggests that the current value of the capital stock q_t depends positively on its expected future value (with weight $q_1 = \beta\gamma^{-\sigma_c}(1-\delta)$), as well as the expected real rental rate on capital $E_t r_{t+1}^k$ and on the ex ante real interest rate and the risk premium disturbance.

Eq. (5) is the first one of the supply side block. It describes the aggregate production function, which maps output to capital (k_t^s) and labor services (l_t). The parameter α captures the share of capital in production, and the parameter ϕ_p is one plus the share of fixed costs in production, reflecting the presence of fixed costs in production.

Eq. (6) suggest that the newly installed capital becomes effective with a one-period delay, hence current capital services in production are a function of capital installed in the previous period k_t and the degree of capital utilization z_t. As stressed by eq. (7), the degree of capital utilization is a positive function of the rental rate of capital, $z_t = z_1 r_t^k$, where $z_1 = (1-\psi)/\psi$ and ψ is a positive function of the elasticity of the capital utilization adjustment cost function normalized to belong to the [0,1] domain.

Eq. (8) describes the accumulation of installed capital k_t, featuring the convolutions:

$$k_1 = (1-\delta)/\gamma \tag{26}$$

$$k_2 = \left[1-\left(1-\frac{\delta}{\gamma}\right)\right]\left(1+\beta\gamma^{1-\sigma_c}\right)\gamma^2\varphi \tag{27}$$

Installed capital is a function not only of the flow of investment but also of the relative efficiency of these investment expenditures as captured by the investment-specific technology disturbance ε_t^i, which follows an autoregressive, stationary process.

Eq. (9) relates to the monopolistic competitive goods market. Cost minimization by firms implies that the price mark-up μ_t^p, defined as the difference between the average price and the nominal marginal cost or the negative of the real marginal cost, is equal to the difference between the marginal product of labor and the real wage w_t, with the marginal product of labor being itself a positive function of the capital-labor ratio and total factor productivity.

Profit maximization by price-setting firms gives rise to the New-Keynesian Phillips curve, i.e., eq. (10), with the convolutions being:

$$\pi_1 = \frac{\iota_p}{1+\beta\gamma^{1-\sigma_c}\iota_p}, \tag{28}$$

$$\pi_2 = \frac{\beta\gamma^{1-\sigma_c}}{1+\beta\gamma^{1-\sigma_c}\iota_p}, \tag{29}$$

$$\pi_3 = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}\iota_p}\frac{(1-\beta\gamma^{1-\sigma_c}\xi_p)(1-\xi_p)}{\xi_p\left[(\phi_p-1)\varepsilon_p+1\right]}. \tag{30}$$

Notice that, in maximizing their profits, firm have to face price stickiness à la Calvo (1983). Firms that cannot reoptimize in a given period index their prices to past inflation as in Smets and Wouters (2003). In equilibrium, inflation π_t depends positively on past and expected future inflation, negatively on the current price mark-up, and positively on a price mark-up disturbance ε_t^p. The price mark-up disturbance is assumed to follow an ARMA(1,1) process. The inclusion of the MA term is to grab high-frequency fluctuations in inflation. When the degree of price indexation $\iota_p = 0$, $\pi_1 = 0$ and eq.

(10) collapses to the purely forward-looking, standard NKPC. The assumption that all prices are indexed to either lagged inflation or trend inflation ensures the verticality of the Phillips curve in the long run. The speed of adjustment to the desired mark-up depends, among others, on the degree of price stickiness ξ_p, the curvature of the Kimball goods market aggregator ε_p, and the steady-state mark up, which in equilibrium is itself related to the share of fixed costs in production $(\phi_p - 1)$ via a zero-profit condition. In particular, when all prices are flexible $(\xi_p = 0)$ and the price mark-up shock is zero at all times, eq. (10) reduces to the familiar condition that the price mark-up is constant, or equivalently that there are no fluctuations in the wedge between the marginal product of labor and the real wage. Cost minimization by firms also implies that the rental rate of capital is negatively related to the capital-labor ratio and positively to the real wage (both with unitary elasticity) (see eq. (11)).

Similarly, in the monopolistically competitive labor market, the wage mark-up will be equal to the difference between the real wage and the marginal rate of substitution between working and consuming, an equivalence captured by eq. (12), where σ is the elasticity of labor supply with respect to the real wage and λ is the habit parameter in consumption. Eq. (13) shows that real wages adjust only gradually to the desired wage mark-up due to nominal wage stickiness and partial indexation, the convolutions related to this equation being:

$$w_1 = \frac{1}{1+\beta\gamma^{1-\sigma_c}} \tag{31}$$

$$w_2 = \frac{1+\beta\gamma^{1-\sigma_c}\iota_w}{1+\beta\gamma^{1-\sigma_c}} \tag{32}$$

$$w_3 = \frac{\iota_w}{1+\beta\gamma^{1-\sigma_c}} \tag{33}$$

$$w_4 = \frac{\iota_w}{1+\beta\gamma^{1-\sigma_c}} \frac{(1-\beta\gamma^{(1-\sigma_c)}\xi_w)(1-\xi_w)}{\xi_w\left[(\phi_w-1)\varepsilon_w+1\right]} \tag{34}$$

Notice that if wages are perfectly flexible $(\xi_w = 0)$, the real wage is a constant mark-up over the marginal rate os substitution between consumption and leisure. When wage indexation is zero ($\iota_w = 0$), real wages do not depend on lagged inflation. Notice that, symmetrically with respect to the pricing scheme analyzed earlier, also the wage-mark up disturbance follows an ARMA(1,1) process.

The model is closed by eq. (14), which is a flexible Taylor rule postulating a systematic reaction by policymakers to current values of inflation, the output gap, and output growth. In particular, one of the objects policymakers react to is the output gap, defined as a difference between actual and potential output (in logs). Consistently with the DSGE model, potential output is defined as the level of output that would prevail under flexible prices and wages in the absence of the two mark-up shocks. Then, policymakers engineer movements in the short-run policy rate r_t, movements which happen gradually given the presence of interest rate smoothing ρ. Stochastic departures from the Taylor rate, i.e. the rate that would realize in absence of any policy rate shocks, are triggered by a stochastic AR(1) process.

Finally, eqs. (15)-(18) define the stochastic processes of the model, which features, as already pointed out, seven shocks (total factor productivity, investment specific technology, risk premium, exogenous spending, price mark-up, wage mark-up, and monetary policy).

Notice that the model features a deterministic growth rate driven by labor-augmenting technological progress, so that the data do not need to be detrended before estimation.

Tables A1 and A2 document our prior and posterior densities. Figure A7 shows that our results are robust to the employment of the Fry and Pagan (2011) search of the model closest to our pointwise medians.

References

Adjemian, S., H. Bastani, M. Juillard, F. Mihoubi, G. Perendia, M. Ratto, and S. Villemot (2011): "Dynare: Reference Manual, Version 4," Dynare Working Paper No. 1, April.

Brooks, S., and A. Gelman (1998): "General Methods for Monitoring Convergence of Iterative Simulations," *Journal of Computational and Graphical Statistics*, 7(4), 434–455.

Calvo, G. (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12, 383–398.

Del Negro, M., F. Schorfheide, F. Smets, and R. Wouters (2007): "On the Fit of New-Keynesian Models," *Journal of Business and Economic Statistics*, 25(2), 124–162.

Fry, R., and A. Pagan (2011): "Sign Restrictions in Structural Vector Autoregressions: A Critical Review," *Journal of Economic Literature*, 49(4), 938–960.

Giordani, P. (2004): "An Alternative Explanation to the Price Puzzle," *Journal of Monetary Economics*, 51, 1271–1296.

Haario, H., E. Saksman, and J. Tamminen (2001): "An Adaptive Metropolis Algorithm," *Bernoulli*, 7(2), 223–242.

Kimball, M. (1995): "The quantitative analytics of the basic Neomonetarist model," *Journal of Money, Credit and Banking*, 27(4), 1241–1277.

Smets, F., and R. Wouters (2003): "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, 1, 1123–1175.

——— (2007): "Shocks and Frictions in US Business Cycle: A Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

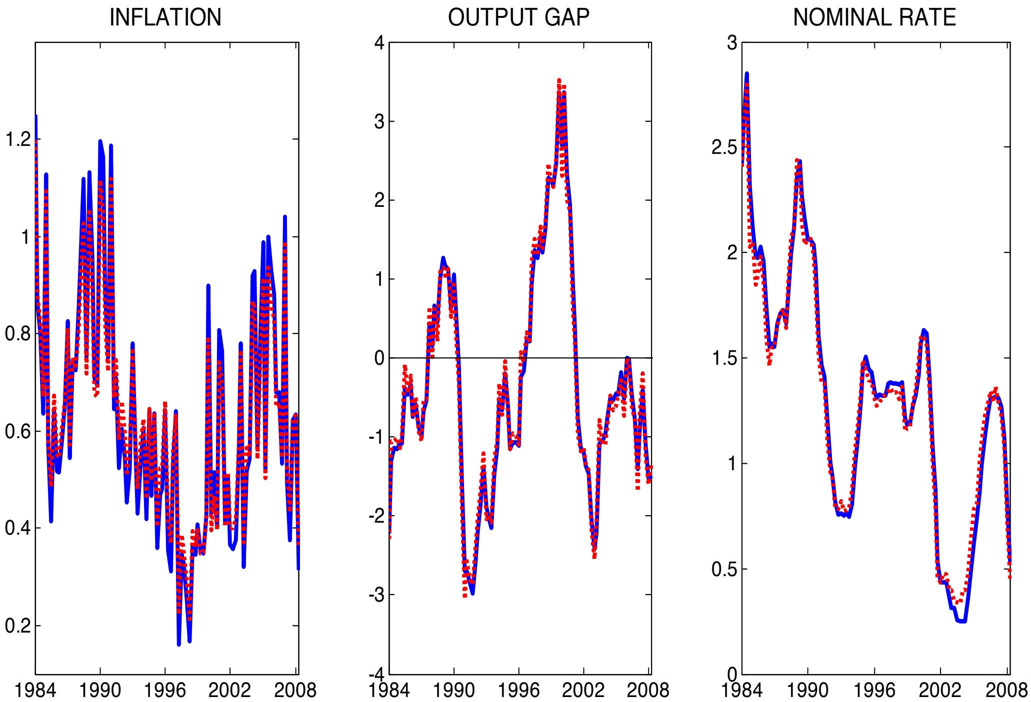

Figure A1: **Actual series vs. DNK's one-step ahead forecasts.** *Solid blue line: Actual series. Dotted red lines: DSGE framework's one-step-ahead predictions.*

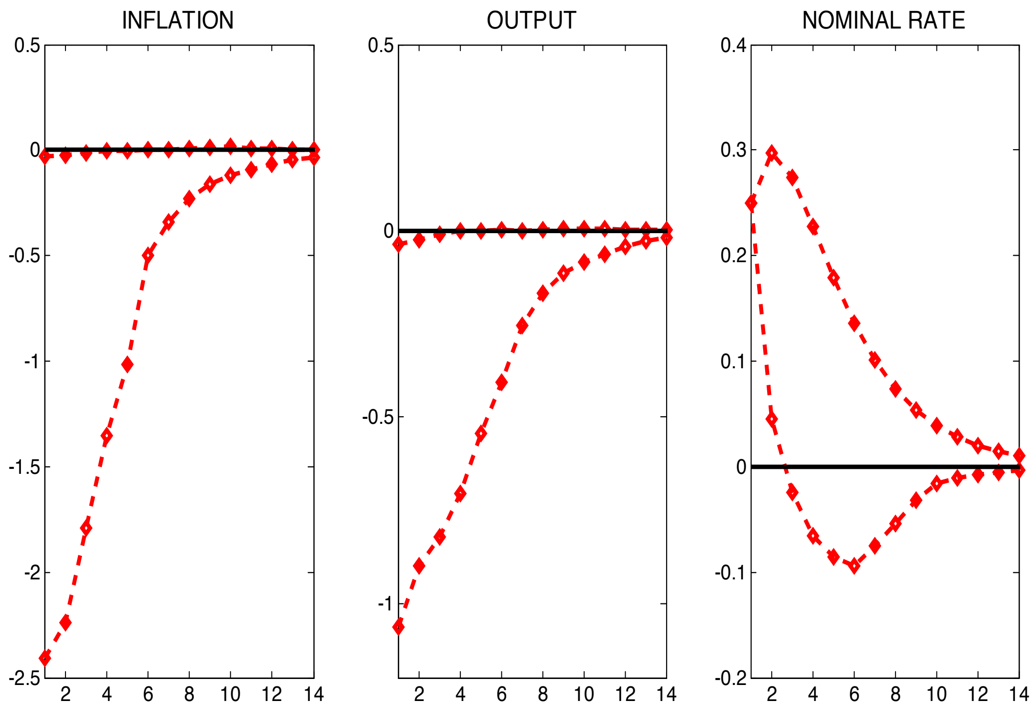

Figure A2: **Robustness of our sign restrictions in the sense of Canova and Paustian (2011) - small scale model.** *Pictures identifying 90% sets obtained by drawing 1,000 different calibrations of our small scale model. Calibrations obtained by drawing from uniform densities with a wide support as in Canova and Paustian (2011).*

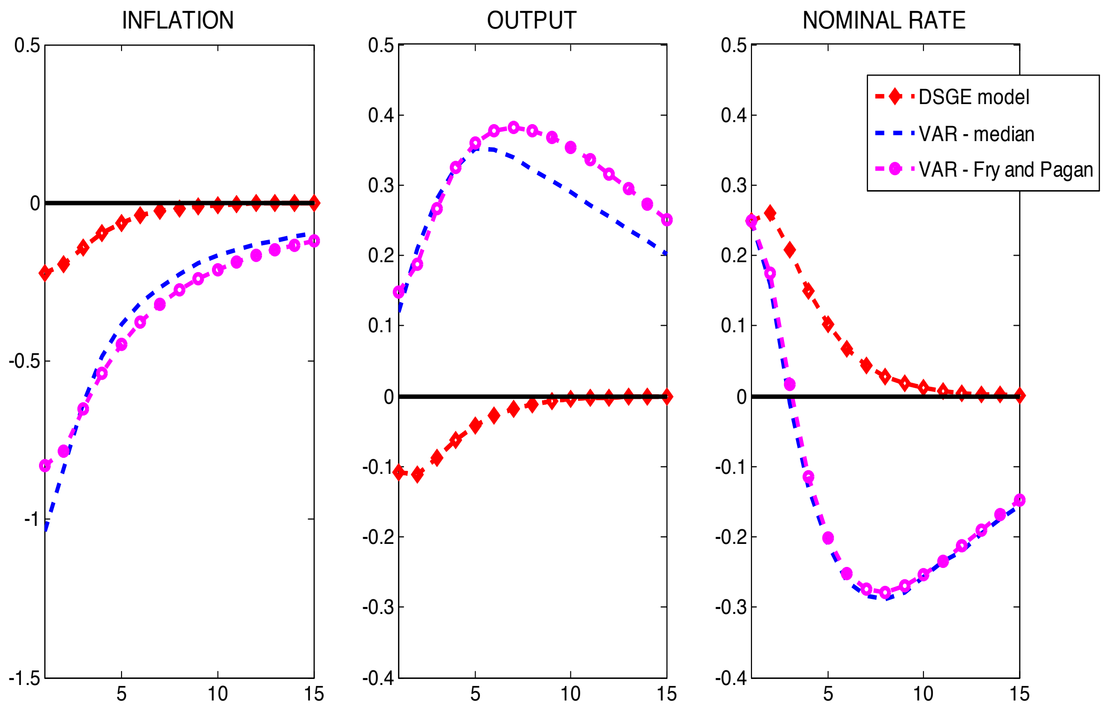

Figure A3. **Fry and Pagan (2011) impulse responses to a monetary policy shock.** *Realizations conditional on sign restrictions imposed for K=2 and concerning the monetary policy shock only. Red dashed lines with diamonds identify the reaction to a monetary policy shock conditional on the DSGE model calibrated with posterior-mean values. Blue dashed lines represent the median response across all the VAR impulse responses meeting the imposed sign restrictions. Magenta lines with circles represent the median VAR responses computed as suggested by Fry and Pagan (2011). Figure based on 1,000 draws.*

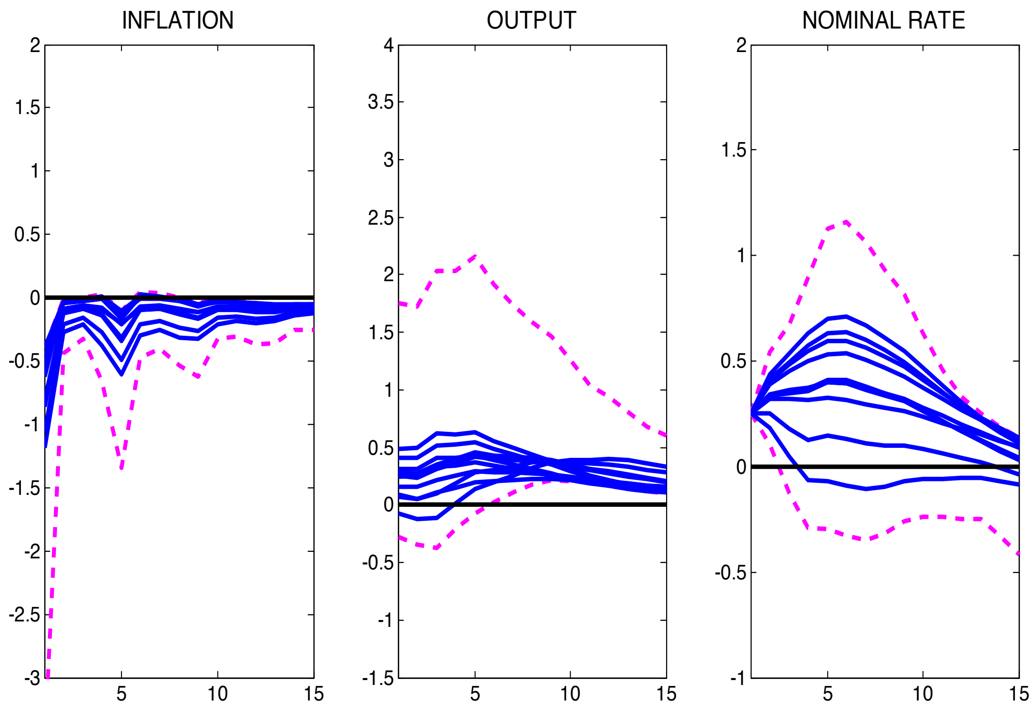

Figure A4. **Impulse response functions to a monetary policy shock identified with sign restrictions - actual U.S. data.** *Realizations conditional on sign restrictions imposed for K=2 and concerning the monetary policy shock only. Blue solid lines represent 10 randomly selected impulse responses meeting the imposed sign restrictions. Dashed magenta lines identify the 5th and 95th percentiles of the distribution. Figure based on 1,000 draws.*

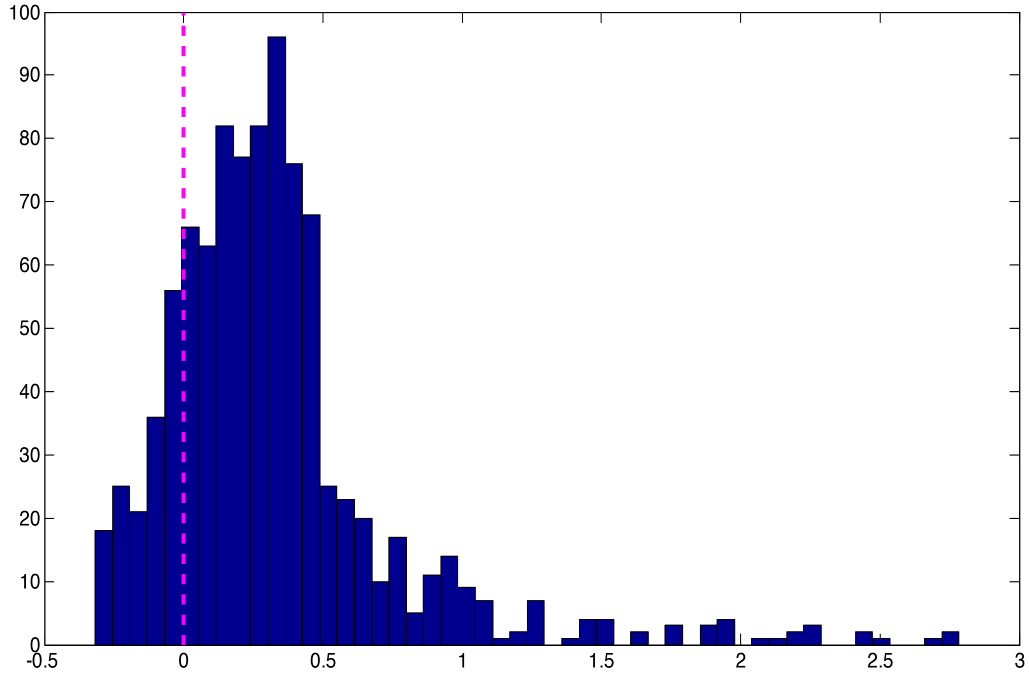

Figure A5. **On impact impulse response function of output to a monetary policy shock - actual U.S. data.** *Realizations conditional on sign restrictions imposed for K=2 and concerning the monetary policy shock only. On impact realizations (i.e., at horizon 0) only. Outliers excluded by trimming the realizations not belonging to the [2.5th,97.5th] percentiles interval out. Figure based on 1,000 draws.*

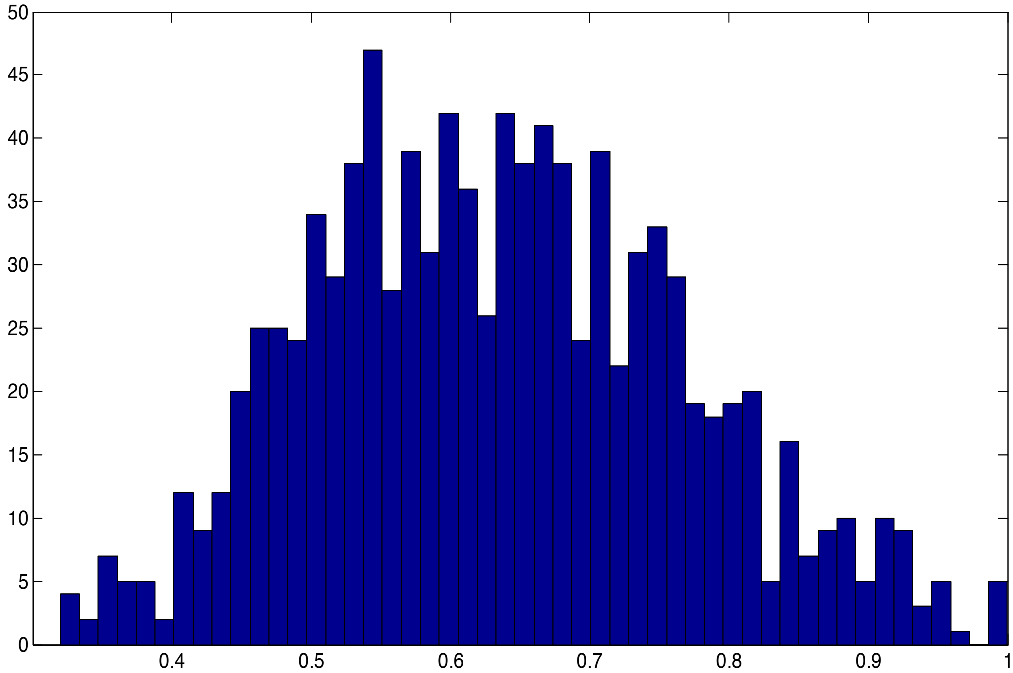

Figure A6. **Density of shares of positive on-impact reactions of output to a monetary policy shock.** *Realizations conditional on sign restrictions imposed for K=2 and concerning the monetary policy shock only. Density constructed on the basis of 1,000 different shares of positive on-impact reactions of output to a monetary policy shock. Per each given dataset (total number of the datasets: 1,000), the share of positive realizations of output is computed by considering 1,000 accepted rotation matrices. Median value of the realized shares: 0.63. [5th,95th] percentiles: [0.43,0.87].*

Param.	*Interpretation*	*Priors*	*Posterior Means* [$5h$,$95th$]
φ	Capital adj. cost elasticity	$\mathcal{N}(4, 1.5)$	6.06 [4.22,7.96]
σ_c	Risk aversion	$\mathcal{N}(1.5, 0.375)$	1.39 [1.16,1.62]
h	Habit formation	$\mathcal{B}(0.7, 0.1)$	0.63 [0.50,0.75]
ξ_w	Wage stickiness	$\mathcal{B}(0.5, 0.1)$	0.64 [0.49,0.79]
σ_l	Elast. lab. supply	$\mathcal{N}(2, 0.75)$	1.76 [0.78,2.74]
ξ_p	Price stickiness	$\mathcal{B}(0.5, 0.1)$	0.71 [0.62,0.80]
ι_w	Wage indexation	$\mathcal{B}(0.5, 0.15)$	0.52 [0.28,0.76]
ι_p	Price indexation	$\mathcal{B}(0.5, 0.15)$	0.40 [0.20,0.59]
ψ	Capacity utiliz. elast.	$\mathcal{B}(0.5, 0.15)$	0.69 [0.54,0.85]
$\Phi - 1$	Fixed c. in prod. (share)	$\mathcal{N}(0.25, 0.125)$	0.44 [0.30,0.57]
r_π	T. Rule, inflation	$\mathcal{N}(1.5, 0.25)$	2.10 [1.78,2.43]
ρ	T. Rule, inertia	$\mathcal{B}(0.75, 0.10)$	0.83 [0.80,0.87]
r_y	T. Rule, output gap	$\mathcal{N}(0.125, 0.05)$	0.05 [0.02,0.09]
$r_{\Delta y}$	T. Rule, output growth	$\mathcal{N}(0.125, 0.05)$	0.16 [0.11,0.20]
$\overline{\pi}$	St. state inflation rate	$\mathcal{G}(0.625, 0.10)$	0.64 [0.55,0.73]
$100(\beta^{-1} - 1)$	St. state interest rate	$\mathcal{G}(0.25, 0.10)$	0.25 [0.10,0.40]
$\bar{l}$	St. state hours worked	$\mathcal{N}(0, 2)$	0.87 [−0.84,2.57]
$\overline{\gamma}$	Trend growth rate	$\mathcal{N}(0.4, 0.1)$	0.42 [0.37,0.47]
α	Share of capital in prod.	$\mathcal{N}(0.3, 0.05)$	0.32 [0.25,0.39]

Table A1: **Bayesian estimates of the Smets and Wouters' (2007) DSGE model - Structural Parameters.** *1984:I-2008:II U.S. data. Legend: (N, B, G, IG) stand for (Normal, Beta, Gamma, Inverse Gamma) densities. Prior densities: Figures indicate the (mean,st.dev.) of each prior distribution. Posterior densities: Figures reported indicate the posterior mean and the [5th,95th] percentile of the estimated densities. Details on the estimation procedure provided in the text.*

Param.	*Interpretation*	*Priors*	*Posterior Means* [5h,95th]
σ_a	TFP shock, st.dev.	$\mathcal{IG}(0.1,2)$	0.41 [0.36,0.46]
σ_b	Risk-premium shock, st.dev.	$\mathcal{IG}(0.1,2)$	0.16 [0.10,0.21]
σ_g	Gov. spending shock, st.dev.	$\mathcal{IG}(0.1,2)$	0.41 [0.36,0.46]
σ_I	Invest.-specific tech. shock, st.dev.	$\mathcal{IG}(0.1,2)$	0.35 [0.27,0.42]
σ_r	Mon. policy shock, st.dev.	$\mathcal{IG}(0.1,2)$	0.12 [0.10,0.14]
σ_p	Price mark-up shock, st.dev.	$\mathcal{IG}(0.1,2)$	0.10 [0.08,0.12]
σ_w	Wage mark-up shock, st.dev.	$\mathcal{IG}(0.1,2)$	0.29 [0.23,0.35]
ρ_a	TFP shock, AR(1) coeff.	$\mathcal{B}(0.5,0.2)$	0.95 [0.92,0.97]
ρ_b	Risk-premium shock, AR(1) coeff.	$\mathcal{B}(0.5,0.2)$	0.32 [0.04,0.62]
ρ_g	Gov. sp. shock, AR(1) coeff.	$\mathcal{B}(0.5,0.2)$	0.95 [0.93,0.97]
ρ_I	Invest.-spec. tech. shock, AR(1) coeff.	$\mathcal{B}(0.5,0.2)$	0.74 [0.65,0.85]
ρ_r	Mon. pol. shock, AR(1) coeff.	$\mathcal{B}(0.5,0.2)$	0.30 [0.15,0.44]
ρ_p	Price mark-up shock., AR(1) coeff.	$\mathcal{B}(0.5,0.2)$	0.89 [0.81,0.98]
ρ_w	Wage mark-up shock, AR(1) coeff.	$\mathcal{B}(0.5,0.2)$	0.93 [0.88,0.98]
μ_p	Price mark-up shock, MA(1) coeff.	$\mathcal{B}(0.5,0.2)$	0.66 [0.47,0.85]
μ_w	Wage mark-up shock, MA(1) coeff.	$\mathcal{B}(0.5,0.2)$	0.71 [0.53,0.88]
ρ_{ga}	Gov.spending-TFP shocks, correlation	$\mathcal{B}(0.5,0.2)$	0.44 [0.28,0.61]

Table A2: **Bayesian estimates of the Smets and Wouters' (2007) DSGE model - Shocks' persistence and variance.** *1984:I-2008:II U.S. data. Legend: (N, B, G, IG) stand for (Normal, Beta, Gamma, Inverse Gamma) densities. Prior densities: Figures indicate the (mean,st.dev.) of each prior distribution. Posterior densities: Figures reported indicate the posterior mean and the [5th,95th] percentile of the estimated densities. Details on the estimation procedure provided in the text.*

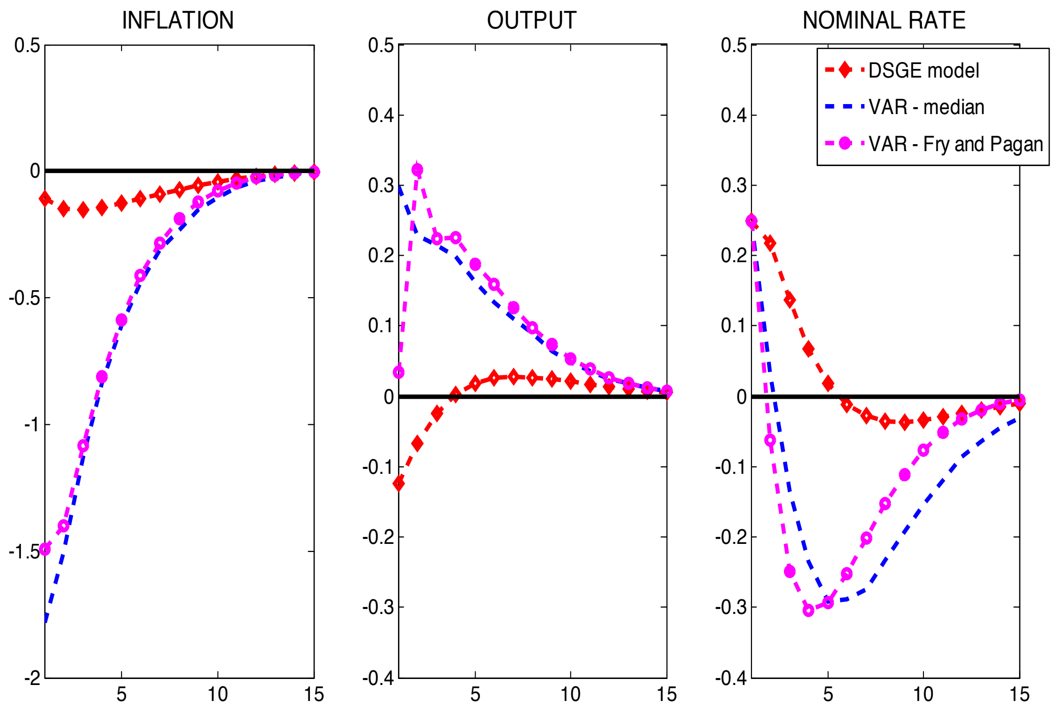

Figure A7. **Impulse responses to a monetary policy shock: Smets and Wouters' DSGE model vs. VAR.** *Realizations conditional on sign restrictions imposed for K=2 and concerning the monetary policy shock only. Red dashed lines with diamonds identify the reaction to a monetary policy shock conditional on the DSGE model calibrated with posterior-mean values. Blue dashed lines represent the median response across all the VAR impulse responses meeting the imposed sign restrictions. Magenta lines with circles represent the median VAR responses computed as suggested by Fry and Pagan (2011). Figure based on 1,000 draws.*