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MONETARY POLICY INDETERMINACY AND IDENTIFICATION FAILURES IN THE U.S.: RESULTS FROM A ROBUST TEST

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Monetary Policy Indeterminacy and Identification Failures in the U.S.: Results from a Robust Test*

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Abstract

We propose a novel robust test to assess whether an estimated new-Keynesian model is consistent with a unique stable solution, as opposed to multiple equilibria. Our strategy is designed to handle identification failures as well as the misspecification of the relevant propagation mechanisms. We invert a likelihood ratio test for the cross-equation restrictions (CER) that the new-Keynesian system places on its reduced form solution under determinacy. If the CER are not rejected, we rule out the occurrence of sunspot-driven expectations from the model equilibrium and accept the structural model. Otherwise, we move to a second-step and invert, using the same grid considered in the first-step, an Anderson and Rubin-type test for the orthogonality restrictions (OR) implied by the system of Euler equations. We accept the hypothesis of indeterminacy if the OR are not rejected. We investigate the finite sample performance of the suggested two-steps testing strategy by some Monte Carlo experiments. Finally, we apply our robust test to a new-Keynesian AD/AS model estimated with actual U.S. post-WWII data. In spite of some evidence of weak identification as for the 'Great Moderation' period, our results offer formal support to the hypothesis of a switch from indeterminacy to uniqueness occurred in the late 1970s.

Keywords: Confidence set, Determinacy, Identification failure, Indeterminacy, Misspecification, new-Keynesian business cycle model, VAR system.

J.E.L.: C31, C22, E31, E52.

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1 Introduction

The U.S. inflation and output growth processes have experienced dramatic breaks in the post-WWII. In particular, a marked reduction of the U.S. macroeconomic volatilities has been documented by Stock and Watson (2002), who coined the popular term 'Great Moderation' to indicate this stylized fact. A possible explanation for such phenomenon hinges upon the hypothesis of the switch to an aggressive monetary policy conduct occurred with the appointment of Paul Volcker as Chairman of the Federal Reserve at the end of the 1970s. With his appointment, the argument goes, the Fed moved from a weakly aggressive reaction to inflation to a much stronger one. Such a switch anchored private sector's inflation expectations, therefore leading the U.S. economy to move from an indeterminate equilibrium to determinacy. This story, popularized by Clarida *et al.* (2000), has subsequently been supported by Lubik and Schorfheide (2004), Boivin and Giannoni (2006), Benati and Surico (2009), Mavroeidis (2010), and Inoue and Rossi (2011*a*).

The above mentioned contributions implicitly assume the new-Keynesian model one works with to be correctly specified and, with the remarkable exception of Mavroeidis (2010), to feature identifiable parameters. As concerns the first issue, albeit new-Keynesian models can display several types of misspecifications (An and Schorfheide, 2007), the omission of propagation mechanisms from the structural equations is a major concern in the empirical assessment of determinacy/indeterminacy. As discussed by Lubik and Schorfheide (2004) and Fanelli (2012), indeterminacy generally entails a richer correlation structure of the data. Therefore, the risk run by an econometrician is to confound a determinate case in which relevant propagation mechanisms are not embedded by the structural model at hand with the indeterminate scenario. In conducting their Bayesian analysis, Lubik and Schorfheide (2004) tackle this issue by analyzing versions of a small-scale new-Keynesian model featuring different dynamic structures, while Fanelli (2012) proposes a frequentist test of determinacy/indeterminacy that explicitly controls for the omission of propagation mechanisms from the specified structural equations.

As concerns the identifiability of the structural parameters, aside from Mavroeidis (2010), who adopts a single-equation 'limited-information' approach, all existing empirical contributions in which the determinacy/indeterminacy issue of U.S. monetary policy is investigated assume that the structural parameters are identifiable. In general, both finite sample and asymptotic distributions for estimators and tests can be strongly affected if identification conditions are not satisfied, see e.g. Sargan (1983), Phillips (1989), Staiger and Stock (1997) and Stock and Wright (2000). Many authors have recently argued that estimated new-Keynesian systems like or similar to the one considered in this paper can be affected by 'weak identification' issues. Identification problems in a system of variables featuring highly nonlinear restrictions may involve the rank condition of the information matrix or suitable transformation of moments (Iskrev, 2008, 2010; Komunjer and Ng, 2011), or the relationship between the structural parameters and the sample objective function, which may display 'small' curvature in certain regions of the parameter space, see e.g. Canova and Sala (2009). The former concept of identification is also referred to as 'population identification' (Canova and Sala, 2009), as opposed to the latter, often termed 'sample identification', because it is specific to a particular dataset and sample size. Our paper is concerned with this second phenomenon, which we characterize as the situation in which the criterion used to estimate the structural parameters and test hypotheses on these parameters exhibit 'little curvature' in all or some directions of the parameter space with the consequence of being nearly uninformative about these parameters. Weak identification of all or part of the estimated parameters can affect negatively the finite sample performances of the testing procedures commonly used by 'frequentist' practitioners. Robust inference under possible identification failure has received increasing attention by the literature on dynamic stochastic general equilibrium (DSGE) models, see e.g. Canova and Sala (2009), Dufour *et al.* (2009, 2013), Kleibergen and Mavroeidis (2009), Mavroeidis (2005, 2010), Guerron-Quintana *et al.* (2013), Qu (2011) and Andrews and Mikusheva (2012), among others.[1]

This paper's contribution is twofold. On the methodological side, we propose a novel test for determinacy/indeterminacy in new-Keynesian monetary policy business cycle models that (i) can be applied regardless of the strength of identification of the model's structural parameters, and (ii) that controls for the case of 'dynamic misspecification', where by this term we mean the omission of relevant propagation mechanisms from the specified system of structural Euler equations. On the empirical side, we use the small scale new-Keynesian model discussed in Benati and Surico (2009) and apply the proposed identification-robust method to post-WWII U.S. data to investigate monetary policy determinacy/indeterminacy on our selected 'pre-Volcker' and 'Great Moderation' samples.

As regards the methodological contribution, the proposed testing strategy is based on two steps. In the first-step, we use an identification-robust 'full-information' method to test the cross-equation restrictions (CER) that the new-Keynesian model places on its unique stable reduced form solution under determinacy. This requires the (numerical) inversion of a likelihood-ratio test for the CER implied by the new-Keynesian model along the lines recently suggested by Guerron-Quintana *et al.* (2013) and Dufour *et al.* (2013). If the CER are not rejected, we can rule out the occurrence of sunspot-driven expectations and arbitrary nuisance parameters from the model's equilibrium. Importantly, in this case we cannot rule out the possibility of a

[1] Inoue and Rossi (2011*b*) and Andrews and Cheng (2012, 2013) tackle the issue from a more general perspective but their analysis can be adapted to the context of DSGE models.

Minimum State Variable (MSV) equilibrium (McCallum, 1983), i.e. a solution nested within the class of indeterminate equilibria but that is observationally equivalent to the determinate reduced form, see Evans and Honkapohja (1986), Lubik and Schorfheide (2004), and Fanelli (2012). Notably, however, the non-rejection of the CER amounts to an implicit acceptance of the hypothesis of correct specification of the new-Keynesian system. If instead the CER are rejected, we move to a second-step to determine whether the outcome obtained in the first-step depends on the multiple equilibria hypothesis, or to the omission of relevant propagation mechanisms from the specified structural equations. To accomplish this task, we apply an identification-robust 'limited-information' method and invert a test for the orthogonality restrictions (OR) implied by the system of Euler equations under the rational expectations hypothesis (and the assumption of correct specification), using the same grid employed in the first-step. In principle, if the new-Keynesian system is correctly specified, the OR are valid irrespective of whether the implied equilibrium is determinate or indeterminate. However, conditional on the result in first-step, in our framework the non-rejection of the OR is evidence of indeterminacy while their rejection suggests that the specified structural equations do not capture the dynamic properties of the data adequately. The test inverted in this second-step is an Anderson Rubin-type (Anderson and Rubin, 1949) test that can be implemented in our multivariate framework following Dufour *et al.* (2009, 2013).[2]

The tests involved in our two-step methodology are based on asymptotically pivotal test statistics which have correct size regardless of the strength of identification of the model's structural parameters. Overall, the suggested testing strategy is asymptotically correctly sized and consistent against the multiple equilibria hypothesis. We investigate its finite sample performance and its practical usefulness by some Monte Carlo experiments and find that the procedure displays reasonable empirical size and reassuring power against some specified indeterminate equilibria in finite samples.

As regards the empirical contribution, the application of our testing strategy on U.S. quarterly data leads us to the following findings. Our identification-robust test for the CER computed in the first step leads us to reject the hypothesis of determinacy on the 'pre-Volcker' sample. Conditional on this first step, our identification-robust test for the OR computed in the second-step does not lead us to reject the new-Keynesian framework at hand. Therefore, our results support the multiple equilibria scenario, which acknowledges a role for self-fulfilling expectations as a driver of the U.S. macroeconomic dynamics during the 1970s. Instead, the

[2] Alternatively, one can apply the 'S-test' approach by Stock and Wright (2000) or the 'K-LM test' approach by Kleibergen (2005), which require the evaluation of the criterion function associated with the continuos-updating version of the generalized method of moments. Some computational issues, discussed in detail in the Appendix, make us prefer the approach by Dufour *et al.* (2009, 2013).

identification-robust test for the CER computed in the first-step clearly supports the CER implied by the hypothesis of determinacy when considering our 'Great moderation' sample. While being unable to interpret this result as conclusive evidence of determinacy (recall the observational equivalence between the determinate and the indeterminate MSV solution), the case of sunspot shocks-driven expectations is clearly ruled out by the data. In line with Mavroeidis (2010), our 'limited information'-based second step delivers wider projected confidence intervals for the estimated policy parameters during the 'Great Moderation' as opposed to those computed for the 'Great Inflation' period. If taken in isolation, the projected confidence intervals of the policy parameters would be considered as uninformative as for the issue of determinacy. Differently, our full-system inferential approach enables us to interpret such evidence as consistent with an economic system under determinacy, hence not affected by sunspot shocks. This is so because our first step does not lead us to reject the structure of the system under investigation when post-1985 data are taken into account. Therefore, our testing procedure is inherently more informative than a single-equation approach (even when the latter is designed to deal with weak identification), in that it allows the econometrician to go a step further in assessing (and, in this case, ruling out) the role of sunspot fluctuations as possible drivers of the U.S. economic dynamics.

The remained of this paper is organized as follows. Section 2 introduces the reference small scale new-Keynesian structural model and discusses its reduced form solutions under determinacy and indeterminacy, respectively. Section 3 summarizes the testing strategy. Section 4 investigates the finite sample performance of the testing strategy by some simulation experiments. Section 5 presents our empirical results obtained on U.S. quarterly data. Section 6 relates our work to the literature, and Section 7 contains some concluding remarks. Additional methodological details are confined in the Appendix. To save space, we have treated some issues regarding the solution properties of the new-Keynesian model in a Technical Supplement.

2 Model

This Section presents the reference small-scale new-Keynesian business cycle model and discusses its time series representations under determinacy and indeterminacy, respectively.

2.1 Structural system

Our reference new-Keynesian model is taken from Benati and Surico (2009). It features the following three equations:

$$\tilde{y}_t = \gamma E_t \tilde{y}_{t+1} + (1-\gamma)\tilde{y}_{t-1} - \delta(R_t - E_t \pi_{t+1}) + \omega_{\tilde{y},t} \tag{1}$$

$$\pi_t = \frac{\beta}{1+\beta\alpha} E_t \pi_{t+1} + \frac{\alpha}{1+\beta\alpha}\pi_{t-1} + \kappa \tilde{y}_t + \omega_{\pi,t} \tag{2}$$

$$R_t = \rho R_{t-1} + (1-\rho)(\varphi_\pi \pi_t + \varphi_{\tilde{y}} \tilde{y}_t) + \omega_{R,t} \tag{3}$$

where

$$\omega_{x,t} = \rho_x \omega_{x,t-1} + \varepsilon_{x,t} \quad , \text{-}1<\rho_x<1 \quad , \varepsilon_{x,t} \sim \text{WN}(0, \sigma_x^2) \quad , \quad x = \tilde{y}, \pi, R \tag{4}$$

and expectations are conditional on the information set $\mathcal{F}_t$, i.e. $E_t \cdot := E(\cdot \mid \mathcal{F}_t)$. The variables $\tilde{y}_t$, π_t, and R_t stand for the output gap, inflation, and the nominal interest rate, respectively; γ is the weight of the forward-looking component in the intertemporal IS curve; α is price setters' extent of indexation to past inflation; δ is households' intertemporal elasticity of substitution; κ is the slope of the Phillips curve; ρ, φ_π, and $\varphi_{\tilde{y}}$ are the interest rate smoothing coefficient, the long-run coefficient on inflation, and that on the output gap in the monetary policy rule, respectively; finally, $\omega_{\tilde{y},t}$, $\omega_{\pi,t}$ and $\omega_{R,t}$ in eq. (4) are the mutually independent, autoregressive of order one disturbances and $\varepsilon_{\tilde{y},t}$, $\varepsilon_{\pi,t}$ and $\varepsilon_{R,t}$ are the structural (fundamental) shocks. This or similar small-scale models have successfully been employed to conduct empirical analysis concerning the U.S. economy. Clarida *et al.* (2000) and Lubik and Schorfheide (2004) have investigated the influence of systematic monetary policy over the U.S. macroeconomic dynamics; Boivin and Giannoni (2006), Benati and Surico (2009), and Lubik and Surico (2010) have replicated the U.S. Great Moderation, Benati (2008) and Benati and Surico (2008) have investigated the drivers of the U.S. inflation persistence; Castelnuovo and Surico (2010) have replicated the VAR dynamics conditional on a monetary policy shock in different sub-samples; Inoue and Rossi (2011*a*) have analyzed the role of parameter instabilities as drivers of the Great Moderation.

We compact the system composed by eq.s (1)-(4) in the representation

$$\Gamma_0 X_t = \Gamma_f E_t X_{t+1} + \Gamma_b X_{t-1} + \omega_t \tag{5}$$

$$\omega_t = \Xi \omega_{t-1} + \varepsilon_t \ , \ \ \varepsilon_t \sim \text{WN}(0, \Sigma_\varepsilon) \tag{6}$$

$$\Xi := dg(\rho_{\tilde{y}}, \rho_\pi, \rho_R) \quad , \quad \Sigma_\varepsilon := dg(\sigma_{\tilde{y}}^2, \sigma_\pi^2, \sigma_R^2)$$

where $X_t := (\tilde{y}_t, \pi_t, R_t)'$, $\omega_t := (\omega_{\tilde{y},t}, \omega_{\pi,t}, \omega_{R,t})'$, $\varepsilon_t := (\varepsilon_{\tilde{y},t}, \varepsilon_{\pi,t}, \varepsilon_{R,t})'$ and

$$\Gamma_0 := \begin{bmatrix} 1 & 0 & \delta \\ -\kappa & 1 & 0 \\ -(1-\rho)\varphi_{\tilde{y}} & -(1-\rho)\varphi_\pi & 1 \end{bmatrix}, \Gamma_f := \begin{bmatrix} \gamma & \delta & 0 \\ 0 & \frac{\beta}{1+\beta\alpha} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \Gamma_b := \begin{bmatrix} 1-\gamma & 0 & 0 \\ 0 & \frac{\alpha}{1+\beta\alpha} & 0 \\ 0 & 0 & \rho \end{bmatrix}.$$

Let $\theta{:=}(\gamma,\delta,\beta,\alpha,\kappa,\rho,\varphi_{\tilde{y}},\varphi_{\pi},\rho_{\tilde{y}},\rho_{\pi},\rho_{R},\sigma_{\tilde{y}}^{2},\sigma_{\pi}^{2},\sigma_{R}^{2})'$ be the $m\times 1$ vector of structural parameters ($m{:=}\dim(\theta)$). The elements of the matrices Γ_0, Γ_f, Γ_b and Ξ depend nonlinearly on θ and, without loss of generality, the matrix $\Gamma_0^{\Xi}{:=}(\Gamma_0+\Xi\Gamma_f)$ is assumed to be non-singular. The space of all theoretically admissible values of θ is denoted by $\mathcal{P}$.

For future uses, we consider the partition $\theta{:=}(\theta_s',\ \theta_\varepsilon')'$, where θ_ε contains the non-zero elements of $vech(\Sigma_\varepsilon)$ and θ_s all remaining elements. The 'true' value of θ, $\theta_0{:=}(\theta_{0,s}',\ \theta_{0,\varepsilon}')'$ is assumed to be an interior point of $\mathcal{P}$. Given the partition $\theta{:=}(\theta_s',\ \theta_\varepsilon')'$, we also consider the corresponding partition of the parameter space $\mathcal{P}{:=}\mathcal{P}_{\theta_s}\times\mathcal{P}_{\theta_\varepsilon}$. This distinction is important for two related reasons. First, in the next sub-section we show that the determinacy/indeterminacy of the system depends only on the values taken by θ_s, and not by θ_ε. Second, the sub-vector θ_ε is not directly recoverable (identifiable) from the estimation of the system of Euler equations (5)-(6) through 'limited-information' methods, and our procedure for testing determinacy/indeterminacy also relies on the direct estimation of θ_s from system (5)-(6).

Throughout the paper, we use the notations '$M(\theta)$' and '$M{:=}M(\theta)$' to indicate that the elements of the matrix M depend nonlinearly on the structural parameters θ, hence in our setup $\Gamma_0{:=}\Gamma_0(\theta)$, $\Gamma_f{:=}\Gamma_f(\theta)$, $\Gamma_b{:=}\Gamma_b(\theta)$ and $\Xi{:=}\Xi(\theta)$. Moreover, we call 'stable' a matrix that has all eigenvalues inside the unit disk and 'unstable' a matrix that has at least one eigenvalue outside the unit disk. Thus, denoted with $\lambda_{\max}(\cdot)$ the absolute value of the largest eigenvalue of the matrix in the argument, we have $\lambda_{\max}(M(\theta))<1$ for stable matrices and $\lambda_{\max}(M(\theta))>1$ for unstable ones.

2.2 Reduced form solutions

The solution properties of the system of Euler equations (5)-(6) depend on whether θ_s lies in the determinacy or indeterminacy region of the parameter space. We assume that $\forall\theta_s\in\mathcal{P}_{\theta_s}$, an asymptotically stationary (stable) reduced form solution to system (5)-(6) exists, hence the case of non stationary and 'explosive' (unstable) solutions is automatically ruled out. The whole set of regularity conditions assumed to hold in the specified structural system are reported in out Technical Supplement. The theoretically admissible parameter space $\mathcal{P}_{\theta_s}$ is decomposed into two disjoint subspaces, the determinacy region, $\mathcal{P}_{\theta_s}^{D}$, and its complement $\mathcal{P}_{\theta_s}^{I}{:=}\mathcal{P}_{\theta_s}\backslash\mathcal{P}_{\theta_s}^{D}$. Since we consider only stationary solutions of the new-Keynesian system, $\mathcal{P}_{\theta_s}^{I}$ contains only values of θ_s that lead to multiple stable solutions.

Determinacy/indeterminacy is a system property that depends on all elements in θ_s. There are cases in which the new-Keynesian system is highly restricted and it becomes relatively simple to identify the region $\mathcal{P}_{\theta_s}^{D}$ ($\mathcal{P}_{\theta_s}^{I}$) of the parameter space. For instance, if system (1)-(4) is restricted such that $\gamma{:=}1$, $\alpha{:=}0$, and $\rho{:=}0$, $\rho_x{:=}0$, $x=\tilde{y},\pi,R$, the model collapses to a 'purely forward-looking' model. In this particular case, it can be shown that the inequality

$$\varphi_\pi + \frac{1-\beta}{\kappa}\varphi_{\tilde{y}} > 1 \tag{7}$$

is sufficient and 'generically' necessary (Woodford, 2003, Proposition 4.3, p. 254) for determinacy. Consequently, the determinacy region of the parameter space is given by $\mathcal{P}_{\theta_s}^D := \left\{ \theta_s \in \mathcal{P}_{\theta_s},\, \varphi_\pi + \frac{1-\beta}{\kappa}\varphi_{\tilde{y}} > 1 \right\}$. However, it is in general not possible to work out a set of closed-form inequality constraints from system (5)-(6) that are both necessary and sufficient for determinacy (indeterminacy) and that can potentially be used to test whether $\theta_{0,s}$ lies in $\mathcal{P}_{\theta_s}^D$ or $\mathcal{P}_{\theta_s}^I$.[3]

It is possible to show that, for values of θ_s such that $\lambda_{\max}(G(\theta_s))<1$, where $G(\theta_s):=(\Gamma_0 - \Gamma_f\Phi_1)^{-1}\Gamma_f$ (see Sub-section 2.2), the system (5)-(6) has a unique stable reduced form solution that can be represented as the finite-order VAR[4]

$$[I_3 - \Phi_1(\theta_s)L - \Phi_2(\theta_s)L^2]X_t = u_t \quad , \quad u_t := \Upsilon(\theta_s)^{-1}\varepsilon_t \tag{8}$$

where L is the lag/lead operator ($L^h X_t := X_{t-h}$), X_0 and X_{-1} are fixed initial conditions, $\Phi_1(\theta_s)$, $\Phi_2(\theta_s)$ and $\Upsilon(\theta_s)$ are 3×3 matrices whose elements depend nonlinearly on θ_s and embody the cross-equation restrictions implied by the small new-Keynesian model (Hansen and Sargent, 1980, 1981). More specifically, the matrices $\Phi_1(\theta_s)$ and $\Phi_2(\theta_s)$ in eq. (8) are obtained as the unique solution to the quadratic matrix equation

$$\mathring{\Phi} = (\mathring{\Gamma}_0 - \mathring{\Gamma}_f\mathring{\Phi})^{-1}\mathring{\Gamma}_b \tag{9}$$

where $\mathring{\Gamma}_f$, $\mathring{\Gamma}_0$, $\mathring{\Gamma}_b$ and the stable matrix $\mathring{\Phi}$ are respectively given by

$$\mathring{\Gamma}_0 := \begin{bmatrix} \Gamma_0^{\Xi} & 0_{3\times 3} \\ 0_{3\times 3} & I_3 \end{bmatrix} \quad , \quad \mathring{\Gamma}_f := \begin{bmatrix} \Gamma_f & 0_{3\times 3} \\ 0_{3\times 3} & 0_{3\times 3} \end{bmatrix} , \quad \mathring{\Gamma}_b := \begin{bmatrix} \Gamma_{b,1}^{\Xi} & \Gamma_{b,2}^{\Xi} \\ I_3 & 0_{3\times 3} \end{bmatrix} , \quad \mathring{\Phi} := \begin{bmatrix} \Phi_1 & \Phi_2 \\ I_3 & 0_{3\times 3} \end{bmatrix},$$

where $\Gamma_0^{\Xi} := (\Gamma_0 + \Xi\Gamma_f)$, $\Gamma_{b,1}^{\Xi} := (\Gamma_b + \Xi\Gamma_0)$, $\Gamma_{b,2}^{\Xi} := -\Xi\Gamma_b$ and $\Upsilon(\theta) := (\Gamma_0 - \Gamma_f\Phi_1(\theta))$. The constrained covariance matrix of the reduced form disturbances u_t, denoted with $\tilde{\Sigma}_u$, depends on the entire θ vector and is given by

$$\tilde{\Sigma}_u(\theta) = \Upsilon(\theta_s)^{-1}\Sigma_\varepsilon(\theta_\varepsilon)\Upsilon(\theta_s)'^{-1}. \tag{10}$$

[3] The following example shows that the condition in eq. (7) is not necessary for determinacy if the structural model (1)-(4) involves lags of the variables, other than leads. Consider the system based on β:=0.99, κ:=0.085, δ:=0.40, γ:=0.25, α:=0.05, ρ := 0.95, $\varphi_{\tilde{y}}$:=2, φ_π:=0.77, $\rho_{\tilde{y}}:=\rho_\pi:=\rho_R$:=0.9. In this case, $\varphi_\pi + \frac{1-\beta}{\kappa}\varphi_{\tilde{y}} > 1$, but the rational expectation-solution to system (1)-(4), while being stable, is not unique. Recall that we assume the existence of at least a solution under rational expectations.

[4] A detailed derivation is confined in our Technical Supplement. The Technical Supplement also contains a detailed derivation of the class of indeterminate reduced form solutions reported in eq.s (11)-(12) below.

Equations (9) and (10) define the cross-equation restrictions implied by our new-Keynesian structural model on its reduced form solution under determinacy.

Conversely, for values of θ_s such that $\lambda_{\max}(G(\theta_s))>1$,[5] the class of reduced form solutions associated with the new-Keynesian system (5)-(6) takes the VARMA-type form:

$$[I_3 - \Pi(\theta_s)L][I_3 - \Phi_1(\theta_s)L - \Phi_2(\theta_s)L^2]X_t = [M(\theta_s,\psi) - \Pi(\theta_s)L]V(\theta_s,\psi)^{-1}\varepsilon_t + \tau_t \quad (11)$$

$$\tau_t{:=}[M(\theta_s,\psi) - \Pi(\theta_s)L]V(\theta_s,\psi)^{-1}P(\theta_s)\zeta_t + P(\theta_s)\zeta_t \quad (12)$$

and corresponds to the situation in which the matrix $G(\theta_s)$ has representation

$$G(\theta_s){:=}P(\theta_s)\begin{bmatrix} \Lambda_1 & 0_{n_1\times n_2} \\ 0_{n_2\times n_1} & \Lambda_2 \end{bmatrix} P^{-1}(\theta_s)$$

where $P(\theta_s)$ is a 3×3 non-singular matrix, Λ_1 is the $n_1 \times n_1$ ($n_1 < 3$) Jordan normal block that collects the eigenvalues of $G(\theta_s)$ that lie inside the unit disk and Λ_2 is the $n_2 \times n_2$ ($n_2 \leq 3$) Jordan normal block that collects the eigenvalues of $G(\theta_s)$ that lie outside the unit disk. Notice that $n_1 + n_2{:=}3$, where $n_2{:=}\dim(\Lambda_2)$ determines the 'degree of multiplicity' of solutions. In system (11)-(12), the matrices $\Phi_1(\theta_s)$ and $\Phi_2(\theta_s)$ are defined and constrained likewise the case of determinacy, see eq. (9), while $\Pi(\theta_s)$, $M(\theta_s,\psi)$ and $V(\theta_s,\psi)$ are given by

$$\Pi(\theta_s){:=}P(\theta_s)\begin{bmatrix} 0_{n_1\times n_1} & 0_{n_1\times n_2} \\ 0_{n_2\times n_1} & \Lambda_2^{-1} \end{bmatrix} P^{-1}(\theta_s) \quad , \quad M(\theta_s,\psi){:=}P(\theta_s)\begin{bmatrix} I_{n_1} & 0_{n_1\times n_2} \\ 0_{n_2\times n_1} & \Psi \end{bmatrix} P^{-1}(\theta_s)$$

$$V(\theta_s,\psi){:=}(\Gamma_0(\theta_s) - \Gamma_f(\theta_s)\Phi_1(\theta_s)) - \Xi(\theta_s)\Gamma_f(\theta_s)(I_3 - M(\theta_s,\psi))$$

where Ψ is a $n_2 \times n_2$ matrix ($n_2 \leq 3$) containing arbitrary auxiliary parameters unrelated to θ_s and $\psi{:=}vec(\Psi)$. Finally, the 'additional' moving average term τ_t depends on the 3×1 vector ζ_t of 'sunspot shocks', whose first n_1 elements are zero and the remaining n_2 elements are MDS with respect to $\mathcal{F}_t$ independent on ε_t. We denote with Σ_ζ the covariance matrix of ζ_t; Σ_ζ will be in general singular unless $n_1{:=}0$. We assume that Σ_ζ is time-invariant.

While the determinate equilibrium in eq. (8) depends only on the state variables of the structural system (5)-(6), there are two sources of indeterminacy that characterize the model equilibria in eq.s (11)-(12). The first one is the 'parametric indeterminacy' that stems from the presence of the auxiliary parameters in the vector ψ. Such parameters index solution multiplicity, and they contribute to amplify or dampen the fluctuations of X_t induced by MA part of the reduced form solution. Importantly, such parameters are not identifiable under determinacy. The second on is the 'stochastic indeterminacy' that stems from the presence of the sunspot

[5] The case in which the matrix $G(\theta_s)$ has eigenvalues equal to one is deliberately ignored because it can be associated with the case of non-stationary processes.

shocks in the vector ζ_t (when $\Sigma_\zeta \neq 0_{3\times 3}$). These shocks may arbitrarily alter the dynamics and volatility of the system (see Lubik and Schorfheide (2003, 2004) and Lubik and Surico (2009) for discussions).

Under indeterminacy, the parameter space associated with the new-Keynesian model is 'larger' compared to the case of determinacy. Indeed, in addition to the structural parameters θ, we must consider also the auxiliary parameters in the vectors $\psi{:=}vec(\Psi)$ and σ_ζ^+, where σ_ζ^+ collects the free elements of the covariance matrix Σ_ζ. Both ψ and σ_ζ^+ are unrelated to θ and are not identified under determinacy. Let $\mathcal{N}$ be the open sub-space of $\mathbb{R}^{(n_2)^2}$ of all possible values taken by ψ, and let $\mathcal{Z}$ be the open sub-space of $\mathbb{R}^6$ of all possible values taken by the elements in σ_ζ^+; the 'complete' parameter space associated with the class of VARMA-type indeterminate reduced form solutions generated by the new-Keynesian system is given by

$$\mathcal{I}{:=}\left\{\theta^*{:=}(\theta', \psi', \sigma_\zeta^{+\prime})',\ \theta_s \in \mathcal{P}_{\theta_s}^I,\ \psi \in \mathcal{N},\ \sigma_\zeta^+ \in \mathcal{Z}\right\}. \tag{13}$$

It can be observed that when ψ and σ_ζ^+ are restricted such that

$$\psi{:=}vec(I_{(n_2)^2}) \quad (\Rightarrow M(\theta_s, \psi){:=}I_3) \quad , \quad \sigma_\zeta^+{:=}0_{6\times 1} \quad (\Rightarrow \tau_t{:=}0_{3\times 1} \text{ a.s. } \forall\ t), \tag{14}$$

system (11)-(12) collapses to a MSV solution (McCallum, 1983), i.e. a reduced form solution which has the same representation as the determinate VAR solution in eq. (8), and it is subject to the same set of cross-equation restrictions, see Evans and Honkapohja (1986), Lubik and Schorfheide (2003, 2004), and Fanelli (2012).[6]

3 Testing strategy

Let X_1, ..., X_T be a sample of T observations that is thought of as being generated by a solution of the new-Keynesian system (5)-(6). Our task is to decide whether the observations X_1, ..., X_T support the hypothesis of a unique stable solution or the hypothesis of multiple stable equilibria, controlling for two factors: (i) the possible identification failure, where by this term we mean the case in which the objective functions used to estimate the structural parameters and derive

[6] Observational equivalence between determinate and indeterminate reduced form solutions may be also obtained from system (5) when the vector of structural shocks is absent, i.e. when $\Sigma_\varepsilon{:=}0_{3\times 3}$ ($\varepsilon_t{:=}0_{3\times 1}$ a.s. $\forall\ t$). In this case, under a set of restrictions (including $\Xi{:=}0_{n\times n}$), the structural model can be solved as in eq. (8). Thus there exists a findamental problem of identification which roughly says that an indeterminate equilibrium of an 'exact' DSGE model (i.e. based on $\varepsilon_t{:=}0_{3\times 1}$ and $\Xi{:=}0_{n\times n}$) corresponds to the determinate equilibrium of some more general DSGE model, see Beyer and Farmer (2007) and Fanelli (2012) for a comprehensive discussion. While being interesting from a theoretical standpoint, the case of absence of fundamental disturbances in the structural equations is empirically unpalatable, and will not be considered in our analysis.

the test statistics may be poorly informative about θ or some of its components; (ii) the data adequacy of system (5)-(6).

In principle, an 'ideal' test for the null $H_0 : \theta_{0,s} \in \mathcal{P}^D_{\theta_s}$ against the alternative $H_1 : \theta_{0,s} \in \mathcal{P}^I_{\theta_s}$ should be based on testing the set of inequality restrictions that identify the region $\mathcal{P}^D_{\theta_s}$ ($\mathcal{P}^I_{\theta_s}$) of the parameter space.[7] In our framework, a general characterization of the indeterminacy region of the parameter space $\mathcal{P}^I_{\theta_s}$ is given by $\mathcal{P}^I_{\theta_s}$:=$\{\theta_s \in \mathcal{P}_{\theta_s},\, \lambda_{\max}(G(\theta_s))>1\}$, (see Sub-section 2.2 and the Technical Supplement). Unfortunately, even under strong identification, the condition $\lambda_{\max}(G(\theta_s))>1$ can hardly be used for testing purposes because (aside from very special cases) it is not easy to map inequality restrictions on the eigenvalues of the $G(\theta_s)$ matrix onto a set of 'manageable' restrictions which might be used in practice. Even working out the inequalities associated with the condition $\lambda_{\max}(G(\theta_s))>1$ on a case-by-case basis, the resulting testing problem would involve nonstandard inference, see e.g. Wolak (1989) and Silvapulle and Sen (2005).

Alternatively, one might compare the likelihoods of the determinate and indeterminate reduced forms, under the assumption of correct specification of the new-Keynesian system. The complication, in this case, stems from the already mentioned observational equivalence between the determinate solution and the MSV solution: even in the absence of sunspot shocks (τ_t:=$0_{3\times 1}$ a.s. $\forall\ t$), a classical likelihood-based test would call for the comparison of the 'VAR(2)' in eq. (8) with a 'VARMA(3,1)' system in eq. (11). As it is known, this would entail a well known non-standard inferential problem, see e.g. Lubik and Schorfheide (2004) and Fanelli (2012).

To circumvent the above mentioned difficulties and address the testing problem from another perspective, we follow Fanelli (2012), and consider the two hypotheses

$$H_0' : X_t \text{ is generated by the VAR system (8) under the CER (9)-(10).} \tag{15}$$

and

$$H_1' : X_t \text{ is generated by the VARMA-type system (11)-(12), with } \theta^* \in \mathcal{I}^0 \tag{16}$$

where $\mathcal{I}^0$ is a subset of $\mathcal{I}$ (see eq. (13)) defined by

$$\mathcal{I}^0 := \left\{ \theta^* := (\theta', \psi', \sigma_\zeta^{+\prime})',\, \theta \in \mathcal{P}^I,\, \psi \in \mathcal{N} \setminus \left\{ vec(I_{(n_2)^2}) \right\},\, \sigma_\zeta^+ \in \mathcal{Z} \setminus \{0_{6\times 1}\} \right\} \subset \mathcal{I}. \tag{17}$$

[7]For instance, Mavroeidis (2010) uses the standard 'Taylor principle' condition in eq. (7) to address the determinacy/indeterminacy issue in U.S. monetary policy by estimating a Taylor-type monetary policy rule in isolation from other structural equations. The typical risk with this 'single-equation' approach is that the 'Taylor principle' holds with certainty in the form of eq. (7) only if the structural system (5)-(6) fulfills e.g. the restrictions γ:=1, α:=0, and ρ:=0, ρ_x:=0, $x = \tilde{y}, \pi, R$. Our estimates reported in Section 5 show that these restrictions are invalid. Farmer and Guo (1995) use the inequality restriction that identify the indeterminacy region of the parameter space in their stylized business cycle model, and show that their point estimates of the structural parameters fulfil the restriction. However, they do not provide any inference.

Under H_0', the new-Keynesian system generates the unique stable solution which is, however, indistinguishable from the indeterminate MSV equilibrium nested in the system (11)-(12) when the conditions in eq. (14) is valid. Under H_1', instead, the new-Keynesian system generates indeterminate non-MSV equilibria, see Sub-section 2.2. A key observation here is that the null of determinacy, $H_0 : \theta_{0,s} \in \mathcal{P}_{\theta_s}^D$, implies the hypothesis H_0' in eq. (15), while the converse is not true. Hence, the rejection of H_0' in eq. (15) leads to the rejection of the null of determinacy. The non-rejection of H_0' is sufficient to rule out the occurrence of arbitrary parameters unrelated to θ ('parametric indeterminacy') and of sunspot shocks unrelated to ε_t from the solution set ('stochastic indeterminacy'), but can not be considered as conclusive evidence of determinacy.

We face the problem of testing H_0' in eq. (15) against H_1' in (16) borrowing from the recent literature on inference in weakly identified DSGE models (Dufour *et al.* 2009, 2013; Kleibergen and Mavroeidis 2009; Mavroeidis 2010; Qu, 2011; Andrews and Mikusheva, 2012; Guerron-Quintana *et al.* 2013). In particular, we exploit the idea that the construction of the confidence set is a well known dual problem to hypothesis testing, i.e. confidence sets are obtained by inverting tests, see e.g. Aitchison (1964) and Lehman (1986).

We maintain that only the sub-vector θ_ε of $\theta{:=}(\theta_s', \theta_\varepsilon')'$ is strongly identified, while identification failure may involve the sub-vector θ_s or some of its components. We then consider the reduced form VAR solution of the new-Keynesian model in eq. (8) and the vector $\phi_u^*{:=}(\phi_u', vech(\Sigma_u)')'$, where $\phi_u{:=}vec(\Phi_u)$ and $\Phi_u{:=}[\Phi_1, \Phi_2]$ collects the VAR unrestricted coefficients. In our setup, ϕ_u^* is assumed to be strongly identified. The CER that the new-Keynesian model places on its determinate reduced form solution in eq.s (9) and (10) can conveniently be compacted in the expression

$$f(\phi_u^*, \theta) = 0_{\dim(\phi_u^*)\times 1} \tag{18}$$

where $f(\cdot,\cdot)$ is a continuous, twice differentiable vector function. By the implicit function theorem, the restrictions in eq. (18) can also be written in explicit form as follows (see Iskrev, 2008, and our Appendix):

$$\phi_{\theta_s}^* {=} g(\theta) \tag{19}$$

where $g(\cdot)$ is a nonlinear twice differentiable function and the mapping is valid in a neighborhood of the 'true' parameter values. We have used the notation '$\phi_{\theta_s}^*$' in eq. (19) to remark that the vector $\phi_{\theta_s}^*$, which reads as the constrained counterpart of ϕ_u^*, depends on θ_s (other than the strongly identified parameters in the vector θ_ε). More precisely, $\phi_{\theta_s}^*$ is the vector of VAR coefficients under the CER implied by the hypothesis of determinacy. The log-likelihood function associated with the reduced form VAR solution depends on θ through eq. (19) (see our Appendix).

Our identification-robust testing procedure for H_0' in eq. (15) against H_1' in eq. (16) is based on the two steps presented below.

Step 1: 'Full-information' LR test for the CER. We invert a LR test for the null hypothesis

$$H_{0,cer} : \phi^*_{\breve{\theta}_s} = g(\breve{\theta}_s, \theta_\varepsilon) \quad , \quad \breve{\theta}_s \in \mathcal{P}_{\theta_s} \tag{20}$$

(against the alternative $H_{1,cer} : \phi^*_{\breve{\theta}_s} \neq g(\breve{\theta}_s, \theta_\varepsilon)$) in the context of the reduced form VAR solution in eq. (8). The hypothesis $H_{0,cer}$ specializes the CER in eq. (19) to the case in which θ_s is fixed at the 'guess' $\theta_s := \breve{\theta}_s$ about the 'true' parameter values. Observe that if $H_{0,cer}$ is valid, also the hypothesis H_0' in eq. (15) is valid for $\theta_s := \breve{\theta}_s$. Likewise, if H_0' in eq. (15) is valid for $\theta_s := \breve{\theta}_s$, $H_{0,cer}$ in eq. (20) is automatically valid. Let $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ be the LR test for the hypothesis H_{cer} in eq. (20), where $\hat{\phi}^*_{\breve{\theta}_s}$ is defined by $\hat{\phi}^*_{\breve{\theta}_s} := g(\breve{\theta}_s, \hat{\theta}_\varepsilon^{\breve{\theta}_s})$ and $\hat{\theta}_\varepsilon^{\breve{\theta}_s}$ is the ML estimate of the sub-vector θ_ε obtained for fixed $\theta_s := \breve{\theta}_s$ (in other words, $\hat{\theta}_\varepsilon^{\breve{\theta}_s}$ is the ML estimates of θ_ε obtained from the concentrated log-likelihood function associated with the determinate reduced form solution); computational aspects are discussed in the Appendix. In practice, there are many possible choices $\theta_s := \breve{\theta}_s$ which might not be rejected by the data. Since the components of θ_s typically lie within bounded (theoretically admissible) intervals, one can test $H_{0,cer}$ for any possible choice of $\breve{\theta}_s$ within a fine grid $\mathcal{G}_{\theta_s} \subseteq \mathcal{P}_{\theta_s}$, giving rise to a 'grid testing' procedure.[8] Under $H_{0,cer}$ in eq. (20), the asymptotic null distribution of $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ is $\chi^2_{d_1}$, $d_1 := \dim(\phi^*_u) - \dim(\theta_\varepsilon)$, regardless of the strength of identification, see e.g. Guerron-Quintana *et al.* (2013) and our Appendix. The by-product of the grid testing approach for $H_{0,cer}$ is the identification-robust confidence set for the structural parameters (or acceptance region)

$$\mathcal{C}^{LR}_{1-\eta_1} := \left\{ \breve{\theta}_s \in \mathcal{G}_{\theta_s},\ LR_T(\hat{\phi}^*_{\breve{\theta}_s}) < c^{\eta_1}_{\chi^2_{d_1}} \right\} \tag{21}$$

which has asymptotic coverage $100(1-\eta_1)$, where $c^{\eta_1}_{\chi^2_{d_1}}$ is the η_1-level cut-off point associated with the $\chi^2_{d_1}$ distribution, and $0<\eta_1<1$ is the pre-fixed nominal level of significance (or type-I error) of the test in the first-step.[9] The hypothesis $H_{0,cer}$ is not rejected at the level η_1 if $\mathcal{C}^{LR}_{1-\eta_1}$ is nonempty, and is rejected if $\mathcal{C}^{LR}_{1-\eta_1}$ is empty. In the first case, we accept the hypothesis H_0' in eq. (15) and stop the analysis. In the second case, we move to the next step.

[8] Mikusheva (2010) observes that grid testing makes sense only if one can *a priori* restrict possible values of the parameters to belong to a bounded set, which is our case. She shows an alternative method to invert identification-robust tests which can not be easily applied in our setup.

[9] Dufour *et al.* (2013) propose a slightly different version of the test for the CER, see the Appendix for details.

Step 2: 'Limited-information' test for the OR. Conditional on the confidence set in eq. (23) being empty, we test the hypothesis H_1' in eq. (16) indirectly, i.e. focusing on an auxiliary hypothesis. In particular, we use a 'limited-information' approach and invert an Anderson Rubin-type (Anderson and Rubin, 1949) test for the simple hypothesis

$$H_{0,spec} : \theta_s{=}\breve{\theta}_s \quad , \quad \breve{\theta}_s \in \mathcal{G}_{\theta_s} \tag{22}$$

(against the alternative $H_{1,spec}{:}\theta_s \neq \breve{\theta}_s$) in the context of the system of Euler equations (5)-(6). $H_{0,spec}$ is the hypothesis that the system of Euler equations is valid in correspondence of the point $\theta_s{:=}\breve{\theta}_s \in \mathcal{G}_{\theta_s}$, regardless of whether their reduced form solutions belong to the determinate equilibrium in eq.s (8)-(10) or to the class of indeterminate equilibria in eq.s (11)-(12). Let $AR_T(\breve{\theta}_s)$ denote any asymptotically pivotal test statistic for the simple hypothesis in eq. (22) computed in the context of system (5)-(6).[10] Under correct specification, the (unconditional) asymptotic distribution of $AR_T(\breve{\theta}_s)$ under $H_{0,spec}$ is $\chi^2_{d_2}$ regardless of the strength of identification (the number of degree of freedom, d_2, is discussed in the Appendix), see, *inter alia*, Dufour *et al.* (2010, 2013). However, conditional on the first-step, $H_{0,spec}$ turns into the hypothesis that the system of Euler equations is valid in correspondence of the point $\theta_s{:=}\breve{\theta}_s \in \mathcal{P}^I_{\theta_s}$. Thus, conditional on the first-step, the grid testing approach for $H_{0,spec}$ generates the identification-robust confidence set (or acceptance region):

$$\mathcal{C}^{LR-AR}_{1-\eta_{2\cdot 1}}{:=}\left\{\breve{\theta}_s \in \mathcal{G}_{\theta_s},\, LR_T(\hat{\phi}^*_{\breve{\theta}_s}) \geq c^{\eta_1}_{\chi^2_{d_1}} \text{ and } AR_T(\breve{\theta}_s) < c^{\eta_2}_{\chi^2_{d_2}}\right\} \tag{23}$$

where $c^{\eta_2}_{\chi^2_{d_2}}$ is the η_2-level cut-off point associated with the $\chi^2_{d_2}$ distribution and $\eta_{2\cdot 1}$ is such that $\eta_{2\cdot 1} \leq \eta_2$ (see the Appendix). The set $\mathcal{C}^{LR-AR}_{1-\eta_{2\cdot 1}}$ is asymptotically valid with coverage 100(1-$\eta_{2\cdot 1}$)$\geq$100(1-η_2). All points $\breve{\theta}_s$ which lies within $\mathcal{C}^{LR-AR}_{1-\eta_{2\cdot 1}}$ are such that the hypothesis H_0' in eq. (15) is rejected by the $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test and the hypothesis H_1' in eq. (16) is accepted by the $AR_T(\breve{\theta}_s)$ test. If the set $\mathcal{C}^{LR-AR}_{1-\eta_{2\cdot 1}}$ is empty, meaning that none value of the parameters within $\mathcal{G}_{\theta_s}$ is compatible with the data, we reject the correct specification of the new-Keynesian system (5)-(6).

Hereafter, we conventionally denote the testing strategy obtained by combining the two described steps above with the symbol '$LR_T \rightarrow AR_T$'. We discuss the computational details of the two tests in the Appendix, where we also focus on the asymptotic properties of the

[10] Our notation emphasizes the dependence of $AR_T(\breve{\theta}_s)$ on the fixed value $\theta_s{:=}\breve{\theta}_s$ Our Appendix shows that the $AR_T(\breve{\theta}_s)$ test reads as a test for the OR implied by the system of Euler equations.

procedure. We remark the fact that the hypothesis H_0' in eq. (15) is rejected (and hence determinacy is rejected) if the inversion of the $LR_T(\hat{\phi}^*_{\mathring{\theta}_s})$ test in the first-step provides an empty confidence set. In this case, we move to the second-step to decide whether the rejection of the CER occurs because the hypothesis of indeterminacy H_1' is valid, or because the new-Keynesian system is dynamically misspecified. It turns out that the second-step is run only conditionally on the rejection of the CER in the first-step and must be based on the same grid $\mathcal{G}_{\theta_s}$ used in the first-step. We remark that the non-rejection of the null H_0' is sufficient to rule out the occurrence of sunspot shocks and arbitrary nuisance parameters from the solution set but can not be considered conclusive evidence of determinacy. However, since when an hypothesis is not rejected by a significance test, all hypotheses implied by that hypothesis must also be considered as non-rejected, the non-rejection of H_0' in the first-step amounts to an implicit non-rejection of the system of structural Euler equations (5)-(6). Table 1 summarizes the logic of our '$LR_T \to AR_T$' testing strategy.

It is worth stressing that when the hypothesis $H_{0,spec}$ in eq. (22) is rejected in the second-step, one should think of alternative structural frameworks to capture the richer dynamics of the data. Lubik and Schorfheide (2004) work along this line by augmenting their purely-forward looking baseline new-Keynesian framework with price indexation and habit formation in consumption to check the robustness of their evidence on indeterminacy/uniqueness in the post-WWII U.S. economy. Dynamically rich, distributed-lag small scale models have been employed by Rudebusch (2002), Estrella and Fuhrer (2002,2003), and Fuhrer and Rudebusch (2004), among others. In general, the knowledge of model misspecification per se is a warning for the researcher to explore the space of the models adopted in the literature in order to find empirically supported alternative frameworks. Again, referring to the monetary-macroarea, an alternative to small scale models (like the one we work with in this paper) is represented by medium scale frameworks à la Christiano *et al.* (2005) and Smets and Wouters (2007). These latter models feature more variables (physical capital and its frictions, among others) as well as different CER on the variables in common with small scale models (due, for instance, on non-separable preferences in consumption and labor as in Smets and Wouters, 2007). Hence, while lacking a unique indication on the alternative framework one should scrutinize when model misspecification is detected in the second-stage of our methodology, we believe an econometrician may be willing to implement our testing strategy to have a sense of the reliability of his/her results on determinacy/indeterminacy.

Table 1. Summary of the '$LR_T \rightarrow AR_T$' testing strategy for the new-Keynesian system (5)-(6)

<table>
<tr><td colspan="3">Step 1: $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test rejects the CER ($\mathcal{C}^{LR}_{1-\eta_1}$empty) ?</td></tr>
<tr><td colspan="2">YES</td><td>NO</td></tr>
<tr><td colspan="2">Step 2: $AR_T(\breve{\theta}_s)$ test rejects the OR ($\mathcal{C}^{LR-AR}_{1-\eta_{2\cdot 1}}$ empty) ?</td><td></td></tr>
<tr><td>YES</td><td>NO</td><td>H_0' in eq. (15) is accepted</td></tr>
<tr><td>Omission of
propagation
mechanisms</td><td>H_1' in eq. (16) is accepted
Indeterminacy</td><td>Non conclusive evidence of determinacy
(sunspot shocks ruled out)</td></tr>
<tr><td>new-Keynesian model rejected</td><td>new-Keynesian model accepted</td><td>new-Keynesian model accepted</td></tr>
</table>

By its design, the size (power) of the '$LR_T \rightarrow AR_T$' testing strategy, defined as the probability of rejecting the hypothesis H_0' in eq. (15) when H_0' (H_1') is 'true', depends on the $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test alone, thus η_1 reads as the pre-fixed type-I error. It can be proved that under the null H_0' in eq. (15), the asymptotic size of the '$LR_T \rightarrow AR_T$' testing strategy is η_1 (see Proposition 1 in the Appendix), and that the test is consistent against the hypothesis of indeterminacy H_1' in eq. (16) (see Proposition 2 in the Appendix). However, we can also think of a 'second-step size' associated with the '$LR_T \rightarrow AR_T$' testing strategy, defined as the probability that the test $AR_T(\breve{\theta}_s)$ computed in the second-step erroneously rejects H_1' when H_1' is 'true'. In finite samples, the 'second-step size' of the $AR_T(\breve{\theta}_s)$ test depends on the power of the test $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ against H_1'. It can be proved that under H_1', the asymptotic 'second-step size' of the procedure converges to the quantity $\eta_{2\cdot 1}$ (see eq. (23)) which has η_2 as nominal upper bound (see Proposition 3 and Proposition 4 in the Appendix).

It is worth observing that in Guerron-Quintana *et al.* (2013), whose analysis covers a family of DSGE models larger than ours, the confidence set $\mathcal{C}^{LR}_{1-\eta_1}$ in eq. (21) is proposed as an identification-robust confidence set for θ_s. As the construction of the identification-robust confidence set $\mathcal{C}^{LR}_{1-\eta_1}$ in a 'full-information' setup requires solving the model under determinacy, tighter inference can be achieved if the CER are not rejected compared to the inference based on 'limited-information' methods, see also Dufour *et al.* (2013). Moreover, since Dufour (1997) it is known that the two identification-robust confidence sets $\mathcal{C}^{LR}_{1-\eta_1}$ and $\mathcal{C}^{LR-AR}_{1-\eta_{2\cdot 1}}$ can be empty (when the tests reject all parameter points in $\mathcal{G}_{\theta_s}$) but also unbounded, which in our framework corresponds to the situation $\mathcal{C}^{LR}_{1-\eta_2} = \mathcal{G}_{\theta_s}$ and/or $\mathcal{C}^{LR-AR}_{1-\eta_{2\cdot 1}} = \mathcal{G}_{\theta_s}$ (when θ_s is unidentified).

We finally observe that point estimates of θ_s can be obtained from the (nonempty) confidence sets $\mathcal{C}^{LR}_{1-\eta_1}$ and $\mathcal{C}^{LR-AR}_{1-\eta_{2\cdot 1}}$. Indeed, the quantities

$$\breve{\theta}^*_{s,ML} := \underset{\breve{\theta}_s \in \mathcal{C}^{LR}_{1-\eta_1}}{\arg\min} \; LR_T(\hat{\phi}^*_{\breve{\theta}_s}) \;\; , \;\; \breve{\theta}^*_{s,LI} := \underset{\breve{\theta}_s \in \mathcal{C}^{LR-AR}_{1-\eta_{2\cdot 1}}}{\arg\min} \; AR_T(\breve{\theta}_s) \qquad (24)$$

can be interpreted as the parameter points within the sets $\mathcal{C}^{LR}_{1-\eta_1}$ and $\mathcal{C}^{LR-AR}_{1-\eta_{2\cdot 1}}$ with associated largest p-values (or the 'least rejected' models at the pre-fixed levels η_1 and η_2, respectively).[11]

The main features of our approach are that (i) it is not necessary to identify the set of parametric inequality restrictions that define the sub-regions $\mathcal{P}^D_{\theta_s}$ ($\mathcal{P}^I_{\theta_s}$) of the parameter space, with the advantage of not being committed to the use of nonstandard asymptotic inference; (ii) it is not necessary to specify prior distributions for θ and, notably, for the auxiliary parameters ψ (and σ^+_ζ) governing solution multiplicity in eq.s (11)-(12); (iii) the procedure is asymptotically valid irrespective of the strength of identification, hence it can be applied also under strong

[11] The point estimates in eq. (24) can be interpreted as 'Hodges-Lehmann' estimates of θ_s, see e.g. Dufour *et al.* (2006, 2009, 2010).

identification; (iv) the test is explicitly designed to control for the case of omitted dynamics, other than identification failure, a key issue in the empirical assessment of determinacy/indeterminacy and, more generally, a crucial challenge in the econometric literature on DSGE models, see e.g. the discussion in Lubik and Schorfheide (2004), Del Negro *et al.* (2007), Del Negro and Schorfheide (2009), Cúrdia and Reis (2010) and Schorfheide (2011).

4 Monte Carlo simulations

In this section, we use the Benati and Surico's (2009) new-Keynesian system in eq.s (1)-(4) as model to investigate the finite sample properties of the '$LR_T \rightarrow AR_T$' testing strategy by some Monte Carlo simulations.

Artificial data sets are generated from the reduced form solutions discussed in Sub-section 2.2 which serve as data generating process (DGP). In all experiments, we consider $M = 1,000$ replications and samples of length $T = 100$ (not including initial lags). The chosen sample size corresponds roughly to the number of quarterly observations we consider for the 'pre-Volcker' (1954q1-1979q2) and 'Great Moderation' (1985q1-2008q2) samples in the empirical section using U.S. data (see Section 5). For each generated data set, we treat the output gap as observable, reproducing the situation we face in Section 5.

To evaluate the empirical size of the '$LR_T \rightarrow AR_T$' test, the Monte Carlo design is calibrated to match the model estimated by Benati and Surico (2009) using U.S. data with Bayesian methods. The discount factor β:=0.99 is treated as known and estimation involves 13 free parameters, 10 of which are collected in the sub-vector θ_s and 3 in the sub-vector θ_ε. The 'true' vector $\theta_{0,s}:=(\theta_{0,s}', \theta_{0,\varepsilon}')'$ is calibrated at the medians of the 90% coverage percentiles of the posterior distribution reported in Table 1 of Benati and Surico (2009) (see the 'After the Volcker stabilization' column). The data are generated from the reduced form VAR solution in eq.s (8) subject to the restrictions in eq.s (9)-(10) using a Gaussian distribution for the structural shocks ε_t and a diagonal covariance matrix Σ_ε (hence the elements of the sub-vector $\theta_{0,\varepsilon}$ correspond to the diagonal components of Σ_ε). With this choice of $\theta_{0,s}$, the largest eigenvalue of the matrix $G(\theta_{0,s}):=(\Gamma_0^{\Xi} - \Gamma_f \Phi_1)^{-1}\Gamma_f$ (see Sub-section 2.2 and the Technical Supplement) is equal to $\lambda_{\max}(G(\theta_{0,s}))$=0.964. For each simulated dataset, the numerical inversion of the $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test is obtained by considering 300 points $\breve{\theta}_s$ randomly chosen using the uniform distribution from the grid $\mathcal{G}_{\theta_s}$ described in detail in the caption of Table 1; the inversion procedure (or grid-testing) generates the identification-robust confidence set $\mathcal{C}^{LR}_{1-\eta_1}$ in eq. (38). The empirical size of the '$LR_T \rightarrow AR_T$' test corresponds to the empirical rejection frequency of the $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test and is evaluated by fixing the type-I error of the test at the level η_1:=0.05.

Table 2. Empirical size of the '$LR_T \rightarrow AR_T$' testing strategy when the data are generated from the new-Keynesian business cycle monetary system (28)-(5) under the hypothesis H_0' in eq. (15).

'true' $\theta_{0,s}$		T=100	η_1=0.05	
$\lambda_{\max}(G(\theta_{0,s}))$:=0.964	Interpret.	$\breve{\theta}^*_{s,ML}$	Med. int. length [with true]	
γ_0:=0.744	IS, forward look. term	0.718 (0.159)	0.060 [0.13]	
δ_0:=0.124	IS, inter. elast. of sub.	0.121 (0.031)	0.030 [0.061]	Rej($LR_T(\hat{\phi}^*_{\breve{\theta}_s})$)=0.045
α_0:=0.059	NKPC: index. past infl.	0.061 (0.023)	0.038 [0.069]	
κ_0:=0.044	NKPC: slope	0.043 (0.011)	0.011 [0.021]	
ρ_0:=0.834	Rule, smoothing term	0.777 (0.175)	0.063 [0.377]	
$\varphi_{\tilde{y},0}$:=1.146	Rule, react. to out. gap	0.974 (0.40)	0.563 [1.310]	
$\varphi_{\pi,0}$:=1.749	Rule, react. to inflation	1.557 (0.591)	0.712 [1.87]	
$\rho_{\tilde{y},0}$:=0.796	Out. gap shock, persist.	0.755 (0.166)	0.058 [0.105]	
$\rho_{\pi,0}$:=0.418	Inflation shock, persist.	0.392 (0.105)	0.110 [0.220]	
$\rho_{R,0}$:=0.404	Pol. rate shock, persist.	0.385 (0.106)	0.103 [0.229]	

NOTES. Results are obtained using M=1,000 replications. Each simulated sample is initiated with 200 additional observations to get a stochastic initial state and then are discarded. The structural parameters are calibrated to the medians of the posterior distributions reported in Table 1 of Benati and Surico (2009), column 'After the Volcker stabilization'. The numerical inversion of the $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test for the CER (step 1 of Section 3 and Appendix) is obtained on each generated dataset by considering 300 points $\breve{\theta}_s$ randomly chosen using the uniform distribution from the grid $\mathcal{G}_{\theta_s}$ given by the rectangle formed by the Cartesian product of the following intervals: [0.688, 0.822] for γ, [0.09, 0.16] for δ, [0.03, 0.099] for α, [0.035, 0.056] for κ, [0.515, 0.877] for ρ, [0.383, 1.61] for $\varphi_{\tilde{y}}$, [0.70, 2.57] for φ_π, [0.738, 0.834] for $\rho_{\tilde{y}}$, [0.30, 0.52] for ρ_π and [0.289, 0.518] for ρ_R. '$\breve{\theta}^*_{s,ML}$' is the point estimates of θ_s derived from the identification-robust confidence set $\mathcal{C}^{LR}_{0.95}$, see eq. (24), and the associated values in parentheses are the corresponding Monte Carlo standard errors. 'Med. lengths [with true]' reports the median interval length of the projected 95% confidence intervals across simulations contrasted with the actual interval lengths. Rej($\cdot$) stands for 'rejection frequency'.

The results are reported in Table 2. Here we summarize the rejection frequency of the $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test and the average point estimates of the structural parameters derived from the generated identification-robust confidence sets $\mathcal{C}^{LR}_{0.95}$, see eq. (24), along with the Monte Carlo standard errors. We notice that the $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test is slightly conservative (0.045 in samples of length T=100 as opposed to the nominal size η_1:=0.05) and that the grid-testing procedure delivers point estimates of the structural parameters relatively close to the true values. A reasonable concern here is the role played by the grid used to invert the $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test: in the limiting case of no identification, one would expect appropriately sized intervals to cover the support of the structural parameter, i.e. $\mathcal{C}^{LR}_{1-\eta_1} = \mathcal{G}_{\theta_s}$. To address this issue, the fourth column of Table 1 contrasts the median interval length obtained for the projected parameters with the actual grid length of the intervals for the individual parameters. The results show that the projected identification-robust intervals are often wide, but not excessively so.

To investigate the power of the '$LR_T \rightarrow AR_T$' procedure against the hypothesis H_1' in eq. (16) and the 'second-step size' of the $AR_T(\breve{\theta}_s)$ test under H_1', we must consider specific DGPs obtained from the VARMA-type reduced form solutions in eq.s (11)-(12). We can only provide limited Monte Carlo experimentation because given the structural parameters and the fundamental shocks, the choice of ψ and σ^+_ζ from system (11)-(12) is completely arbitrary. To simplify the analysis, we follow Lubik and Schorfheide (2004) and Fanelli (2012), and focus on the case of 'indeterminacy without sunspots', which corresponds to the situation in which sunspot shocks do not enter the reduced form solution, i.e. σ^+_ζ:=$0_{6\times 1}$ ($\Rightarrow \tau_t$:=$0_{3\times 1}$ a.s. $\forall$ t) in eq.s (11)-(12). The 'true' vector of parameters $\theta_{0,s}$:=$(\theta'_{0,s}, \theta'_{0,\varepsilon})'$ is calibrated at the medians of the 90% coverage percentiles of the posterior distribution reported in Table 1 of Benati and Surico (2009), 'Before October 1979' column. With this choice of $\theta_{0,s}$, the largest eigenvalue of the matrix $G(\theta_{0,s})$ is equal to $\lambda_{\max}(G(\theta_{0,s}))$=1.0051, hence only one eigenvalue lies outside the unit circle and the vector of auxiliary parameters ψ:=$vec(\Psi)$ which governs the 'parametric indeterminacy' of the system collapses to a scalar. We consider three possible values for ψ: 0.95, 1.05 and 0.5, respectively, where 0.95 and 1.05 are relatively close to the point ψ:=1 which generates an indeterminate MSV solution observationally equivalent to the unique stable solution, see Subsection 2.2. Given these three possible choices of ψ, artificial dataset are generated from system (11) which reads as a pure 'VARMA(3,1)' system with highly restricted parameters. Also in this experiment, for each simulated dataset, the numerical inversion of the tests $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ and $AR_T(\breve{\theta}_s)$ is conducted by considering 300 points $\breve{\theta}_s$ randomly chosen by employing the uniform distribution from the same grid $\mathcal{G}_{\theta_s}$ used for the size experiment in Table 2. The $AR_T(\breve{\theta}_s)$ test is computed by following the method described in the Appendix. The empirical power and 'second-step size' of the '$LR_T \rightarrow AR_T$' testing strategy are evaluated by fixing η_1 and η_2

at the levels η_1:=0.05 and η_2:=0.05, respectively. Other than documenting the joint empirical rejection frequency of the $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ and $AR_T(\breve{\theta}_s)$ tests (as required by the second-step of the '$LR_T \rightarrow AR_T$' procedure), we also report the (marginal) empirical rejection frequency of the $AR_T(\breve{\theta}_s)$ test, i.e. computed by disregarding the outcome of the $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test in the first step (see our Appendix). As noticed in the previous section, the 'second-step size' associated with the '$LR_T \rightarrow AR_T$' testing strategy is bounded by construction by the unconditional rejection frequency of the $AR_T(\breve{\theta}_s)$ test under H_1'.

The results are summarized in Table 3. We observe that the power of the test against the hypothesis H_1' in eq. (16) is reasonably good even when the indeterminate equilibrium is close to the MSV solution (the empirical power is 61.5% for ψ:=0.95 and 70.5% for ψ:=1.05). The finite sample rejection frequency of the $AR_T(\breve{\theta}_s)$ test, instead, seems to be influenced to some extent by the value taken by the nuisance parameter ψ which, recall, amplifies or dampens the oscillations of the reduced form solution in addition to what implied by the fundamental shocks through the moving average part of system (11). In samples of size T=100, the empirical size of our computed version of the $AR_T(\breve{\theta}_s)$ test ranges from 0.064 (ψ:=0.50) to 0.025 (ψ:=0.95) as opposed to the pre-fixed nominal size $\eta_{2.1} \leq \eta_2$:=0.05, so that we can conclude that the under(over)-rejection phenomenon is confined to admissible levels.

Overall, the results of our Monte Carlo experiment summarized in Tables 2 and 3 suggest that the '$LR_T \rightarrow AR_T$' testing strategy delivers reasonable empirical size coverage with respect to the null H_0' in eq. (15) and reassuring empirical power against the hypothesis of indeterminacy. Furthermore, also the 'second-step size' coverage of the testing strategy, i.e. its tendency to erroneously reject the hypothesis of indeterminacy, appears under control in samples of lengths typically available to practitioners.

Table 3. Empirical power and 'second-step size' of the '$LR_T \rightarrow AR_T$' testing strategy when the data are generated from the new-Keynesian business cycle monetary system (5)-(6) under the hypothesis H_1' in eq. (16) (with sunspot shocks absent).

'true' $\theta_{0,s}$			
$\lambda_{\max}(G(\theta_{0,s}))$:=1.0051		T=100	η_1=0.05, η_2=0.05
γ_0:=0.744	Indet. param:	ψ:=0.95	
δ_0:=0.124			Rej($LR_T(\hat{\phi}^*_{\breve{\theta}_s})$)=0.615
α_0:=0.059			Rej($AR_T(\breve{\theta}_s)$)=0.033
κ_0:=0.044			Rej($AR_T(\breve{\theta}_s)$; $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$)=0.025
ρ_0:=0.595			
$\varphi_{\widetilde{y},0}$:=0.527			
$\varphi_{\pi,0}$:=0.821		ψ:=1.05	
$\rho_{\widetilde{y},0}$:=0.796			Rej($LR_T(\hat{\phi}^*_{\breve{\theta}_s})$)=0.705
$\rho_{\pi,0}$:=0.418			Rej($AR_T(\breve{\theta}_s)$)=0.038
$\rho_{R,0}$:=0.404			Rej($AR_T(\breve{\theta}_s)$; $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$)=0.03
		ψ:=0.50	Rej($LR_T(\hat{\phi}^*_{\breve{\theta}_s})$)=1
			Rej($AR_T(\breve{\theta}_s)$)=0.064
			Rej($AR_T(\breve{\theta}_s)$; $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$)=0.064

NOTES. Results are obtained using M=1,000 replications. Each simulated sample is initiated with 200 additional observations to get a stochastic initial state and then are discarded. The structural parameters are calibrated to the medians of the posterior distributions reported in Table 1 of Benati and Surico (2009), column 'Before October 1979'. The numerical inversions of the tests $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ (step 1 of Section 3 and Appendix) and $AR_T(\breve{\theta}_s)$ (step 2 of Section 3 and Appendix) are obtained on each generated dataset by considering 300 points $\breve{\theta}_s$ randomly chosen using the uniform distribution from the same grid $\mathcal{G}_{\theta_s}$ used in the size experiment in Table 2. ψ is the auxiliary parameter which governs the 'parametric indeterminacy' of the system, see Sub-section 3.2. $AR_T(\breve{\theta}_s)$ is computed as a quasi-LR test as detailed in the Appendix, using Z_t:=$(X_{t-1}', X_{t-2}', ..., X_{t-r}')'$ and r =6 in the auxiliary multivariate regression system (32). Rej($\cdot$) stands for 'rejection frequency'; Rej($AR_T(\breve{\theta}_s)$; $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$) denotes the joint rejection frequencies of the two tests.

5 Empirical evidence

We now turn to the implementation of our two-step investigation as for the post-WWII U.S. economic system. We employ U.S. quarterly data, sample 1954q3-2008q3, and three observable variables, $X_t{:=}(\tilde{y}_t, \pi_t, R_t)'$. The output gap $\tilde{y}_t$ is computed as percent log-deviation of the real GDP with respect to the potential output estimated by the Congressional Budget Office.[12] The inflation rate π_t is the quarterly growth rate of the GDP deflator. For the short-term nominal interest rate R_t we consider the effective Federal funds rate expressed in quarterly terms (averages of monthly values). The source of the data is the Federal Reserve Bank of St. Louis' web site. The beginning of the sample is due to data availability (in particular, of the effective Federal Funds rate. The end of the sample is justified by our intention to avoid dealing with the 'zero-lower bound' phase began in December 2008, which triggered a series of non-standard policy moves by the Federal Reserve whose effects are hardly captured by our standard new-Keynesian framework.

Our reference structural model is given by the new-Keynesian system (1)-(4).[13] Following most of the literature on the 'Great Moderation', we divide the post-WWII U.S. era in two periods, roughly corresponding to the 'Great Inflation' and the 'Great Moderation' samples. We take the advent of Paul Volcker as Chairman of the Federal Reserve to identify our first sub-sample, i.e. 1954q3-1979q2, which we call 'pre-Volcker' sample. As for the 'Great Moderation' sample, we consider the period 1985q1-2008q3. McConnell and Pérez-Quirós (2000) find a break in the variance of the U.S. output growth in 1984q1. Our empirical investigation deals with a measure of the output gap, inflation, and the federal funds rate. Signs of the 'Volcker disinflation' are still evident in 1984. This is possibly due to the 'credibility build-up' undertaken by the Federal Reserve in the early 1980s, a period during which private agents gradually changed their view on the Federal Reserve's ability to deliver low inflation (Goodfriend and King, 2005). Moreover, the first years of Volcker's tenure (until October 1982) were characterized by non-borrowed reserves targeting. Hence, the fit of our policy rule would substantially worsen if

[12] This measure of the output gap is the one used by, among others, Benati and Surico (2009). In theory, different proxies of the output gap may lead to different answers to our research question. Canova and Ferroni (2011) propose a method to combine different proxies of the business cycle in a likelihood-based estimation of a modern new-Keynesian model for the U.S. economy. We leave the investigation of the impact of alternative proxies of the output gap for the determinacy/indeterminacy issue to future research.

[13] A limit of our investigation is the absence of any consideration regarding the possible impact that fiscal policy may exert on determinacy. Since Leeper's (1991) contribution, we have known that the assessment of equilibrium determinacy in new-Keynesian monetary policy models of the business cycle should involve monetary and fiscal policies jointly. Our test may very well be applied to more sophisticated models dealing with the fiscal-monetary policy mix, an idea that belongs to our research agenda.

we included the Volcker disinflation (Estrella and Fuhrer, 2003; Mavroeidis, 2010), a fact that would carry consequences on the estimates of all parameters of the system.[14] To circumvent this problem, we postpone the beginning of our second sub-sample to 1985q1. A similar choice is undertaken by Christiano *et al.* (2013). Thus, our 'Great Moderation' sample is given by the period 1985q1-2008q3 and will be denoted as 'post-1985' sample throughout this Section.

The first-step of the '$LR_T \rightarrow AR_T$' testing strategy requires the computation of the 'full-information' likelihood-based test of the CER that the new-Keynesian model implies under determinacy, i.e. the $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test discussed in Section 3. As it is common in the literature, we pre-fix the nominal level of significance at the 10% level (η_1:=0.10). We report in Table 4 the results of the $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test on the 'pre-Volcker' and 'post-1985' samples, respectively.

A detailed description of the grid $\mathcal{G}_{\theta_s}$ used to invert the test numerically may be found in the caption of Table 4. In the upper panel of Table 4, we summarize the projected 90% confidence intervals for the individual elements of θ_s derived from the identification-robust confidence set $\mathcal{C}^{LR}_{0.90}$ (see eq. (21)) and the point estimate of θ_s, $\breve{\theta}^*_{s,ML}$, see eq. (24). In the lower panel of Table 4, we indicate whether the grid-testing procedure leads to an empty or nonempty identification-robust confidence set and report the value of $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ associated with $\breve{\theta}^*_{s,ML}$ and corresponding p-value.

[14] Our results, however, are robust to the employment of a shorter 'pre-Volcker' sample (1966q1-1979q2) and, with qualifications, to a longer 'Great Moderation' sample (1979q4-2008q3). The results obtained on these samples are available upon request to the authors.

Table 4. Projected 90% identification-robust confidence intervals, point estimates of the structural parameters θ_s:=$(\gamma, \delta, \alpha, \kappa, \rho, \varphi_{\tilde{y}}, \varphi_\pi, \rho_{\tilde{y}}, \rho_\pi, \rho_R)'$ and results of the first-step of the '$LR_T \rightarrow AR_T$' testing strategy on U.S. quarterly data.

		1954q3-1979q2 'pre-Volcker'		1985q1-2008q3 'Great Moderation'	
Parameter	Interpret.	$\breve{\theta}^*_{s,ML}$	proj. 90% c.i.	$\breve{\theta}^*_{s,ML}$	proj. 90% c.i.
γ	IS, forward look. term	-	-	0.729	0.652-0.772
δ	IS, inter. elast. of sub.	-	-	0.082	0.082-0.154
α	NKPC: index. past infl.	-	-	0.020	0.020-0.059
κ	NKPC: slope	-	-	0.048	0.042-0.098
ρ	Rule, smoothing term	-	-	0.666	0.569-0.697
$\varphi_{\tilde{y}}$	Rule, react. to out. gap	-	-	0.339	0.127-0.479
φ_π	Rule, react. to inflation	-	-	5.439	2.318-.5.445
$\rho_{\tilde{y}}$	Out. gap shock, persist.	-	-	0.920	0.720-0.978
ρ_π	Inflation shock, persist.	-	-	0.925	0.748-0.970
ρ_R	Pol. rate shock, persist.	-	-	0.794	0.730-0.806
identification-robust c.s. $\mathcal{C}^{LR}_{0.90}$		empty		nonempty (card($C^{LR}_{0.90}$)=15)	
$\lambda_{\max}(G(\breve{\theta}^*_{s,ML}))$		−		0.946	
$LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test (first-step)		−		19.54 [0.36]	

NOTES. The projected 90% identification-robust confidence intervals (proj. 90% c.i.) have been obtained from the 90% identification-robust confidence set $\mathcal{C}^{LR}_{0.90}$ (see eq. (21)) by computing, in turn, the smallest and largest values of each parameter included in the set $\mathcal{C}^{LR}_{0.90}$. The set $\mathcal{C}^{LR}_{0.90}$ has been obtained by inverting numerically the $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test (step 1 of Section 3 and Appendix) considering 5,000,000 points $\breve{\theta}_s$ randomly chosen using the uniform distribution with support given by the rectangle formed by the Cartesian product of the following intervals: [0.65, 0.85] for γ, [0.08, 0.16] for δ, [0.02, 0.10] for α, [0.04, 0.10] for κ, [0.50, 0.70] for ρ, [0.05, 1.5] for $\varphi_{\tilde{y}}$, [0.5, 5.5] for φ_π, [0.40, 0.98] for $\rho_{\tilde{y}}, \rho_\pi$ and ρ_R,. '$\breve{\theta}_{s,ML}$' is the point estimate derived from $\mathcal{C}^{LR}_{0.90}$, see eq. (24), and $\lambda_{\max}(\cdot)$ denotes the modulus of the largest eigenvalue of the matrix in the argument. $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ in the lower panel reports the value of the test statistics obtained in correspondence of the 'least rejected' model within $\mathcal{C}^{LR}_{0.90}$. P-values in brackets. Estimation on each sub-period is carried out by considering within-periods initial values and variables are demeaned within each sub-period.

Table 4 suggests two important facts. First, the CER that the new-Keynesian system implies under determinacy are firmly rejected on the 'pre-Volcker' sample (the set $\mathcal{C}_{0.90}^{LR}$ is empty),[15] and are firmly accepted on the 'post-1985' sample by the data (the set $\mathcal{C}_{0.90}^{LR}$ is nonempty and the p-value associated with the 'least rejected' model is 0.36). We thus reject the hypothesis of determinacy on the 'pre-Volcker' sample and do not reject the hypothesis H_0' in eq. (15) on the 'post-1985' sample. Despite we can not interpret the result relative to the chosen 'Great Moderation' regime as conclusive evidence of determinacy (see the discussions in Sub-section 2.2 and Section 3),[16] the inference is sufficient to rule out the scenario according to which the U.S. business cycle was driven by sunspot expectations extraneous to fundamental shocks. Interestingly, the fact that the CER entailed by the hypothesis of determinacy are not rejected on the period 1985q1-2008q3 suggests an implicit non-rejection of the new-Keynesian system (1)-(4) on that sample.[17] Second, the 90% projected identification-robust confidence intervals for the policy (feedback) parameters $\varphi_{\tilde{y}}$ and φ_{π} are surprisingly tighter than the confidence sets documented by other authors using frequentist methods. In particular, the estimation of the value of the parameter φ_{π}, which captures the systematic reaction of the Federal Reserve to inflation, has attracted a lot of attention. The debate has been intense also because of the lack of precision surrounding the estimates of such parameter. A prominent example in the literature is represented by Mavroeidis (2010). He convincingly shows that, in a single-equation context, the estimation of φ_{π} tends to be imprecise, and the formal evidence in favor of an aggressive systematic policy response to inflation is scant. Possible reasons include (a) the absence of sunspot shocks under determinacy, which implies a lower volatility of inflation and output and, therefore, a harder identification of the systematic relationship between the policy rate and the policy relevant-macroeconomic variables, and (b) a higher degree of interest rate smoothing, which limits the reaction of the policy rate in presence of shocks hitting inflation and output.[18]

[15] Also using the level η_1:=0.05 we find that $\mathcal{C}_{0.95}^{LR}$ is empty. Results available upon request.

[16] A merely descriptive indicator that the equilibrium we have estimated and tested on the 'post-1985' sample in Table 4 is not a MSV solution which might occur under the multiple equilibria case is given by the largest eigenvalues of the estimated matrix $G(\breve{\theta}_{s,ML}^{*})$. We obtain $\lambda_{\max}(G(\breve{\theta}_{s,ML}^{*}))$=0.946, a value that would encourage one to rule out the MSV hypothesis and consider the uniqueness scenario seriously.

[17] According to Pesaran (1987), this evidence also implies the non-rejection of the rational expectations hypothesis.

[18] Mavroeidis (2010) also points to the 'Volcker-experiment', which implies a larger volatility of the residuals of the Taylor rule during the period 1979-1982, as another possible reason for the imprecise estimates he finds with his single-equation approach. When experimenting with the sample 1979q3-1997q4, our procedure found evidence in favor of indeterminacy. This result may be due to the fact that the period 1979q3-1997q4 includes the 'Volcker experiment' and the subsequent Volcker disinflation, which are hard to describe with our model under uniqueness. Interestingly, our procedure favors indeterminacy with respect to model misspecification in this case. A possible interpretation is the 'flexibility' of the model under indeterminacy, which features extra shocks and parameters

Interestingly, our result, collected in Table 4, allows us to formally support an aggressive policy conduct by the Federal Reserve in the 'post-1985' period ruling out the role of sunspot fluctuations on the one hand, and under a fair amount of interest rate smoothing (ranging from 0.73 to 0.81, according to our 90% confidence interval) on the other hand. Hence, our analysis suggests that 'full-information' methods designed to deal with identification failure provide more precise information than 'limited-information'/single-equation approaches. Importantly, our identification-robust approach does not lead us to reject the correct specification of the specified new-Keynesian model during the 'Great Moderation'. Our findings are particularly important in light of a recent paper by Cochrane (2011), who has argued that the parameters of Taylor-type rules like that in eq. (3) are not identifiable in prototypical new-Keynesian models. Cochrane (2011), however, considers formulations of the new-Keynesian system which are 'less involved', from a dynamic standpoint, than our 'hybrid' specification in eq.s (1)-(4). Our results in Table 4 show that the 'full-information' approach delivers relatively tight confidence sets not only for $\varphi_{\tilde{y}}$ and φ_{π}, but also for δ (intertemporal elasticity of substitution), α (indexation to past inflation) and κ (slope of the NKPC), which are notoriously difficult to estimate precisely from the data.

We then proceed with the second-step of the '$LR_T \rightarrow AR_T$' testing strategy, which requires the inversion of the Anderson and Rubin-type $AR_T(\breve{\theta}_s)$ test for the OR implied by the system of Euler equations (1)-(4) on the 'pre-Volcker' sample. Note, indeed, that the CER implied by the new-Keynesian model under the hypothesis of determinacy have been rejected by the data on the 'pre-Volcker' sample. Therefore the second-step 'limited-information' evaluation approach is conducted to establish whether the rejection of the hypothesis of determinacy must be ascribed to the multiple equilibria hypothesis or to the inability of the estimated system to capture the propagation mechanisms at work in the data. For completeness, we invert the $AR_T(\breve{\theta}_s)$ test not only on the 'pre-Volcker' sample but also on the 'post-1985' sample, albeit this calculation would not be required by the '$LR_T \rightarrow AR_T$' testing strategy (recall that we have accepted the new-Keynesian system on the 'post-1985' sample in the previous step).

and a richer dynamic structure.

Table 5. Projected 90% identification-robust confidence intervals, point estimates of the structural parameters θ_s:=$(\gamma, \delta, \alpha, \kappa, \rho, \varphi_{\tilde{y}}, \varphi_\pi, \rho_{\tilde{y}}, \rho_\pi, \rho_R)'$ and results of step 2 of the '$LR_T \rightarrow AR_T$' procedure on U.S. quarterly data.

		1954q3-1979q2 'pre-Volcker'		1985q1-2008q3 'Great Moderation'	
Parameter	Interpret.	$\breve{\theta}^*_{s,LI}$	proj. 90% c.i.	$\breve{\theta}^*_{s,LI}$	proj. 90% c.i.
γ	IS: forward look. term	0.841	0.660-0.845	0.821	0.650-0.85
δ	IS: inter. elast. of sub.	0.088	0.084-0.160	0.132	0.080-0.160
α	NKPC: index. past infl.	0.025	0.020-0.070	0.097	0.020-0.099
κ	NKPC: slope	0.042	0.040-0.058	0.087	0.040-0.10
ρ	Rule, smoothing term	0.520	0.500-0.698	0.699	0.500-0.700
$\varphi_{\tilde{y}}$	Rule, react. to out. gap	0.138	0.05-0.325	0.295	0.05-1.043
φ_π	Rule, react. to inflation	0.687	0.50-0.906	2.123	0.50-5.499
$\rho_{\tilde{y}}$	Out. gap shock, persist.	0.900	0.620-0.964	0.911	0.400-0.980
ρ_π	Inflation shock, persist.	0.578	0.414-0.793	0.907	0.400-0.980
ρ_R	Pol. rate shock, persist.	0.798	0.565-0.916	0.795	0.674-0.980
identification-robust c.s. $\mathcal{C}^{LR-AR}_{0.90}$ ($\mathcal{C}^{AR}_{0.90}$)		nonempty (card($\mathcal{C}^{LR-AR}_{0.90}$)=26)		nonempty (card($\mathcal{C}^{AR}_{0.90}$)=41891)	
$\lambda_{\max}(G(\breve{\theta}^*_{s,LI}))$		1.012		0.965	
$AR_T(\breve{\theta}^*_{s,LI})$ test (second-step)		24.44 [0.14]		19.27 [0.37]	

NOTES. The projected 90% identification-robust confidence intervals (proj. 90% c.i.) have been obtained from the 90% identification-robust confidence set $\mathcal{C}^{LR-AR}_{0.90}$ (see eq. (23)) computed on the 'pre-Volcker sample' and from the 90% identification-robust confidence set $\mathcal{C}^{AR}_{0.90}$ (see eq. (38) in the Appendix) computed on the 'Great Moderation' sample by considering, in turn, the smallest and largest values of each parameter included in the set $\mathcal{C}^{LR-AR}_{0.90}$ ($\mathcal{C}^{AR}_{0.90}$). These confidence sets have been obtained by inverting the test $AR_T(\breve{\theta}_s)$ (step 2 of Section 3 and Appendix); in practice, $AR_T(\breve{\theta}_s)$ is computed as a quasi-LR test using Z_t:=$(X'_{t-1}, X'_{t-2})'$ in the auxiliary multivariate regression system (32), considering 5,000,000 points $\breve{\theta}_s$ randomly chosen using the uniform distribution with support given by the rectangle formed by the Cartesian product of the same intervals as in Table 4. '$\breve{\theta}^*_{s,LI}$' is the point estimate derived from $\mathcal{C}^{LR-AR}_{0.90}$, see eq. (24), and $\lambda_{\max}(\cdot)$ denotes the modulus of the largest eigenvalue of the matrix in the argument. $AR_T(\breve{\theta}^*_{s,LI})$ reports the value of the test statistics obtained in correspondence of the 'least rejected' model within $\mathcal{C}^{LR-AR}_{0.90}$. P-values in brackets. Estimation on each sub-period is carried out by considering within-periods initial values and variables are demeaned within each sub-period.

In this latter case, however, the identification-robust confidence set resulting from the inversion of the $AR_T(\breve{\theta}_s)$ test must be interpreted as detailed in eq. (38) of the Appendix. We pre-fix η_2 at the level η_2:=0.10 and invert the test using the same grid $\mathcal{G}_{\theta_s}$ employed to invert the test for the CER in the first-step (recall that $\eta_{2\cdot 1} \leq \eta_2$). The $AR_T(\breve{\theta}_s)$ test is computed as detailed in the Appendix.

The results of the second-step are summarized in Table 5. In the upper panel, we report the projected confidence intervals for the individual elements of θ_s derived from the identification-robust confidence set $\mathcal{C}_{1-\eta_{2\cdot 1}}^{LR-AR}$ produced by the grid-testing procedure (see eq. (23)) along with the point estimate (see eq. (24)). In the lower panel, we indicate whether the grid-testing procedure leads to an empty or nonempty identification-robust confidence set and, in the second case, we report the value of the test statistic associated with the point estimate $\breve{\theta}_{s,LI}^{*}$ and corresponding p-value.

Table 5 shows that the new-Keynesian model is not rejected by the $AR_T(\breve{\theta}_s)$ test on the 'pre-Volcker' sample (the set $\mathcal{C}_{0.90}^{LR-AR}$ is nonempty and the p-value associated with the 'least rejected' model is 0.14). As expected, we also find that the new-Keynesian model is not rejected by the $AR_T(\breve{\theta}_s)$ test on the 'post-1985' sample (the set $\mathcal{C}_{0.90}^{LR-AR}$ is nonempty and the p-value associated with the 'least rejected' model is 0.37). This is a 'reassuring' result, as it corroborates the outcome obtained with the $LR_T(\hat{\phi}_{\breve{\theta}_s}^{*})$ test in the first-step. Moreover, if we compare the projected identification-robust confidence intervals built with the 'full-information' method (sixth column of Table 4) with the corresponding intervals built with the 'limited-information' method (sixth column of Table 4), we find that the former are remarkably more informative than the latter.

By combining the evidence in Table 5 with that in Table 4 we conclude that the '$LR_T \rightarrow AR_T$' testing strategy leads us to accept the hypothesis of indeterminacy (H_1' in eq. (16)) on the 'pre-Volcker' sample on which the set $\mathcal{C}_{0.90}^{LR}$ is empty and the set $\mathcal{C}_{0.90}^{LR-AR}$ is nonempty, see Table 1, and not to reject the hypothesis H_0' in eq. (15) on the 'Great Moderation' sample on which the set $\mathcal{C}_{0.90}^{LR}$ is nonempty, see Table 1. Thus, our conclusions point towards a policy switch in the late 1970s. This result is not new in the literature, as it corroborates the one proposed by Clarida *et al.* (2000), Lubik and Schorfheide (2004), Boivin and Giannoni (2006), Benati and Surico (2009), Mavroeidis (2010), and Inoue and Rossi (2011*a*), among others. The novelty of our analysis is that our conclusions have been derived with a formal testing strategy with (i) a robustness check for identification failure and model misspecification, a crucial information when conducting inference in the class of monetary policy new-Keynesian models, and on (ii) a full-system context without appealing to any a-priori distribution or any calibration of nuisance parameters. Clearly, our prior-free approach maximizes the role attached to the data in determining our results.

An approximate and purely indicative measure of the extend of the change characterizing the parameters of the model across the two regimes can be broadly obtained by comparing the identification-robust confidence intervals and the point estimates reported in Table 4 and Table 5. For instance, we find that as for the parameters δ (intertemporal elasticity of substitution) α (indexation to past inflation), φ_π (long run reaction to inflation) and ρ_π (inflation shock persistence), the 'full-information' point estimates computed on the 'post-1985' sample (see the fifth column of Table 4) do not lie within (or lie on the border of) the corresponding 'limited-information' identification-robust confidence intervals computed on the 'pre-Volcker' sample (see the fourth column of Table 5). Evidence of instability in the parameters of the private sector, other than the policy parameters, has also been found, among others, by Canova (2009), Inoue and Rossi (2011a), Canova and Menz (2011), Canova and Ferroni (2012), Castelnuovo (2012a), and Cantore *et al.* (2013).

6 Relation to the literature

Our findings support the role played by variations in the structure of the economic system in driving the U.S. economy from indeterminacy to a unique equilibrium under rational expectations. Both policy and non-policy parameters are found to be unstable across sub-samples. Our evidence on variations of the policy parameters provides fresh support to the story popularized by Clarida *et al.* (2000) on the aggressive monetary policy implemented by the Federal Reserve to engineer a deflation in the early 1980s and maintain inflation to low levels afterwards. Our results, which are obtained with a frequentist approach and by dealing with identification issues in a full-system context, confirm the findings obtained by Lubik and Schorfheide (2004) with their Bayesian analysis. Recent empirical investigations show that changes in both policy and non-policy structural parameters may be behind the U.S. 'Great Moderation'. Our estimates, obtained with the 'limited-information' approach, also suggest the possibility of variations in our private sector's parameters. Hence, our findings are consistent with those documented by previous contributions on the instability in the private sector's parameters in the U.S. economy (see, among others, Canova, 2009; Inoue and Rossi, 2011a; Canova and Menz, 2011; Canova and Ferroni, 2012; Castelnuovo, 2012a; Cantore *et al.*, 2013). Admittedly, our analysis does not take any stand on the relevance of the 'good luck' explanation supported by, among others, Stock and Watson (2002), Sims and Zha (2006), Smets and Wouters (2007), Justiniano and Primiceri (2008), Canova *et al.* (2009), and Canova and Ferroni (2012). Such interpretation suggests a reduction in the volatility of the shocks hitting the economy. An elaboration of our proposal aimed at identifying the relative importance of these two drivers of the 'Great Moderation' is left to future research.

On the methodological side, our paper is connected to the recent work of Guerron-Quintana *et al.* (2013) on 'full-information' frequentist inference in DSGE models, see also Dufour *et al.* (2013). Indeed, the first step of our testing procedure is essentially based on the pointwise inversion of the likelihood ratio test proposed by these authors as a tool to build identification-robust confidence sets for the structural parameters of DSGE models. In our setup, the likelihood ratio test is used to obtain an identification-robust 'acceptance region' for the structural parameters which fulfil the CER the new-Keynesian system places on its reduced form solution under determinacy. Likewise, our methodology is connected to the contributions by Stock and Wright (2000), Kleiberger and Mavroeidis (2009) and Dufour *et al.* (2006, 2009, 2010, 2013), among others. Indeed, conditional on the first-step, the second-step of the suggested testing strategy requires the pointwise inversion of an Anderson Rubin-type (Anderson and Rubin, 1949) test for the OR implied by the system of Euler equations. Finally, compared to Fanelli (2012), who also proposes a test for determinacy/indeterminacy in new-Keynesian models controlling for the omission of propagation mechanisms, our procedure is robust to identification failures and can be applied regardless of the strength of identification.

On the empirical side, Lubik and Schorfheide (2004) test for determinacy in the U.S. economy with a model similar to ours by undertaking a Bayesian investigation in which posterior weights for the determinacy and indeterminacy regions of the parameter space are constructed and compared. Our paper implements a frequentist approach, which neither requires the use of a-prior distributional assumptions nor the commitment to non-standard inference. In particular, we are not forced to choose a prior distribution for some arbitrary auxiliary parameters that index the multiplicity of solutions under rational expectations as in Lubik and Schorfheide (2004). With respect to Boivin and Giannoni (2006), our method is based on the direct estimation of the structural new-Keynesian model and provides a direct control for the cases of identification failure and dynamic misspecification. Hence, we need not minimize the distance between some selected impulse responses taken from a VAR modeling the macroeconomic variables of interest and the structural model-based responses, a methodology which is unfortunately bias-prone as for expectations-based models like ours (Canova and Sala, 2009). More importantly, we need not make restrictive assumptions on the solution under indeterminacy, as opposed to the MSV solution assumed by Boivin and Giannoni (2006). While being plausible, such solution is anyhow arbitrary, and it may importantly affect the simulated moments of interest (Castelnuovo, 2012b). Mavroeidis (2010) applies identification-robust 'limited-information' methods to investigate the determinacy/indeterminacy of U.S. monetary policy conditional on the estimation of the policy rule in isolation. Compared to Mavroeidis (2010), we (i) investigate the issue of macroeconomic stability of U.S. monetary policy by using a fully specified 'hybrid new-Keynesian model' of

Benati and Surico (2009), and (ii) apply a testing strategy which is robust to identification failure. Mavroeidis (2010) conjectures that the difference between the (precise) confidence intervals in the 'pre-Volcker' period and the (imprecise) ones in the 'post-Volcker' phase may be interpreted as (a) absence of sunspot fluctuations during the 'Great Moderation'; (b) increase in the policy inertia; (c) larger variability of the policy shocks during the first years of the Volcker era. Our methodology formally shows that sunspot fluctuations are unlikely to have played a role during the 'Great Moderation'. We therefore offer statistical support to Mavroeidis' conjecture (a). Differently, we do not find clear evidence in favor of an increase in the policy inertia when moving from our first to our second sub-sample. However, the confidence interval surrounding the point estimate of the degree of interest rate smoothing during the 'Great Moderation' does not exclude Mavroeidis' second conjecture (b) either. Finally, our 'Great Moderation' sub-sample begins in 1985, i.e., after the end of the 'Volcker experiment' related to the targeting of non-borrowed reserves by the Federal Reserve. Hence, our results are not necessarily driven by a large volatility of the policy shocks, whose volatility has drastically reduced since 1985 (see Mavroeidis (2010), Figure 3 - left panel). More importantly, however, we show that, when applying a system based 'full-information' approach designed to handle weak identification, the precision of the estimates obtained for the 'Great Moderation' sample is higher than the one achieved via a single-equation approach.

7 Concluding remarks

This paper has proposed and implemented a novel approach to test for monetary policy determinacy/indeterminacy in the United States in the context of a fully specified small-scale new-Keynesian model. Importantly, our test is robust to identification failures and model dynamic misspecification. Identification failures are characterized as the situation in which the criterion function used to estimate the new-Keynesian model is nearly uninformative (or even uninformative) about the structural parameters, whereas dynamic misspecification is interpreted as the omission of relevant propagation mechanisms from the specified structural system, which may render the identification of indeterminacy (vs. determinacy) unreliable. The proposed testing strategy relies on the principle that the construction of identification-robust confidence sets for the structural parameters requires the inversion of asymptotically pivotal test statistics which have correct asymptotic size regardless of the strength of identification. Our methodology can be applied regardless of the strength of identification of the structural parameters and requires neither the use of prior distributions nor that of nonstandard inference, hence the degree of arbitrariness of our empirical results is substantially reduced. The provided Monte

Carlo experimentation suggest that the '$LR_T \to AR_T$' testing strategy can fruitfully be applied in empirical work.

When applied to U.S. data using the new-Keynesian framework à la Benati and Surico (2009) as reference structural model, our testing strategy finds formal support in favor of a switch from indeterminacy to a framework consistent with uniqueness roughly corresponding to the advent of Paul Volcker as Chairman of the Federal Reserve. Importantly, we obtain formal evidence in favor of an aggressive monetary policy during the Great Moderation period by controlling for the effects of weak identification and the possible misspecification of the propagation mechanisms of the shocks. This result differs from the one documented by Mavroeidis (2010) with his single-equation approach. We attribute this difference to the 'full-information' nature of the first step of our robust test and to the fact that the estimated new-Keynesian model is not rejected by the data on the 'Great Moderation' period.

Our findings, which line up with a number of previous contributions in the literature, are consistent with, but not necessarily pointing to, the 'good policy' explanation of the U.S. Great Moderation. We plan to elaborate further on our methodology to assess whether other drivers, in addition to the change in the conduct of monetary policy documented in this paper, have contributed significantly to the 'Great Moderation' phenomenon.

In light of the recent financial crisis, the uniqueness scenario supported by our analysis as for the period mid-1980s-onwards may very well be over. When enough data become available, our methodology will help to shed further light on this issue. We also believe our methodology may be fruitfully applied to understand if other economic realities have experienced changes in their macroeconomic environment of the type documented here. Benati (2008) documents a reduction in inflation persistence in a variety of countries under stable monetary regimes with clearly defined nominal anchors, e.g., official inflation targeters. We plan to apply our formal testing strategy to these countries in future research.

Appendix

In this Appendix, we focus on some computational aspects and asymptotic distributions of the tests $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ and $A_T(\breve{\theta}_s)$ discussed in Section 3, and then formalize the asymptotic properties of the '$LR_T \to AR_T$' testing strategy.

Computation issues and asymptotic properties: the $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test

The test $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ for the CER obtained under determinacy is computed as follows. Consider the VAR model of lag order two

$$X_t = \Phi_u Z_t + u_t, \tag{25}$$

where Φ_u:=$[\Phi_1, \Phi_2]$, Φ_1 and Φ_2 are unrestricted 3×3 matrices, Z_t:=$(X'_{t-1}, X'_{t-2})'$ and u_t is assumed to obey a 3-dimensional Gaussian white noise process with covariance matrix Σ_u. System (25) reads as the unrestricted counterpart of the determinate reduced form solution of the new-Keynesian model in eq. (8). Assuming Gaussian disturbances, the log-likelihood function of the unrestricted VAR system is given by

$$\log L_T(\phi_u^*):=\text{const} - \frac{T}{2}\log(\det(\Sigma_u)) - \frac{1}{2}\sum_{t=1}^{T}(X_t - \Phi_u Z_t)'\Sigma_u^{-1}(X_t - \Phi_u Z_t) \tag{26}$$

where ϕ_u^*:=$(\phi_u', \text{vech}(\Sigma_u)')'$ and ϕ_u:=vec(Φ_u). Obviously, in our framework ϕ_u^* is strongly identified. We refer to Guerron-Quintana *et al.* (2013) and Andrews and Mikusheva (2012) for the analysis of more involved situations, especially for cases in which not all components of X_t are observed.

As observed in Section 3, the CER that the new-Keynesian system implies under the hypothesis of determinacy can conveniently be compacted in implicit form as in eq. (18). It can be proved that, in our setup, the following condition holds (see Iskrev, 2008; Fanelli, 2011):

$$rank\left[\frac{\partial f(\phi_{0,u}^*, \theta_0)}{\partial \phi_u^{*\prime}}\right] = \dim(\phi_u^*) \tag{27}$$

(in correspondence of neighborhoods of the 'true' values of ϕ_u^* and θ). By the implicit function theorem, the CER can be represented in explicit form as in eq. (19), i.e. $\phi_{\theta_s}^*$=$g(\theta)$, where the nonlinear function $g(\cdot)$ is differentiable. Thus, the log-likelihood of the reduced form solution of the new-Keynesian model under the hypothesis H_0' in eq. (15) is obtained by replacing $\phi_u^{*\prime}$ with $\phi_{\theta_s}^*$=$g(\theta)$, i.e. by imposing the restrictions that the structural model places on on Φ_u and Σ_u by eq.s (9)-(10). The log-likelihood function of the constrained VAR is denoted with $\log L_T(\phi_{\theta_s}^*)$.

First we assume temporarily that θ (and hence $\phi_{\theta_s}^*$) is strongly identified as in Fanelli (2012). The maximization of $\log L_T(\phi_{\theta_s}^*)$ requires the use of numerical (iterative) techniques based on various approximations of the quadratic matrix equation in (9). Departures from the normality assumption imply that the estimator of θ is actually a quasi-ML (Q-ML) estimator. Given the Q-ML estimate of θ, $\hat{\theta}$, the (quasi-)LR test for the cross-equation restrictions is given by

$$LR_T:= -2(\log L_T(\hat{\phi}_{\theta_s}^*) - \log L_T(\hat{\phi}_u^*)) \tag{28}$$

where $\hat{\phi}_{\theta_s}^*$:=$g(\hat{\theta})$ is the ML estimator of the VAR coefficients subject to eq.s (9)-(10). Under the assumptions reported in the Technical Supplement and the null that the CER in eq. (19) are valid, the test statistic LR_T is asymptotically $\chi^2(\dim(\phi_u^*) - \dim(\theta))$-distributed, see Fanelli (2012).

We now relax the assumption of strong identification of θ_s and consider the situation in which it is known that only the sub-vector θ_ε of θ:=$(\theta_s', \theta_\varepsilon')'$ is strongly identified. In this framework, the shape of $\log L_T(\phi^*_{\breve{\theta}_s})$ might be poorly informative (or uninformative) about θ_s or some of its components, violating one the standard regularity conditions underlying ML estimation, see, *inter alia*, Andrews and Mikusheva (2012). However, for fixed $\theta_s{:=}\breve{\theta}_s$, the CER correspond to the hypothesis $H_{0,cer}$ in eq. (20) and the (concentrated) log-likelihood function $\log L_T(\phi^*_{\breve{\theta}_s})$ depends on θ_ε alone. Moreover, under the assumptions reported in the Technical Supplement, $\log L_T(\phi^*_{\breve{\theta}_s})$ fulfills the conditions discussed in e.g. Guerron-Quintana *et al.* (2013). We denote with $\hat{\phi}^*_{\breve{\theta}_s}$ the ML estimator of the constrained VAR coefficients given by $\hat{\phi}^*_{\breve{\theta}_s}{:=}g(\breve{\theta}_s, \hat{\theta}^{\breve{\theta}_s}_\varepsilon)$, where $\hat{\theta}^{\breve{\theta}_s}_\varepsilon$ is the ML estimate of the sub-vector θ_ε obtained for fixed $\theta_s{:=}\breve{\theta}_s$ (in other words, $\hat{\theta}^{\breve{\theta}_s}_\varepsilon$ is the ML estimates of θ_ε obtained from the concentrated log-likelihood function associated with the VAR system (8)). If $\theta_s{:=}\breve{\theta}_s{:=}\theta_{0,s} \in \mathcal{P}^D_{\theta_s}$ (which implies that it is valid the hypothesis H'_0 in eq. (15)), by Proposition 3a in Guerron-Quintana *et al.* (2013) we have

$$LR_T(\hat{\phi}^*_{\breve{\theta}_s}){:=}-2(\log L_T(\hat{\phi}^*_{\breve{\theta}_s}) - \log L_T(\hat{\phi}^*_u)) \to \chi^2(d_1) \tag{29}$$

where $d_1{:=}\dim(\phi^*_u)$-$\dim(\theta_\varepsilon)$. If instead $\theta_s{:=}\breve{\theta}_s{:=}\theta_{0,s} \in \mathcal{P}^I_{\theta_s}$ (i.e. it is valid the hypothesis of indeterminacy H'_1 in eq. (16)) or $\theta_s{:=}\breve{\theta}_s \neq \theta_{0,s} \in \mathcal{P}^I_{\theta_s}$ (i.e. the selected vector of parameter does not fulfil the CER), $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ is $O_p(T)$. Indeed, in the first case the DGP is given by the VARMA-type system (11)-(12) and accordingly the VAR in eq. (8) omits relevant propagation mechanisms; in the second case, instead, the constrained reduced form VAR solution obtained for $\theta_s{:=}\breve{\theta}_s$ is not consistent with the data.

Dufour *et al.* (2013) have proposed another identification-robust 'full-information' approach for the structural parameters of DSGE models based on the (numerical) inversion of a test for zero coefficients in the multivariate regression of the quantity $u_t(\breve{\theta}_s){:=}[I_3{-}\Phi_1(\breve{\theta}_s)L{-}\Phi_2(\breve{\theta}_s)L^2]X_t$ (which correspond to the disturbance of the VAR system (8) under the CER) on the regressors $Z_t{:=}(X'_{t-1}, X'_{t-2})'$.

Computation issues and asymptotic properties: the $AR_T(\breve{\theta}_s)$ test

The test $AR_T(\breve{\theta}_s)$ for the hypothesis $H_{0,spec}$ in eq. (22) can be computed by following the approach proposed by Dufour *et al.* (2006, 2010) in a single-equation framework and then extended by Dufour *et al.* (2009, 2013) in the multi-equational setup. Alternatively, one can use the 'S-test' method by Stock and Wright (2000), or the 'K-LM test' by Kleibergen (2005), both based on the evaluation of the criterion function corresponding to the continuos-updating version of generalized method of moments. Some computational issues make us prefer the approach in Dufour *et al.* (2009, 2013).[19]

[19] Kleibergen and Mavroeidis (2009) discuss weak instrument robust statistics for testing hypotheses on θ_s or its subset in the GMM framework and then apply these methods to the new-Keynesian Phillips curve.

The test works as follows. By simple algebraic manipulations, we re-write the system of Euler equations (5)-(6) in the form

$$\Gamma_0^{\Xi} X_t = \Gamma_f X_{t+1} + \Gamma_{b,1}^{\Xi} X_{t-1} + \Gamma_{b,2}^{\Xi} X_{t-2} + \Xi \Gamma_f \xi_t + \varepsilon_t - \Gamma_f \xi_{t+1},$$

where $\xi_t := X_t - E_{t-1} X_t$ is a vector MDS. Then we define the 3×1 vector function

$$v(X_t, \theta_s) := \Gamma_0^{\Xi} X_t - \Gamma_f X_{t+1} - \Gamma_{b,1}^{\Xi} X_{t-1} - \Gamma_{b,2}^{\Xi} X_{t-2}. \tag{30}$$

Under correct specification, the vector $v(X_t, \theta_{0,s})$ in eq. (30) follows a VMA(1)-type process and fulfills the OR:

$$E\left(v(X_t, \theta_{0,s}) \mid \mathcal{F}_{t-1}\right) = 0_{3\times 1}. \tag{31}$$

Given eq. (31), consider the hypothesis $H_{0,spec}$ in eq. (22) and the multivariate linear regression model

$$v(X_t, \breve{\theta}_s) = \Pi_{\breve{\theta}_s} Z_t + \epsilon_t \quad , \quad Z_t \in \mathcal{F}_{t-1} \quad , \, t = 1, ..., T \tag{32}$$

where $\Pi_{\breve{\theta}_s}$ is a $3\times r$ matrix of coefficients, Z_t is a $r \times 1$ vector of regressors selected from the information set $\mathcal{F}_{t-1}$ (i.e. contains variables dated $t-1$ and earlier) and ϵ_t is a term whose covariance matrix, Σ_ϵ, can possibly be non-diagonal. We have used the notation '$\Pi_{\breve{\theta}_s}$' for the regression coefficients to remark that a system like that in eq. (32) is associated to the choice $\theta_s := \breve{\theta}_s$. Under $H_{0,spec}$:$\theta_s := \breve{\theta}_s := \theta_{0,s}$, additional information from predetermined variables should be irrelevant in the multivariate regression system (32), hence the associated problem

$$H_{0,spec}^{*} : \Pi_{\breve{\theta}_s} := 0_{3\times r} \quad \text{against} \quad H_{1,spec}^{*} : \Pi_{\breve{\theta}_s} \neq 0_{3\times r} \tag{33}$$

should lead us to accept $H_{0,spec}^{*}$. We have thus transformed the problem of testing $H_{0,spec}$ in eq. (22) in the context of the structural new-Keynesian system (5)-(6) into the problem of testing the hypotheses in eq. (33) in the context of the multivariate linear regression system (32), for which standard asymptotic distributional theory applies irrespective of whether the structural parameters are identifiable or not.

More precisely, if system (5)-(6) is correctly specified and $H_{0,spec}$ is valid we have

$$AR_T(\breve{\theta}_s) \to \chi^2(d_2) \quad , \quad d_2 := 3r \tag{34}$$

irrespective of whether $\theta_{0,s} \in \mathcal{P}_{\theta_s}^{D}$ (determinacy) or $\theta_{0,s} \in \mathcal{P}_{\theta_s}^{I}$ (indeterminacy). Obviously $AR_T(\breve{\theta}_s)$ is $O_p(T)$ if the system (5)-(6) is affected by an 'omitted variable' issue. Given the rejection of the CER in the first-step, the inversion of the test at the level of significance η_2 generates the identification-robust confidence set reported in eq. (23).

In practice, the $AR_T(\breve{\theta}_s)$ can be a Wald-type, a Lagrange Multiplier or (quasi-)LR test.[20] Since the ϵ_t term follows a VMA-type process in system (32), HAC-type (Newey and West, 1997) versions of the tests can be applied as suggested by Dufour *et al.* (2013). Our simulation studies (part of which are reported in Table 2) show that even in the case of 'indeterminacy without sunspots' (see Sub-section 2.2), the finite sample rejection frequency of the $AR_T(\breve{\theta}_s)$ ($AR_T^c(\breve{\theta}_s)$) test may be to some extent affected by the values of the nuisance parameters $\psi{:=}vec(\Psi)$ in eq.s (11)-(12), which are the only drivers of the multiplicity of the solutions when sunspot shocks are absent, as well as by the chosen number of lags r in the auxiliary multivariate regression system (32).

Asymptotic properties of the '$LR_T \rightarrow AR_T$' testing strategy

Let $P^{LR}_{\breve{\theta}_s,T}[\cdot]$ be the probability measure associated with the distribution of the $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test in eq. (29) in a sample of length T. Thus, the asymptotic size of the '$LR_T \rightarrow AR_T$' testing strategy is given by

$$\eta_{1,\infty}{:=}\limsup_{T\rightarrow\infty} \eta_{1,T} \quad , \quad \eta_{1,T}{:=}\sup_{\breve{\theta}_s\in\mathcal{P}^D_{\theta_s}} P^{LR}_{\breve{\theta}_s,T}[LR_T(\hat{\phi}^*_{\breve{\theta}_s}) \geq c_T^{\eta_1}] \tag{35}$$

where $\eta_{1,T}$ is the size in a sample of length T and $c_T^{\eta_1}$ is the critical value of the test at nominal level $0{<}\eta_1{<}1$ in a sample of length T. The next proposition establishes that the test has correct size asymptotically.

Propostion 1 [Asymptotic size] Consider the new-Keynesian system (5)-(6) and the '$LR_T \rightarrow AR_T$' testing strategy. Under the null H_0' in eq. (15) and for $\theta_s{:=}\breve{\theta}_s{:=}\breve{\theta}_{0,s} \in \mathcal{P}^D_{\theta_s}$, $\eta_{1,\infty} = \eta_1$.

Proof. The result follows trivially from eq. (29) and eq. (35).■

Borrowing notation from Mikusheva (2010) and Andrews and Cheng (2012), an equivalent way of stating the result in Proposition 1 is

$$\liminf_{T\rightarrow\infty} \inf_{\breve{\theta}_s\in\mathcal{P}^D_{\theta_s}} P^{LR}_{\breve{\theta}_s,T}[\text{hypothesis } H_{0,cer}{:}\phi^*_{\breve{\theta}_s}{=}g(\breve{\theta}_s,\theta_\varepsilon) \text{ is accepted}] = 1-\eta_1$$

which is equivalent to the claim that the confidence set $\mathcal{C}^{LR}_{1-\eta_1}$ defined in eq. (21) (or the acceptance region of the test) has asymptotic coverage $1-\eta_1$.

Likewise, the finite sample power of the '$LR_T \rightarrow AR_T$' testing strategy against the hypothesis H_1' is given by

$$p_{1,\infty}{:=}\limsup_{T\rightarrow\infty} p_{1,T} \ , \quad p_{1,T}{:=}\sup_{\breve{\theta}_s\in\mathcal{P}^I_{\theta_s}\subset\mathcal{I}^0} P^{LR}_{\breve{\theta}_s,T}[LR_T(\hat{\phi}^*_{\breve{\theta}_s}) \geq c_T^{\eta_1}] \tag{36}$$

[20] Dufour *et al.* (2013) discuss F-type approssimations for the asymptotic null distribution of the $AR_T(\breve{\theta}_s)$ test.

where recall that the set $\mathcal{I}^0$ is defined in eq. (17) and contains all model's parameters (including the auxiliary parameters ψ and σ_ζ^+) which lead to multiple equilibria but the MSV equilibrium, see Sub-section 2.2. The next proposition establishes the consistency of the test.

Proposition 2 [Consistency against indeterminacy] Consider the new-Keynesian system (5)-(6) and the '$LR_T \to AR_T$' testing strategy. Under the hypothesis H_1' in eq. (16) and for $\theta_s{:=}\breve{\theta}_s{:=}\breve{\theta}_{0,s} \in \mathcal{P}^I_{\theta_s}$, $p_{1,\infty} = 1$.

Proof. Under H_1' and for $\theta_s{:=}\breve{\theta}_s{:=}\breve{\theta}_{0,s} \in \mathcal{P}^I_{\theta_s}$, $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ is $O_p(T)$; the result follows from eq. (36).■

To analyze the asymptotic behaviour of the '$LR_T \to AR_T$' testing strategy in the second-step, we temporarily focus on the unconditional behaviour of the $AR_T(\breve{\theta}_s)$ test, i.e. we ignore the fact that the test $AR_T(\breve{\theta}_s)$ is computed conditional on the rejection of the CER by the $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ test in the first-step.

Let $P^{AR}_{\breve{\theta}_s,T}[\cdot]$ be the probability measure associated with the distribution of the $AR_T(\breve{\theta}_s)$ test in a sample of length T. Given the null $H_{0,spec}$ in eq. (22), the asymptotic size of the $AR_T(\breve{\theta}_s)$ test is given by

$$\eta_{2,\infty}{:=}\limsup_{T\to\infty} \eta_{2,T} \quad , \quad \eta_{2,T}{:=} \sup_{\breve{\theta}_s \in \mathcal{P}_{\theta_s}} P^{AR}_{\breve{\theta}_s,T}[AR_T(\breve{\theta}_s) \geq c_T^{\eta_2}] \tag{37}$$

where $\eta_{2,T}$ is the size in a sample of length T and $c_T^{\eta_2}$ is the critical value of the test at nominal level $0{<}\eta_2{<}1$ in a sample of length T. The next proposition establishes that the test $AR_T(\breve{\theta}_s)$ is correctly sized.

Proposition 3 [Asymptotic size of the $AR_T(\breve{\theta}_s)$ test] Consider the new-Keynesian system (5)-(6) and assume that its reduced form solution belongs either to the finite order VAR system in eq. (8) or to the class of VARMA-type solutions in eq.s (11)-(12). Under the null $H_{0,spec}$ in eq. (22), $\eta_{2,\infty} = \eta_2$.

Proof. The result follows trivially from eq. (34) and eq. (37).■

Also in this case, an equivalent way of stating the result in Proposition 3 is that

$$\liminf_{T\to\infty} \inf_{\breve{\theta}_s \in \mathcal{P}_{\theta_s}} P^{AR}_{\breve{\theta}_s,T}[\text{hypothesis } H_{0,spec}{:}\theta_s{:=}\breve{\theta}_s \text{ is accepted}] = 1 - \eta_2$$

which is equivalent to the claim that the confidence set (acceptance region)

$$\mathcal{C}^{AR}_{1-\eta_2}{:=}\left\{\breve{\theta}_s \in \mathcal{G}_{\theta_s},\ AR_T(\breve{\theta}_s) < c^{\eta_2}_{\chi^2_{d_2}}\right\} \tag{38}$$

has asymptotic coverage $1-\eta_2$.

Now we take into account the fact that the second-step of the '$LR_T \to AR_T$' testing strategy is run conditional on the rejection of the CER in the first-step. Let $P^{LR,AR}_{\breve{\theta}_s,T}[\cdot\,;\cdot]$ be the probability measure associated with the joint distribution of the test statistics $LR_T(\hat{\phi}^*_{\breve{\theta}_s})$ and $AR_T(\breve{\theta}_s)$ in a sample of length T. The proposition that follows shows that the asymptotic 'second-step size' of the test is bounded by the fixed η_2.

Proposition 4 ['second-step' asymptotic size] Consider the new-Keynesian system (5)-(6) and the '$LR_T \to AR_T$' testing strategy. Under the hypothesis H_1' in eq. (16) and in particular for $\theta_s{:=}\breve{\theta}_s{:=}\breve{\theta}_{0,s} \in \mathcal{P}^I_{\theta_s}$, the asymptotic 'second-step size' of the $AR_T(\breve{\theta}_s)$ test, denoted $\eta_{2\cdot 1}$, is such that $\eta_{2\cdot 1} \leq \eta_2$.

Proof. It holds the inequality

$$\sup_{\breve{\theta}_s \in \mathcal{P}^I_{\theta_s} \subset \mathcal{I}^0} P^{LR,AR}_{\breve{\theta}_s,T}[LR_T(\hat{\phi}^*_{\breve{\theta}_s}) \geq c_T^{\eta_1}\ ;\ AR_T(\breve{\theta}_s) \geq c_T^{\eta_2}] \leq \sup_{\breve{\theta}_s \in \mathcal{P}^I_{\theta_s} \subset \mathcal{I}^0} P^{AR}_{\breve{\theta}_s,T}[AR_T(\breve{\theta}_s) \geq c_T^{\eta_2}] \tag{39}$$

where the term on the left-hand-side denotes the 'second-step size' in a sample of length T. By applying the limsup to both sides of the inequality in eq. (39) and using Proposition 3 one has

$$\limsup_{T\to\infty} \sup_{\breve{\theta}_s \in \mathcal{P}^I_{\theta_s} \subset \mathcal{I}^0} P^{LR,AR}_{\breve{\theta}_s,T}[LR_T(\hat{\phi}^*_{\breve{\theta}_s}) \geq c_T^{\eta_1}\ ;\ AR_T(\breve{\theta}_s) \geq c_T^{\eta_2}] \leq \limsup_{T\to\infty} \sup_{\breve{\theta}_s \in \mathcal{P}^I_{\theta_s} \subset \mathcal{I}^0} P^{AR}_{\breve{\theta}_s,T}[AR_T(\breve{\theta}_s) \geq c_T^{\eta_2}]$$

and the result is obtained.■

From Proposition 4 it follows that the identification-robust confidence set $\mathcal{C}^{LR-AR}_{1-\eta_{2\cdot 1}}$ defined in eq. (23) has asymptotic coverage $1-\eta_{2\cdot 1} \geq 1-\eta_2$.

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