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# ENVIRONMENTAL INNOVATION, WAR OF ATTRITION AND INVESTMENT GRANTS

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# Environmental Innovation, War of Attrition and Investment Grants*

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## Abstract

The paper analyses the timing of spontaneous environmental innovation when second-mover advantages, arising from the expectation of declining investment costs, increase the option value of waiting created by investment irreversibility and uncertainty about private payoffs. We then focus on the design of public subsidies aimed at bridging the gap between the spontaneous time of technological change and the socially desirable one. Under network externalities and incomplete information about firms' switching costs, auctioning investment grants appears to be a cost-effective way of accelerating pollution abatement, in that it allows targeting grants instead of subsidizing the entire industry indiscriminately



**JEL: Q28; O38.**

**Keywords: Environmental innovation, Investment irreversibility, Network externalities, Investment grants, Second-price auction.**

# 1 Introduction

Since pollution abatement generally requires investment expenditures, profit-maximizing firms do not spontaneously[1] improve their environmental performance[2] unless costs are offset by some expected private benefits. Following the literature on so-called voluntary approaches, these benefits may come from better use of inputs (e.g. energy or material savings, abatement of waste disposal costs), sales increase (consumers may be willing to pay more for environment-friendly products or for goods produced by a firm which has acquired a green reputation) and/or regulatory gains (preemption of more stringent mandatory regulation or regulatory capture) (Brau and Carraro, 1999; Carraro and Léveque, 1999).

The performance of self-regulation has been analysed across different dimensions, including its impacts upon market competition and environmental effectiveness. As far as the latter is concerned, Carraro and Léveque (1999) cite two frequent sources of concern about the actual contribution of volun-

[1] By spontaneous pollution control ("self-regulation") we mean control efforts which are neither imposed by explicit directives ("command-and-control regulations) nor are driven by "marked-based" regulations that encourage firms to undertake pollution abatement (e.g. pollution charges or tradable permits).

[2] Firms may improve their environmental performance either by undertaking process innovations or changes in product design which involve pollution abatement during the product life cycle.

tary approaches to environmental quality improvements. One is that firms may not respect their commitments. The second cause concerns the low ambition of pollution abatement targets.

A third potential cause of ineffectiveness, addressed in this paper, relates to the timing of environmental innovation. Although firms have discovered potentially profitable green investment opportunities, voluntary process innovations or changes in product design may occur too slowly, i.e. they may not prevent undesirable levels of pollutant accumulation and environmental damage.

Why would firms, which have discovered a green investment opportunity whose costs are counter-balanced by expected private gains, postpone environmental innovation? The real options approach to investment decisions provides a possible answer. For instance, this approach teaches that when an agent does not face a now-or-never investment decision, an option value of waiting emerges before undertaking a project involving sunk costs and uncertain payoffs (*irreversibility effect*). In other words, the agent may find it profitable to delay the investment, despite the project exhibiting a positive net present value.[3]

[3]Obviously, not all green investment decisions meet the conditions required in order to apply the conclusions of the real options approach: for example, these conclusions do

This standard result stems from the analysis of investment decisions for a single agent in isolation, i.e. without regarding to the potential impact of other firms' investment strategies. For instance, recent developments of the real options approach show that when these decisions take place in a competitive environment, strategic interactions between firms may either decrease or further increase the option value of waiting (Lambrecht and Parraudin, 2003; Grenadier, 2002; Mason and Weeds, 2001; Moretto, 2000).

The value of waiting may significantly decrease if the investment payoffs depend on the number of firms which have already improved their environmental performance and there is an advantage in being first. For example, preemption can hasten pollution abatement when firms interpret self-regulation as a product differentiation strategy aimed at differentiating their product or process from those of other firms in the industry in order to increase their market share. In other words, the risk of foregone competitive advantages may counter-balance the benefits of waiting for additional information about consumers' response to the supply of green products.

However, instead of hastening environmental innovation, strategic interactions may further increase the option value of waiting. This may occur

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not apply when firms are able to recover investment expenditures should the payoffs (e.g. consumers' willingness to pay for green products) turn out to be worse than anticipated.

when there is an expectation of declining switching costs, due to the diffusion of green technologies, whilst the investment payoffs are not negatively correlated (e.g. when market demand shifts upward when green products are sold in the market) or are independent of the number of firms which have improved their environmental performances (e.g. when the investment payoffs are expected to come from input savings or from avoidance of future costs of forthcoming public regulations that firms cannot influence).

Both strategic interactions typically involve an inefficient time pattern of private investment decisions. However, if we adopt a narrow view and focus on the environmental effectiveness of self-regulation, the most critical scenario is the one where, because of second-mover advantages, strategic interactions exacerbate, rather than mitigate, the irreversibility effect. In particular, the expectation of declining investment costs may involve a *war of attrition* whose effect is to further delay pollution abatement.

The purpose of this paper is twofold. First, we illustrate the impacts of the war of attrition upon the option value of waiting and, consequently, upon the private time of pollution abatement. Secondly, assuming that a public authority has somehow arbitrarily pre-identified the desirable time for technological change, we focus on the design of policy instruments - namely, investment grants - aimed at bridging the gap between the spontaneous time of environmental innovation and the "socially" desirable one.

We do this by extending and generalizing the continuous-time model of environmental policy adoption of Dosi and Moretto (1997; 1998). In markets with no large investors, Dosi and Moretto (1997) stressed that, in order to enhance the effectiveness of environmental policies, regulators should account for the option value firms face when deciding the time of an investment involving sunk costs and uncertain returns. In particular, the optimal subsidy must be selected to compensate the firm's value of waiting. However, in the case of large investors the problem faced by policy-makers becomes more complicated because they have to consider the impact of other firms' investment strategies. In this respect, Dosi and Moretto (1998) analysed the impacts of declining switching costs in a duopoly model and argued that regulators may accelerate environmental innovation by auctioning investment grants.

Here we generalize the above papers by providing a general solution approach for deriving the firms' equilibrium investment strategies in a second-mover-advantages framework. We consider both the irreversibility effect and network externalities on the investment costs in a $N+1$ agents-model and we model the competition for the investment grant as a Vickrey auction where firms simultaneously submit their bids, the subsidy is granted to the most efficient firm and it is priced according to the second-best bidder. Moreover, in order to illustrate the properties of the model and get some quantitative idea of the effect of the second-mover-advantage on the firms' adoption decision, we calibrate the model following as far as possible the indications given in the real option literature (Dixit and Pindyck, 1994).

The rest of the paper is organised as follows. Section 2 presents a model in which $N+1$ firms, belonging to the same industry, face the same opportunity of undertaking an irreversible green investment involving stochastic payoffs; each agent's timing of technological change is influenced by the investment decision of the other, because switching costs are negatively correlated to the number of firms which have adopted the green technology. Section 3 deals with the war of attrition game that emerges; we show that if switching costs are private knowledge, the free-riding attitude induced by the expectation of network benefits may significantly increase the option value of waiting and, consequently, the investment delay. Section 4 focuses on the design of investment grants aimed at bridging the gap between the expected private time of innovation and the socially desirable time; we examine the properties of a second-price auction, in which agents bid for the right to obtain public funds for use in financing the technological change. Section 5 concludes, and the Appendix contains the proofs omitted in the text.

# 2 The model

Consider a situation where $N+1$ ($N > 0$) risk neutral firms, belonging to the same industry, can abandon, at any time, their present (*polluting*) production process, in order to adopt a new (*green*) one, by affording a sunk switching cost $C_n$, $n = 1, 2.....N + 1$.

The instantaneous green investment payoff at time $t$, $x_t$, is stochastic and evolves according to a geometric Brownian motion:

$$dx_t = \alpha x_t dt + \sigma x_t dz_t \qquad \text{with } \alpha, \sigma > 0 \text{ and } x_0 = x. \tag{1}$$

where $dz_t$ is the increment of a standard Wiener process, satisfying the conditions that $E(dz_t) = 0$ and $E(dz_t^2) = dt$; both the drift parameter $\alpha$ and the volatility parameter ($\sigma$) are constant over time. Therefore $E(dx_t) = \alpha x_t dt$ and $E(dx_t^2) = (\sigma x_t)^2 dt$, i.e. starting from the initial value $x_0$, the random position of the instantaneous payoff $x_t$ at time $t > 0$ has lognormal distribution with mean $x_0 e^{\alpha t}$ and variance $x_0^2(e^{\sigma^2 t} - 1)$ which increases as we look further and further into the future. The process has no memory, i.e. i) at any point in time $t$,the observed $x_t$ is the best predictor of future profits, ii) $x_t$ may next move upwards or downwards with equal probability.

Whilst the investment payoff is independent of the number of green firms, we assume that agents' switching cost, $C_n$, depends on the number of firms $q$ that have adopted the green technology:[4]

$$C_n(\theta, q) = \theta_n k(q), \qquad n = 1, 2...N+1 \quad \text{and } q = 1, 2...N+1$$

where $k(q)$ stands for the pure capital cost which is common knowledge, and $\theta_n \in [0 \leq \underline{\theta}, \bar{\theta} \leq \infty]$ is a private valuation parameter reflecting agent $n$'s perception of foregone alternative investment opportunities in the future.

[4] As anticipated, the aim of this paper is to focus on situations where strategic interactions exacerbate the impacts of investment irreversibility and uncertainty. However, the model could be easily expanded to explore the impacts of preemption upon environmental innovation time. For instance, if the instantaneous investment payoffs decline with the number of green firms, a first-mover advantage will emerge whose effect is to reduce the second-mover advantage resulting from the expectation of declining investment costs. See for example Murto and Keppo (2002), Grenadier (2002) and Lambrecht and Perraudin (2003) for preemption models and Moretto (2000) for an application of both effects to a duopoly model.

We assume that:[5]

$$k(q) = \begin{cases} k & \text{for } q = 1 \\ k - \Delta k & \text{for } q \neq 1 \end{cases} \quad \text{and } 0 < \Delta k < k$$

Since $0 < \Delta k < k$ there is an the advantage in coordinating or *joining a network*: the higher the agent's investment opportunity cost, $\theta_n$, the greater its share value of the network benefit.[6]

According to the classical real-option based models (Dixit and Pindyck, 1994) the firms' optimal investment rule is that the new technology's benefits must outweigh its costs, where the latter consist of the individual strike price $C_n$ plus the value of the option exercised by undertaking the investment. As, at any time $t$, all information about the future evolution of $x$ is summarized in the current value $x_t$, the optimal decision rule relies on a realization of $x$ that is necessary and sufficient to stop waiting and undertake the project. In

[5]For the sake of simplicity, we consider a quite extreme form of war of attrition in that all the firm but the first gain the same network benefit. Although this may be fairly realistic for investment costs reduction that are expected to come from input savings, our model may be generalized to situations in which the network benefit increase as long as more firms adopt the new technology.

[6]For different reasons we exclude both $\Delta k = 0$ and $\Delta k = k$. If $\Delta k = 0$ there would be no strategic interaction and each firms' problem could be solved separately. On the other hand, if $\Delta k = k$ the model reduces to a game of private provison of a pure public goood (except for the fact that if two or more firms provide the good at the same time, their provisions costs would fall to zero), with stochastic flow benefits (see Bliss and Nalebuff, 1984).

other words, the firms will invest if the current flow of income $x_t$ has crossed from below an upper single trigger value $\bar{x}_n, \ n \ 1, 2, ...N+1$.

The agent $n's$ investment option can be written as follows:

$$V_n(\bar{x}_n^*, C_n; x) \quad = \quad E_0 \left\{ e^{-rT_n} \left( \int_{T_n}^{\infty} x_t e^{-rt} dt - \theta_n k \right) \mid \ x_0 = x \right\} \quad \forall n, \quad (2)$$

where $r > \alpha$ is the constant risk-free rate of interest[7], and $E_0(.)$ is the operator expectation conditional on the information available at time $t = 0$. Furthermore, $T_n = \inf(t > 0 \mid x_t = \bar{x}_n^*)$, is the future random starting time at which firm $n$ finds it optimal to go first and $\bar{x}_n^*$ is the income threshold that triggers it.

# 3 The war of attrition

Firms' time of investment is affected by two sources of inertia. On the one hand, because of sunk costs, environmental innovation is slowed by the un-

[7] Alternatively we can use a discount rate that includes an appropriate adjustment for risk and take the expectation with respect to a distribution for $x$ that is adjusted for risk neutrality (see Cox and Ross, 1976; Harrison and Kreps, 1979).

certainty about the investment payoffs (*irreversibility effect*). On the other hand, innovation is decelerated by the second-mover advantage resulting from declining switching costs. In particular, as far as the second source is concerned, the uncertainty about the other firms' opportunity cost makes it advisable to wait in order to see how things go for the others before switching (*war of attrition effect*). If this does not happen and the rivals are reluctant to adopt the green technology, the agent may eventually decide to switch first.

At each time $t$ firms observe the realization of the state variable $x_t$, and, depending on their private valuation parameter $\theta$, decide whether to invest. Secondly, there is a Bayesian learning process where agents learn by observing the rivals' behaviour. A Nash equilibrium will then be the solution of a pair of linked stopping time problems, where each agent solves its switching problem by taking account of the rivals' possible actions and learning about the rivals' valuation parameters from the fact that they have not switched up to that moment.

Specifically, each agent $n$ will optimally select an upper trigger level $\bar{x}_n^*$, $n = 1, 2....N + 1$. Thus, if at time $t$ $x_t \geq \bar{x}_n^*$ and the rivals have not yet switched, the agent $n$ will unilaterally innovate. Otherwise, if any one of its rivals has already switched at $x_t < \bar{x}_n^*$, agent $n$ learns that it can adopt the green technology by paying $k - \Delta k$ and with him, all the others.

Note, however, that the certainty of being second does not imply switching immediately. As the switching cost depends on $\theta$, and $x$ is assumed to be independent of the number of green firms, a lower trigger level $\bar{x}_n^{**} < \bar{x}_n^*$ always exists, below which the only dominant strategy is to keep the option to invest alive, and wait longer before exercising it. Only when $x_t$ crosses $\bar{x}_n^{**}$ do the agents consider the possibility of switching second.

As long as $\bar{x}_n^{**} < x_t < \bar{x}_n^*$ each firm waits for the others to change technology first. During this period of *excess inertia* (Farrell and Saloner, 1985) each firm experiences both costs (foregone expected cash flows) and benefits of delaying: the latter come from the hope of getting additional information about the investment payoffs and by second-mover advantages. In continuous time, this countervailing interest can be represented by the following *bandwagon strategy*:

$$a_n = \begin{cases} (a) & if \quad 0 < x < \bar{x}_n^{**} \\ (b) & if \quad \bar{x}_n^{**} \leq x < \bar{x}_n^{*} \\ (c) & if \quad x \geq \bar{x}_n^{*} \end{cases} \quad \text{for } \forall n. \tag{3}$$

where:

(a) do not switch, regardless of the rivals' behaviour;

(b) switch only if a rival has already switched, i.e. jumping on the bandwagon;

(c) unilaterally switch, i.e. initiating the bandwagon.

### 3.1 The optimal private trigger values

Consider the optimal trigger value $\bar{x}_n^*$ for agent $n$ (by symmetry the same results hold for all $N+1$ agents as well). We assume that firm $n$ has rational conjectures about the distribution of the other firms' triggers. More specifically, we assume that each firm's investment trigger is continuously distributed and drawn independently from a common distribution function $F(\bar{x}_n^*)$ which is strictly increasing on the interval $[\bar{x}^l, \infty)$ and has a continuous differentiable density $f(\bar{x}_n^*)$.

As long as the $N+1$ firms are independent, what matters for the firm $n$ is the event $\min[\bar{x}_j^*,\ j \neq n]$ and, consequently, the joint distribution:

$$F^{(N)}(\bar{x}_n^*) \equiv \Pr\left\{\min[\bar{x}_j^*,\ j \neq n]\ \leq \bar{x}_n^*\right\} \equiv 1 - (1 - F(\bar{x}_n^*))^N$$

which is the cumulative distribution (with density $f^{(N)}(\bar{x}_n^*)$) of the minimum of the $N$ rivals' triggers (i.e. the probability that all the other $N$ firms have lower triggers than $n$) on the same support $[\bar{x}^l, \infty)$.

Let's now derive the optimal investment rule for firm $n$, taking account of the other firms' behaviour as exogenously given. Firm $n$'s option value at time zero to adopt the green technology at time $T_n$ if the other firms are still using the polluting technology is given by:

$$V_n(x; \bar{x}_n^*) =$$

$$E_{\min_{j\neq n}(T_j)}\left\{E_0\left\{e^{-r\min_{j\neq n}(T_j)}\left(\int_{\min_{j\neq n}(T_j)}^{\infty} x_t e^{-rt}dt - \theta_n(k - \Delta k)\right)\right\} \mid T_n \geq \min_{j\neq n}(T_j)\right\}$$

$$+E_0\left\{e^{-rT_n}\left(\int_{T_n}^{\infty} x_t e^{-rt}dt - \theta_n k\right)\right\}\Pr(T_n < \min_{j\neq n}(T_j))$$

In other words, firm $n$'s option value of investing is given by the sum of the option value to go as second at cost $\theta_n(k - \Delta k)$ when a firm has already adopted at time $\min_{j\neq n}(T_j) = \inf(t > 0 \mid x_t = \min\left\{\bar{x}_j^*,\ j \neq n\ \right\})$, plus the option value of not investing until time $T_n$ and then going first.

$T_n = \inf(t > 0 \mid x_t = \bar{x}_n^*)$ is then the switching time at which agent $n$ decides unilaterally to adopt the green technology (strategy c).

Furthermore, as $x_t$ moves randomly over time, the firm $n$ will update its conjecture. In particular, as time goes by and $x_t$ hits new upper levels without the rivals switching, agent $n$ learns that the rivals' triggers lie in a smaller, higher interval. A sufficient statistic that captures this information is given by $u_t = \sup_{0<s<t}(x_t)$ which denotes the maximum level of payoff up to time $t$ without one of the firms having adopted the green technology. The firm $n$ then observes the realization of the state variable $x_t$, updates its conjecture on the rivals' thresholds by using $F^{(N)}(\bar{x}_n^*; u_t) = \frac{F^{(N)}(\bar{x}_n^*) - F^{(N)}(u_t)}{1 - F^{(N)}(u_t)}$,which is strictly increasing on the interval $[u_t, \infty)$, and instantaneously considers when it is profitable to invest by maximizing:

$$V_n(x_t; \bar{x}_n^*) = \tag{4}$$

$$\int_{u_t}^{\bar{x}_n^*} E_t \left\{ e^{-r(\min_{j \neq n}(T_j) - t)} \left( \int_{\min_{j \neq n}(T_j)}^{\infty} x_s e^{-rt} dt - \theta_n (k - \Delta k) \right) \right\} dF^{(N)}(\bar{x}; u_t)$$
$$+ E_t \left\{ e^{-r(T_n - t)} \left( \int_{T_n}^{\infty} x_s e^{-rt} dt - \theta_n k \right) \right\} (1 - F^{(N)}(\bar{x}_n^*; u_t))$$

The following proposition describes the properties of the stationary strategy (3) resulting from maximization of (4).

**Proposition 1** *(i) If a threshold level* $\bar{x}_n^* \in [\bar{x}^l, \infty)$ *exists, such that* $0 < \bar{x}_n^{**} < \bar{x}_n^*$, *then a perfect equilibrium involves each firm playing the following stationary strategy:*

$$a_n(F^{(N)}) = \begin{cases} \textit{Strategy } (a) & if \quad 0 < x < \bar{x}_n^{**} \\ \textit{Strategy } (b) & if \quad \bar{x}_n^{**} \leq x < \bar{x}_n^* \\ \textit{Strategy } (c) & if \quad x \geq \bar{x}_n^* \end{cases} \qquad \forall n$$

*where the optimal trigger values are:*

$$\bar{x}_n^{**} = \frac{\beta}{\beta - 1}(r - \alpha)\theta_n(k - \Delta k), \qquad \forall n \tag{5}$$

$$\bar{x}_n^* = \frac{\beta}{\beta - 1}(r - \alpha)\theta_n k + \frac{\beta}{\beta - 1}(r - \alpha)\theta_n \Delta k \frac{\bar{x}_n^* N h(\bar{x}_n^*)}{\beta}, \quad \forall n \tag{6}$$

$h(\bar{x}_n^*) \equiv \frac{f(\bar{x}_n^*)}{1 - F(\bar{x}_n^*)}$ *is the hazard rate and* $\beta > 1$ *is the positive root of the quadratic equation* $\Phi(\beta) \equiv \frac{1}{2}\sigma^2\beta(\beta - 1) + \alpha\beta - r = 0$.

*(ii) The optimal triggers are monotonically increasing in* $\theta_n$.

**Proof.** See Appendix.

According to (4) and (6), although the value of the green investment depends on both the current value of $x_t$ and on the statistic $u_t$, the threshold that triggers the technological change does not because the the hazard rate $Nh(\bar{x}_n^*)$ is independent of both $x_t$ and $u_t$ (see Appendix).

Since the hazard rate is defined as the likelihood of an event occurring in the next instant, given that the event has not occurred up to that instant, in (6) it measures the likelihood of the firm $n$ investing at $\bar{x}_n^*$. The hazard rate is zero when there is no probability of one firm going first and goes to infinity when $u_t$ and/or $N$ goes to infinity.

Following Lambrecht and Perraudin (2003), let's consider what happens as the incomplete information case reduces to one with complete information. If $\theta_n$ is common knowledge and the degree of asymmetry in the firm-specific parameters is low (i.e. the interval $[\underline{\theta}, \bar{\theta}]$ is small), firms have no interest in going unilaterally. They will be better-off coordinating and choosing to invest at the time when the firm with the highest cost parameter $\theta_n$ switches. The unique Nash equilibrium in pure strategies is characterized as a war of attrition where the firms try to invest as late as possible and, thereby, at the optimal trigger of the less efficient firm. That is, there is a common trigger value:

$$\bar{x}^{**} = \sup_{n \in [1,N+1]} (\bar{x}_n^{**}) \equiv \frac{\beta}{\beta - 1}(r - \alpha)(k - \Delta k) \sup_{n \in [1,N+1]} (\theta_n)$$

above which firms switch to the green technology.[8] Consequently, unlike in Farrell and Saloner (1985), there might be excess inertia even under complete information. The firms with lower cost parameters will find it optimal to wait until technological change becomes profitable for some of their rivals and then coordinate adoption. The loss due to waiting is more than compensated by the reduction in investment cost deriving from coordination.[9]

For the latter limit case, suppose that an upper trigger $\bar{x}^u$ exists so that $\bar{x}_n^* \in [\bar{x}^l, \bar{x}^u]$. As $u_t \to \bar{x}^u$ and no firms have adopted yet, the firm $n$ knows that at least one of its rivals will act almost certainly in the next few instants, which causes the hazard rate to explode to infinity. The trigger value for firm $n$ should therefore also explode to infinity which contradicts the fact of having an upper bound $\bar{x}^u < \infty$.

Finally, a third interesting and related limiting case occurs when the

[8] If the degree of asymmetry is high there exist an equilibrium in which always one firm invest before the others (Sparla, 2000).

[9] In the symmetric case $\theta_n = \theta$ for all $n$, the social optimum is always obtained. A unique threshold $\bar{x}^{**}$ exists beyond which all the firms find it optimal to move simultaneously (see Moretto, 2000).

number of competing firms goes to infinity. Since $\lim_{N\to\infty} Nh(\bar{x}_n^*) = \infty$, also the trigger $\bar{x}_n^*$ converges to infinity. This is a straightforward consequence of the war of attrition; as $N$ increases each firm knows almost certainly that at least one of its rivals will go first. Each firm takes this opportunity, delaying the investment indefinitely.

The following corollary illustrates the effect of the war of attrition on the strategic option trigger:

**Corollary 1** *The strategy (c)'s optimal trigger is situated between infinite and the non-strategic trigger which, in turn, is above the second-mover trigger, i.e.:*

$$\bar{x}_n^{**} \leq \bar{x}_n^{+} \leq \bar{x}_n^{*} \leq \infty$$

*where* $\bar{x}_n^{+} \equiv \frac{\beta}{\beta-1}(r-\alpha)\theta_n k$.

*The upper bound is reached when* $h(\bar{x}_n^*) \to \infty$ *or* $N \to \infty$, *while when* $h(\bar{x}_n^*) \to 0$ *the optimal trigger converges to* $\bar{x}^{**} = \sup_{n\in[1,N+1]}(\bar{x}_n^{**})$.

In short, whilst $\bar{x}_n^{+}$ reflects the *irreversibility effect*, the second term on the r.h.s. of (6) reflects the *war of attrition effect* which exacerbates the impacts of investment irreversibility and uncertainty about private benefits, i.e. increases the optimal trigger value and the investment delay.

Furthermore, proposition 1 shows that the higher $\theta_n$ the greater the instantaneous investment payoff at which it becomes profitable to invest: the optimal trigger $\bar{x}_n^*(\theta_n)$ is an increasing mapping function of $\theta_n$, in the support $[\bar{x}^l(\underline{\theta}), \bar{x}^u(\bar{\theta}) = \infty)$.[10] Therefore, even without making use of a discrete-time model, we can also have sequential investments depending on the wedge in agents' valuation parameter $\theta_n$. Specifically, if the firm $n$ is the leader, we get the following result.

**Corollary 2** *Sequential investment ('diffusion') exists if* $\bar{x}_j^{**}(\theta_j) > \bar{x}_n^*(\theta_n)$, *for some* $j \neq n$.

## 3.2 Numerical results

To illustrate the properties of the above model and get some quantitative ideas of the impact exercised by the war of attrition on the competitive adoption of the new technology, in this section we provide some numerical solutions of (5) and (6). The choice of parameters was made in the interest of sim-

[10] Using a model of preemption Lambrecht and Perraudin (2003) show that asymmetric information on costs results in the optimal trigger value $\bar{x}_n^*$ being a unique continuous increasing mapping function of $\theta_n$, i.e. $\bar{x}_n^* = \bar{x}_n^*(\theta_n) \in [\bar{x}^l(\underline{\theta}), \bar{x}^u(\bar{\theta}) = \infty)$, with $\frac{\partial \bar{x}_n^*(\theta_n)}{\partial \theta_n} > 0$.

plicity, respecting as far as possible some indications found in other studies (Dixit and Pindyck, 1994; Mauer and Ott, 1995; Lambrecht and Perraudin, 2003). The base parameters take the values: $r = 0.05, \alpha = 0.03,\ \sigma = 0.2$, $N = 4,\ k = 10$ and $\Delta k = 5,\ 2.5$. The choice of $\alpha$ is made to guarantee the firms' average waiting time positive. Figures 1 and 2 show numerical solutions for $\bar{x}_n^{**}(\theta_n)$ and $\bar{x}_n^{*}(\theta_n)$ within the interval $\theta_n \in [0, 2]$, when $F(\bar{x}_n^*)$ is a Pareto distribution of the form $1 - \left(\frac{\bar{x}_n^*}{\bar{x}^l}\right)^{-\gamma}$, with $\gamma = 1$ and $\bar{x}^l = 0.094$.

Figure 1 about here

Figure 2 about here

The triggers shown include: (i) the strategic trigger $\bar{x}_n^*$; the non-strategic trigger $\bar{x}_n^+$; and the second-mover trigger $\bar{x}_n^{**}$, for cost reduction of 50% and 25% respectively. In both cases the solution starts at the origin and increases monotonically for all the interval $[0, 2]$. The second-mover trigger is always far below the optimal trigger under the war of attrition. In addition, the ratio between the strategic trigger and the non-strategic trigger, $\bar{x}_n^*/\bar{x}_n^+$, equals 2.48 for $\Delta k = 5$ and 1.74 for $\Delta k = 2.5$ respectively. Thus current investment payoffs have to rise more than double the level that ensures a positive net benefit for a single firm in isolation before the war of attrition ceases to be worth playing by the firms (*war of attrition effect*). If, to this effect, we add the *irreversible effect* measured by the multiplier $\frac{\beta}{\beta-1} = 3.85$ (i.e. $\beta = 1.35$), we get a total effect of 5 to 6 times the point in which the total expected discounted investment payoffs equals the cost of investment, i.e. the Marshallian trigger $\bar{x}_n^M \equiv (r - \alpha)\theta_n k$ Therefore, even if the cost of capital is as low as 5% per year, the value of waiting with network externalities can quite easily lead to adjusted hurdle rates of 20 to 30 per cent.

# 4 Auctioning investment grants

Let's now consider an agency which, on the grounds of available information on firms' pollutant emissions, accumulation processes and consequent social damage, has identified $\hat{T}$ as the date by which all firms should abandon the polluting technology and adopt the green one. Moreover, let's assume that the agency is unable or unwilling to adopt mandatory regulations and, if necessary, intends to accelerate environmental innovation by subsidizing green investment expenditures. Subsidies will be granted if, and only if, the agency believes that firms face a value of waiting, before undertaking the green investment, greater than the one faced by society as a whole. However, since the private switching time $T$ is a stochastic variable, the agency has to set a policy-rule referring to $T$'s probability distribution. For the sake of simplicity, we assume the following simple rule:[11]

$$E(T) = \hat{T} \tag{7}$$

By (1) and the definition of $T$, (7) may be reformulated in terms of the instantaneous investment payoff, $x$, at which the technological change should take place in order to satisfy the agency's environmental objective. We denote with $\hat{x}$ the *social trigger value* such that $E[\inf(t > 0 \mid x_t = \hat{x})] = \hat{T}$.[12]

[11] Depending on different assumptions about the agency's risk aversion, the policy-rule can be made more stringent by giving different weights to different moments of the private switching time distribution.

[12] As the instantaneous payoffs are driven by (1), the first passage time $T$ from $x$ to $\hat{x}$ is a stochastic variable with first moment $E(T) = m^{-1} \ln(\frac{\hat{x}}{x})$, with $m \equiv (\alpha - \frac{1}{2}\sigma^2)$ so that $\hat{x} = xe^{\pi\hat{T}}$ (Cox and Miller, 1965, p. 221-222).

To solve the optimization problem the environmental agency has to find an optimal compensation function. In order to optimize this compensation function for all possible functions we apply the *revelation principle* which reduces the possible set of grant-aided schemes to those where lying is not profitable. We organize the model as an auction of the Vickrey-type where each firm simultaneously reports their respective optimal private triggers, without seeing each other's bid, and the subsidy is given to the firm that reports the lowest one (Laffont and Tirole 1993 pp.314-320).

Before describing the grant-aided scheme, it is worth underlying two important features of our model. First, since the evolutionary pattern of $x$ is a Markov process (Harrison, 1985, pp. 80-81), the agency's announcement of $\hat{T}$ (or, equivalently, $\hat{x}$) does not affect the firms' waiting game played prior to $\hat{T}$. Secondly, while the second-mover advantage, resulting from $\Delta k$, slows down the spontaneous technological change, the existence of network benefits provides the agency with the opportunity to adopt a targeted policy. For instance, by subsidizing the firm with the lower trigger $\bar{x}_n^*$ (the *leader firm*), i.e. by anticipating initiation of the bandwagon, the agency may accelerate the technological change throughout the entire industry.

In particular, we will show that the subsidy received by the leader is formed by the sum of a fixed payment function - defined according to the difference between the announced trigger $\tilde{x}_n^*$ and the social trigger $\hat{x}$ - plus a linear sharing of overruns which depends on the announced trigger value. If this subsidy is incentive-compatible it will be sufficient to induce the leader to announce the true trigger $\tilde{x}_n^* = \bar{x}_n^*$, and to adopt the green technology when $x$, randomly fluctuating, hits the social trigger $\hat{x}$. Although granting a subsidy only to the leader firm may not be enough to achieve the policy objective, by creaming the industry the proposed grant-aided scheme allows the agency to induce the followers to jump on the bandwagon without paying informational rents.[13]

The rationale behind the proposed grant-aided scheme can be summarised as follows. Since the war of attrition which will emerge within the industry can be interpreted as a sequence of (all-pay) second-price auctions[14], granting

[13]By the *revelation principle* instead of having the firms submit their bid as a function of $\bar{x}_n^*$ and then applying the rules of the auction mechanism to choose who receives the subsidy, we could directly ask the firms to report their values $\bar{x}_n^*$ and then make sure that the outcome is the same as if they had submitted bids.

[14]Referring to the literature of auctions, what has just been described as a war of attrition can be interpreted as a sequence of all-pay second-price auctions (Hirshleifer and Riley, 1992, ch.10). For instance, at each time $t$, it is as if agents bid the value of their opportunity to invest (4), $V_n(x_t; \bar{x}_n^*)$, and compare the relative merit of dropping out immediately (investing first) or staying in (delaying the decision) and bidding a further amount. Agents bid by deciding upon a maximum (stochastic) number of periods over which to compete which is determined by their optimal trigger levels $\bar{x}_n^*$. Thus, as long as

a subsidy to the leader firm implies that the agent with the lowest investment opportunity cost, whilst losing the war of attrition, will be the winning bidder in the public auction.[15] By contrast, the followers will gain the network benefit, but will not receive public subsidies, unless their investment opportunity costs are so high that a public grant is still required in order to avoid an undesirable time lag between the leader's and the followers' innovation time.

## 4.1 The agency's optimization problem

Let's assume that the environmental agency acts as a utilitarian regulator interested in accelerating environmental innovation.

Since $\bar{x}_n^*$ is private information, in order to exploit the potential regulatory benefits resulting from network externalities, the agency has to identify an appropriate incentive mechanism such that the (unknown) leader firm will find it profitable to abandon the polluting technology the first

firms can perfectly observe the rival's actions and immediately respond to them, if after $T_n = \inf(t > 0 \mid x_t = \bar{x}_n^*)$ periods firms $j \neq n$ find that $n$ has abandoned the polluting technology, they adopt the green one by paying less than the rival's bid, i.e.:

$$V_n(\bar{x}_n^*; \bar{x}_n^*) \equiv \frac{\bar{x}_n^*}{r-\alpha} - \theta_n k \leq V_j(\bar{x}_n^*; \bar{x}_j^{**}) \equiv \frac{\bar{x}_n^*}{r-\alpha} - \theta_j(k - \Delta k), \quad \forall j \neq n \tag{8}$$

provided that $\theta_n k \geq \theta_j(k - \Delta k)$. .

[15] See also Bulow and Klemperer (1999)

time $x$, randomly fluctuating, hits the social trigger $\hat{x}$. Therefore, defining $y_n(\bar{x}_n^*; \bar{\mathbf{x}}^*_{-n})$ as the probability that firm $n$ is selected to receive the subsidy, with $\bar{\mathbf{x}}^*_{-n} = (\bar{x}_1^*, \bar{x}_2^*, .\bar{x}_{n-1}^*, ., \bar{x}_{n+1}^*, .\bar{x}_{N+1}^*)$ and $\sum_{n=1}^{N+1} y_n = 1$, the optimal targeted grant-aided scheme, under incomplete information, should emerge maximizing at time $\hat{T}$ a welfare function, the maximand of which is the expectation of:

$$\left(\sum_{n=1}^{N+1} y_n(\bar{x}_n^*; \bar{\mathbf{x}}^*_{-n})\right) B - (1+\lambda) \sum_{n=1}^{N+1} s_n(\bar{x}_n^*; \bar{\mathbf{x}}^*_{-n}) + \sum_{n=1}^{N+1} \pi_n(\bar{x}_n^*; \hat{x}) \tag{9}$$

where $B$ is the estimated social benefit brought about by accelerating environmental innovation (i.e. by lowering firms' optimal trigger value at $\hat{x}$), $s_n(\bar{x}_n^*; \bar{\mathbf{x}}^*_{-n})$ is the subsidy in annuity terms, $\lambda \geq 0$ is the shadow cost of public funds and $\pi_n(\bar{x}_n^*; \hat{x})$ denotes the subsidized firm's rental price:

$$\pi_n(\bar{x}_n^*; \hat{x}) = E_{\bar{\mathbf{x}}_{-n}} \left\{ s_n(\bar{x}_n^*; \bar{\mathbf{x}}^*_{-n}) - y_n(\bar{x}_n^*; \bar{\mathbf{x}}^*_{-n})(\bar{x}_n^* - \hat{x}) \right\}, \quad \text{for } \hat{x} \leq \bar{x}_n^*.$$

Furthermore, without loss of generality, we may assume that the agency knows the firms' conjectural distribution. Therefore, conditional on the information available at the time when the grant-aided scheme is announced, the firms' optimal trigger levels are drawn independently from the same continuous distribution $F(\bar{x}_n^*; u_t)$, with density $f(\bar{x}_n^*; u_t)$ and $u_t = \hat{x}$.

The agency's optimization problem is then:

$$\max_{y_n, \pi_n} E_{\bar{x}_n^*, \bar{\mathbf{x}}_{-n}^*} \left\{ \left( \sum_{n=1}^{N+1} y_n(\bar{x}_n^*; \bar{\mathbf{x}}_{-n}^*) \right) B - (1+\lambda) \sum_{n=1}^{N+1} s_n(\bar{x}_n^*; \bar{\mathbf{x}}_{-n}^*) + \sum_{n=1}^{N+1} \pi_n(\bar{x}_n^*; \hat{x}) \right\}$$

subject to all the $N+1$ firms' optimization problem. The firm's $n$ optimization problem is given by:

$$\max_{\bar{x}_n^*} \pi_n(\bar{x}_n^*; \hat{x}) \geq 0 \qquad \forall n$$

Continuing with agent $n$ as representative, the following proposition indicates the results of the auction.

**Proposition 2** *The firm $n$ will receive the subsidy only if:*

$$\bar{x}_n^* < \min \left\{ \bar{x}_j^*, j \neq n \right\}$$

*and the optimal expected transfer in annuity terms is:*

$$E_{\bar{\mathbf{x}}^*_{-n}}\left\{s_n(\bar{x}^*_n;\bar{\mathbf{x}}^*_{-n})\right\} = (\bar{x}^*_n - \hat{x})(1 - F(\bar{x}^*_n;\hat{x}))^N + \int_{\bar{x}^*_n}^{\infty}(1 - F(\tilde{x}^*_n;\hat{x}))^N d\tilde{x}^*_n$$

**Proof.** See Appendix.

Differentiating the above equation yields:

$$\frac{\partial E_{\bar{\mathbf{x}}^*_{-n}}\left\{s_n(\bar{x}^*_n;\bar{\mathbf{x}}^*_{-n})\right\}}{\partial \bar{x}^*_n} = -N\left(1 - F(\bar{x}^*_n;\hat{x})\right)^{N-1} h(\bar{x}^*_n) < 0$$

which shows that the subsidy is strictly monotone decreasing in $\bar{x}^*_n$, and confirms the efficiency of the auction: the subsidy is given to the most efficient firm.

## 4.2 Implementation

While maximization of (9) determines expected transfers, i.e. the firms' optimal reporting strategies on average given the rivals' strategies through the probability $(1 - F^{(N)}(\bar{x}^*_n;\hat{x}))$, we can construct a dominant strategy auction of a Vickrey type that implements the same investment strategy as the one found from optimizing the welfare function (9), and selects the most efficient firm.[16]

Since, for the Vickrey auction, revelation of the true trigger value $\bar{x}_n^*$ is a dominant strategy but the subsidy is priced according to the second bid (*second-price auction*), in our $N+1$ agents case this implies implementing a subsidisation scheme of the type:

$$
\begin{aligned}
\tilde{s}_n(\bar{x}_n^*;\hat{x}) &= (\bar{x}_n^* - \hat{x}) + (\min\left\{\bar{x}_j^*, j \neq n\right\} - \bar{x}_n^*), \text{ for } \bar{x}_n^* \leq \min\left\{\bar{x}_j^*, j \neq n\right\} \\
\tilde{s}_n(\bar{x}_n^*;\hat{x}) &= 0 \quad \text{otherwise}
\end{aligned} \tag{10}
$$

When agent $n$ wins the auction, the subsidy is equal to the individually rational transfer $(\bar{x}_n^* - \hat{x})$ plus the rent it gets when the conjectural distribution is truncated at the lowest rivals' trigger value $\min\left\{\bar{x}_j^*, j \neq n\right\}$. Since $E_{\bar{x}_j}\left\{\tilde{s}_i(\bar{x}_i^*;\hat{x})\right\} = s_i(\bar{x}_i^*;\hat{x})$, the contract given by (10) costs the same in terms of annuity subsidy as the optimal Bayesian auction (Laffont and Tirole, 1993, pp. 319-320).

Thus competition among the firms implies that the interval of possible private investment triggers $[\bar{x}^l, \infty)$ is truncated to $[\bar{x}^l, \min\left\{\bar{x}_j^*, j \neq n\right\}]$ where

[16] A dominant strategy auction is an auction where each agent has a strategy that is optimal for any bids by its opponents.

$\min\left\{\bar{x}_j^*, j \neq n\right\}$ is the second-lowest bid reported at time $\hat{T}$ when the auction is run.

Alternatively, we can calculate the total subsidy to be transferred to the leader firm as:

$$\begin{aligned} S_n(\bar{x}_n^*; \hat{x}) &= \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (11) \\ &= \left(\frac{\hat{x} + (\bar{x}_n^* - \hat{x}) + (\min\left\{\bar{x}_j^*, j \neq n\right\} - \bar{x}_n^*)}{r - \alpha} - \theta_n k\right) - \left(\frac{\hat{x}}{r - \alpha} - \theta_n k\right) \\ &\equiv \frac{(\min\left\{\bar{x}_j^*, j \neq n\right\} - \hat{x})}{r - \alpha} \end{aligned}$$

Recalling that $x_t$ has lognormal distribution with mean $E_0(x_t) = x_0 e^{\alpha t}$, the first term in the r.h.s. of (11) represents the expected net present value of the payoffs starting at the given initial position $x_{\hat{T}} = \hat{x} + \tilde{s}_n(\bar{x}_n^*; \hat{x})$, whilst the second term is the net present value of the project starting at the initial position $\hat{x}$ without compensation.

Continuing with the numerical solutions of section 3.2, we are able to evaluate the total subsidy (11). Let's assume that the second-lowest bidding firm has a private valuation parameter equal (normalized) to one, i.e. $\min\{\theta_j, j \neq n\} = 1$, so that its optimal trigger values are $\min\left\{\bar{x}_j^*, j \neq n\right\} = 1.34$ for $\Delta k = 2.5$ and $1.91$ for $\Delta k = 5$ respectively.

If $\hat{T} = 20$ years from now and the income starting state is $x = 1$, the social trigger value equals $\hat{x} = 1.22$. Then, provided that $\bar{x}_n^* > 1.22$, the winning firm's total subsidy is equal to $S_n = 6$ with $\Delta k = 2.5$ and 34.5 with a cost reduction $\Delta k = 5$ respectively.[17] Further, if $\hat{T}$ reduces to 10 years and then $\hat{x} = 1.1$, the total subsidy increases substantially from $S_n = 12$ with $\Delta k = 2.5$ to 40.5 with $\Delta k = 5$ respectively.

Although the above results should be viewed as illustrative in nature and limited to giving an initial idea of the magnitude of the network effect, they show that the total subsidy to induce the most efficient firm to adopt the green technology earlier can be considerably higher than the investment cost. This suggests guidelines for more realistic research.

So far, we have considered the case where the network benefit is such that adoption of the green technology by the (subsidized) leader firm is sufficient to induce the other firms to switch immediately afterwards. However, as shown in Corollary 2, we can have diffusion depending on the wedge in firms' opportunity cost $\theta$. In particular, when $n$ goes first, we get sequential adoption if $\bar{x}_j^{**}(\theta_j) > \bar{x}_n^*(\theta_n)$, at least for some $j \neq n$. In this case, granting a subsidy to the leader firm is not enough to induce technological change

[17] The private average waiting time for $\theta = 1$ varies from nearly 30 to 65 years.

throughout the entire industry: in other words, a subsidy should also be granted to other firms. However, under our assumptions, on the basis of the announcement received from the leader firm, the subsidy received by the followers does not involve payment of an informational rent and it will be calculated referring to $\bar{x}_j^{**}$.

# 5 Final remarks

Even when firms have discovered profitable green investment opportunities, various sources of inertia may involve a private time of environmental innovation incompatible with avoidance of undesired levels of pollutant accumulation and social damage. This may occur when investment irreversibility, and the ability to postpone the decision, creates an option value of waiting before undertaking a technological change involving uncertain payoffs.

Strategic interactions may either decrease or further increase this option value. This occurs when the value of an investment depends on the number of firms which have undertaken the technological change, so that each agent's investment time is influenced by the investment decisions of others. In this paper we have examined what appears to be the most critical scenario from an environmental point of view, i.e. a situation where second-mover advantages exacerbate the irreversibility effect and increase the option value of waiting. In particular, we have explored the impacts of second-mover advantages arising from the expectation of declining investment costs due to the diffusion of new green technologies.

Although the expectation of declining investment costs tends to further decelerate voluntary irreversible green investments, the existence of network benefits provides the policy-maker with the opportunity of targeting investment grants to the firm(s) with lower switching costs. In fact, by accelerating initiation of technological change, the regulator may induce the whole industry to switch. However, this policy strategy requires knowledge of the private switching costs. Otherwise, appropriate incentive mechanisms are required to minimize agents' informational rents.

To find a cost-effective grant-aided scheme, we have examined a second-price sealed-bid private value auction where agents are required to announce their optimal trigger values, and a subsidy is granted to the firm which announces the lowest one, i.e. to the agent with the lowest switching cost. However the subsidy is priced according to the second-best bidder. Besides taking into account pure capital expenditures and including informational rents, the subsidy under consideration must compensate the leader firm for killing its option value of waiting. In other words, the firm must be compensated for the loss of benefits from delaying investment, i.e. for the value of waiting for more information about the investment payoffs and for the loss of network benefits.

Granting a subsidy only to the leader firm may prove to be insufficient to induce the other agents to switch immediately afterwards. For instance, simultaneous or sequential environmental innovation may emerge, depending on the wedge in firms' switching cost. However, under the proposed grant-aided scheme, the subsidy received by the followers does not involve payment of an informational rent. In other words, auctioning investment grants may prove to be a cost-effective way of creaming the industry, and accelerating environmental innovation, under incomplete information about private switching costs.

# A Appendix

## A.1 proof of proposition 1

The first part of the proof consists in identifying the optimal choice of the pure strategies' trigger levels for all players as a function of the state variable $x$ and of the conjectural distribution $F$, and then looking for the stationary Nash equilibrium strategies. Let's begin with strategy (b). As investment payoffs do not depend on the number of green firms, agent $n$ does not need to know his rivals' valuation parameter $\theta$ to follow strategy (b). He will consider switching only if $x_t \geq \bar{x}_n^{**}$ which is obtained by maximizing:

$$V_n(\bar{x}_n^{**}; x) \equiv E_0 \left\{ e^{-rT_n} \left( \int_{T_n}^{\infty} x_t e^{-rt} dt - \theta_n (k - \Delta k) \right) \mid \; x_0 = x \right\} \tag{12}$$

By using standard results (McDonald and Siegel, 1986; Dixit and Pindyck, 1994), it easy to write (12) as:

$$V_n(\bar{x}_n^{**}; x) \equiv (\frac{\bar{x}_n^{**}}{r - \alpha} - \theta_n (k - \Delta k)) \left( \frac{x}{\bar{x}_n^{**}} \right)^{\beta} . \tag{13}$$

where $\beta > 1$ is the positive root of the quadratic equation $\Phi(\beta) \equiv \frac{1}{2}\sigma^2\beta(\beta - 1) + \alpha\beta - r = 0$.

Finally, taking the derivative of the above expression with respect to $\bar{x}_n^{**}$ and solving it, we obtain (5) and the value of the option to go second becomes:

$$V_n(\bar{x}_n^{**}; x) = \begin{cases} \left(\frac{\bar{x}_n^{**}}{r-\alpha} - \theta_n(k - \Delta k)\right) \left(\frac{x}{\bar{x}_n^{**}}\right)^\beta & \text{for } \ x < \bar{x}_n^{**} \\ \\ \frac{x}{r-\alpha} - \theta_n(k - \Delta k) & \text{for } \ x \geq \bar{x}_n^{**} \end{cases} \tag{14}$$

where $\bar{x}_n^{**}$ is the point at which $V_n(x; \bar{x}_n^{**})$ smoothpastes to the exercise line $\frac{x}{r-\alpha} - \theta_n(k - \Delta k)$ (Dixit and Pindyck, 1994, p. 183)

Let's continue with strategy (c). If agent $n$ decides to invest unilaterally, taking account of the probability of being anticipated, the value at time $t$ of adopting the green technology is given by (4). As stated in the text, using Bayes' rule, the relationship between $F^{(N)}(\bar{x}_n^*)$ and $F^{(N)}(\bar{x}_n^*; u_t)$ for $t > 0$ can be described by:

$$F^{(N)}(\bar{x}_n^*; u_t) = \frac{F^{(N)}(\bar{x}_n^*) - F^{(N)}(u_t)}{1 - F^{(N)}(u_t)} \qquad \text{where } u_t = \sup_{0<s<t}(x_t). \tag{15}$$

In addition, indicating $h(\bar{x}_n^*) = \frac{f(\bar{x}_n^*)}{1-F(\bar{x}_n^*)}$ as the current value of the hazard rate, it can be easily seen that it is independent of $u_t$, that is:

$$\frac{f^{(N)}(\bar{x}_n^*; u_t)}{1 - F^{(N)}(\bar{x}_n^*; u_t)} = \frac{f^{(N)}(\bar{x}_n^*)}{1 - F^{(N)}(\bar{x}_n^*)} = \frac{Nf(\bar{x}_n^*)}{1 - F(\bar{x}_n^*)} = Nh(\bar{x}_n^*) \tag{16}$$

Therefore, making use of (13) and (15), the option value (4) can be rewritten as:

$$V_n(\bar{x}_n^*; x_t) = \left(\frac{\bar{x}_n^{**}}{r-\alpha} - \theta_n(k - \Delta k)\right)\left(\frac{x_t}{\bar{x}_n^{**}}\right)^\beta \int_{u_t}^{\bar{x}_n^{**}} dF^{(N)}(\bar{x}; u_t) + \left(\frac{\bar{x}_n^*}{r-\alpha} - \theta_n k\right)\left(\frac{x_t}{\bar{x}_n^*}\right)^\beta$$

$$+ \left[\int_{\bar{x}_n^{**}}^{\bar{x}_n^*} \left[\left(\frac{\bar{x}}{r-\alpha} - \theta_n(k - \Delta k)\right)\left(\frac{\bar{x}_n^*}{\bar{x}}\right)^\beta - \left(\frac{\bar{x}_n^*}{r-\alpha} - \theta_n k\right)\right] dF^{(N)}(\bar{x}; u_t)\right]\left(\frac{x_t}{\bar{x}_n^*}\right)^\beta . \tag{17}$$

The first term accounts for the case in which $u_t < \bar{x}_n^{**}$. In this case, the agent does not invest even if it knows that it will pay $k - \Delta k$. The second term is the usual option value of a single firm, and finally the third term is the expected gain by fighting before adopting. Firm $n$'s optimal trigger value can be obtained by maximizing (17). The first order condition requires:

$$\frac{\partial V_n(\bar{x}_n^*; x_t)}{\partial \bar{x}_n^*} = \frac{1-\beta}{(r-\alpha)\bar{x}_n^*}\left(\frac{x_t}{\bar{x}_n^*}\right)^{\beta}\left(1 - F^{(N)}(\bar{x}_n^*; u_t)\right) \times \tag{18}$$

$$\left[(\bar{x}_n^* - \bar{x}_n^+) - (\bar{x}_n^+ - \bar{x}_n^{**})\frac{\bar{x}_n^* f^{(N)}(\bar{x}_n^*; u_t)}{\beta(1 - F^{(N)}(\bar{x}_n^*; u_t))}\right] = 0.$$

where $\bar{x}_n^+ = \frac{\beta}{\beta-1}(r-\alpha)\theta_n k$ is the trigger value of going first without strategic behavior (or if firms do not expect a network benefit, i.e. $\Delta k = 0$). Looking for a maximum of $V_n(x_t; \bar{x}_n^*)$ also requires the square-bracketed term below to be positive:

$$\frac{\partial^2 V_n(\bar{x}_n^*; x_t)}{\partial (\bar{x}_n^*)^2} = \frac{(1-\beta)}{\beta(r-\alpha)\bar{x}_n^*}\left(\frac{x_t}{\bar{x}_n^*}\right)^{\beta}\left(1 - F^{(N)}(\bar{x}_n^*; u_t)\right) \times \tag{19}$$

$$\left[\beta - (\bar{x}_n^+ - \bar{x}_n^{**})Nh(\bar{x}_n^*) - (\bar{x}_n^+ - \bar{x}_n^{**})\bar{x}_n^* N\frac{dh(\bar{x}_n^*)}{d\bar{x}_n^*}\right] < 0$$

where the assumption that $h(\bar{x}_n^*)$ is increasing in $\bar{x}_n^*$ assures the sufficiency.[18] Rearranging (18) we obtain the following implicit form for the trigger level $\bar{x}_n^*$:

$$\begin{aligned} \bar{x}_n^* &= \bar{x}_n^+ + (\bar{x}_n^+ - \bar{x}_n^{**}) \frac{\bar{x}_n^* f^{(N)}(\bar{x}_n^*; u_t)}{\beta(1 - F^{(N)}(\bar{x}_n^*; u_t))} \\ &= \bar{x}_n^+ + (\bar{x}_n^+ - \bar{x}_n^{**}) \frac{\bar{x}_n^* N h(\bar{x}_n^*)}{\beta} \end{aligned} \tag{20}$$

Although $\bar{x}_n^*$ is invariant to the current value of the state variable $x$, in general it is not so with respect to $u_t$. The agent cannot credibly commit itself to the trigger level $\frac{\bar{x}_n^*}{\theta_n}$ as $x_t$ increases, and the bandwagon optimal rule defined in (3) and (20) is a contingent plan of how to play each time $t$ for possible realization of the state $x$, which summarizes the entire history of the game up to that point. However, as the hazard rate (16) is independent of $u_t$, the trigger value also becomes independent of the information variable $u_t$. This makes the optimal operating rule $a_n$ stationary.

Finally, by (17) and (20) we are able to write the value of the option to

[18] This assumption is satisfied by standard distributions as uniform, negative exponential, Weibull and Pareto.

invest first at time $t$ as:

$$V_n(\bar{x}_n^*; x_t) = \begin{cases} A(\bar{x}_n^{**}) F^{(N)}(\bar{x}_n^{**}; u_t) x_t^{\beta} + A(\bar{x}_n^*) x_t^{\beta} + B(\bar{x}_n^*) x_t^{\beta} & \text{for } x_t < \bar{x}_n^* \\ \\ \frac{x_t}{r-\alpha} - \theta_n k & \text{for } x_t \geq \bar{x}_n^* \end{cases} \tag{21}$$

where $A(\bar{x}_n^{**}) \equiv \left(\frac{\bar{x}_n^{**}}{r-\alpha} - \theta_n(k - \Delta k)\right)(\bar{x}_n^{**})^{-\beta}$, $A(\bar{x}_n^*) \equiv \left(\frac{\bar{x}_n^*}{r-\alpha} - \theta_n k\right)(\bar{x}_n^*)^{-\beta}$ and $B(\bar{x}_n^*) \equiv \left[\int\limits_{\bar{x}_n^{**}}^{\bar{x}_n^*} \left[\frac{\bar{x}}{r-\alpha} - \theta_n(k - \Delta k)\left(\frac{\bar{x}_n^*}{\bar{x}}\right)^{\beta} - \left(\frac{\bar{x}_n^*}{r-\alpha} - \theta_n k\right)\right] dF^{(N)}(\bar{x}; u_t)\right](\bar{x}_n^*)^{-\beta}$. That is, the stationary trigger $\bar{x}_n^*$ is the point at which the envelope function $V_n(u_t; u_t)$ smoothpastes to the exercise line $\frac{x_t}{r-\alpha} - \theta_n k$ (Moretto, 2000; Lambrecht and Perraudin, 2003). This concludes the first part of the proposition.

For the second part, applying the implicit function theorem to (6) we obtain:

$$\frac{d\bar{x}_n^*}{d\theta_n} = \frac{\left(\bar{x}_n^+ + (\bar{x}_n^+ - \bar{x}_n^{**})\bar{x}_n^* N h(\bar{x}_n^*)\right)^2}{\theta_n \left(\bar{x}_n^+ - (\bar{x}_n^+ - \bar{x}_n^{**})(\bar{x}_n^*)^2 N \frac{dh(\bar{x}_n^*)}{d\bar{x}_n^*}\right)} > 0$$

Positivity of the above expression is guaranteed by the second order condition for a maximum (19).

## A.2 Proof of proposition 2

We look for an incentive-compatible mechanism $[s_n(.), y_n(.)]$, $n = 1, 2...N+1$ that induces a truth-telling Bayesian Nash equilibrium. Defining with $s_n(\tilde{x}_n^*; \tilde{\mathbf{x}}_{-n}^*)$ the firm $n$'s subsidy per unit of time, required to induce adoption of the green technology at $\hat{x}$, as a function of the announced trigger levels $\tilde{x}_n^*$ and the rivals' announcement $\tilde{\mathbf{x}}_{-n}^* = (\tilde{x}_1, \tilde{x}_2, ..\tilde{x}_{n-1}, ., \tilde{x}_{n+1}, ..\tilde{x}_{N+1})$, its expected rental price can be expressed as:

$$\pi_n(\bar{x}_n^*, \tilde{x}_n^*; \hat{x}) = E_{\tilde{\mathbf{x}}_{-n}} \left\{ s_n(\tilde{x}_n^*; \tilde{\mathbf{x}}_{-n}^*) - y_n(\tilde{x}_n^*; \tilde{\mathbf{x}}_{-n}^*)(\bar{x}_n^* - \hat{x}) \right\}, \quad \text{for } \hat{x} \leq \tilde{x}_n^*. \tag{22}$$

We refer to (22) as the firm $n$'s profit function, and $y_n(\tilde{x}_n^*; \tilde{\mathbf{x}}_{-n}^*)$ is the probability that firm $n$ is selected to receive the subsidy, with $\sum_{n=1}^{N+1} y_n(\tilde{x}_n^*; \tilde{\mathbf{x}}_{-n}^*) = 1$.

A necessary condition for truth-telling is that the derivatives of firms' profit with respect to the agent $n$'s announcement $\tilde{x}_n^*$, and evaluated at the true trigger value, i.e. $\tilde{x}_n^* = \bar{x}_n^*$, is nil.

$$\frac{\partial \pi_n}{\partial \tilde{x}_n^*} = E_{\bar{\mathbf{x}}_{-n}^*} \left\{ \frac{\partial s_n}{\partial \tilde{x}_n^*} - \frac{\partial y_n}{\partial \tilde{x}_n^*}(\bar{x}_n^* - \hat{x}) \right\} = 0, \quad \forall n \tag{23}$$

Then, letting $\pi_n(\bar{x}_n^*; \hat{x})$ be firm $n$'s profit function when telling the truth, by the envelope theorem, (22) and (23) we obtain:

$$\frac{d\pi_n(\bar{x}_n^*; \hat{x})}{d\bar{x}_n^*} = -E_{\tilde{\mathbf{x}}_{-n}^*} \left\{ y_n(\bar{x}_n^*; \bar{\mathbf{x}}_{-n}^*) \right\} < 0, \quad \forall n \tag{24}$$

That is, at the optimum the profit function is nonincreasing in $\bar{x}_n^*$. It follows that the firm $n$'s individual rationality (participation constraint) is satisfied if it is satisfied at $x = \bar{x}^u \leq \infty$.Finally, by using (22) and (23) to integrate (24), we obtain:

$$\pi_n(\bar{x}_n^*; \hat{x}) = \pi_n(\bar{x}^u; \hat{x}) + \int_{\bar{x}_n^*}^{\bar{x}^u} E_{\tilde{\mathbf{x}}_{-n}^*} \left\{ y_n(\tilde{x}_n^*; \bar{\mathbf{x}}_{-n}^*) \right\} d\tilde{x}_n^*, \qquad \forall n \tag{25}$$

and the sufficient condition for truth-telling requires (Fudenberg and Tirole 1991, theorem 7.2 p. 260):

$$E_{\bar{\mathbf{x}}_{-n}^*} \left\{ \frac{\partial y_n}{\partial \bar{x}_n^*} \right\} \leq 0, \quad \forall n. \tag{26}$$

From (9) and the above arguments, the environmental agency's ex ante objective function can be expressed as:

$$\left(\sum_{n=1}^{N+1} y_n(\bar{x}_n^*;\bar{\mathbf{x}}_{-n}^*)\right) B+(1+\lambda)\sum_{n=1}^{N+1} y_n(\bar{x}_n^*;\bar{\mathbf{x}}_{-n}^*)(\bar{x}_n^*-\hat{x})-\lambda\sum_{n=1}^{N+1}\pi_n(\bar{x}_n^*;\hat{x}) \quad (27)$$

Since the agency's objective function is decreasing in $\pi_n$, and from (24) the profit function is decreasing in $\bar{x}_n^*$, the individual participation constraint will be tight at the highest trigger value $\bar{x}^u$. That is, assuming that, outside the relationship with the regulator, each firm has opportunities normalized to zero, we get: $\pi_n(\bar{x}^u;\hat{x}) = 0$, for all $n$.

The agency's optimization problem under incomplete information can be expressed as follows:

$$\max_{y_n,\pi_n} E_{\bar{x}_n^*,\bar{\mathbf{x}}_{-n}^*}\left\{\left(\sum_{n=1}^{N+1} y_n(\bar{x}_n^*;\bar{\mathbf{x}}_{-n}^*)\right) B+(1+\lambda)\sum_{n=1}^{N+1} y_n(\bar{x}_n^*;\bar{\mathbf{x}}_{-n}^*)(\bar{x}_n^*-\hat{x}) \right. \quad (28)$$

$$\left. -\lambda\sum_{n=1}^{N+1}\pi_n(\bar{x}_n^*;\hat{x})\right\}$$

subject to an Incentive Constraint, a Participation Constraint and a Sufficient Condition:

$$\frac{d\pi_n(\bar{x}_n^*;\hat{x})}{d\bar{x}_n^*} = -E_{\bar{\mathbf{x}}_{-n}^*}\left\{y_n(\bar{x}_n^*;\bar{\mathbf{x}}_{-n}^*)\right\} < 0, \quad \forall n$$

$$\pi_n(\bar{x}^u = \infty;\hat{x}) = 0, \qquad \forall n.$$

$$E_{\bar{\mathbf{x}}_{-n}^*}\left\{\frac{\partial y_n}{\partial \bar{x}_n^*}\right\} \leq 0, \quad \forall n$$

$$\sum_{n=1}^{N+1} y_n(\bar{x}_n^*;\bar{\mathbf{x}}_{-n}^*) = 1, \quad \text{for any } \bar{x}_n^* \text{ and } \bar{\mathbf{x}}_{-n}^*.$$

As is usual in the regulatory theory under asymmetry of information, we first ignore the second-order condition to check later that it is indeed satisfied at the optimum. As $\pi_n$ is considered the state variable in the above maximization, we can substitute (25) in the agency's objective function and solve for the optimal $y_n$. Integrating by parts (28) for given $\bar{\mathbf{x}}_{-n}^*$, the objective function can be rewritten as follows:

$$E_{\bar{x}_n^*,\bar{\mathbf{x}}_{-n}^*}\left\{\sum_{n=1}^{N+1} y_n(\bar{x}_n^*;\bar{\mathbf{x}}_{-n}^*)\left[B-(1+\lambda)(\bar{x}_n^*-\hat{x})-\lambda\frac{F(\bar{x}_n^*;\hat{x})}{f(\bar{x}_n^*;\hat{x})}\right]\right\}$$

Recalling the learning process (15), we simplify the agency's objective function as:

$$E_{\bar{x}_n^*,\bar{\mathbf{x}}_{-n}^*}\left\{\sum_{n=1}^{N+1} y_n(\bar{x}_n^*;\bar{\mathbf{x}}_{-n}^*)R(\bar{x}_n^*,\hat{x};x,\lambda)\right\}, \qquad \forall n \tag{29}$$

where:

$$R(\bar{x}_n^*, \hat{x}; x, \lambda) = \left[B - (1+\lambda)\left((\bar{x}_n^* - \hat{x}) + \frac{\lambda}{1+\lambda}\frac{F(\bar{x}_n^*) - F(\hat{x})}{f(\bar{x}_n^*)}\right)\right]$$

By the monotone hazard rate assumption the term $R(\bar{x}_n^*, \hat{x}; x, \lambda)$ is nonincreasing in $\bar{x}_n^*$, therefore the optimal choice by the regulator would be:

$$y_n(\bar{x}_n^*; \bar{\mathbf{x}}_{-n}^*) = 1 \quad if \quad \bar{x}_n^* \leq \min_{j \neq n} \bar{x}_j^*$$

$$y_n(\bar{x}_n^*; \bar{\mathbf{x}}_{-n}^*) = 0 \quad if \quad \bar{x}_n^* > \min_{j \neq n} \bar{x}_j^*$$

Hence $E_{\bar{\mathbf{x}}_{-n}^*}\left\{y_n(\bar{x}_n^*; \bar{\mathbf{x}}_{-n}^*)\right\}$ is nonincreasing almost everywhere which implies that the second order condition (26) is always satisfied. Finally, from (22), (24) and (25), the optimal Bayesian auction-based grant-aided scheme is such that:

$$\begin{aligned} E_{\bar{\mathbf{x}}_{-n}^*}\left\{s_n(\bar{x}_n^*; \bar{\mathbf{x}}_{-n}^*)\right\} &= \pi_n(\bar{x}_n^*; \hat{x}) + E_{\bar{\mathbf{x}}_{-n}^*}\left\{y_n(\bar{x}_n^*; \bar{\mathbf{x}}_{-n}^*)(\bar{x}_n^* - \hat{x})\right\}, \\ &= E_{\bar{\mathbf{x}}_{-n}^*}\left\{y_n(\bar{x}_n^*; \bar{\mathbf{x}}_{-n}^*)(\bar{x}_n^* - \hat{x})\right\} + \int_{\bar{x}_n^*}^{\infty} E_{\bar{\mathbf{x}}_{-n}^*}\left\{y_n(\tilde{x}_n^*; \bar{\mathbf{x}}_{-n}^*)\right\} d\tilde{x}_n^* , \end{aligned}$$

for all $n$. Finally as long as the probability of being the lowest bidder is $E_{\bar{\mathbf{x}}_{-n}^*}\left\{y_n(\bar{x}_n^*; \bar{\mathbf{x}}_{-n}^*)\right\} = 1 - F^{(N)}(\bar{x}_i^*; \hat{x}) \equiv (1 - F(\bar{x}_i^*; \hat{x}))^N$, we get the subsidy in the text. This concludes the proof.

# References


[1] Bliss, C. and B. Nalebuff, (1984), "Dragon-Slaying and Ballroom Dancing: The Private Supply of a Public Good", *Journal of Public Economics,* 25, 1-12.

[2] Bolow, J. and P. Klemperer, (1999), "The Generalized War of Attrition". *American Economic Review* 89, 175-189.

[3] Brau, R. and C.Carraro, (1999), "Voluntary Approaches, Market Structure and Competition", FEEM Note di Lavoro 53.99, Milan.

[4] Carraro, C. and F. Léveque, (1999), "Introduction: The Rationale and Potential of Voluntary Approaches", in C. Carraro and F. Leveque, eds., *Voluntary Approaches in Environmental Policy*, Dordrecht: Kluwer Academic Publishers, 1-15.

[5] Cox, D.R. and H.D. Miller, (1965) *The Theory of Stochastic Process* (Chapman and Hall, London).

[6] Cox, J.C. and S.A. Ross, (1976), "The Valuation of Options for Alternative Stochastic Process", *Econometrica*, 53, 385-408.

[7] Dixit, A. and R.S. Pindyck, (1994), *Investment under Uncertainty* (Princeton University Press, Princeton)

[8] Dosi, C. and M. Moretto, (1997), "Pollution Accumulation and Firm Incentives to Promote Irreversible Technological Change under Uncertain Private Benefits", *Environmental & Resource Economics* 10, 285-300.

[9] Dosi, C. and M. Moretto, (1998), "Auctioning Green Investment Grants as a Means of Accelerating Environmental Innovation", *Revue d'Economie Industrielle*, 83, 99-110.

[10] Farrell, J., and G., Saloner, (1985), "Standardization, Compatibility and Innovation", *RAND Journal of Economics,* 16, 70-83.

[11] Grenadier, S.R., (2002), "Option Exercise Games: An Application to the Equilibrium Investment Strategies of Firms", *The Review of Financial Studies*, 15, 691-721.

[12] Harrison, J.M., (1985), *Brownian Motion and Stochastic Flow Systems,* New York (John Wiley & Sons, New York, NY).

[13] Harrison J.M., and D. Kreps, (1979), "Martingales and Arbitrage in Multiperiod Securities Markets", *Journal of Economic Theory*, 20, 381-408.

[14] Hirshleifer, J., and J.G. Riley, (1992), *The Analytics of Uncertainty and Information,* (Cambridge University Press, Cambridge, UK).

[15] Laffont, J.J., and J. Tirole, (1993), *A Theory of Incentives in Procurement and Regulation*, (The MIT Press, Cambridge, MA)

[16] Lambrecht, B., and W., Perraudin, (2003), "Real Options and Preemption under Incomplete Information", *Journal of Economic Dynamics & Control*, 27, 619-643.

[17] Mason, R., and H. Weeds (2001), "Irreversibility Investment with Strategic Interactions", CEPR Working Paper n.3013.

[18] Mauer, D.C., e S.H. Ott, (1995), "Investment under Uncertainty: The Case of Replacement Investment Decisions", *Journal of Financial and Quantitative Analysis*, 30, 4, pp.581-605.

[19] McDonald R., and D. Siegel (1986), "The Value of Waiting to Invest", *Quarterly Journal of Economics*, 101, 707-727.

[20] Moretto, M., (2000), "Irreversible Investment with Uncertainty and Strategic Behavior", *Economic Modelling,* 17, 589-617.

[21] Murto, P. and J. Keppo (2002), "A Game Model of Irreversible Investment under Uncertainty", *International Game Theory Review*, 4, 127-140.

[22] Sparla, T., (2000), "Closure Options in Duopoly: The Case of Second-Mover Advantages", University of Dortmund, Discussion Paper n.01-06.

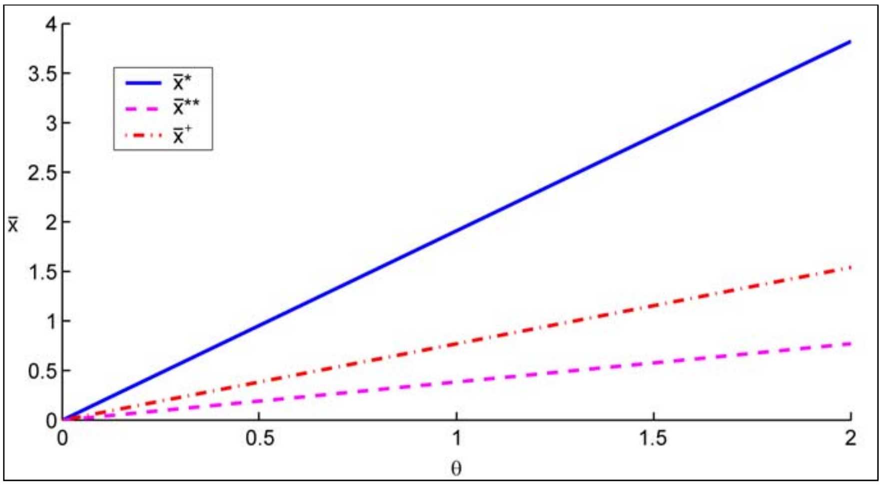


Figure 1. Network effect $\Delta k = 5$

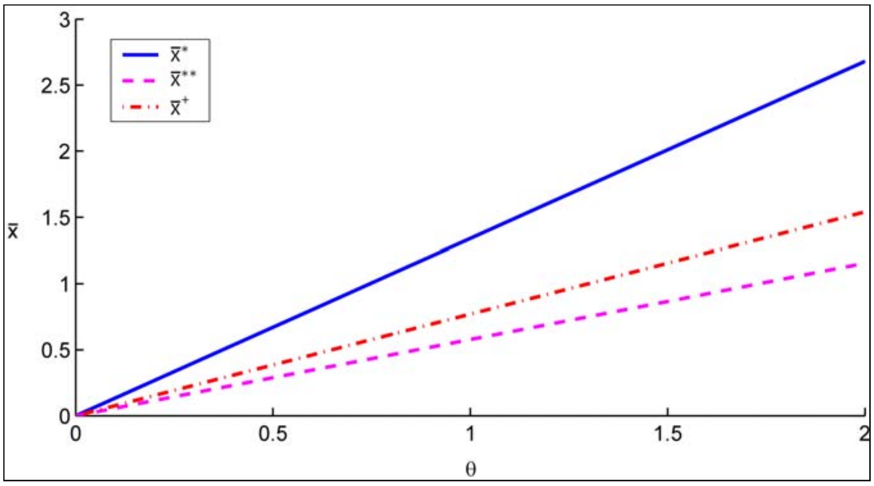


Figure 2. Network effect $\Delta k = 2.5$