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ON THE PREDICTABILITY OF STOCK PRICES:
A CASE FOR HIGH AND LOW PRICES

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On the Predictability of Stock Prices: a Case for High and Low Prices*

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Abstract

Contrary to the common wisdom that asset prices are barely possible to forecast, we show that that high and low prices of equity shares are largely predictable. We propose to model them using a simple implementation of a fractional vector autoregressive model with error correction (FVECM). This model captures two fundamental patterns of high and low prices: their cointegrating relationship and the long memory of their difference (i.e. the range), which is a measure of realized volatility. Investment strategies based on FVECM predictions of high/low US equity prices as exit/entry signals deliver a superior performance even on a risk-adjusted basis.

Keywords. high and low prices; predictability of asset prices; range; fractional cointegration; exit/entry trading signals; chart/technical analysis.

JEL Classifications: G11; G17; C53; C58.

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1   Introduction

The common wisdom in the financial literature is that asset prices are barely predictable (e.g. Fama, 1970, 1991). The rationale for this idea is the *Efficient Market Hypothesis* (EMH) in which asset prices evolve according to a random walk process. In this paper, we argue that this principle does not hold for the so-called "high" and "low" prices, i.e. the maximum and minimum price of an asset during a given period. By focusing on asset price predictability rather than assessing the EMH paradigm, we address three main questions in this paper: are high and low prices of equity shares predictable? How can we model them? Do forecasts of high and low prices provide useful information for asset and risk management?

There are three respects in which high and low prices can provide valuable information for their predictability. First, they inform people's thinking. Kahneman and Tversky (1979) show that when forming estimates, people start with an initial arbitrary value, and then adjust it in a slow process. In more general terms, behavioral finance studies have shown that agents' behavior generally depends on reference levels. As in a self-reinforcing mechanism, these forms of mental accounting and framing plus previous highs and lows typically represent the reference values for future resistance and support levels.

Second, high and low prices actually shape the decisions of many kinds of market participants, in particular technical analysts.[1] More generally, any investor using a path-dependent strategy typically tracks the past history of extreme prices. Thus, limit prices in pending stop-loss orders often match the most extreme prices in a previous representative period. Moreover, as highlighted in the literature on market microstructure, high and low prices also convey information about liquidity provision and the price discovery process.[2]

Finally, extreme prices are highly informative as a measure of dispersion. The linear difference between high and low prices is known as the range. Since Feller (1951), there have been many studies on the range, starting from the contribution of Parkinson (1980) and Garman and Klass (1980) among many others.[3] The literature shows that the range-based estimation of volatility is highly statistically efficient and robust with respect to many microstructure frictions (see e.g. Alizadeh, Brandt, and Diebold, 2002).

In order to answer the question of why high and low prices of equity shares are predictable, we present a simple implementation of a fractional vector autoregressive model with error correction (hereafter referred to as *FVECM*) between high and low prices. The rationale for this modeling strategy is twofold. First, it captures the cointegrating relationship between high and low prices, i.e. they may temporarily diverge but they have an embedded convergent path in the long run. This motivates the error correction mechanism between high and low prices. Second, the difference between the high and low prices, i.e. the range, displays long memory that can be well captured by the fractional autoregressive technique. Combining the cointegration between highs and lows with the long memory of the range naturally leads to model high and low prices in an FVECM framework.

The long-memory feature of the range is consistent with many empirical studies on the predictability of the daily range, a proxy of the integrated volatility, see for example Gallant,

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[1]Recently, academics documented that technical analysis strategies may succeed in extracting valuable information from typical chartist indicators, such as candlesticks and bar charts based on past high, low, and closing prices (e.g. Lo, Mamaysky, and Wang, 2000), and that support and resistance levels coincide with liquidity clustering (Kavajecz and Odders-White (2004)). The widespread use of technical analysis especially for short time horizons (intraday to one week) is documented in, e.g., Allen and Taylor (1990). Other papers on resistance levels are Curcio and Goodhart (1992), DeGrauwe and Decupere (1992), and Osler (2000).

[2]For instance, Menkhoff (1998) shows that high and low prices are very informative when it comes to analyzing the order flow in foreign exchange markets.

[3]See also Beckers (1983), Ball and Torous (1984), Rogers and Satchell (1991), Kunitomo (1992), and more recently Andersen and Bollerslev (1998), Yang and Zhang (2000), Alizadeh, Brandt, and Diebold (2002), Brandt and Diebold (2006),Christensen and Podolskij (2007), Martens and van Dijk (2007), Christensen, Zhu, and Nielsen (2009).

Hsu, and Tauchen (1999) which fits a three-factor model to the daily range series to mimic the long-memory feature in volatility, or Rossi and Santucci de Magistris (2009) which finds fractional cointegration between the daily range computed on futures and spot prices. This evidence allows the predictability of variances to be embedded in a model for the mean dynamic of extreme prices. The model we propose thus captures the predictability of extreme prices by means of the predictability of second-order moments, which is a widely accepted fact since the seminal contribution of Engle (1982) and following the ARCH models literature. We thus provide an affirmative answer to the first and second questions initially posed: first, there is compelling evidence of the predictability of high and low prices; second, it suffices to apply a linear model that captures the fractional cointegration between the past time series of highs and lows.

To answer our third question, i.e. do forecasts of high and low prices provide useful information for asset management, we analyze intraday data of the stocks forming the Dow Jones Industrial Average index over a sample period of eight years. The sample period is pretty representative since it covers calm and liquid markets as well as the recent financial crisis. We find strong support for the forecasting ability of FVECM, which outperforms any reasonable benchmark model. We then use the out-of-sample forecasts of high and low prices to implement some simple trading strategies. The main idea is to use high and low forecasts to determine entry and exit signals. Overall, the investment strategies based on FVECM predictions deliver a superior performance even on a risk-adjusted basis.

The present paper is structured as follows. Section 2 discusses the integration and cointegration properties of high and low daily prices in a non-parametric setting. At this stage, our analysis is purely non-parametric and employs the most recent contributions of the literature (such as Shimotsu and Phillips (2005), Robinson and Yajima (2002), and Nielsen and Shimotsu (2007)). Section 3 presents the FVECM that is an econometric specification consistent with the findings of the previous section, i.e. the fractional cointegration relationship between high and low prices. After reporting the estimation outputs, Section 4 provides an empirical application of the model forecast in a framework similar to the technical analysis. Section 5 concludes the paper.

2 Integration and Cointegration of Daily High and Low Prices

Under the EMH, the daily closing prices embed all the available information. As a result, the best forecast we could make for the next day's closing price is today's closing price. This translates into the commonly accepted assumption of non-stationarity for the closing prices or, equivalently, the hypothesis that the price evolution is governed by a random walk process (which is also referred to as an integrated process of order 1, or a unit-root process). Consequently, price movements are due only to unpredictable shocks.[4] Furthermore, the random walk hypothesis is also theoretically consistent with the assumption that price dynamics are driven by a geometric Brownian motion, which implies normally distributed daily log-returns. Finally, the EMH implies that the prices are not affected by some short-term dynamics such as autoregressive (AR) or moving average (MA) patterns.[5]

However, it is also commonly accepted that daily log-returns strongly deviate from the hypothesis of log-normality, thus casting some doubts on the law of motion hypothesis, in particular with respect to the distributional assumption. In fact, extreme events are more likely to occur compared with the Gaussian case, and returns are asymmetric, with a density shifted to the

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[4]In more general terms, if the EMH holds, future prices cannot be predicted using past prices as well as using past values of some covariates. We do not consider here the effects of the introduction of covariates, but focus on the informative content of the price sequence. As a result, we focus on the "weak-form efficiency" as presented in Fama (1970).

[5]The short-term dynamic could co-exist with the random walk within an ARIMA structure, where autoregressive (AR) and moving average (MA) terms are coupled with unit root (integrated) components.

left (negative asymmetry). Many studies analyze market efficiency by testing the stationarity of daily closing prices or market values at given points of time, see Lim and Brooks (2011).[6]

Although market efficiency should hold for any price, by their very nature high/low prices differ from closing prices in two main respects. First, liquidity issues are more relevant for high and low prices. Intuitively, high (low) prices are more likely to correspond to ask (bid) quotes, thus transaction costs and other frictions such as price discreteness, the tick size (i.e. the minimal increments) or stale prices might represent disturbing factors. Second, highs and lows are more likely to be affected by unexpected shocks such as (unanticipated) public news announcements. Then, some aspects such as market resiliency and quality of the market infrastructure can be determinant. In view of these considerations, we pursue a conservative approach by considering the predictability of highs and lows per se as weak evidence of market inefficiency. A more rigorous test is the analysis of the economic implications arising from the predictability of highs and lows. More specifically, we assess whether their prediction provides superior information to run outperforming trading rules.

We should also stress that this attempt will be limited to the evaluation of the serial correlation properties of those series, without the inclusion of the information content of other covariates. Our first research question is thus to analyze the stationarity properties of daily high and daily low prices in order to verify whether the unpredictability hypothesis is valid if applied separately to the two series. If the unpredictability hypothesis holds true, both high and low prices should be driven by random walk processes, and should not have relevant short-term dynamic components (they should not be governed by ARIMA processes). To tackle this issue we take a purely empirical perspective: at this stage we do not make assumptions either on the dynamic process governing the evolution of daily high and low, nor on the distribution of prices, log-prices or returns over high and low. Such a choice does not confine us within the unrealistic framework of geometric Brownian motions, and, more relevantly, does not prevent us from testing the previous issues. In fact, when analyzing the non-stationarity of price sequences, a hypothesis on the distribution of prices, log-prices or returns is generally not required. Given the absence of distributional hypotheses and of assumptions on the dynamic, we let the data provide some guidance. We thus start by analyzing the serial correlation and integration properties of daily high and low prices. Furthermore, given the link between high and low prices and the integrated volatility presented in the introduction, we also evaluate the serial correlation and integration of the difference between high and low prices, the range. In the empirical analyses we focus on the 30 stocks belonging to the Dow Jones Industrial Average index as of end of December 2010.

We consider the daily high log-price, $p_t^H = log(P_t^H)$, and the daily low log-price, $p_t^L = log(P_t^L)$. Our sample data covers the period January 2, 2003 to December 31, 2010, for a total of 2015 observations. The plots of the daily high and low prices show evidence of a strong serial correlation, typical of integrated processes, and the Ljung-Box test obviously strongly rejects the null of no correlation for all lags.[7] Therefore, we first test the null hypothesis of unit root for the daily log-high, log-low and range by means of the Augmented Dickey Fuller (ADF) test. In all cases, the ADF tests cannot reject the null of unit root for the daily log-high and log-low prices.[8] TThis result is robust to the inclusion of the constant and trend, as well as different choices of lag. The outcomes obtained by a standard approach suggest that the daily high and daily low price sequences are integrated of order 1 or, equivalently, that they are governed by random walk processes (denoted as I(1) and I(0), respectively). At first glance, this finding seems to support the efficient market hypothesis. This is further supported by the absence of a long- and short-range dependence on the first differences of daily high and daily low log-prices. Moreover, there is clear evidence that daily range is not an I(1) process, but

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[6]To our knowledge, Cheung (2007) represents the only noticeable exception that analyzes high and low prices.
[7]Results not reported but available on request.
[8]Using standard testing procedures based on the ADF test, we also verified that the integration order is 1, given that on differenced series the ADF test always rejects the null of a unit root.

should be considered an I(0) process. Based on these results, one can postulate that daily high and daily low prices are cointegrated, since there exists a long-run relation between highs and lows that is an I(0) process. However, this finding must be supported by proper tests, since the existence of a cointegrating relation between daily high and daily low potentially allows for the construction of a dynamic bivariate system which governs the evolution of the two series, with a possible impact on predictability. These results confirm the findings in Cheung (2007). Besides the previous result, Andersen and Bollerslev (1997) and Breidt, Crato, and de Lima (1998), among others, find evidence of long memory (also called long range dependence) in asset price volatility. This means that shocks affecting the volatility evolution produce substantial effects for a long time. In this case, volatility is said to be characterized by a fractional degree of integration, due to the link between the integration order and the memory properties of a time series, as we will see in the following. In our case, the autocorrelation function, ACF, in Figure 1 seems to suggest that the daily range is also a fractionally integrated process, provided that it decays at a slow hyperbolic rate. In particular, the ACF decays at a slow hyperbolic rate, which is not compatible with the I(0) assumption made on the basis of ADF tests. Unfortunately, the ADF is designated to test for the null of unit root, against the I(0) alternative, and it is also well known, see Diebold and Rudebusch (1991), that the ADF test has very low power against fractional alternatives. Therefore, we must investigate the integration order of daily high and low prices and range in a wider sense, that is in the fractional context. We also stress that the traditional notion of cointegration is not consistent with the existence of long-memory. In order to deeply analyze the dynamic features of the series at hand, we resort to more recent and non-standard tests for evaluating the integration and cointegration orders of our set of time series. Our study thus generalizes the work of Cheung (2007) since it does not impose the presence of the most traditional cointegration structure, and it also makes the evaluation consistent with the findings of long memory in financial data.

We thus investigate the degree of integration of the daily high and low prices, and of their difference, namely the *range*, in a fractional or long-memory framework. This means that we assume that we observe a series, $y_t \sim I(d)$, $d \in \Re$, for $t \geq 1$, is such that

$$(1 - L)^d y_t = u_t \tag{1}$$

where $u_t \sim I(0)$ and has a spectral density that is bounded away from zero at the origin. Differently from the standard setting, the integration order d might assume values over the real line and is not confined to integer numbers. Note that if $d = 1$ the process collapses on a random walk, whereas if $d = 0$ the process is integrated of order zero, and thus stationary. The econometrics literature on long-memory processes distinguishes between type I and type II fractional processes. These processes have been carefully examined and contrasted by Marinucci and Robinson (1999), and Davidson and Hashimzade (2009). The process y_t reported above is a type II fractionally integrated process, which is the truncated version of the general type I process, since the initial values, for $t = 0, -1, -2, ...$ are supposed to be known and equal to the unconditional mean of the process (which is equal to zero)[9]. In this case, the term $(1 - L)^d$ results in the truncated binomial expansion

$$(1 - L)^d = \sum_{i=0}^{T-1} \frac{\Gamma(i - d)}{\Gamma(-d)\Gamma(i + 1)} L^i \tag{2}$$

so that the definition in (1) is valid for all d, see Beran (1994) among others. In particular, for $d < 0$, the process is said to be anti-persistent, while for $d > 0$ it has long memory. When dealing with high and low prices, our interest refers to the evaluation of the integration order d for both high and low, as well as of the integration order for the range. Furthermore, if the daily high and daily low time series have a unit root while the high-low range is a stationary

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[9]In contrast, type I processes assume knowledge of the entire history of y_t

but long-memory process, a further aspect must be clarified. In fact, as already mentioned, the presence of a stationary linear combination (the high-low range) of two non-stationary series opens the door to the existence of a cointegrating relation. However, the traditional tests of cointegration are not consistent with the memory properties of the high-low range. Therefore, we chose to evaluate the fractional cointegration across the daily high and daily low time series.

In the context of long-memory processes, the term fractional cointegration refers to a generalization of the concept of cointegration, since it allows linear combinations of $I(d)$ processes to be $I(d-b)$, with $0 < b \leq d$. The term fractional cointegration underlies the idea of the existence of a common stochastic trend, that is integrated of order d, while the short period departures from the long-run equilibrium are integrated of order $d-b$. Furthermore, b stands for the fractional order of reduction obtained by the linear combination of $I(d)$ variables, which we call *cointegration gap*. We first test for the presence of a unit root in the high and low prices, so that $d = 1$, and, as a consequence, the fractional integration order of the range becomes $1-b$. In order to test the null hypothesis of a unit root and of a fractional cointegration relation between daily high and low prices, we consider a number of approaches and methodologies. First, we estimate the fractional degree of persistence of the daily high and low prices by means of the univariate local exact Whittle estimator of Shimotsu and Phillips (2005). Notably, their estimator is based on the type II fractionally integrated process. The univariate local exact Whittle estimators for high and lows ($\hat{d}_H$ and $\hat{d}_L$, respectively) minimizes the following contrast function

$$Q_{m_d}(d_i, G_{ii}) = \frac{1}{m_d} \sum_{j=1}^{m_d} \left[\log(G_{ii}\lambda_j^{-2d}) + \frac{1}{G_{ii}}I_j \right] \quad i = H, L \tag{3}$$

which is concentrated with respect to the diagonal element of the 2×2 matrix G, under the hypothesis that the spectral density of $U_t = [\Delta^{d_H}p_t^H, \Delta^{d_L}p_t^L]$ satisfies

$$f_U(\lambda) \sim G \quad as \quad \lambda \to 0. \tag{4}$$

Furthermore, I_j is the coperiodogram at the Fourier frequency $\lambda_j = \frac{2\pi j}{T}$ of the fractionally differenced series U_t, while m_d is the number of frequencies used in the estimation. The matrix G is estimated as

$$\hat{G} = \frac{1}{m_d} \sum_{j=1}^{m_d} Re(I_j) \tag{5}$$

where $Re(I_j)$ denotes the real part of the coperiodogram. Table 1 reports the exact local Whittle estimates of d^H and d^L for all the stocks under analysis. As expected, the fractional orders of integration are high and generally close to 1. Given the estimates for the integration orders, we test for equality according to the approach proposed in Nielsen and Shimotsu (2007) that is robust to the presence of fractional cointegration. The approach resembles that of Robinson and Yajima (2002), and starts from the fact that the presence or absence of cointegration is not known when the fractional integration orders are estimated. Therefore, Nielsen and Shimotsu (2007) propose a test statistic for the equality of integration orders that is informative independently from the existence of the fractional cointegration. In the bivariate case under study, the test statistic is

$$\widehat{T}_0 = m_d \left(S\hat{d} \right)' \left(S\frac{1}{4}\hat{D}^{-1}(\hat{G} \odot \hat{G})\hat{D}^{-1}S' + h(T)^2 \right)^{-1} \left(S\hat{d} \right) \tag{6}$$

where $\odot$ denotes the Hadamard product, $\hat{d} = [\hat{d}^H, \hat{d}^L]$, $S = [1, -1]'$, $h(T) = \log(T)^{-k}$ for $k > 0$, $D = \text{diag}(G_{11}, G_{22})$. If the variables are not cointegrated, that is the cointegration rank r is zero, $\widehat{T}_0 \to \chi_1^2$, while if $r \geq 1$, the variables are cointegrated and $\widehat{T}_0 \to 0$. A significantly large value of $\widehat{T}_0$, with respect to the null density χ_1^2, can be taken as an evidence against the equality of the integration orders. The estimation of the cointegration rank r is obtained by calculating the eigenvalues of the matrix $\hat{G}$. Since G does not have full rank when p_t^H and p_t^L

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are cointegrated, then G is estimated following the procedure outlined in Nielsen and Shimotsu (2007, p. 382) which involves a new bandwidth parameter m_L. In particular, $\hat{d}_H$ and $\hat{d}_L$ are obtained first, using (3) with m_d as bandwidth. Given $\hat{d}$, the matrix G is then estimated, using m_L periodogram ordinates in (5), such that $m_L/m_d \to 0$. The table reports the estimates for $m_d = T^{0.6}$ and $m_L = T^{0.5}$, while results are robust to all different choices of m_d and m_L.

Let $\hat{\delta}_i$ be the ith eigenvalue of $\hat{G}$, it is then possible to apply a model selection procedure to determine r. In particular,

$$\hat{r} = \arg \min_{u=0,1} L(u) \tag{7}$$

where

$$L(u) = v(T)(2-u) - \sum_{i=1}^{2-u} \hat{\delta}_i \tag{8}$$

for some $v(T) > 0$ such that

$$v(T) + \frac{1}{m_L^{1/2} v(T)} \to 0. \tag{9}$$

The previous tests are concordant in suggesting that the integration order of p_t^H and p_t^L are equal and close to unity. This is the case for 26 out of 30 equities of the DJIA index. Furthermore, Table 1 shows that $L(1) < L(0)$ in all cases considered, and this can be taken as strong evidence in favor of fractional cointegration between p_t^H and p_t^L, so that equation 8 is minimized in correspondence of $r = 1$. The results reported are relative to the case where $v(T) = m_L^{-0.35}$ and remain valid for alternative choices of $v(T)$ (not reported to save space). Note that, in the bivariate case, equation 8 is minimized if $v(T) > \min(\delta_i)$, where $\min(\delta_i)$ is the smallest eigenvalue of $\hat{G}$ (or alternatively of the estimated correlation matrix $\hat{P} = \hat{D}^{-1/2} \hat{G} \hat{D}^{-1/2}$). Provided that the correlation between p^H and p^L is approximately 1 in all cases, then $\hat{P}$ is almost singular and the smallest eigenvalue of $\hat{P}$ is very close to 0. Therefore $v(T) \approx v(T) - \min(\delta_i) > 0$ and this explains why $L(1)$ has a similar value across all the assets considered. As a preliminary data analysis, we also carry out the univariate Lagrange Multiplier (LM) test of Breitung and Hassler (2002) to verify the null hypothesis of unit roots against fractional alternatives. Unfortunately, we cannot use the extension of Nielsen (2005), since the multivariate LM test of fractional integration order, which is based on the type II fractionally integrated process, always rejects the null of $d = 1$, when series are fractionally cointegrated. Therefore, it is impossible to know if the null has been rejected due to a cointegration relation or because one of the variables is $I(1)$. Table 1 reports the $p-$values of the univariate LM tests of Breitung and Hassler (2002) for p_t^L and p_t^H. In all cases the null hypothesis cannot be rejected at the 5% significance level, meaning that the high and low log-prices can be considered unit root processes, so that $[\Delta p_t^H, \Delta p_t^H] \sim I(0)$. As a further check, provided that the daily range is an estimate of the daily integrated volatility (Parkinson (1980)), and given several empirical studies showing that the daily realized range has long-memory, we consider the evaluation of the fractional parameter of the daily range series. Note that we refer to the range, $R_t = p_t^H - p_t^L$, as the rescaled root square of the Parkinson estimator, that is $RG_t^2 = 0.361 \cdot (p_t^H - p_t^L)^2$. The Breitung and Hassler (2002) test on R_t confirms the results of the Nielsen and Shimotsu (2007) procedure, since the linear combination $p_t^H - p_t^L$ significantly reduces the integration order in almost all cases, and the local exact Whittle estimates of d^R are approximately $0.6 - 0.7$ in all cases, that is in the non-stationary region. Furthermore, the LM test for R_t rejects the null of unit root in 28 out of 30 cases at 10% significance level. As expected, the range has a fractional integration order that is significantly below 1, and in the next section we will show how to incorporate this evidence in a parametric model which exploits the long-run relationship between daily high and low prices. These results are in line with the well-known stylized fact that volatility of financial returns is characterized by long-range dependence, or long memory, see, for instance, Baillie (1996), Bollerslev and Mikkelsen (1996), Dacorogna, Muller, Nagler, Olsen, and Pictet (1993), Ding, Granger, and Engle (1993), Granger and Ding (1996), Andersen, Bollerslev, Diebold, and Ebens

(2001). In a recent contribution, Rossi and Santucci de Magistris (2009) find evidence of long memory and fractional cointegration between the daily ranges of the spot and futures prices of the S&P 500 index.

3   Modeling Daily High and Low Prices

As we showed in the previous section, the high and low prices have an embedded convergent path in the long run, so that they are (fractionally) cointegrated. For intuitive reasons[10], we expect that high and low prices can deviate in the short run from their long-run relation. The concept of cointegration has been widely studied during the last three decades, since the original paper by Granger (1981). Most of the analysis has concentrated on the special case where a linear (or nonlinear) combination of two or more $I(1)$ variables is $I(0)$. Tests for $I(1)/I(0)$ cointegration are carried out in a regression setup, as proposed by Engle and Granger (1987), or investigate the rank of the cointegration matrix in a system of equations following the Johansen (1991) procedure. A similar approach has been followed by Cheung (2007). When a cointegration relation exists across two variables, the bivariate system can be re-written in a Vector Error Correction (VEC) form, where the first difference of the cointegrated variables is a function of the cointegrating relation. Such a model has an interesting economic interpretation since deviations from the long-run equilibrium (given by the cointegrating relation) give rise to an adjustment process that influences the future values of the modeled variables. In our framework, changes in the actual high and low prices that make them inconsistent with the long-run relation will produce an effect on future high and low prices. However, the long memory must also be taken into account, differently from what has been considered in Cheung (2007). In a recent contribution, Johansen (2008) proposes a generalization of the VEC model to the fractional case. Such an extension allows for a representation (through a so-called Granger representation theorem) which, in turn, enables us to distinguish between cofractional relations and common trends. Johansen (2008) suggests studying the fractional cointegration relation in the following system representation

$$\Delta^d X_t = (1 - \Delta^b)(\Delta^{d-b}\alpha\beta' X_t - \mu) + \sum_{j=1}^{K} \Gamma_j \Delta^d L_b^j X_t + \epsilon_t \tag{10}$$

which is based on the new *generalized* lag operator $L_b = 1 - (1 - L)^b$. In our set-up, the vector X_t includes the daily high and low log-prices, $X_t = (p_t^H, p_t^L)'$, and $\epsilon_t = (\epsilon_t^H, \epsilon_t^L)'$ is assumed to be i.i.d. with finite eight moments, mean zero and variance Ω. Furthermore, $\alpha = (\alpha_H, \alpha_L)$ is the vector of the loadings, while $\beta = (1, \gamma)$ is the cointegration vector. The first term on the right can be represented as $\alpha(1 - \Delta^b)\Delta^{d-b}(p_t^H + \gamma p_t^L - \mu)$ where the core component $(1 - \Delta^b)\Delta^{d-b}(p_t^H + \gamma p_t^L - \mu)$ defines the cointegration relation. Differently, the element α_i in the vector α measures the single period response of variable i to the shock on the equilibrium relation. In the following, we restrict the cointegration parameter γ to be equal to -1, so that the cointegration relation can be interpreted in terms of the high-low range. We note that in preliminary estimates where this restriction was not imposed, the estimated cointegration parameter was generally very close to one, and we were not rejecting the null of $\gamma = 1$. Moreover, imposing the condition $d = 1$ in model (10) implies that, following the definition of Hualde and Robinson (2010), in the case of *strong* fractional cointegration, i.e. $b > 1/2$, the range would be stationary and integrated of order $d - b < 0.5$. On the other hand, the case of *weak* fractional cointegration, where $0 < b < 1/2$ see Hualde and Robinson (2010), leads to a non-stationary fractionally integrated range. We also introduce a constant term, μ, in the cointegration relation, which represents the mean value of the range; μ must be positive, provided

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[10]High and low prices can deviate temporarily because of information motives, liquidity factors or other microstructure effects such as bid-ask spread bounces, price discreteness, trading pressure.

that $p_t^H \geq p_t^L$ by definition. Assuming that the rank of the matrix $\alpha\beta$, the cointegration rank r, is known already, model (10) is estimated following the procedure outlined in Lasak (2008) and Johansen and Nielsen (2010a). In particular, model (10) is estimated via a maximum likelihood technique analogous to that developed by Johansen (1991) for the standard VEC model[11]. The asymptotic distribution of the FVECM model estimators is studied in Lasak (2008) and Johansen and Nielsen (2010b,a), while this estimation procedure has been employed by Rossi and Santucci de Magistris (2009), which show the finite sample properties of the estimators through a Monte Carlo simulation. In a recent contribution, Johansen and Nielsen (2011) show that the asymptotic distribution of all the FVECM model parameters is Gaussian when $b < 1/2$. On the other hand, when $b > 1/2$, the asymptotic distribution of β is mixed Gaussian, while the estimators of the remaining parameters are Gaussian. On the basis of the results in the previous section, it seems natural to estimate model (10) on the log high and low prices for the 30 stocks under analysis. One lag in the generalized autoregressive term is assumed to be sufficient in all cases for a good description of the short term component. Table 2 reports the estimation results of model (10) on the 30 series. The estimated parameter b is lower than 0.5 in 27 cases, meaning that, as expected, daily range is generally non-stationary. In particular, the estimated degree of long range dependence of daily range implied by the FVECM model is between 0.4611 and 0.7467, meaning that the persistence of the range is very high, while the null hypothesis $d = 1$ cannot be rejected in most cases. It is noteworthy to stress that the estimated degree of long memory of the range which is implied by the FVECM model is very close to the values obtained with the semiparametric estimators, in Section 2. Moreover, α_H and α_L are significant and with the expected signs, so that it is possible to conclude that high and low prices are tied together by a common long-memory stochastic trend toward which they converge in the long run at significant rates.

Interestingly, the deviations from the attractor (the long-run relation) have an economic interpretation in terms of volatility proxy, that is the range. Therefore, a parametric model for the high-low prices incorporates significant information on the degree of dispersion of the prices. Hence, we are able to exploit the long-memory feature of the price dispersion in order to obtain better forecasts of future high and low log-prices based on past values. Figure 1 reports the high and low log-prices series of IBM and the common stochastic trend constructed from the estimates of model 10 and the Granger representation theorem for fractional VAR in Johansen (2008). It is also clear that the common stochastic trend closely follows the dynamics of the high and low log-prices, provided that they are tied together by a strong cointegration relation. The deviations from the long run relationship, which is the high-low distance or *range*, are also plotted. The range is a highly persistent series characterized by long periods above or below the unconditional mean. For example, the recent financial crisis in 2008-2009 is characterized by a larger discrepancy between the high and low prices, that is the result of a higher market risk. In the next section, we will show a possible trading strategy that can benefit from better forecasts of daily high-low bands.

4   Forecasting and Trading

Given the estimates of model (10), a natural exercise would be based on the forecasts of the high and low prices, using them to implement a trading strategy based on the predicted so-called high-low bands. Our focus is purely illustrative, and we provide a reasonable application of our modeling approach, thereby extending the study of Cheung (2007) in that direction. In doing so, we consider an expanding estimation period, which is used to evaluate the model and produce the one-step-ahead forecasts. We base our analysis on a subset of 20 assets of the Dow Jones index, for which we dispose of high frequency prices (i.e. one-minute frequency) for a total of 390 intradaily prices per day. The sample covers the period from January 2, 2003 to

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[11]The VEC is obtained as a special case of the FVECM when d and b are fixed to be equal to 1

December 31, 2009, for a total of $T = 1762$ observations. We provide out-of-sample forecasts for the last 251 sample data points, the year 2009. Therefore, the estimation sample length includes a minimum of $S = 1511$ observations and a maximum of $S = T - 1$ data points. At each step, model (10) is estimated on the first S observations, and a one-step-ahead forecast of the high and low prices is obtained; we call these forecasts FC bands. Note that, in order to guarantee that the predicted high-low bands are robust to the overnight activity, the overnight return $\log O_t - \log C_{t-1}$ is included in model (10) as an additional explanatory variable (O_t and C_{t-1} are the opening and closing prices, respectively). The introduction of the overnight return log mimics the regular operating activity of a trader who, using the model, produces the day t forecasts for high and low, including the opening price of time t in the information set. Given the bands, the trader uses them within day t regular trading hours. Figure 2 plots the Japanese candlesticks, based on the observed prices of JPM, with the corresponding predicted high-low bands. It is clear that the FC bands provide a better out-of-sample fit of the high-low dispersion with respect to the high-low bands based on a 5-day moving average of high and low, MA_5, which represents a tool often used by technical analysts. Table 3 reports the p-values of the Diebold and Mariano (1995) test for the out-of -ample forecasts of the high and low log-prices, obtained using the $FVECM$. As competing models we chose the random walk, RW, the 5-day moving average, MA_5, and the 22-day moving average, MA_{22}, which correspond to weekly and monthly averages respectively.[12] We correct the predicted high-low bands of RW, MA_5 and MA_{22} by adding the difference between the opening price of day t, O_t, and the closing price of day $t - 1$, C_{t-1}. Such a choice realigns those competing forecasts with the information set used in the $FVECM$ model where the opening price of time t is taken into account. The Diebold and Mariano (1995) test for the forecasting superiority of $FVECM$ is carried out focusing on the mean squared error (MSE) and on the mean absolute error (MAE) of the forecasts, where the error of model i at date t is defined as the difference, $\epsilon_{i,t}$, between the observed high in the period $t + 1$[13], and the corresponding forecast provided by model i:

$$\epsilon_{t+1,i}^H = p_{t+1}^H - \hat{p}_{t+1,i}^H \quad i = FVECM, RW, MA_5, MA_{22} \tag{11}$$

Specifically, the interest is on the measure of the relative forecasting performance of the different model specifications, testing the superiority of model i over FVECM, which is the benchmark, with a t-test for the null hypothesis of a zero mean in

$$
\begin{aligned}
\epsilon_{t+1,FVECM}^2 - \epsilon_{t+1,i}^2 &= c_i + \nu_t \\
|\epsilon_{t+1,FVECM}| - |\epsilon_{t+1,i}| &= d_i + \xi_t
\end{aligned}
\tag{12}
$$

where negative estimates of c_i and d_i indicate support for the $FVECM$, provided that all models are compared to the $FVECM$. The Diebold and Mariano (1995) test is the t-statistics for the nullity of the estimates of c_i and d_i. Overall, Table 3, depicts a situation where the forecast errors associated with the FVECM are significantly smaller than those of the competing models. As expected, the p-value of the test for MA_5 and MA_{22} is highly significant in all cases. This is due to the fact that moving averages are used in practice to identify local trends in the prices and not for a point estimate of the prices. On the other hand, for what concerns the RW model, the t-test based on MSE is always negative and the p-value is often lower than 5% for both the p_t^H and p_t^L. This result suggests that the fractional cointegration model significantly improves the forecasts of high and low log prices, with respect to the model that, consistently with the strong efficiency of the markets, assumes that the best forecast of future prices is the actual price.

We then implement two simple trading rules that make use of the model outcomes. At first, we consider a trading rule based on the predicted high-low bands. We define buy and sell

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[12] The alternative specifications all consider separately the high and low prices for the construction of high and low bands. By contrast, our model provides the high and low forecasts within a unified framework.

[13] Analogously for the low log-price

signals comparing the intradaily evolution of the equity price and the high-low bands. If the price crosses the high band, we interpret it as a buy signal. Conversely, if the price crosses the low band we have a sell signal, see Murphy (1999). This can be interpreted as a *trend-following* strategy, where the upper line is used as a *bull*-trendline in a uptrend, while the lower line is used as a *bear*-trendline in a downtrend. We also implement the *contrarian* strategy: if the price crosses the high band, we interpret it as a sell signal. Conversely, if the price crosses the low band we have a buy signal.

It is therefore possible to carry out a realistic trading strategy based on the high-low bands, as described above. In particular, the previous rules define the creation of long or short positions. Those positions created within day t can be closed or maintained in the following days. Once we enter a new position, two additional bands are created in order to guarantee a desired minimum level of profit or a maximum acceptable loss. These additional bands, called the *stop-loss* and *take-profit* bands, are proportional to the high-low distance and are created as follows. Suppose that we have a *trend-following* strategy, and the predicted upper bound, $\hat{P}_t^H$, is crossed by the price at a given time $t < \tau_0 < t + 1$. In this case, we open a long position and the *stop-loss* and *take-profit* thresholds are created as $SL_t = \hat{P}_t^H - \delta \hat{R}_t$ and $TP_t = \hat{P}_t^H + \delta \hat{R}_t$, respectively, where δ determines the size of the new bands and $\hat{R}_t = \hat{P}_t^H - \hat{P}_t^L$ is the predicted daily range. If the price crosses the SL_t or TP_t in $\tau_0 < \tau < t + 1$, then the long position is closed. On the other hand, if an open position is not closed during day t, then it is maintained until $t + 1$. By design, positions can also be closed at the opening price for two possible reasons: first, during the market closing period, relevant news might be released with a potentially elevating impact on prices; second, the FVECM model does not guarantee that the opening price at time $t + 1$ is inside the high/low bands of time $t + 1$. Suppose that at time t we have an opened long position in a trend following strategy. At the beginning of $t + 1$, the position is closed if $O_{t+1} < \max([\hat{P}_{t+1}^L; SL_t])$ or if $O_{t+1} > \min([\hat{P}_{t+1}^H; TP_t])$. Such a choice allows us to avoid too risky strategies which might be taken in case of large variations of the prices over two consecutive days. If a position is not closed during the market opening, then $\hat{P}_{t+1}^L$ and $\hat{P}_{t+1}^H$ are used as predicted H-L bands for $t + 1$. Similarly we can define stop-loss and take-profit bands for the short positions and for the *contrarian* strategy. The payoffs of this trading system, based on two couples of high-low bands, are reported in Table 4. In order to consider a realistic trading strategy, it is assumed that an initial amount, A_0, of 20,000 US dollars can be invested in the strategy. A fixed cost of 5 dollars is charged on each transaction. During day t, the investor can buy or sell an amount of stocks that is equal to $N_t = [A_t/P_{\tau_0}]$, where $[x]$ indicates the nearest integer less than or equal to x and τ_0 is the time in which the price crosses the bands.

It is immediately evident that the strategies are symmetric, so that when the *trend-following* strategy produces a positive payoff, then in most cases the corresponding payoff of the *contrarian* strategy is lower than 20000 dollars. We notice in general that, across the strategies considered, the payoff from the *contrarian* strategy is generally higher than the payoff obtained with *trend following*. This is probably due to the fact that, when the price exceeds a given threshold, there is a higher probability that it will revert, tending to move toward the initial value, rather than moving further toward more extreme values. The reverting behavior of the extremes has also been noted by Cheung (2007).

Relatively to the other bands employed, the FC bands have similar performances in terms of final outcome. The FC bands provide less frequently the worst performance with respect to the alternative choices. Moreover, when the contrarian strategy is employed with the FC bands, we observe 12 and 11 positive outcomes with $\delta = 0.25$ and $\delta = 0.5$. In particular, the strategy based on the FC bands has good relative performances when the *contrarian* strategy is applied with $\delta = 0.5$, which increases the probability of maintaining the position opened until $t + 1$. In that case, the FC bands have the best performances in 7 cases. Except for the case of BAC and JPM, the profits and losses associated with FC are rarely above 3000 dollars (or less than -3000), so we can conclude that in general it seems that the payoff variability associated with

FC is lower than the variability associated with the other strategies.

An alternative trading system could be based on a different construction of the bands. Now we focus on the predictions of the model (10) as a possible way of recovering a forecast of the range, $\hat{R}_t$. This is automatically obtained by the difference between the predicted high and low prices. Therefore a range-based strategy can be employed. For MA_5 and MA_{22}, these are the averages of the daily range with different horizons, while RW simply takes the range at time $t-1$ as the predicted range at time t. In this case, the daily H-L bands for day t, are centered on the open price, so that the upper band is equal to $O_t + \hat{R}_t$ and $O_t - \hat{R}_t$. Therefore, the size of the band is $2\hat{R}_t$. Once the bands have been created, the trend-following (or the contrarian) strategy could be implemented similarly to as described above, with the corresponding SL and TP bands which are constructed with $\delta = 0.5$. In this case, the position, eventually opened during day t, is not maintained until $t+1$ and must be closed before (or in correspondence of) the last trade of day t. Table 5 reports the results of a *contrarian* strategy based on the daily range bands. Table 5 reports the value of the investment at the end of the sample, say on December 31, 2009. In 12 out of 20 cases the strategy based on the range, obtained from model (10), reports positive profits, while the strategy based on buy and hold, fifth column, has positive profits in 11 out of 20 cases. With respect to the other strategies, the FC bands perform generally better than RW and MA_5, while having similar performances to MA_{22}. This is due to the fact that MA_{22} is able to well approximate the long-run dependence of the range series and, in this particular strategy, the difference in terms of profits is not relevant. This evidence can be well understood from Figure 3 which plots the forecasts of the range based on different model specifications. It is clear that RW provides a very noisy forecast of the range, while MA_{22} is extremely smooth and it is close to the forecast of the $FVECM$ that is based on the maximum likelihood estimates of b, which governs the persistence of the range. Relative to the other strategies, the strategy based on $FVECM$ has a lower number of days of trading, that is approximately 40, compared to RW and MA_5, which trade more frequently. Moreover, the variance associated with the daily payoff of the various strategies is the lowest for the $FVECM$ in 12 out of 20 cases. The variance of the *buy & hold* strategy, $B\&H$, is always greater than the variance of the strategies based on intradaily trading. Finally, the Sharpe ratio associated with the FC strategy often takes the highest value. This means that, when the returns of the strategies are adjusted by their riskiness, the FC strategy outperforms the others. This is probably due to the fact that the FVECM is able to provide good forecasts of the range, and therefore more precise timing for investments.

5   Conclusion

The main contribution of this paper is to show that high and low prices of equity shares are partially predictable and that future extreme prices can be forecast simply by using past high and low prices that are readily available. Apparently, their predictability is at odds with the difficulty of forecasting asset returns when prices are taken at a fixed point in time (at closing, say) and with a long tradition of empirical work (e.g. Fama (1970 and 1991)) supporting the efficient market hypothesis.

We propose to model high and low prices using a simple implementation of a fractional vector autoregressive model with error correction (FVECM). This modeling strategy is consistent with two main stylized patterns documented in this paper: first, high and low prices are cointegrated, i.e. they may temporarily diverge but they have an embedded convergent path in the long run. Second, the difference between them, i.e. the range, displays a long persistence that can be well captured by the fractional autoregressive technique.

The analysis of some simple trading strategies shows that the use of predicted high and low prices as entry and exit signals improves the investment performance. More specifically, we find that out-of-sample trading strategies based on FVECM model forecasts outperform any

reasonable benchmark model, even on a risk-adjusted basis.

Future research might extend our work in many respects. An interesting question to address is how good predictions of high and low prices can enhance risk analysis and management.

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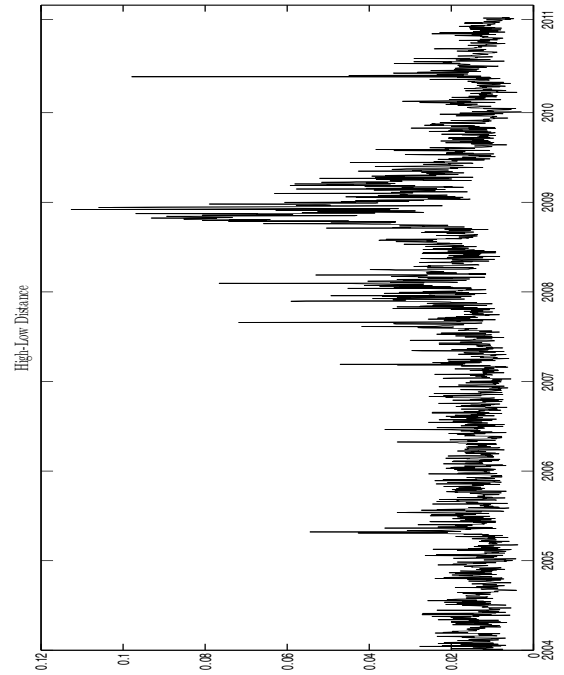
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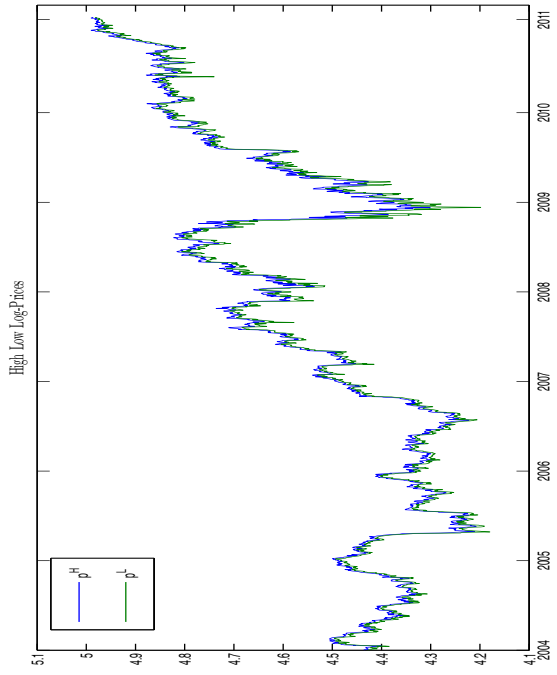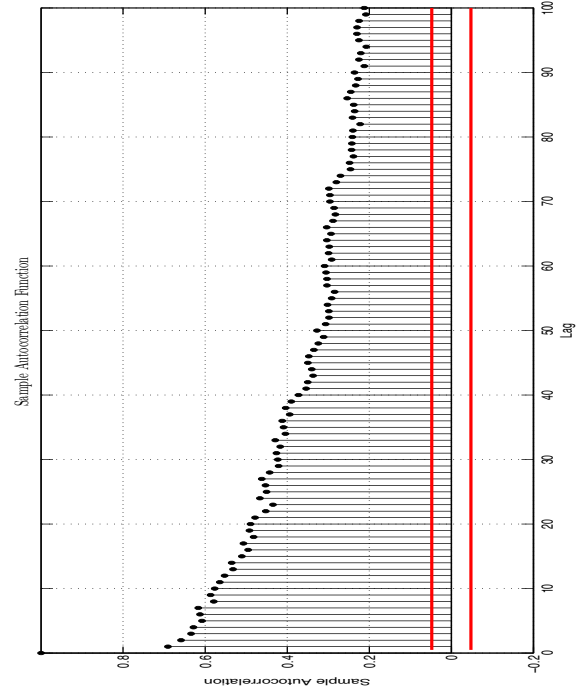
(a) High and Low Log-Prices of IBM
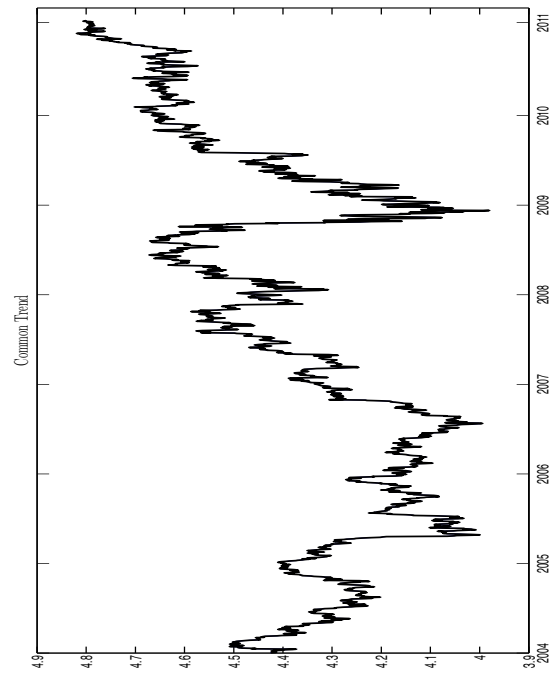
(b) Common Trend of IBM

(c) High and Low Distance of IBM

(d) Autocorrelation Function of the range of IBM.

Figure 1: High and Low log prices, Common trend, Range and its autocorrelation function

	d^H	d^L	$\hat{T}_0$	d^R	$L(0)$	$L(1)$	ADF^H	ADF^L	ADF^R	BH^H	BH^L	BH^R	BH^{HH}	BH^{LL}
AA	1.0848	1.0654	0.5331	0.5637	-1.2991	-1.6468	0.6349	0.5552	0.0000	0.4041	0.3038	0.0221	0.4077	0.3064
AXP	1.1273	0.9824	0.3747	0.6218	-1.2991	-1.6421	0.3249	0.3055	0.0000	0.2751	0.2512	0.0244	0.2809	0.2432
BA	1.0016	0.9733	0.5297	0.6330	-1.2991	-1.6463	0.4173	0.4191	0.0000	0.4009	0.6812	0.1268	0.3366	0.6325
BAC	1.1124	0.9596	0.0882	0.6770	-1.2991	-1.6444	0.6339	0.5353	0.0000	0.2177	0.4334	0.0611	0.2100	0.4161
CAT	1.0309	1.0827	0.1955	0.6235	-1.2991	-1.6453	0.4374	0.4546	0.0000	0.2588	0.1719	0.0545	0.3441	0.4593
CSCO	1.0137	0.9823	0.1120	0.5523	-1.2991	-1.6443	0.7602	0.7546	0.0000	0.1970	0.1625	0.0105	0.2695	0.7133
CVX	0.7924	0.8389	7.7693	0.5582	-1.2991	-1.6412	0.5075	0.4934	0.0000	0.2902	0.5934	0.0107	0.26951	0.71334
DD	1.0618	1.0307	0.0013	0.5807	-1.2991	-1.6460	0.4586	0.3933	0.0000	0.4644	0.4325	0.0400	0.4885	0.4236
DIS	0.9645	0.9619	0.3197	0.6862	-1.2991	-1.6444	0.2269	0.2252	0.0000	0.7260	0.3328	0.0532	0.5480	0.1356
GE	1.0880	0.9651	0.4989	0.6104	-1.2991	-1.6439	0.5884	0.5573	0.0000	0.3391	0.3984	0.0195	0.3375	0.3933
HD	0.9284	1.0163	0.2995	0.6913	-1.2991	-1.6441	0.8446	0.8474	0.0000	0.3364	0.1888	0.0169	0.1114	0.3143
HPQ	0.9452	0.9877	0.0243	0.5963	-1.2991	-1.6440	0.5117	0.4852	0.0000	0.2433	0.2144	0.0324	0.2017	0.0732
IBM	1.0808	1.0018	2.6934	0.6623	-1.2991	-1.6440	0.8250	0.7689	0.0000	0.7496	0.3154	0.0150	0.5617	0.3417
INTC	1.0784	1.0290	0.2405	0.6488	-1.2991	-1.6460	0.7680	0.7687	0.0000	0.6995	0.7717	0.0334	0.6168	0.6883
JNJ	0.9426	0.9450	0.3649	0.5684	-1.2991	-1.6406	0.3885	0.3074	0.0000	0.7268	0.4298	0.0402	0.8001	0.4969
JPM	1.0021	0.8836	4.0912	0.5837	-1.2991	-1.6374	0.0226	0.0068	0.0000	0.1176	0.2276	0.0402	0.1117	0.2051
KFT	0.8719	0.9699	0.3984	0.5472	-1.2991	-1.6437	0.6771	0.6770	0.0000	0.1382	0.7410	0.0158	0.3522	0.9141
KO	1.0685	1.0098	0.0809	0.6524	-1.2991	-1.6422	0.9368	0.9242	0.0000	0.3775	0.1756	0.0177	0.3055	0.1499
MCD	0.9238	0.9946	0.1789	0.5451	-1.2991	-1.6435	0.5975	0.5990	0.0000	0.3930	0.1961	0.0108	0.1248	0.6076
MMM	0.8719	0.9699	0.3984	0.5472	-1.2991	-1.6437	0.2188	0.1782	0.0000	0.2930	0.1466	0.0143	0.2640	0.1362
MRK	1.0639	0.9716	1.2338	0.5397	-1.2991	-1.6428	0.2260	0.1866	0.0000	0.5058	0.9536	0.0164	0.4936	0.2856
MSFT	1.0163	0.9729	1.5089	0.5581	-1.2991	-1.6432	0.1189	0.1121	0.0000	0.1844	0.2010	0.0436	0.1800	0.1962
PFE	0.9626	0.9120	0.0034	0.5221	-1.2991	-1.6448	0.3415	0.3029	0.0000	0.2156	0.1942	0.0141	0.0961	0.1937
PG	0.9439	1.0036	5.1116	0.4385	-1.2991	-1.6363	0.3527	0.2142	0.0000	0.2087	0.2184	0.0057	0.7931	0.1609
T	1.0061	0.9731	4.4517	0.6318	-1.2991	-1.6390	0.5508	0.4881	0.0000	0.3233	0.3027	0.0128	0.2979	0.2811
TRV	0.8134	0.8893	0.1183	0.7296	-1.2991	-1.6233	0.3598	0.3376	0.0000	0.2446	0.1160	0.1137	0.2072	0.1035
UTX	0.9708	0.9769	0.2015	0.6304	-1.2991	-1.6433	0.3999	0.3807	0.0000	0.5784	0.4849	0.0477	0.5443	0.2838
VZ	0.9093	0.8937	0.3750	0.6228	-1.2991	-1.6430	0.2418	0.1657	0.0000	0.7232	0.1651	0.0118	0.3188	0.1189
WMT	0.8327	0.8767	1.2432	0.5688	-1.2991	-1.6401	0.7634	0.7711	0.0000	0.0895	0.0684	0.0197	0.0888	0.0676
XOM	0.8810	0.8968	0.2200	0.5532	-1.2991	-1.6353	0.3639	0.3612	0.0000	0.5522	0.6749	0.0473	0.5922	0.4737

Table 1: Long memory and fractional cointegration estimation. Table reports the long memory estimates of d of p^H and p^L based on the exact local Whittle estimator of Phillips and Shimotsu (2005). The T_0 is the test statistic for the equality of the integration order as in Nielsen and Shimostu(2007). $L(0)$ and $L(1)$ is the value of loss function of Nielsen and Shimostu(2007), evaluated in correspondence of a cointegration rank equal to 0 and 1, respectively. ADF^H, ADF^L and ADF^R are the p-values of the augmented Dickey-Fuller test of unit root for p^H, p^L and p^R. Table also reports the p-value of the Breitung and Hassler (2002) test, BH, for unit root against fractional alternatives, for p_t^H, p_t^L and R_t. The last two columns reports the Breitung and Hassler (2002) test for the null hypothesis that Δp_t^H and Δp_t^L are I(0), against fractional alternatives.
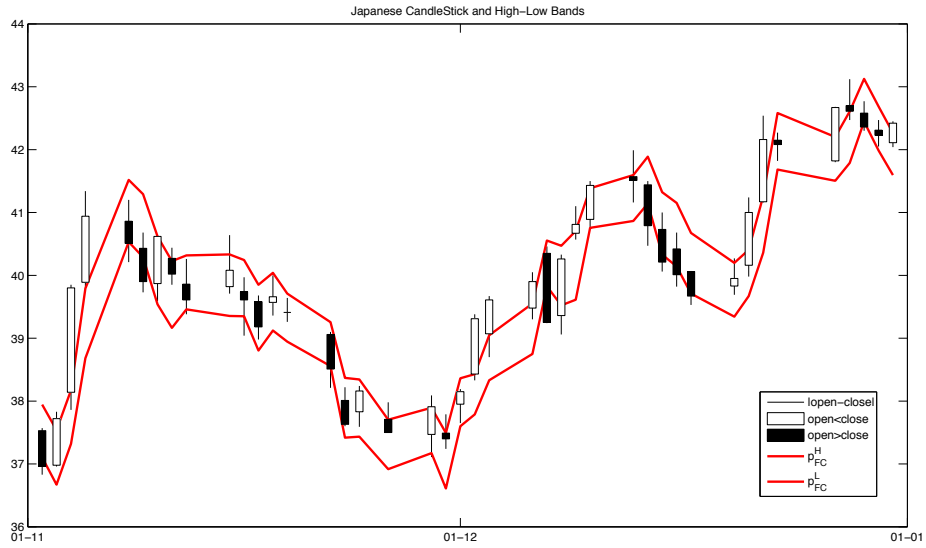
(a) FC bands
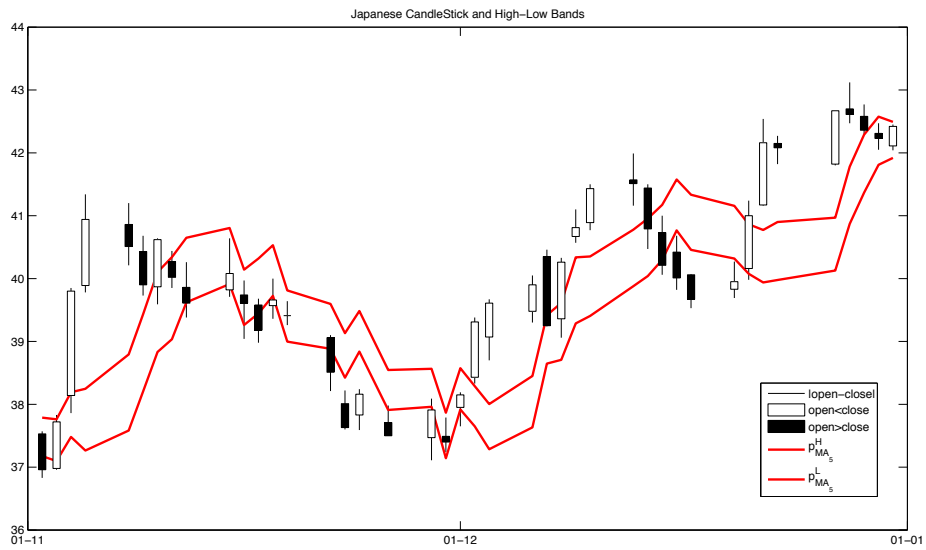
(b) MA$_5$ bands
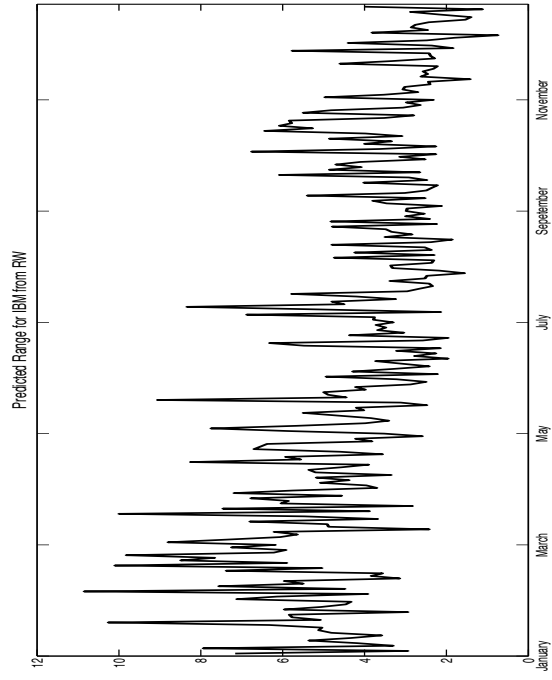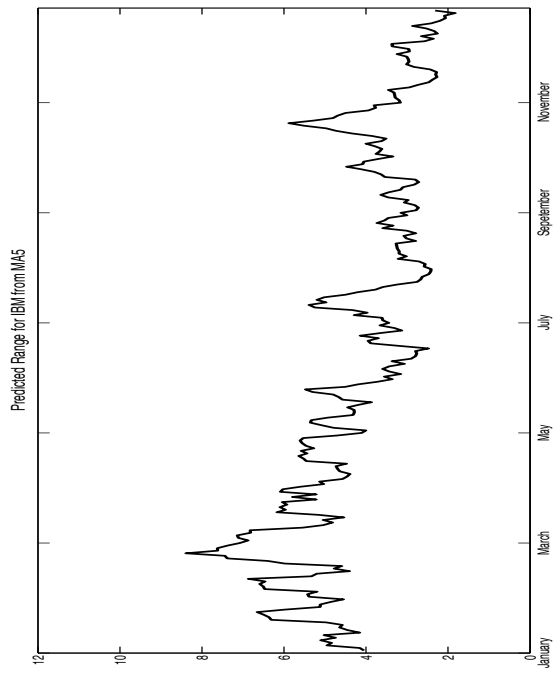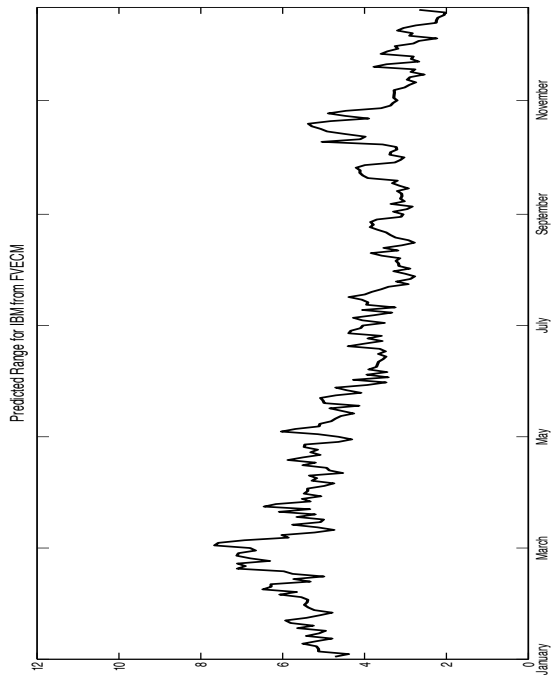
Figure 2: Japanese CandleStick with predicted high-low bands. Panel a), the red lines are the high-low band based on model (10). Panel b) the red lines are the high-low bands based on the 5 days moving average. The sample covers the period from November 1, 2010 to December 31, 2010.
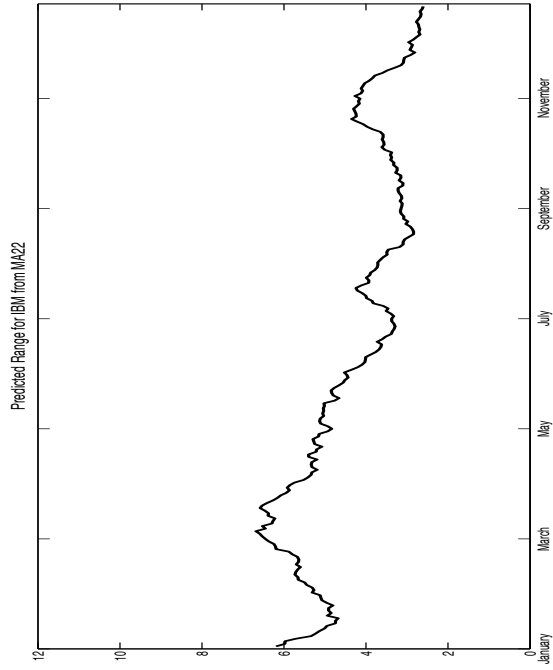
(a) FVECM

(b) RW

(c) MA$_5$

(d) MA$_{22}$

Figure 3: Forecasts of daily range based on different models.

	b	α_H	α_L	μ	γ_{11}	γ_{12}	γ_{21}	γ_{22}	R^2_H	R^2_L	$d=1$
AA	**0.3802**	**−0.7876**	**1.9877**	0.0291	0.1187	0.8603	**0.3370**	1.3752	0.0782	0.0590	0.0795
AXP	**0.3502**	**−0.7125**	**1.8281**	0.0393	0.2689	0.1149	−0.0569	**0.4921**	0.0298	0.1560	0.5162
BA	**0.3375**	**−1.0255**	**2.4771**	**0.0306**	0.2686	−0.8925	0.0422	1.4337	0.0577	0.0873	0.3629
BAC	**0.3844**	**0.2369**	**2.2214**	0.0372	−0.2898	−0.5099	**0.6056**	**1.1072**	0.0330	0.1097	0.0852
CAT	**0.2697**	**−2.3315**	**2.4985**	**0.0467**	1.2326	−0.5665	−0.9508	1.1358	0.0547	0.0777	0.1407
$CSCO$	**0.4275**	**−0.6787**	**1.4959**	**0.0229**	**0.1058**	−0.3997	0.2773	0.7573	0.0612	0.0662	0.0905
CVX	**0.3575**	**−0.5380**	**1.9635**	**0.0469**	0.0715	−0.4240	0.1927	**0.9602**	0.0382	0.0829	0.8450
DD	**0.3777**	**−0.8042**	**1.8971**	**0.0268**	−0.0255	−0.6022	0.2900	**1.1052**	0.0832	0.0772	0.2250
DIS	**0.3063**	**−1.5021**	**2.1362**	0.0354	1.0623	−0.1047	−0.8818	0.7130	0.0212	0.1153	0.0058
GE	**0.3147**	**−0.8360**	**2.2335**	0.0266	0.0995	−0.6979	0.2625	1.3624	0.0721	0.0842	0.0516
HD	**0.2948**	**−1.1819**	**2.9520**	0.0281	0.7487	−0.6773	−0.4761	03550	0.0242	0.1580	0.0682
HPQ	**0.5045**	**−0.5667**	**1.1586**	**0.0245**	0.0301	−0.0405	0.1366	**0.4246**	0.0579	0.0899	0.2885
IBM	**0.2716**	**−1.0091**	**3.6272**	**0.0375**	0.2703	−1.2317	0.0306	**2.0732**	0.0420	0.1185	0.2019
$INTC$	0.3111	−0.7790	**3.3368**	0.0199	−0.0398	−1.5638	0.3408	2.0540	0.0498	0.0838	0.0193
JNJ	**0.3511**	−0.4001	**2.1323**	0.0187	−0.1219	−0.5973	0.2461	1.0202	0.0265	0.0945	0.9027
JPM	**0.4055**	**0.6632**	**2.3961**	**0.0365**	**−0.9959**	**−1.0623**	**1.1665**	**1.6600**	0.0404	0.1285	0.3737
KFT	**0.3837**	**−1.1606**	**2.1448**	**0.0190**	**0.6585**	−0.5196	−0.4301	**1.0009**	0.0433	0.1368	0.8407
KO	**0.4179**	**−0.7837**	**1.2345**	**0.0187**	0.1014	0.0076	0.0849	**0.4602**	0.0657	0.0921	0.4340
MCD	**0.2533**	**−1.5769**	**4.2245**	**0.0238**	0.5656	−1.9158	−0.4243	**2.3400**	0.0404	0.1090	0.7580
MMM	**0.3837**	**−1.1606**	**2.1448**	**0.0190**	0.6585	−0.5196	**−0.4301**	**1.0009**	0.0433	0.1368	0.1370
MRK	**0.3772**	**−0.9899**	**1.9819**	0.0236	0.2362	−0.6770	−0.0323	**1.0499**	0.0708	0.0784	0.2805
$MSFT$	**0.2948**	**−2.3810**	**1.9808**	0.0230	1.3380	−0.3381	−1.2334	0.7385	0.0464	0.0817	0.2973
PFE	**0.5044**	**−0.6663**	**1.1935**	**0.0197**	0.1622	−0.2072	0.0241	**0.4410**	0.0726	0.0796	0.0810
PG	**0.5389**	**−0.1512**	**1.8463**	**0.0179**	−0.0526	−0.3057	**0.0960**	**0.4942**	0.0431	0.3229	0.9057
T	0.2686	−1.4082	3.4427	0.0420	0.1897	−1.8229	−0.0263	2.2960	0.0738	0.0783	0.3577
TRV	**0.2631**	−1.7507	1.7828	0.0475	0.6894	−0.0560	−0.5969	0.4932	0.0567	0.0876	0.6518
UTX	**0.3402**	**−0.6000**	**2.3287**	**0.0318**	0.1768	−0.2775	0.0241	0.8860	0.0257	0.1457	0.1235
VZ	**0.3611**	−0.3482	**2.4522**	0.0210	−0.0092	−0.6247	0.2592	1.1408	0.0324	0.1306	0.6763
WMT	**0.4395**	**−0.7569**	**1.1748**	**0.0167**	**0.4527**	0.2228	−0.2677	0.1232	0.0296	0.1328	0.9219
XOM	**0.4183**	−0.0364	**1.6712**	**0.0309**	**−0.6596**	−0.6936	**0.6496**	**0.9401**	0.0471	0.0576	0.3069

Table 2: FVECM estimation. Bold means significance at 5%.

	p_t^H						p_t^L					
	MSE			MAE			MES			MAE		
	RW	MA_5	MA_{22}	RW	MA_5	MA_{22}	RW	MA_5	MA_{22}	RW	MA_5	MA_{22}
AXP	0.0089	0.0000	0.0000	0.0076	0.0000	0.0000	0.0042	0.0000	0.0000	0.1359	0.0000	0.0000
BA	0.0111	0.0000	0.0000	0.3395	0.0000	0.0000	0.0021	0.0000	0.0000	0.0442	0.0000	0.0000
BAC	0.3427	0.0000	0.0000	0.7831	0.0000	0.0000	0.6205	0.0000	0.0000	0.6517	0.0000	0.0000
CAT	0.0032	0.0000	0.0000	0.0442	0.0000	0.0000	0.0022	0.0000	0.0000	0.0015	0.0000	0.0000
CVX	0.1224	0.0000	0.0000	0.9611	0.0000	0.0000	0.0012	0.0000	0.0000	0.0077	0.0000	0.0000
DD	0.0041	0.0000	0.0000	0.0858	0.0000	0.0000	0.0025	0.0000	0.0000	0.0456	0.0000	0.0000
GE	0.0695	0.0000	0.0000	0.6094	0.0000	0.0000	0.7163	0.0000	0.0000	0.4283	0.0000	0.0000
HD	0.2005	0.0000	0.0000	0.7307	0.0000	0.0000	0.1004	0.0000	0.0000	0.4059	0.0000	0.0000
HPQ	0.0009	0.0000	0.0000	0.0028	0.0000	0.0000	0.0006	0.0000	0.0000	0.0305	0.0000	0.0000
IBM	0.0146	0.0000	0.0000	0.0042	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
JNJ	0.0051	0.0000	0.0000	0.0031	0.0000	0.0000	0.0003	0.0000	0.0000	0.0045	0.0000	0.0000
JPM	0.2140	0.0000	0.0000	0.1722	0.0000	0.0000	0.0372	0.0000	0.0000	0.1714	0.0000	0.0000
KFT	0.0198	0.0000	0.0000	0.2294	0.0000	0.0000	0.0095	0.0000	0.0000	0.1471	0.0000	0.0000
MCD	0.4937	0.0000	0.0000	0.5012	0.0000	0.0000	0.0455	0.0000	0.0000	0.1225	0.0000	0.0000
PFE	0.0141	0.0000	0.0000	0.0102	0.0000	0.0000	0.0149	0.0000	0.0000	0.0577	0.0000	0.0000
PG	0.3179	0.0000	0.0000	0.5565	0.0000	0.0000	0.3134	0.0000	0.0000	0.7674	0.0000	0.0000
T	0.0017	0.0000	0.0000	0.0005	0.0000	0.0000	0.0160	0.0000	0.0000	0.2384	0.0000	0.0000
VZ	0.1094	0.0000	0.0000	0.0822	0.0000	0.0000	0.2841	0.0000	0.0000	0.8604	0.0000	0.0000
WMT	0.3155	0.0000	0.0000	0.3800	0.0000	0.0000	0.0400	0.0000	0.0000	0.0624	0.0000	0.0000
XOM	0.1278	0.0000	0.0000	0.4371	0.0000	0.0000	0.0033	0.0000	0.0000	0.0043	0.0000	0.0000

Table 3: Diebold Mariano test for out-of-sample forecast: table reports the *p*-value of the Diebold-Mariano test based on the mean-squared error, MES, and on the mean-absolute error, MAE. The forecast period includes the last year of transactions.

Strategy Perfomances

| | Trend-Following | | | | | | | | Contrarian | | | | | | | |
| | $\delta = 0.25$ | | | | $\delta = 0.5$ | | | | $\delta = 0.25$ | | | | $\delta = 0.5$ | | | |
	FC	RW	MA_5	MA_{22}	FC	RW	MA_5	MA_{22}	FC	RW	MA_5	MA_{22}	FC	RW	MA_5	MA_{22}
AXP	25131	16588	17674	18503	17923	12414	15299	21831	15248	22947	21737	21248	19080	27421	22381	17224
BA	21226	19832	20419	19640	24645	18318	19458	19030	18534	19807	19378	20148	15247	20610	19639	20535
BAC	14049	13672	10429	24399	44110	15439	7462	31268	25074	25887	34157	15891	5880	17548	40545	11860
CAT	22955	20843	21467	18215	15541	23830	28227	15391	16840	18540	18273	21752	23311	15337	13106	25040
CVX	19395	18710	21198	18612	15705	19643	24690	18091	20387	21052	18726	21374	24470	19497	15650	21559
DD	17100	18106	20874	20320	18250	18158	25736	23411	22736	21592	18920	19543	20736	20664	14794	16589
GE	18010	21738	17975	20522	20217	23109	15309	22454	21244	17758	21691	19350	17639	15427	23566	17416
HD	14890	15643	19447	18480	21882	18362	25504	21302	26321	25120	20304	21465	17417	20479	14939	18255
HPQ	21511	19293	20660	18471	21941	19839	21858	16823	18348	20416	19137	21487	17535	19333	17711	23278
IBM	21316	18705	18596	19086	21316	18705	18596	19086	18597	21214	21372	20849	19901	22475	22487	19146
JNJ	17094	17339	18506	19346	19378	17857	20782	18470	23268	22905	21479	20577	20260	21938	18797	21278
JPM	19921	21708	21107	21474	10448	10405	23608	22750	19067	17633	18454	18210	31943	31478	14993	16698
KFT	21035	21168	18663	19253	14538	19193	16864	15867	18820	18697	21242	20712	26583	20303	23136	24705
MCD	19460	18583	20061	18597	15650	18297	14424	19379	20422	21357	19828	21410	24973	21317	26891	20206
PFE	19563	19134	20492	19309	22410	17043	21907	18161	20175	20560	19346	20569	17227	22676	17576	21549
PG	19809	18606	19826	20003	21055	21417	20475	23351	20043	21319	20037	19901	18560	18259	19064	16933
T	19857	18122	17307	19198	19029	15849	14312	19530	19842	21769	22860	20750	20214	24236	26879	20055
VZ	19806	19125	19277	19521	18869	17834	18982	21297	19998	20674	20501	20407	20583	21664	20425	21335
WMT	20832	21853	18713	18719	21603	20154	19523	18363	19038	18173	21207	21239	18074	19414	19891	18454
XOM	19373	18018	20499	18900	16942	17960	17787	15145	20405	21955	19407	21062	22877	21490	21642	25861

Table 4: Stop-loss and Contarian Strategy performances based on different H-L bands. *FC* refers to the strategy based on the predicted high-low bands from model (10). *RW* refers to the strategy based on the predicted high-low bands from a random walk model. MA_5 and MA_{22} refers to the strategy based on the predicted high-low bands from a moving average model of the last 5 and 22 days, respectively. The initial monetary amount is 20000 dollars.

	Payoff					Number of Transactions				Payoff Variance					Sharpe Ratio				
	FC	RW	MA_5	MA_{22}	B&H	FC	RW	MA_5	MA_{22}	FC	RW	MA_5	MA_{22}	B&H	FC	RW	MA_5	MA_{22}	B&H
AXP	17780	15819	19214	19331	42408	41	61	53	46	2.3969	2.1412	1.9014	2.6758	104.12	–	–	–	–	8.2698
BA	23240	21797	22891	23499	23776	48	66	52	48	1.1327	1.2431	1.3403	1.3340	21.538	12.054	7.7032	10.257	12.031	3.7596
BAC	15139	17081	12649	12912	21015	36	55	49	39	10.600	3.3298	6.3379	3.4755	75.367	–	–	–	–	–
CAT	19281	18857	20792	20158	24343	43	65	49	41	1.8900	2.1414	1.8975	1.9650	26.083	–	–	2.6885	0.7994	4.2094
CVX	20082	19441	19446	19322	20207	39	52	40	38	0.3049	0.3441	0.2215	0.2622	10.481	0.7207	–	–	–	0.9740
DD	17847	15881	17101	19177	25828	43	62	50	43	0.8208	0.6618	0.8267	1.1014	31.542	–	–	–	–	4.8798
GE	23825	28488	19198	22227	17625	48	62	50	51	2.1932	4.3626	2.0740	3.6149	23.125	10.406	15.211	18.961	4.8961	–
HD	24779	21998	24354	24301	24141	42	54	51	43	0.6102	0.9362	0.8297	0.7642	18.213	23.947	8.2744	–	19.271	3.909
HPQ	19490	19273	18855	19104	28131	44	61	53	49	0.7340	0.5161	0.6358	0.5462	18.274	–	–	–	–	7.4841
IBM	20195	20591	19978	19406	30263	47	65	53	43	0.4362	0.5118	0.4672	0.5858	15.595	4.0053	3.2041	–	–	10.248
JNJ	20805	20955	20668	20132	21329	37	56	40	33	0.1472	0.2202	0.1974	0.1474	4.1811	8.2920	8.2037	6.1025	1.4464	3.0183
JPM	26914	23154	26441	27094	27111	35	59	52	38	1.9902	3.3331	1.7588	2.5489	80.732	16.982	9.0347	18.334	15.187	5.2394
KFT	19480	19495	18969	19568	19905	44	68	51	45	0.3281	0.4180	0.4070	0.3429	9.3886	–	–	–	–	–
MCD	20763	20633	20418	21361	19717	44	69	51	45	0.3839	0.4088	0.3934	0.3898	6.3828	5.0193	4.059	2.7998	8.7672	–
PFE	19399	21330	20532	20106	19761	39	62	44	41	0.7179	0.8730	0.8422	0.7808	11.010	–	5.7428	2.5382	0.6980	–
PG	20461	19486	21021	20003	19467	42	62	48	39	0.2706	0.4075	0.2840	0.2902	6.8057	1.5370	1.0763	1.0270	1.3817	–
T	20758	18868	20217	21736	19480	39	62	50	42	0.4013	0.4452	0.3493	0.4434	8.7918	4.9675	1.1209	–	–	–
VZ	21666	21070	19020	22260	19668	46	64	49	50	0.7179	0.8200	0.5025	0.8101	7.9600	7.9862	4.9179	–	10.246	–
WMT	20743	20076	20574	20747	18708	37	65	45	41	0.1726	0.3474	0.1955	0.2444	6.0072	7.3423	0.6598	5.3207	6.1568	–
XOM	19381	19859	19114	19135	16770	38	59	42	44	0.2554	0.4162	0.3104	0.3344	8.2115	–	–	–	–	–

Table 5: Strategy performances based on different daily range models. FC refers to the strategy based on the predicted range from model (10). RW refers to the strategy based on the predicted range from a random walk model. MA_5 and MA_{22} refers to the strategy based on the predicted range from a moving average model of the last 5 and 22 days, respectively. $B\&H$ is the payoff of a strategy based on buy at the initial date and sell at the final date. The initial monetary amount is 20000 dollars. The variance of the portfolio increments is calculated as $var(\Delta A_t/100)$. The Sharpe Ratio is calculated as $SR = \frac{\mu}{\sigma} \cdot 100$, where $\mu = mean(\Delta A_t/A_{t-1})$ and $\sigma = std(\Delta A_t/A_{t-1})$. The Sharpe ratio is only reported for positive values of μ.