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DO SMART GRIDS BOOST INVESTMENT IN PHOTOVOLTAICS? THE PROSUMER INVESTMENT DECISION

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January 2016

"MARCO FANNO" WORKING PAPER N.203

Do Smart Grids Boost Investment in Photovoltaics? The Prosumer Investment Decision

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Abstract

In Italy and in many EU countries, the last decade was characterized by a large development of distributed generation power plants. Their presence determined new critical issues for the design and management of the overall energy system and the electric grid due to the presence of discontinuous production sources. It is commonly agreed that contingent problems that affect local grids (e.g. inefficiency, congestion rents, power outages, etc.) may be solved by the implementation of a "smarter" electric grid.

The main feature of smarts grid is the great increase in production and consumption flexibility. Smart grids give producers and consumers, the opportunity to be active in the market and strategically decide their optimal production/consumption scheme. The paper provides a theoretical framework to model the prosumer's decision to invest in a photovoltaic power plant, assuming it is integrated in a smart grid. To capture the value of managerial flexibility, a real option approach is implemented. We calibrate and test the model by using data from the Italian energy market.

1 Introduction

Growing concern about GHG emissions and future availability of traditional energy sources motivated national governments to promote renewable energy distributed generation.

In Italy, the last decade was characterized by a large development of distributed generation power plants, mostly biomass and photovoltaic power plants: private investments in these sectors were boosted through incentives, that made them particularly attractive for both institutional investors and (small) private investors.

Private participation in the photovoltaic sector was favored by implementation of high feed-in tariff remuneration schemes[1]. These incentives, on the one hand, allowed for a faster development photovoltaic technology, by guaranteeing free-risk payoffs for large initial investments, but on the other hand, they caused an increase in public expenditure, due to both monetary disbursement to pay incentives and to additional system costs born to manage a significant number of energy production sources not efficiently integrated. Photovoltaic plants, actually, have a relevant responsibility for grid costs increase: in 2012, the installed photovoltaic capacity reached a power amount of more than 16,4 GW, through 478.331 plants, that generated an increase in power of 28,5%

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[1]The "Conto Energia" programme, started in Italy in 2005.

with respect to 2011 (GSE, 2013)[2]. Total power amount and fragmented subdivision of the plants have a considerable impact on the electric system, provided that the grid is not design to support peripheral inflows, and especially those instable coming from unpredictable production.

It is undeniable that photovoltaics might have a considerable role in future energy supply in Italy, due to particularly favorable geographical conditions. The increasing number of investments in photovoltaic power plants, as other discontinuous and distributed energy production sources, generated problems that affected local grids (e.g. inefficiency, congestion rents, power outages, etc.), part of which might be solved by the implementation of a "smarter" electric grid. Smart grids represent de facto the evolution of electrical grids and their implementation is challenging the electric market organization and management. To favor the development of photovoltaic energy production in a sustainable way, the electric system need to be balanced and efficiently managed. This objective could be reached by implementing the so called Smart Grid. Smart Grids allow for an instantaneous interaction between agents and the grid: depending on its needs, the grid can send signals (through prices) to the agents, and the agents have the possibility to respond to the signals and obtain a monetary gain. In this way, the system can allow for better integration of renewables – that in turn contribute to keep the grid stable - and for photovoltaic development in the absence of costly monetary incentives. In addition, due to the possibility to gain revenues by direct grid management, the investor has more positive flows to account for in the investment evaluation and this may accelerate the process to private investments sustainability.

The main feature of the smart grid is the great increase in production and consumption flexibility. Smart grids give producers and consumers, the opportunity to be active in the market and, eventually, to match their needs with the neighbors' ones in a complementary way. Smart grids generate managerial flexibilities that prosumers (i.e. subjects that both produce and consume electric energy) can exploit when deciding to invest in photovoltaics.

This flexibility gives prosumers the option to strategically decide the optimal production/consumption scheme and can significantly contribute to energy saving and hedging of investment risk. In other words, if optimally exercised, operational flexibility can be economically relevant and its value is strongly related to the prosumers ability to decide their investment strategy and planned course of action in the future, given then-available information.

Traditional capital budgeting techniques fail to capture the value of this managerial flexibility.

It is widely recognized that the Net Present Value rule fails because it cannot properly capture managerial flexibility to adapt and revise later decisions in response to unexpected market events.

As new information arrives and uncertainty about future cash flows is gradually resolved, management may have valuable flexibility to alter its initial operating strategy in order to capitalize on favorable future opportunities. The real option approach, by endogenizing the optimal operating rules and explicitly capturing the value of flexibility, provides contextually for a consistent treatment of investment risk. The paper provides a theoretical framework to model the prosumer's decision to invest in a photovoltaic power plant, assuming it is integrated in a smart grid. The paper remainder is organized as follows. Section 2 describes the model set up. Section 3 and 4 provide the model on the optimal investment sizing and timing respectively. Section 5 introduces the model parameter estimations from empirical data driven from the Italian energy market and Section 6 provides simulations and sensitivity analyses to calibrate the model and illustrate theoretical results. Section 7 concludes.

[2] Other non dispatchable energy sources (e.g. wind) provide for less than half of photovoltaic power capacity (8,1 GW) and this power is concentrated in 807 plants.

2 Model set up

We consider an agent that has to decide when and whether to invest in a photovoltaic (PV) plant to cover his demand of energy. However, although his energy load is his primary target, he may decide to be connected to a local energy market through a smart grid with the possibility of selling, totally or partially, the energy produced by the plant. Since the agent plays the role of consumer and producer, we call the agent prosumer (Toffler, 1980; Karnouskos, 2011; Da Silva et al.2014).[3].

The decision to sell energy is based on the selling price that prevails in the local market at that time. The agent always keeps the option to call for energy from the national grid at a contractually fixed price.

We also assume that it is not possible for the agent to buy energy from the local grid. This is motivated by the fact that he is *de facto* acting into two separate markets: as a consumer, he can buy energy from the national grid at a fixed price or consume the energy produced by the PV plant; as a producer, he can be called by the regulator of the local market for collaborating to grid equilibrium by selling partially or entirely the energy produced.

Since the prosumer's objective is to minimize energy costs, the investment decision will depend on his energy demand and on the ratio between the buying and selling price of energy.

Let's introduce some simplifying assumptions:

Assumption 1 The agent's demand of energy per unit of time $t > 0$ (i.e. day, week, month, year...) is normalized to 1 (i.e. 1 KWh, 1 MWh, etc.). This is can be represented as:

$$1 = \xi\alpha_1 + \alpha_2 \tag{1}$$

where $\alpha_1 > 0$ is the expected production of the power plant per unit of time, $\xi \in [0, 1]$ is the production quota used for self-consumption and $0 < \alpha_2 \leq 1$ is the energy quota bought from the national grid.

For example, if we consider as unit measure of time a day (i.e. 24 hours), then $\xi\alpha_1 + \alpha_2 \equiv \int_0^{24} l(s)ds = 1$ where $l(s)$ denotes the consumption of energy at time $s \in [0, 24]$. In this case α_2 is the quota of the energy demand that must necessarily be bought from the national grid, since it is required during the interval of plant inactivity (i.e. when solar radiations are not available), while $\xi\alpha_1$ is the prosumed energy, when the plant is in operation and producing. This also implies that $(1 - \xi)\alpha_1$ is the quota the agent can sell on the local energy market.

Assumption 2 The agent receives information on the selling price at the beginning of each time interval t and, on the basis of this information, he makes the decision on how much of the produced energy to consume and how much to sell in the local market.

This latter assumption simplifies the analysis and does not seems overly restrictive. Though smart grids allow for istantaneous exchange of energy flows and information on energy prices, due to the small dimension of our agent, it is reasonable to assume that he cannot rapidly change his consumption pattern $l(s)$.

Assumption 3 The prosumer cannot buy energy from the local grid.

[3]While referring to the prosumer, we will use prosumption to identify production with the consequent consumption of the energy produced by the prosumer itself, and the verb to prosume to express prosumer's activity.

This is a crucial assumption in our analysis. Although, the possibility that agents produce energy and inject it in the grid is actually one of the reasons for implementing smart grid tecnologies, here we assume that the reverse is not possible. The market related to the local grid that we are considering is not for direct consumption but for the general magagement of the electric system. Since there are events in which the demand for power is higher than the supply (i.e. there is less energy on the grid than requested), the prosumer is called for being active in the market and increasing the level of reliability of the system by selling part of the energy he produces. This helps to reduce system costs caused by unpredictable energy inflows coming from distributed and non-dispatchable energy sources, and this in turn makes more challenging system balancing activities. Information on grid needs are delivered through the buying price to solve balancing needs, local congestions or sudden black outs.

Assumption 4 Storage is not possible.

This is consistent with the assumption of $\alpha_2 > 0$. In other words, no batteries are included in the PV plant. This reduces managerial flexibilities, since energy must be used as long as it is produced[4].

According to Assumptions 1- 4, indicating by c the fixed contract price (buying price) of energy, a the per unit cost paid to produce energy by the PV plant and v the selling price of energy, we can write the prosumer's net cost of energy per unit of time as:

$$\begin{aligned} C &= \min\left[c-\alpha_1(v-a), \quad \xi\alpha_1 a+(1-\xi\alpha_1)c-(1-\xi)\alpha_1(v-a)\right] \\ &= c-\alpha_1(v-a)+\min[\xi\alpha_1(v-c), \quad 0] \end{aligned} \tag{2}$$

The first term inside the square bracket is the net cost in the absence of self-consumption (i.e. energy is totally sold in the market), the second term indicates the net cost in the presence of self-consumption. Notice that the energy costs paid by the agent depends on the possibility of choosing between selling energy in the market or prosuming PV energy. In the first case, the agent pays c and earns $\alpha_1 v$, minus the production cost of α_1, and the prosumer sells production from photovoltaics in the local market. In the second case, part of the energy produced by the plant is prosumed ($\xi\alpha_1$, at the production cost a), part of the energy required $\alpha_2 = 1-\xi\alpha_1$ is bought at the contract fixed price c and the energy produced but not consumed is sold in the local market at price v.

We conclude the set up by introducing one more assumption:

Assumption 5 The maximum prosumed energy quota is caped from above, i.e. $\xi\alpha_1 \leq \bar{\alpha} < 1$.

Although households energy management is widely recognized[5] as a priority to reach an overall cost-saving by PV generation systems, nowadays consumers' load during the day is still particularly high during the evening[6], while the quota of energy consumed in the morning and/or in the afternoon is still quite low. Consequently, it is reasonable to assume that the energy quota

[4] We leave to further research the analysis on investigating the investment decision in the presence of batteries, that generates new investment opportunities for the producer.

[5] See Ciabattoni et al. (2014) among others.

[6] According to the analysis performed by the Italian National Authority for Electricity, Gas and Water Services (AEEGSI) in 2009, the higher peak load demanded by residential users occurs in the evening, between 8:00 p.m. and 10:00 p.m. (AEEGSI, 2009).

self-consumed, $\xi\alpha_1$, will not exceed a percentage of total energy demand[7]. We take into account the agent's load curve by setting $\xi\alpha_1 = \bar{\alpha}$, i.e. by fixing $\bar{\alpha}$ the prosumed quota ξ is endogenously determined by choosing the plant size α_1. Active households energy management may increase $\bar{\alpha}$, this in turn may induce investors to install higher size plants.

Finally, for sake of simplification, we assume that the buying price c is constant over time and the marginal cost of internal production $a = 0$. On the contrary, the selling price v is stochastic and driven by the following Geometric Brownian Motion[8]:

$$dv(t) = \gamma v(t)dt + \sigma v(t)dz(t) \qquad \text{with } v(0) = \nu_0 \tag{3}$$

where $dz(t)$ is the increment of a Wiener process[9], σ is the istanataneous volatility and γ is the drift term lower than the market (i.e. risk adjusted) discount rate r, i.e. $\gamma \leq r$[10]. By (3), we implicitly assume that $v(t)$ does not depend on the agent's supply, this is again justified by our emphasis on the investment decision of a small prosumer, unable to influence the market.

3 The value of the PV plant

Once installed and connected to the local market through the smart grid, according to (2), the plant allows for a flexible choice between two polar cases. Whenever $v(t) > c$ the agent minimizes energy costs by selling to the local market the entire production, i.e. $\xi = 0$, and satisfying his demand by buying energy from the national grid. Whereas, whenever $v(t) < c$ the agent minimizes energy costs via a positive prosumption quota $\xi > 0$.

Then, for any $\xi > 0$, the present value of energy costs with the embedded flexibility to switch form self-consumption to "total" selling, is given by the solution of the following dynamic programming problems (Dixit and Pindyck, 1994; Moretto, 1996; Hu and Oksendal, 1998)[11]:

$$\Gamma C^0(v(t);\xi,\alpha_1) = -[c - \alpha_1 v(t) + \xi\alpha_1(v(t) - c)], \qquad for \;\; v(t) < c \tag{4.1}$$

and

$$\Gamma C^1(v(t);\xi,\alpha_1) = -[c - \alpha_1 v(t)], \qquad for \;\; v(t) > c, \tag{4.2}$$

where Γ indicates the differential operator: $\Gamma = -r + \gamma v \frac{\partial}{\partial v} + \frac{1}{2}\sigma^2 v^2 \frac{\partial^2}{\partial v^2}$. The solution of the differential equations (4.1) and (4.2) is subject to the two following boundary conditions:

$$\lim_{v \to 0} \left\{ C^0(v(t);\xi,\alpha_1) - \frac{(1-\xi\alpha_1)c}{r} + \frac{(1-\xi)\alpha_1 v(t)}{r-\gamma} \right\} = 0 \tag{5.1}$$

and

[7] Many technical reports and contribution in the literature show that this quota ranges between 30% and 40%. See as an example Ciabattoni et al. (2014).

[8] Alternative dynamic frameworks may be used, such as mean reverting process. Conclusions would not change, but it would not be possible to determine a close form solution.

[9] The process $dz(t)$ has mean $E(dz) = 0$ and variance $E(dz^2) = dt$. Therefore, $E(dv(t))/v(t) = \gamma dt$ and $E(dv(t)/v(t))^2 = \sigma^2 dt$, i.e. starting from the initial value v_0, the random position of the price $v(t)$ at time $t > 0$ has a normal distribution with mean $v_0 e^{\gamma t}$ and variance $v_0^2(e^{\sigma^2 t} - 1)$.

[10] This assumption is due to guarantee convergence.

[11] A PV plant has generally a very long technical life that rages between 20 and 25 years. Then, without loss in generality, in (6) we approximate the technical life to infinite.

$$\lim_{v\to\infty}\left\{C^1(v(t);\xi,\alpha_1)-\frac{c}{r}+\frac{\alpha_1 v(t)}{r-\gamma}\right\}=0, \tag{5.2}$$

In (5.1) the term $\frac{(1-\xi\alpha_1)c}{r}-\frac{(1-\xi)\alpha_1 v(t)}{r-\gamma}$ represents the present value of operating costs meanwhile the prosumer uses the PV plant for self-consumption, whereas in (5.2) the term $\frac{c}{r}-\frac{\alpha_1 v(t)}{r-\gamma}$ indicates the present value of operating costs when selling the whole energy produced. By the linearity of (4.1) and (4.2) and according to (5.1) and (5.2) we obtain:

$$C(v(t);\xi,\alpha_1)=\begin{cases} C^0(v(t);\xi,\alpha_1)=\frac{(1-\xi\alpha_1)c}{r}-\frac{(1-\xi)\alpha_1 v(t)}{r-\gamma}+\hat{A}v(t)^{\beta_1} & \text{if } v(t)<c \\ \\ C^1(v(t);\xi,\alpha_1)=\frac{c}{r}-\frac{\alpha_1 v(t)}{r-\gamma}+\hat{B}v(t)^{\beta_2} & \text{if } v(t)>c. \end{cases} \tag{6}$$

where $\beta_2<0$ and $\beta_1>1$ are the negative and the positive roots of the characteristic equation $\Phi(\beta)\equiv\frac{1}{2}\sigma^2\beta(\beta-1)+\gamma\beta-r$ respectively.

In (6), the additional terms $\hat{A}v(t)^{\beta_1}$ and $\hat{B}v(t)^{\beta_2}$ represent the value of the option to switch from self-consumption to energy selling if $v(t)$ increases, and the value of the option to switch the other way round if $v(t)$ decreases, respectively. Finally, imposing the value matching and the smooth pasting conditions at $v(t)=c$, we obtain:

$$\begin{cases} \hat{B}=\xi\alpha_1 cB\equiv\xi\alpha_1 c\frac{1}{(r-\gamma)}\frac{r-\gamma\beta_2}{r(\beta_2-\beta_1)} \\ \hat{A}=\xi\alpha_1 cA\equiv\xi\alpha_1 c\frac{1}{(r-\gamma)}\frac{r-\gamma\beta_1}{r(\beta_2-\beta_1)}. \end{cases} \tag{7}$$

which are always non-positive and both linear in $\xi\alpha_1$.

4 The optimal size and investment timing

We are now ready to calculate the value of the option to invest in the PV plant (i.e. the ex-ante value of the plant), as well as its optimal size. The opportunity to invest must be considered with respect to the alternative that, in our case, is to satisfy the entire demand by buying energy from the national grid at the contracted price c. The agent will invest if and only if the plant generates a payoff (in term of lower costs), greater than the *status quo*, i.e.:

$$\Delta C(v(t);\xi,\alpha_1)\equiv\frac{c}{r}-C(v(t);\xi,\alpha_1)=\begin{cases} \frac{\xi\alpha_1 c}{r}+\frac{(1-\xi)\alpha_1 v(t)}{r-\gamma}-\hat{A}v(t)^{\beta_1} & \text{if } v(t)<c \\ \\ \frac{\alpha_1 v(t)}{r-\gamma}-\hat{B}v(t)^{\beta_2} & \text{if } v(t)>c. \end{cases} \tag{8}$$

The agent's problem is to choose the optimal size by maximizing (8) with respect to α_1, net of the investement cost. In addition, since we focus on a prosumer, the optimal size is given by:

$$\alpha_1^*(v(t))=\arg\max\left[\Delta C(v(t);\xi,\alpha_1)-I(\alpha_1)\right] \tag{9}$$

where $I(\alpha_1)$ represents the plant's sunk investment cost and ΔC represents the agent's payoff when $v(t)<c$.

The cost of a PV plant is, in general, related to the maximum power of the plant measured in kWp[12]. However, referring to the characteristics of the plant as well as to the photovoltaic panel

[12] kWp stands for "kilowatt peak", and indicates the nominal power of the plant (or of the panel). It is calculated with respect to specific standard environmental conditions: 1000 W/m^2 light intensity, cell positioned at latitude 35° N, reaching a temperature of 25° C (International IEC standard 904-3, 1989).

production curve, it is possible to have an estimate of the plant's cost depending on the size of the plant. In particular, we model the cost of the plant as a Cobb-Douglas, with increasing cost-to-scale and quadratic in α_1[13]:

$$I(\alpha_1) = \frac{K}{2}\alpha_1^2. \tag{10}$$

Equation (10) captures capital costs (panel costs, inverters and cables), the on-going system-related costs (operating and maintenance costs) and insurance, along with the (estimated) amount of electricity produced during the lifetime of the plant, and converts them into the common metric α_1. Note that if $\alpha_1 = 0$ the cost is null and it grows as the size of the plant increases. The convexity of (10) captures the efficency losses caused by the system depreciation during its production life[14].

By substituting (10) into (9), and according to Assumption 5, we get:

$$\alpha_1^*(v(t)) = \arg\max\left[NPV(\alpha_1; v(t))\right]$$

where $NPV(\alpha_1; v(t)) \equiv \frac{\bar{\alpha}c}{r} + \frac{(\alpha_1-\bar{\alpha})v(t)}{r-\gamma} - \bar{\alpha}cAv(t)^{\beta_1} - \frac{K}{2}\alpha_1^2$ is the net present value of the project. In the specific $\frac{\bar{\alpha}c}{r}$ indicates the energy saving and $\frac{(\alpha_1-\bar{\alpha})v(t)}{r-\gamma}$ the expected revenues from selling the quota exceeding prosumption, i.e. $\alpha_1 - \bar{\alpha}$. The term $\bar{\alpha}cAv(t)^{\beta_1}$ represents the revenues generated by the option to sell the entire production to the local market.

From the first order condition, it follows[15]:

$$\alpha_1^*(v(t)) = \max\left[\frac{\frac{v(t)}{(r-\gamma)}}{K},\ \bar{\alpha}\right]. \tag{1}$$

The plant's optimal size is given by the ratio between the expected discounted flow of revenues produced by an additional unit of capacity and the marginal cost of this unit. Note that, as $\alpha_1$is a function of the current value of $v(t)$, the selling price must be sufficiently high to make profitable to invest in a plant of a size greater than $\overline{\alpha}$. Otherwise, by Assumption 5, the optimal choice is to set $\xi = 1$.

Let's now turn to the optimal investment strategy. Denoting by v^* the selling price that triggers the investment, the agent's ex-ante value of the plant is given by the solution of the following dynamic programming problem[16]:

$$\Gamma F(v(t)) = 0, \qquad \text{for } \nu_0 < v(t) < v^* \tag{12}$$

In particular, assuming that the current value of $v(t)$ is sufficiently low so that it is not optimal to invest immediately, the general solution of (12) is:

$$F(v(t)) = Mv(t)^{\beta_1} \quad \text{for } \nu_0 < v(t) < v^* \tag{13}$$

where $\beta_1 > 1$ is the positve root of $\Phi(\beta)$ and M is a constant to be determined. Imposing the value matching and the smooth pasting conditions at $v(t) = v^*$, it is easy to show that:

[13] The sunk cost is assumed to be quadratic only for the sake of simplicity. None of the results were altered if the investment cost is represented by a more general formulation $I(\alpha) = K\alpha^\delta$ with $\delta > 1$.

[14] Lorenzoni et al. (2009) assume that the system depreciation rate varies between 1% to 1.5% per year. Ciabbattoni et al. (2014) record that PV module producers guarantee at least 80% of their initial performance after 20 years. This is equivalent to assume that α_1 should be increased each year to maintain production constant.

[15] As a matter of fact, if $v(t) = 0$, the NPV reduces to $NPV(\alpha_1; 0) \equiv \frac{\bar{\alpha}c}{r} - \frac{K}{2}\alpha_1^2$. In order the investment to be profitable, it is necessary that $\frac{\bar{\alpha}c}{r} \geq \frac{K}{2}\bar{\alpha}^2$.

[16] Whenever $\nu_0 < v^* < v(t)$ it would be optimal for the prosumer to invest immediately (i.e. the agent takes the investment decision according to the NPV rule and the option value to wait is null).

Proposition 1 *Provided that $v^* < c$,*

i) if $\beta_1 < 2$, the optimal investment trigger is given by:

$$\frac{v^*}{r-\gamma} = \frac{\beta_1 - 1}{\beta_1 - 2}\left(\frac{1}{2}\bar{\alpha}K\right) + \sqrt{(\frac{\beta_1 - 1}{\beta_1 - 2})^2\left(\frac{1}{2}\bar{\alpha}K\right)^2 - \frac{\beta_1}{\beta_1 - 2}\frac{\bar{\alpha}c}{r}K} \tag{14.1}$$

ii) if $\beta_1 \geq 2$, and $\overline{\alpha}\frac{c}{r} - \frac{(\beta_1-1)^2}{\beta_1(\beta_1-2)}\frac{K}{2}(\overline{\alpha})^2 > 0$ the option to sell energy to the local market is so high, that will never be optimal for the agent to become a prosumer, it would be optimal to enter as producer.

and the constant is given by:

$$M = -\overline{\alpha}cA + \left[\frac{1}{2K}(\frac{v^*}{r-\gamma})^2 - \overline{\alpha}(\frac{v^*}{r-\gamma} - \frac{c}{r})\right] v^{*-\beta_1} \tag{14.2}$$

Proof. See Appendix A ■

An empirical application may better illustrate the relationship between the optimal plant dimension, the investment trigger and the value of being connected a local smart grid. To do this, in the next section, we calibrate the model using data related to the Italian market.

5 Parameter estimations from empirical data

In order to perform the sensitivity analysis in term of optimal plant dimension and investment timing, we estimate the parameters of the model referring to the Italian market.

Let's start with the contracted energy price c, the input cost a, and the selling price v:

- c is the fixed buying price of energy, and it is representative of the average price paid by household consumers over the period 2013 and 2014. The average basic energy price paid by household consumers in 2013 and 2014 was $c = 160,00$ €/MWh net of taxes and levies (Eurostat, 2015).

- a is the photovoltaic production cost. The production input for photovoltaic production – solar radiations – is for free, and marginal production costs for the photovoltaic power plant can be considered negligible and equal to zero.

- v is the price at which the prosumer sells energy to the local market. We use as a proxy for v the "Italian zonal prices" recorded between 2010 and 2014. We built a dataset starting from hourly data provided by Terna S.p.A., the Italian Transmission System Operator[17]. Then,

[17] PV plants receive a payment for the energy sold to the grid, whose price depends on the plant's location. The Italian electric system is divided into different zones, among which physical energy exchanges are limited due to system security needs. The GME glossary provides a summary of the zones the Italian market is divided into. These zones are grouped into: a) geographical zones; b) national virtual zones; c) foreign virtual zones; and d) market zones. Geographical zones represent a geographical portion of the national grid and are respectively classified into northern area, northern-central area, southern area, southern-central area, Sicily and Sardinia. National virtual zones identify limited production poles: Monfalcone, Rossano, Brindisi, Priolo and Foggia. Foreign virtual zones represent points where the nationa grid connects to adjoining Countries: France, Switzerland, Austria, Slovenia, BSP (a Slovenian electricity market zone, connected to IPEX by market coupling mechanisms), Corsica, and Greece. Finally market zones are aggregation of geographical and virtual zones in which energy flows respect the limits imposed by the Italian Transmission System Operator (Terna S.p.a.). It is worth note that zonal prices determines PUN level, because the final PUN for each hour of the day is the result of zonal prices averages, weighted by energy exchanges. Differences in zonal prices are determined by differences on transmission capacity, consumers' behavior (Gianfreda and Grossi, 2009) and different distributed production patterns. It can be assumed that zonal prices give a measure of the local congestion of the grid at every time of the day.

from each day we estracted the interval between 8 a.m. and 7 p.m., assuming that - on average - it can be considered as the interval of photovoltaic activity[18]. We calculated the average price within the above interval and we estimated the average montly price according to the photovoltaic dayly averages. Next, we validated the 59 monthly seasonally adjusted observations, verified that they are distributed as a Geometric Brownian Motion (GBM) and estimated the process parameters following the procedure proposed by Chen (2012) and Biondi and Moretto (2015). We performed the test for lognormality and for the presence of unit root (Dickey Fuller test)[19]. The estimates of γ and σ for the geographical areas North, North-Central, South and South-Central are reported in Table 1.

Geographical areas	σ (%)	γ (%)
North	32.07	5.14
North- Central	30.35	4.60
South	31.12	4.84
South-Central	29.83	4.45

Table 1 - Estimated values for γ and σ for four of the Italian geographical zones North, North-Central, South and South Central.

- Finally, as starting value ν_0 in each geographical zone we assume the average yearly selling prices recorded in the time interval 2013-2014 (GME, 2015) as summarized inTable 2:

Year	**Yearly average zonal prices** (€/MWh)			
	North	North- Central	South	South-Central
2013	63.66	61.60	55.88	59.69
2014	53.21	50.88	47.00	49.65

Table 2 - Yearly averge zonal prices in 2013 and 2014

Other inputs are the following:

- T indicates the investment life time, equal to 20 or 25 years;

- r is the risk adjusted discount rate. According to the Capital Asset Pricing Method (CAPM) , $r = r_f + B(MRP)$, where MRP is the market risk premium, B measures the systematic risk and r_f is the risk free interest rate. Following Fernandez et al. (2011; 2013), we use 5.0% for the Italian market risk premium. For the risk-free interest rate we take the average of the interest rates on the Italian BTP (maturity 20 and 25 years) as published by the Italian "Dipartimento del Tesoro", i.e. $r_f = 2.28\%$ and 3.07% respectively. Finally, for the Beta B of the photovoltaic sector we use 0.65 (Capizzani, 2012; Biondi and Moretto, 2015). Putting all this information together we set $r = 7.0\%$[20].

[18] That corresponds to F1 time-of-use tariff (Ciabattoni, 2014).

[19] See the Appendix.

[20] Ciabattoni et al. (2014) consider the value of the discount rate r equal to the Weighted Average Cost of Capital (WACC). They set the WAAC to 5% comparing the investment of the PV plant to a 20 year government bond and considering that the investor wants to earn a 1% more than investing in Italian Treasury Bonds.

To calibrate the cost function (10), we refer to the photovoltaic Levelized Cost Of Electricity (LCOE). It represents a life cycle cost per kWh and it can interpreted as the minimum price per kWh that an electricity generating plant would have to obtain in order to break-even on its investment over its entire life cycle (Kost et al., 2013). Specifically we calculate $K = 2\frac{LCOE}{r}(1 - e^{-rT})$, where:

- $LCOE$ is set equal to 180 €/MWh and 250 €/MWh. Values of 180 €/MWh and 250 €/MWh are nowadays reachable in Italy[21][22]. Based on these values of LCOE, Table 3 summarizes K values with reference to the here considered four geographical areas.

	T (year)	K (€)
LCOE=180 €/MWh	20	3,874.64
	25	4,249.16
LCOE=250 €/MWh	20	5,381.45
	25	5,901.65

Table 3 - Investment costs K for T=20, 25 years and LCOE=180, 250 €/MWh

Finally,

- $\bar{\alpha}$ represents the percentage of electric energy consumption that can be concentrated during the photovoltaic interval by the prosumer. Simulations are made for $\bar{\alpha}$ equal to 30%, 50% and 70%. The smaller value is near to actual average percentage of daily energy usage (Ciabbatoni et al. 2014). Self consumption percentages of 50% and 70% are performed to consider the effects of being connected to a smart grid in terms of energy management[23].

6 Simulations and sensitivity analysis

In order to test the model, we perform the analysis for the four geographical zones North, North-Central, South and South-Central the Italian market is grouped into.

First of all we are interested in calculating $v^*$and α_1^* according to the parameter estimates illustrated in Table 1. In Table 4 there are presented the results found by varying the life time T and the self consumption rate $\bar{\alpha}$ and adopting as starting values v_0 for each geographical zone, the average yearly zonal prices recorded in 2013 (Table 2).

[21] These values are consistent with recent contributions in the literature referred to Italy (Kost et al., 2013; Ossenbrink et al., 2013).

[22] Being solar irradiation one of the critical values to estimate the LCOE level, it's worth to note that for different geographical zones in Italy we should use different LCOE levels. However, many factors impact on LCOE definition, so we assume the same value for the four zones.

[23] Ciabbatoni et al. (2014), in Table 5 suggest that energy management actions are able to empower grid agents with tools and mechanisms that optimize consumption patterns up to 50%.

LCOE=180

		North		North Central		South		South Central	
T	$\overline{\alpha}$	α_1*	**v***	α_1*	**v***	α_1*	**v***	α_1*	**v***
20	0,3	0,647	46,598	0,669	62,195	0,658	55,099	0,676	66,793
20	0,5	0,814	58,689	0,834	77,540	0,825	69,021	0,840	83,003
20	0,7	0,944	68,063	0,959	89,193	0,952	79,697	0,964	95,230
25	0,3	0,615	48,603	0,635	64,763	0,626	57,419	0,642	69,514
25	0,5	0,774	61,143	0,790	80,611	0,783	71,826	0,796	86,233
25	0,7	0,896	70,843	0,908	92,606	0,903	82,843	0,910	98,589

LCOE=250 **rate**

		North		North Central		South		South Central	
T	$\overline{\alpha}$	α_1*	**v***	α_1*	**v***	α_1*	**v***	α_1*	**v***
20	0,3	0,540	54,089	0,555	71,744	0,548	63,745	0,560	76,894
20	0,5	0,678	67,827	0,688	88,902	0,683	79,430	0,692	94,926
20	0,7	0,783	78,385	0,788	101,768	0,786	91,329	0,789	108,337
25	0,3	0,514	56,376	0,527	74,632	0,521	66,371	0,531	79,940
25	0,5	0,643	70,599	0,652	92,308	0,648	82,568	0,654	98,485
25	0,7	0,742	81,500	0,745	105,508	0,744	94,812	0,746	112,158

Table 4 - $v^*$and α_1^* for the geographical zones North, North-Central, South and South-Central

Note that whenever the optimal investment tirgger v^* is within the interval (v_0, c), the option to wait to invest is positive. In other words, in these cases, without the introduction of tax incentives it is currently not profitable to invest in a PV plant. This confirms the results obtained in other recent contributions (Ciabbatoni et al. 2014; Biondi and Moretto, 2015).

Moreover, it is worth noting that since $\alpha_1^* > \bar{\alpha}$, the possibility to sell energy in the local market favours the agent to invest in a plant of bigger size if compared to the one needed for self-consumption. This result also holds when, due to improved actions in terms of energy management, the prosumption energy quota increases up to satisfying the entire demand (i.e. 1 MWh/y). Finally, as we expected, there exists a positive relation between α_1^* and v^*: the greater the plant's size the greater is the selling price that triggers the investment.

Further, in line with the real option theory, an increase in LCOE generates an increase in the investment timing and a reduction in the plant's size[24]. If we take into consideration the plant's useful life T, we can observe that for increasing T, ceteris paribus, the plant size decreases and the selling price that triggers the investment increases (i.e. the agent waits longer to invest). Intuitively, higher Ts imply higher investment costs (i.e. higher annual maintenance costs, insurance costs, higher depreciation rate...) that in turn determine a generalized investment delay. This delay reduces by reducing the plant size.

Tables 5 and 6 present the optimal size α_1^* and the optimal trigger v^* in the geographical zones North and South respectively for different T=20, 25 years, LCOE=180, 250 €/MWh and different values of $\overline{\alpha}$ and σ.

[24] In our setting LCOE is constant over the geographical zones. Therefore we can capture exclusively the effect on the plant size. According to our results, the optimal size in the South is greater and the selling price that triggers the investment is greater (i.e. there is a greater delay in undertaking the investment). Nonetheless it is worth noting that in the South of Italy LCOE is smaller than in other areas, and this in turn might reduce the delay.

NORTH								
LCOE=180			σ					
			0,30		0,35		0,40	
	T	$\overline{\alpha}$	α_1*	**v***	α_1*	**v***	α_1*	**v***
	20	0,30	0,65	46,86	0,64	46,26	0,64	45,77
	20	0,50	0,82	58,92	0,81	58,39	0,80	57,95
	20	0,70	0,95	68,24	0,94	67,84	0,94	67,50
	25	0,30	0,62	48,86	0,61	48,27	0,60	47,78
	25	0,50	0,78	61,36	0,77	60,86	0,76	60,44
	25	0,70	0,90	71,00	0,89	70,64	0,89	70,35
LCOE=250			σ					
			0,30		0,35		0,40	
	T	$\overline{\alpha}$	α_1*	**v***	α_1*	**v***	α_1*	**v***
	20	0,30	0,54	54,34	0,54	53,77	0,53	53,30
	20	0,50	0,68	68,01	0,68	67,60	0,67	67,26
	20	0,70	0,78	78,47	0,78	78,28	0,78	78,13
	25	0,30	0,52	56,62	0,51	56,06	0,51	55,61
	25	0,50	0,64	70,76	0,64	70,40	0,64	70,10
	25	0,70	0,74	81,54	0,74	81,44	0,74	81,36

Table 5 - Optimal size α_1^* and optimal trigger v^* in the North for T=20, 25 years, LCOE=180, 250 €/MWh, $\gamma = 5.14\%$ and different values of $\overline{\alpha}$ and σ

SOUTH								
LCOE=180			σ					
			0,30		0,35		0,40	
	T	$\overline{\alpha}$	α_1*	**v***	α_1*	**v***	α_1*	**v***
	20	0,30	0,66	55,31	0,65	54,45	0,64	53,75
	20	0,50	0,83	69,20	0,82	68,45	0,81	67,83
	20	0,70	0,95	79,84	0,95	79,26	0,94	78,80
	25	0,30	0,63	57,63	0,62	56,77	0,61	56,08
	25	0,50	0,78	72,00	0,78	71,28	0,77	70,70
	25	0,70	0,90	82,96	0,90	82,46	0,89	82,05
LCOE=250			σ					
			0,30		0,35		0,40	
	T	$\overline{\alpha}$	α_1*	**v***	α_1*	**v***	α_1*	**v***
	20	0,30	0,55	63,94	0,54	63,13	0,54	62,47
	20	0,50	0,68	79,57	0,68	78,99	0,68	78,52
	20	0,70	0,79	91,39	0,78	91,13	0,78	90,91
	25	0,30	0,52	66,56	0,52	65,77	0,51	65,13
	25	0,50	0,65	82,69	0,64	82,18	0,64	81,77
	25	0,70	0,74	94,85	0,74	94,70	0,74	94,58

Table 6 - Optimal size α_1^* and optimal trigger v^* in the South for T=20,25 years, LCOE=180, 250 €/MWh, $\gamma = 4.84\%$ and different values of $\overline{\alpha}$ and σ

By direct inspection of Table 5 and 6, we can observe that ceteris paribus, the greter the uncertainty the greater the option value to wait and the greater the uncertainty the smaller the plant size. This trend is amplified, ceteris paribus, for higher LCOEs: in other words, ceteris paribus, the greater LCOE the smaller the plant size and the greater the investment deferral.

Tables 7 and 8 display the optimal size α_1^* and the optimal trigger v^* in the geographical zones North and South respectively for different T=20, 25 years, LCOE=180, 250 and different values of $\overline{\alpha}$ and γ.

North							
LCOE=180							
			γ				
			1%		3%		5%
	T	$\overline{\alpha}$	α_1*	v*	α_1*	v*	α_1*	v*
	20	0,3	0,842	195,812	0,728	112,816	0,651	50,454
	20	0,5	0,975	226,592	0,884	137,032	0,818	63,413
	20	0,7	1,059	246,262	0,996	154,410	0,947	73,419
	25	0,3	0,790	201,379	0,688	116,973	0,619	52,606
	25	0,5	0,910	232,069	0,834	141,698	0,777	66,036
	25	0,7	0,986	251,476	0,937	159,333	0,899	76,379
LCOE=250								
			γ					
			1%		3%		5%	
	T	$\overline{\alpha}$	α_1*	v*	α_1*	v*	α_1*	v*
	20	0,3	0,667	215,653	0,595	128,061	0,543	58,490
	20	0,5	0,761	245,807	0,715	153,988	0,680	73,168
	20	0,7	0,819	264,347	0,800	172,170	0,784	84,392
	25	0,3	0,625	221,205	0,562	132,558	0,516	60,939
	25	0,5	0,709	251,030	0,673	158,906	0,645	76,119
	25	0,7	0,760	269,160	0,751	177,252	0,743	87,693

Table 7 - Optimal size α_1^* and optimal trigger v^* in the North for T=20, 25 years, LCOE=180, 250 €/MWh, $\sigma = 32.07\%$ and different values of $\overline{\alpha}$ and γ

South								
LCOE=180								
			γ					
			1%		3%		5%	
	T	$\overline{\alpha}$	α_1*	v*	α_1*	v*	α_1*	v*
	20	0,3	0,857	199,259	0,734	113,692	0,653	50,594
	20	0,5	0,986	229,152	0,889	137,759	0,820	63,537
	20	0,7	1,067	247,957	0,999	154,936	0,949	73,513
	25	0,3	0,803	204,705	0,693	117,834	0,621	52,746
	25	0,5	0,919	234,413	0,838	142,379	0,778	66,153
	25	0,7	0,992	252,899	0,940	159,786	0,899	76,461
LCOE=250								
			γ					
			1%		3%		5%	
	T	$\overline{\alpha}$	α_1*	v*	α_1*	v*	α_1*	v*
	20	0,3	0,677	218,590	0,599	128,861	0,545	58,623
	20	0,5	0,767	247,525	0,718	154,520	0,681	73,263
	20	0,7	0,821	265,015	0,801	172,398	0,785	84,435
	25	0,3	0,632	223,959	0,565	133,324	0,517	61,068
	25	0,5	0,713	252,476	0,675	159,365	0,646	76,203
	25	0,7	0,761	269,514	0,751	177,376	0,743	87,717

Table 8 - Optimal size α_1^* and optimal trigger v^* in the South for T=20, 25 years, LCOE=180, 250 €/MWh, $\sigma = 31.12\%$ and different values of $\overline{\alpha}$ and γ

According to the simulation results illustrated in Table 7 and 8, we can observe that for increasing values of γ the value of deferring the investment decreases. In other words the greater γ the lower the selling price that triggers the investment. Simultaneously, the greater γ the smaller the plant size. When γ the profitability to invest decreases and the optimal investment strategy is waiting to invest and install a big size plant. There exists a negative trade-off between the investment timing and the size plant. When the agent accelerate investment, he installs a smaller size plant as a precautionary measure against future reductions in the energy price trend. Whereas, when γ is small, the agent waits longer to invest, but when undertaking the investment he will install a bigger size plant due to the information on prices acquired in the meanwhile. Finally, ceteris paribus, the greater $\bar{\alpha}$, the greater the optimal size and the delay.

7 The value of being connected to a smart grid

We conclude the empirical example by determining the contribution to the investment value due to the conncetion to a smart grid. We do this by comparing (13) to the value of the option to invest in a plant of fixed dimension $\alpha_1 = 1$ in the absence of the option to decide whether and when to sell the energy produced by the PV plant in the local market [25]. In this case revenues are

[25] This is equivalent to eliminate Assumption 2. The agent cannot use the information received on the selling price in order to decide what to do of his energy production.

fixed: $\frac{\bar{\alpha}c}{r}$ still represents the energy saving and $\frac{(1-\bar{\alpha})v(t)}{r-\gamma}$ is the expected revenues from selling the extra-production $(1-\bar{\alpha})$[26].

Denoting by v^{**} the selling price that triggers the investment in this case, analogously to Section 4, the agent's ex-ante value of the plant is given by:

$$\hat{F}(v(t)) = Nv(t)^{\beta_1} \quad \text{for} \quad \nu_0 < v(t) < v^{**} \tag{15}$$

where $\beta_1 > 1$ is the positve root of $\Phi(\beta)$, $v^{**} = \frac{\beta}{\beta-1}(\frac{r-\gamma}{1-\overline{\alpha}})(\frac{K}{2} - \frac{\overline{\alpha}c}{r})$ and N is the new constant equal to $N = (\frac{1-\overline{\alpha}}{r-\gamma})\frac{\nu^{**(1-\beta_1)}}{\beta_1}$.

The difference between $\hat{F}(v_0)$ and $F(v_0)$ identifies the value of being connected to a smart grid for a given starting value of the selling price. In particular, in Table 9 we show the ratio between M and N for the four geographical zones North, North-Central, South and South-Central. M is always greater than N, therefore being connected to a smart grid always increases the investment value and its profitability.As an example, when LCOE=180 €/MWh, T=20 years and $\overline{\alpha} = 30\%$, ceteris paribus, investing in the North in a PV plant connected to a smart grid is worth 13 times more that in the absence of connection (Table 9). By observing Table 9, we note that ceteris paribus, for increasing values of $\overline{\alpha}$, LCOE and T respectively, the ratio M/N increases. In other words, ceteris paribus, the longer the plant useful life, the greater the value of flexibility generated by the connection to the smart grid; similarily the greater the LCOE or $\overline{\alpha}$, the greater the value of flexibility. In the geographical zone South-Central we identify the greatest ratios, ceteris paribus. This is due to higher values of flexibility that characterizes this zone with respect to the others.

			M/N			
	T	$\overline{\alpha}$	North	North Central	South	South Central
LCOE=180	20	0,3	13,106	22,648	17,717	26,473
	25	0,3	13,281	23,245	18,081	27,269
LCOE=250	20	0,3	13,710	24,736	18,982	29,271
	25	0,3	13,875	25,313	19,328	30,050
LCOE=180	20	0,5	28,524	49,778	38,809	58,261
	25	0,5	29,323	52,045	40,268	61,220
LCOE=250	20	0,5	31,073	57,234	43,548	68,060
	25	0,5	31,684	59,111	44,718	70,556
LCOE=180	20	0,7	61,190	105,389	82,729	122,720
	25	0,7	65,450	116,371	90,026	136,827
LCOE=250	20	0,7	72,854	136,800	103,228	163,509
	25	0,7	75,100	143,340	107,364	172,165

Table 9 - M/N ratios for the four geographical zones North, North-Central, South and South-Central for T=20, 25 years, LCOE=180, 250 €/MWh and different values of $\overline{\alpha}$

[26] This PV system looks like the new Italian energy contract scheme known as "scambio sul posto", where the agent gets credits for the value of the excess of electricity fed into the grid over a time period. In this case the GSE pays a contribution calculated on the min(*Injected*, *Withdrawn*) electricity in kWh/y (see http://www.autorita.energia.it/allegati/docs/14/612-14.pdf).

8 Final remarks (preliminary)

The development of distributed power plants, in the future, shall be managed through a system that allow for a better integration of renewable energy plants, calling for private actions helping grid management.

In this paper we modelled the investment decision of a prosumer in a PV plant connected to a smart grid. Our findings show that the possibility to sell energy in the local market favours the agent to invest in a plant of bigger size if compared to the one needed for self-consumption and there exists a positive relation between the optimal size and the optimal investment timing. The greater the variance over selling price the shorter the delay in undertaking the investment and the smaller the plant size. In other words, the agent might enter the market relatively earlier, but with smaller size plants. In this respect, it is reasonable to expect that in those area where the grid suffers from congestions or high degrees of production unpredictability, the involvement of the prosumers in the grid management might push investments, making agents do an extra effort to provide the grid with private services on response to price signals: in these zones, actually, the prosumer expects to be called more frequently to contribute to grid management i.e. higher prices/higher volatility are expected. The possibility to sell energy to the local market via the smart grid, increases the investment value. The connection to the smart grid, in turn, increases managerial flexibility: the agent can optimally exercise the option to decide the prosumption quota and switch from prosumption to production, thus increasing the investment value.

As far as further research is concerned, to complicate the analysis, and better capture the value of time in the investment decision, it is possible to consider the buyng price of energy c as a stochastic variable. If expectations on price c enter the analysis, they might strongly affect the decisions whether or not to undertake the investment and on the investment timing. On the one hand, if the energy price c is expected to incraese in the future, the opportunity to invest, ceteris paribus, becomes more valuable, due to increasing savings obtained by prosumption; on the other hand, if price drops are expected, the prosumer might decide to wait and see future price realizations – and not to kill the waiting to invest option.

A Appendix A

Form (9), (10) and substituting α_1^*, we get:

$$\begin{aligned} NPV(v(t)) &= \alpha_1^* \frac{v(t)}{r-\gamma} - \overline{\alpha}(\frac{v(t)}{r-\gamma} - \frac{c}{r}) - (\overline{\alpha} c A v(t)^{\beta_1} + \frac{K}{2}(\alpha_1^*)^2) \\ &= \frac{c}{r}\overline{\alpha} + \frac{v(t)}{r-\gamma}\left(\frac{1}{2K}(\frac{v(t)}{r-\gamma}) - \overline{\alpha}\right) - \overline{\alpha} c A v(t)^{\beta_1} \end{aligned}$$

with $NPV(0) = \frac{c}{r}\overline{\alpha}$ and $NPV'(0) = -\frac{1}{r-\gamma}\overline{\alpha}$.

In order to determine the optimal trigger v^*, we impose the following matching value and smooth pasting conditions:

$$M v^{*\beta_1} = \frac{1}{2K}(\frac{v^*}{r-\gamma})^2 - \overline{\alpha}(\frac{v^*}{r-\gamma} - \frac{c}{r}) - \overline{\alpha} c A v^{*\beta_1} \tag{A.1}$$

$$M \beta_1 v^{*\beta_1 - 1} = \frac{1}{K}(\frac{v^*}{r-\gamma})\frac{1}{r-\gamma} - \overline{\alpha}\frac{1}{r-\gamma} - \overline{\alpha} c A \beta_1 v^{*\beta_1 - 1} \tag{A.2}$$

These can be rearranged as follows:

$$\begin{aligned} Mv^{*\beta_1} &= \frac{1}{2K}(\frac{v^*}{r-\gamma})^2 - \overline{\alpha}(\frac{v^*}{r-\gamma} - \frac{c}{r}) - \overline{\alpha}cAv^{*\beta_1} \\ Mv^{*\beta_1} &= \frac{1}{\beta_1 K}(\frac{v^*}{r-\gamma})\frac{v^*}{r-\gamma} - \frac{\overline{\alpha}}{\beta_1}\frac{v^*}{r-\gamma} - \overline{\alpha}cAv^{*\beta_1} \end{aligned}$$

Let $y = \frac{v^*}{r-\gamma}$,we can reduce the above expression as:

$$y^2(\frac{\beta_1 - 2}{\beta_1}) - 2(\frac{\beta_1 - 1}{\beta_1})K\overline{\alpha}y + 2\overline{\alpha}\frac{c}{r}K = 0$$

Let's rearrange it as:

$$y^2 - 2K\overline{\alpha}y(\frac{\beta_1 - 1}{\beta_1 - 2}) + \frac{\beta_1}{\beta_1 - 2}2K\overline{\alpha}\frac{c}{r} = 0 \tag{A.3}$$

Define now

$$J(y) = y^2 - 2K\overline{\alpha}y(\frac{\beta_1 - 1}{\beta_1 - 2}) + \frac{\beta_1}{\beta_1 - 2}2K\overline{\alpha}\frac{c}{r} \tag{A.4}$$

The function $J(y)$ is convex in y and $J(0) = \frac{\beta_1}{\beta_1 - 2}2\overline{\alpha}\frac{c}{r}K$. The minimum $y^{\min}$ solves the following equation:

$$\begin{aligned} 2y^{\min} - 2(\frac{\beta_1 - 1}{\beta_1 - 2})K\overline{\alpha} &= 0 \\ y^{\min} &= (\frac{\beta_1 - 1}{\beta_1 - 2})K\overline{\alpha} \end{aligned}$$

Note that for $\beta_1 < 2$ we have $y^{\min} < 0$ and $J(0) < 0$, this implies that we have a negative and positive solution to the equation above. The optimal one is the positive solution:

$$\frac{v^*}{r-\gamma} = \frac{\beta_1 - 1}{\beta_1 - 2}\left(\frac{1}{2}\bar{\alpha}K\right) + \sqrt{(\frac{\beta_1 - 1}{\beta_1 - 2})^2\left(\frac{1}{2}\bar{\alpha}K\right)^2 - \frac{\beta_1}{\beta_1 - 2}\frac{\bar{\alpha}c}{r}K} \tag{A.5}$$

For $\beta_1 \geq 2$ we have $y^{\min} > 0$ and $J(0) \geq 0$, this implies that we may have: 1) two solutions, namely $0 \leq y_1 < y_2$ where the first should be the optimal one, 2) one solution, $0 < y_1$ or 3) no solution at all. Let's check the condition for $J(y^{\min}) \geq 0$. By (A.4) it follows that:

$$\begin{aligned} J(y^{\min}) &= (\frac{\beta_1 - 1}{\beta_1 - 2})^2(K\overline{\alpha})^2 - 2K\overline{\alpha}(\frac{\beta_1 - 1}{\beta_1 - 2})K\overline{\alpha}(\frac{\beta_1 - 1}{\beta_1 - 2}) + \frac{\beta_1}{\beta_1 - 2}2K\overline{\alpha}\frac{c}{r} \\ &= K\overline{\alpha}(\frac{\beta_1 - 1}{\beta_1 - 2})\left[-K\overline{\alpha}(\frac{\beta_1 - 1}{\beta_1 - 2}) + \frac{\beta_1}{\beta_1 - 1}2\frac{c}{r}\right] \end{aligned}$$

that is positve if:

$$\overline{\alpha}\frac{c}{r} - \frac{(\beta_1 - 1)^2}{\beta_1(\beta_1 - 2)}\frac{K}{2}(\overline{\alpha})^2 > 0 \tag{A.6}$$

Then, since $\overline{\alpha}\frac{c}{r} > \frac{K}{2}(\overline{\alpha})^2$, condition (A.6) always holds true if $\frac{(\beta_1 - 1)^2}{\beta_1(\beta_1 - 2)}$ is close to one.

B Appendix B

The procedures for validating a Geometric Brownian Motion (GBM) and for estimating the parameters as well as the results are provided as follows:

Step 1: We test for normality of prices returns plotting the sample data of log returns $s(t) = \frac{\ln v(t+1)}{\ln v(t)}$ against the standard normal distribution. This test is automatically provided by statistic software: here we present those performed by STATA on the monthly averages calculated on the photovoltaic interval between 2010 and the first half of 2014 for the geographical zones: North, South, North-Central and South-Central.

Graphical evidences support the hypothesis of lognormality for all the four areas under analysis. Graphical evidences has been supported also by the Shapiro-Wilk test on the variables:

Shapiro-Wilk W test for normal data

Variable	Obs	W	V	z	Prob>z
denord	59	0.97260	1.469	0.829	0.20363
decnor	59	0.93437	3.520	2.710	0.00337
desud	59	0.94570	2.912	2.302	0.01068
decsud	59	0.93358	3.562	2.736	0.00311
desici	59	0.97398	1.395	0.717	0.23655
desard	59	0.97085	1.564	0.962	0.16792

Shapiro-Wilk test

Step 2: We test the presence of the unit root with the Dickey Fuller test, automatically provided by the statistic software STATA.

```
. reg rnord lnnord1

      Source |       SS       df       MS              Number of obs =      59
-------------+------------------------------           F(  1,    57) =    5.86
       Model |  .06759621     1  .06759621           Prob > F      =  0.0187
    Residual |  .657827772    57  .011540838           R-squared     =  0.0932
-------------+------------------------------           Adj R-squared =  0.0773
       Total |  .725423981    58  .01250731            Root MSE      =  .10743

------------------------------------------------------------------------------
       rnord |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lnnord1 |   -.185281   .0765576    -2.42   0.019     -.338585   -.031977
       _cons |   .7769859   .3222351     2.41   0.019     .1317213   1.422251
------------------------------------------------------------------------------

. reg rcnor lncnor1

      Source |       SS       df       MS              Number of obs =      59
-------------+------------------------------           F(  1,    57) =    4.24
       Model |  .048574221     1  .048574221           Prob > F      =  0.0441
    Residual |  .653262101    57  .011460739           R-squared     =  0.0692
-------------+------------------------------           Adj R-squared =  0.0529
       Total |  .701836322    58  .012100626           Root MSE      =  .10705

------------------------------------------------------------------------------
       rcnor |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lncnor1 |  -.1368447   .0664709    -2.06   0.044    -.2699503   -.0037391
       _cons |   .5720771    .278919     2.05   0.045     .0135516    1.130603
------------------------------------------------------------------------------
```

Dickey Fuller test (part a)

```
. reg rsud lnsud1

      Source |       SS       df       MS              Number of obs =      59
-------------+------------------------------           F(  1,    57) =    3.54
       Model |  .043769325     1  .043769325           Prob > F      =  0.0650
    Residual |  .704528812    57  .012360155           R-squared     =  0.0585
-------------+------------------------------           Adj R-squared =  0.0420
       Total |  .748298138    58  .012901692           Root MSE      =  .11118

------------------------------------------------------------------------------
        rsud |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      lnsud1 |   -.118092   .0627549    -1.88   0.065    -.2437564    .0075725
       _cons |   .4861411    .258215     1.88   0.065    -.0309255    1.003208
------------------------------------------------------------------------------

. reg rcsud lncsud1

      Source |       SS       df       MS              Number of obs =      59
-------------+------------------------------           F(  1,    57) =    3.53
       Model |  .036607667     1  .036607667           Prob > F      =  0.0654
    Residual |  .591075631    57  .010369748           R-squared     =  0.0583
-------------+------------------------------           Adj R-squared =  0.0418
       Total |  .627683298    58  .010822126           Root MSE      =  .10183

------------------------------------------------------------------------------
       rcsud |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lncsud1 |  -.1151142    .061267    -1.88   0.065    -.2377993     .007571
       _cons |   .4796751   .2562917     1.87   0.066    -.0335402    .9928904
------------------------------------------------------------------------------
```

Dickey Fuller test (part b)

Step 3: If $v(t)$ is provided to be a GBM process, the volatility can be calculated by $\sigma = \sum_{i=1}^{n} \frac{(s_i - \widehat{s})^2}{n}$, where $\widehat{s}$ is the sample mean of $s(t)$ and the drift term γ can be estimated by performing the regression analysis of $s(t) = \beta t + \varepsilon(t)$ where $\beta = \gamma - \frac{\sigma^2}{2}$ and $\varepsilon(t) = \sigma(z(t+1) - z(t))$. The results are illustrated in Table 1.

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