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ARE HOUSEHOLD PORTFOLIOS EFFICIENT? AN ANALYSIS CONDITIONAL ON HOUSING

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Abstract: In this paper we argue that standard tests of portfolio efficiency are biased because they neglect the existence of illiquid wealth. In the case of household portfolios, the most important illiquid asset is housing: if housing stock adjustments are costly and therefore infrequent, we show how the dynamic optimization problem produces optimal portfolios in periods of no adjustment that are affected by housing price risk (through a hedge term). When the housing stock is not adjusted, we argue that tests for portfolio efficiency of financial assets must then be run conditionally upon housing wealth.

In our application, we use Italian household portfolio data from SHIW 1998 and time series data on financial asset and housing stock returns to assess whether actual portfolios are efficient. We first consider purely financial portfolios and portfolios that also treat the housing stock as another asset. We then consider the consequences of treating the housing stock as given and test for efficiency in this framework. Our empirical results support the view that the presence of housing wealth plays an important role in determining whether portfolios chosen by home-owners are efficient.

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1. Introduction

There has been an increased interest in recent years in household portfolio choice. A number of country studies have looked at the way households allocate their financial wealth across different financial instruments and found that a decreasing but still sizeable proportion of households fail to invest in the stock exchange (Guiso, Haliassos and Jappelli, 2002).

Households allocate their wealth into financial and real assets, but the portfolio allocation problem has typically been addressed empirically focusing solely on financial assets. A few studies have extended the analysis to cover other forms of household wealth, notably own business (Heaton and Lucas, 2000) and housing equity (Flavin and Yamashita, 2002, and Cocco, 2005). Both assets are illiquid, that is subject to non-negligible trading costs. These trading costs are likely to be particularly high for the housing stock for homeowners. When consumption and investment needs differ, and rental markets are imperfect (Henderson and Ioannides, 1983), short run adjustments can be all but impossible. Flavin and Yamashita (2002) stress that in this sense "demand for housing is over-determined", and investment considerations may be of secondary importance.

In this paper we address the issue of efficiency of household portfolios when illiquid housing wealth is also considered. This issue has been investigated by Grossman and Laroque (1990) and more recently by Flavin and Nakagawa (2004). Grossman and Laroque show that the standard CAPM holds in a dynamic setting when households derive utility from just one good that is durable and illiquid (and therefore infrequently adjusted). In their model there are risky financial assets, and also a risk-free asset: given that the numeraire is the durable good, this implies that the nominal return on this asset has unit correlation with the housing return. Flavin and Nakagawa's paper extends Grossman and Laroque's model by allowing for the presence of two goods in the utility function. In their model, there is no correlation between housing returns and financial asset returns. Flavin and Nagakawa prove that over those periods where the housing stock is not adjusted, all households hold a single optimal portfolio of risky assets (the standard Markowitz optimal risky portfolio), the standard CAPM holds and housing wealth affects portfolio allocations only through the relative risk aversion of individual investors. A number of recent papers have produced micro evidence on the role of housing on portfolio allocations within this framework, in which housing wealth contributes to background risk (Flavin and Yamashita, 2002, Kullman and Siegel, 2003, Yamashita, 2003, LeBlanc and Lagarenne, 2004, Cauley, Pavlov and Schwartz, 2005, and Cocco, 2005)

We extend the analysis to cover the case where returns are correlated, and show how efficient financial portfolios should be after allowance is made for the presence of a given housing stock. In these portfolios housing wealth affects the optimal shares in two distinct ways: indirectly, via risk aversion, and directly, via a hedge motive. In particular, we observe that all households will hold as single optimal portfolio of risky assets (the standard Markowitz optimal portfolio) and a hedge term covering house price risk.

On the basis of our theoretical analysis, we expect optimally chosen financial portfolios not to be mean-variance efficient in the standard sense when asset and housing returns are correlated. Also, if the housing stock is not frequently adjusted, we also expect the overall portfolios (that include financial assets and housing wealth) not to be mean-variance efficient. However, we show that optimal portfolios should be conditionally mean-variance efficient, that is mean-variance efficient when housing wealth is treated as given but stochastic. Our analysis provides the economic rationale for implementing the conditional test of mean-variance efficiency that treats housing wealth as predetermined suggested by Gourieroux and Jouneau (1999).

Our paper builds upon recent work by Flavin and Yamashita (2002), but differs from it in a number of important respects. Flavin and Yamashita characterize the efficiency frontier for house owners, when the house cannot be changed in the short run and there are non-negativity constraints on all assets. But they consider the case where financial returns are not correlated with housing returns, and therefore the main effect of housing is to change the background risk faced by investors. We instead allow for non-zero correlation, and show that even without imposing non-negativity constraints the optimal portfolio changes, because investors who are house owners hedge housing price risk. We also formally test for the efficiency of household portfolios, and are able to show that many financial portfolios that appear inefficient when housing is neglected are instead efficient, but many others that appear efficient when the hedge term is neglected are instead inefficient.

Our paper is also closely related to Cocco (2005). Cocco numerically derives solutions of an intertemporal optimization problem that includes a risk-free asset, one risky asset, housing and human capital under borrowing and short sale restrictions. That paper is ideally suited to address the issue of limited participation in the risky assets market, but does not investigate how short-term financial portfolio decisions should be made to hedge housing risk. Not only does Cocco limit the investment set to just one risky asset, but he also assumes zero correlation between housing and financial asset returns, thus ruling our hedging motives. Our paper complements Cocco's analysis, by showing how financial portfolios should be chosen at a given point in time, when housing wealth is given, and investigating whether household portfolios are optimally chosen in the presence of housing wealth risk.

To our knowledge, our paper is the first that formally tests for the efficiency of household portfolios and investigates the role played by housing wealth in making portfolios more or less efficient. In particular, our paper shows to what extent households use financial assets to hedge the risk posed by their housing position.

In our application, we use Italian household portfolio data from the Bank of Italy Survey on Household Income and Wealth (SHIW) for 1998 and time series data on financial assets returns as well as housing stock returns to test the hypothesis that observed portfolios are efficient. We show that in our data there are significant partial correlations between financial and housing returns, and argue that similar patterns can be found in other European countries and also in the US.

The paper is organized as follows: section 2 presents the theory, section 3 discusses the test statistic and econometric issues, section 4 describes the data used, sections 5-7 report estimation and test results, section 8 presents results from robustness analysis and section 9 concludes.

2. Theory

In this section we show how housing wealth can be introduced in the standard mean-variance one-period model – in the Appendix we provide conditions under which our analysis is valid in a multi-period model where housing is not just an investment good but also provides consumption services.

We shall now derive an equation for optimal financial assets holdings in the static mean-variance analysis framework, if the existing housing stock is treated as an additional constraint to the optimization problem (see Mayers (1973) and Anderson and Danthine, 1981, for the general case where an asset is constrained).

Let us consider a market with a risk-less asset, n unconstrained and one constrained risky assets. Denote the first two moments of asset returns as $\underline{m} + r_f$ (where $\underline{m} = \begin{pmatrix} \underline{\mu} \\ \mu_H \end{pmatrix}$ and $\underline{\mu}$ is the expected excess return) and Ω. The variance covariance matrix for excess returns can be decomposed in four blocks, corresponding to the n unconstrained risky assets and the constrained risky asset as follows:

(1) $$\Omega = \begin{bmatrix} \Sigma & \Gamma_{b_i,P} \\ \Gamma'_{b_i,P} & \sigma_P^2 \end{bmatrix}$$

Consider an investor whose portfolio allocation in the risky assets is:

(2) $$Z = \begin{pmatrix} \underline{x_0} \\ h_0 \end{pmatrix}$$

where $\underline{x_0} \equiv \frac{\underline{X_0}}{W_0}$ and $h_0 \equiv \frac{H_0 P_0}{W_0}$

and $(1\text{-}Z)^T \underline{1}$ in the risk-less asset ($\underline{1}$ is an $n+1$ vector of ones). Assume that this investor is constrained in his h_0 (that is h_0 is given and equal to $\bar{h}_0$), but otherwise behaves according to the mean-variance model. The investor problem becomes:

(3) $$\begin{cases} \min\limits_{Z} Z^T \Omega Z \\ s.t. \begin{cases} Z^T \underline{m} + r_f = m^* \\ h_0 = \bar{h}_0 \end{cases} \end{cases}$$

where m^* is a given level of expected return.

The problem can be solved by defining the lagrangian:

(4) $$\Lambda = \left(\underline{x_0} \Sigma \underline{x_o^T} + h_0^2 \sigma_P^2 + 2 h_0 \underline{x_0} \Gamma_{bP} \right) - 2\gamma \left[\underline{x_0} \underline{\mu} + h_0 \mu_H + r_f - m^* \right]$$

The first order conditions are:

(5) $$\frac{\partial \Lambda}{\partial \underline{x_0}} = \left(2\Sigma \underline{x_o^T} + 2h_0\Gamma_{bP}\right) - 2\gamma\left[\underline{\mu}\right] = 0$$

(6) $$\frac{\partial \Lambda}{\partial \gamma} = \underline{x_0}\underline{\mu} + h_0\mu_H + r_f - m^* = 0$$

The solution is:

(7) $$\underline{x_0} = \gamma\Sigma^{-1}\underline{\mu} - h_0\Sigma^{-1}\Gamma_{bP}$$

where γ is the Lagrange multiplier of the constraint on the expected return, that has the standard relative risk aversion interpretation (Samuelson, 1970).

This result means that investors have to be efficient with respect to the risky financial assets and choose the efficient Markowitz portfolio according to their risk aversion (see Markowitz (1992)). However, they also use the risky financial assets to hedge their exposure on the constrained asset. If Γ_{bP}=0 the hedge term vanishes and portfolio choice can be separated between financial and real assets.

3. Econometric Issues

In section 2 we have seen that the notion of efficiency of household portfolios depends on the assumption we make on the nature of housing investment. If housing investment is costless, then the efficient frontier should be computed using all financial assets returns as well as the return on housing[1]. If transaction costs affect housing investment, then the analysis differs according to the correlation between housing and financial returns. If this correlation is zero, household portfolios will be mean-variance efficient in the usual sense (i.e.: with respect to the standard financial assets frontier). If this correlation is instead non-zero, household portfolios will be mean-variance efficient once we condition on the value of the housing stock, as shown in equation (17).

In this section we show how we can test for the efficiency of the observed household portfolios in all cases discussed above. In order to do this, we use time series data on asset returns for a period prior to the survey to estimate the mean variance frontier, taking into account the theoretical assumptions of rational expectations and normal return distributions. In particular, we use weighted sample means and covariances in order to estimate expected excess returns and risk (i.e. the first two unconditional moments). The weights are a declining function of the time distance from the end of the sample period.[2]

In the vast literature on efficient portfolios, only a few papers incorporate real estate as an asset. Goetzmann and Ibbotson (1990) and Goetzmann (1993) used regression estimates of real estate price appreciation, and Ross and Zisler (1991) calculated returns from real estate investment trust funds, to characterize the risk and return to the real estate investment. Flavin and Yamashita (2002) use data from the 1968-1992 waves of the Panel Study of Income Dynamics that contain records on the owner's estimated value of the house and compute rates of return from regional real estate price data.

[1] Housing can be neglected if its return is spanned by financial assets.

[2] Another way is to consider the first two conditional moments from a time series model of the returns data that allows for time-varying conditional heteroskedasticity, as in Blake (1996). This modeling framework requires long time series.

Mean-variance efficiency is usually assessed on the basis of a graphical comparison. However, Jobson and Korkie (1982,1989) and Gibbons, Ross and Shanken (1989) have proposed a test of the significance of the difference between the actual portfolio held by an investor and a corresponding efficient portfolio. This test is based on the difference between the slopes of arrays from the origin through the two portfolios in the expected return-standard deviation space. If the actual portfolio is an efficient portfolio, the two slopes will be the same; if the actual portfolio is inefficient, the slope of the efficient portfolio will be significantly greater.

Gourieroux and Jouneau (1999) derive efficiency tests for the conditional or constrained case, i.e. for the case where a subset of asset holdings is potentially constrained (housing in our case). They define the Sharpe ratio of the unconstrained risky financial assets portfolio as:

$$S_1 = \underline{\mu}^T \Sigma^{-1} \underline{\mu} \tag{8}$$

The Sharpe ratio for the observed (constrained) portfolio made of the first n (financial) assets is defined in this notation as:

$$S_1(Z) = \frac{\left[\underline{\mu}^T v_1\right]^2}{v_1^T \Sigma v_1} \tag{9}$$

where $v_1^T = \underline{x}_0^T + h_0 \Sigma^{-1} \Gamma_{bP}$ (see equation 7), that is the actual risky financial asset portfolio after eliminating the hedge term.

When all asset returns are normally distributed, Gourieroux and Jouneau show that the Wald statistic

(10) $$\xi_1 = T\frac{\hat{S}_1 - \hat{S}_1(Z)}{1+\hat{S}_1(Z)\dfrac{Z^T\Omega Z}{v_1^T\Sigma v_1}}$$

is distributed as a $\chi^2(n\text{-}1)$ under the null hypothesis that the risky financial assets portfolio (after eliminating the hedge term) lies on the financial efficient frontier[3].

Gourieroux and Jouneau also show that a test for the efficiency of the whole portfolio can be derived as a special case by setting $v_1 = Z$. The test statistic becomes

(11) $$\xi_e = T\frac{\hat{S} - \hat{S}(Z)}{1+\hat{S}(Z)}$$

where $\hat{S} = \underline{m}^T\Omega^{-1}\underline{m}$ and $\hat{S}(Z) = \dfrac{\left[\underline{m}^T Z\right]^2}{Z^T\Omega Z}$.

ξ_e is distributed as a $\chi^2(n)$ under the null hypothesis that mean and standard deviation of the observed portfolio lie on the efficient frontier. In this special case, this test is asymptotically equivalent to the test derived by Jobson and Korkie (1982,1989) and Gibbons, Ross and Shanken (1989).

The intuition behind the conditional (constrained) test is the following. The standard test for portfolio efficiency is based on (the square of) the Sharpe ratio. The Sharpe ratio is in fact the same along the whole efficient frontier (with the exception of the intercept), that is along the capital market line. This test breaks down when one asset is taken as given, because the efficient frontier in the mean-variance space

[3] For the sake of simplicity we do not stress in our notation that the test statistic is defined as a function of sample estimates of the first two moments of the rates of return distribution and takes observed portfolio shares as given.

corresponding to all assets is no longer a line, rather a curve. However, equation (7) implies that we can go back to the standard case when the analysis is conducted conditioning on a particular asset, once the hedge term component is subtracted from the observed portfolio. That is, a Sharpe ratio can be used to test for efficiency in the mean variance space corresponding to the "unconstrained" assets, after allowance has been made for the presence of the same hedge term in all efficient portfolios.

It is worth stressing that the test statistic is based on the square of the Sharpe ratio, thus portfolios with Sharpe ratios of the same magnitude but opposite sign are treated in the same way. In our empirical application of the constrained case we shall treat as inefficient those portfolios that have a negative excess return. [4]

In our empirical analysis, we compute efficiency test statistics (either ξ_e or ξ_1) for each household in our sample. In particular, we compute the standard test (ξ_e) twice: once for the financial portfolio (as in standard practice), and once for the whole portfolio (inclusive of housing). In this latter case, we also compute the constrained test (ξ_1).

We use the computed test statistics in two different ways. First, we show what proportions of household portfolios fail the efficiency tests for a range of possible test sizes (from .10 to .01)[5]. Second, we regress the computed test statistic (ξ_1) on household characteristics, income and housing wealth, as a way to investigate possible causes for inefficient portfolio allocations.

[4] We do this after checking that portfolios with the same standard deviation and excess returns just above zero are indeed counted as inefficient by the formal test.
[5] Throughout the paper, we use the term "test size" to denote the probability of type-I error (probability of rejecting the null hypothesis when the null is true). This is sometimes known as significance level.

4. Application.

To show the implications of our theoretical analysis we use data on Italian asset returns and household portfolios. Italy provides a good test case to study the effect of housing on portfolios because home ownership is wide spread and household stock market participation is relatively low but has much increased in recent years. As we shall see, in Italy housing returns unambiguously correlate with financial returns, thus providing the need for a hedge term in house owners portfolios. Finally, an attractive feature of Italy for our purposes is that pension wealth, whose amount is typically not recorded in survey data, is still almost entirely provided by the public pay-as-you-go social security system and is therefore both out of individual investors' control and not directly related to the financial markets performance.

Italian households traditionally have held poorly diversified financial portfolios (Guiso and Jappelli, 2002). In the 1980s and even more in the 1990s, though, the stock exchange has grown considerably and mutual funds have become a commonly held financial instrument. Household financial accounts reveal that the aggregate financial portfolio share in stocks and funds amounted to 16.15% in 1985, 20.69% in 1995 and rose to an unprecedented 46.95% in 1998. This growth in the equity market paralleled the sharp decrease in importance of bank accounts and short-term government debt in household portfolios. These aggregate statistics are uninformative on the participation issue, though. To this end, an analysis of survey data is required. The most widely used Italian survey data, the Bank of Italy-run Survey on Household Income and Wealth (SHIW), shows direct or indirect participation in equity markets (broadly defined to include life insurance, private pensions and own business) to have increased from 26.43% in 1985 to 38.19% in 1995 and to 48.24% in 1998. For comparison, the percentage of homeowners in the same sample hovered around 63-65% over the period. Finally, the share of financial to total wealth in SHIW was 11.7% in 1991 and rose to 14.59% in 1998 – housing wealth accounted for a 68.91% of total wealth in 1991 and fell slightly to 65.81% in 1998 (50.11% to 48.84% if we consider the principal residence only).

These summary statistics clearly show that household financial portfolios have changed a great deal over the years, and that a key role in total household wealth is played by real estate. It makes sense to consider the interaction of housing and financial wealth holdings when assessing the efficiency of household portfolios, as stressed by Flavin and Yamashita (2002). A financial portfolio may deviate from the mean variance frontier for financial assets simply as a result of its covariance properties with the return on housing equity. This is a relevant issue whether housing wealth is treated as liquid or instead as an illiquid asset.

In our application we use household portfolio data for 1998 and asset return data for the period 1989-1998.

The 1998 SHIW wave contains detailed information on asset holdings of 7115 households as of 31.12.1998, as well as self assessed value of their housing stock (both principal residence and other real estate) and actual or imputed rent for each dwelling. For each household we also know the region of residence and a number of demographic characteristics (that are used to characterize departures from efficiency). The survey does not over sample the very rich, and it therefore captures about a third of total household financial wealth. It does cover a relatively large number of assets, including individual pension funds: these are still remarkably unimportant in Italy, though, partly because of inadequate tax incentives. Occupation pension schemes are also relatively minor, even though recent reforms of the Italian Social Security system (particularly the Dini reform of 1995) imply that they should become wide-spread.[6]

Asset return data cover five major assets: short term government bonds (3-month BOT), medium term government bonds, long-term government bonds (BTP), corporate bonds and equity (the MSCI Italy stock index)[7]. We treat the short-term bond as risk free, and assume that this is the relevant return on bank deposits, once account is taken of non-pecuniary benefits. For medium term, long term and corporate bonds we derive the holding period returns by standard methods. In particular, for medium term we use the RENDISTAT index (the index of the medium term government bonds yields) and we determine the holding period return by assuming a duration of two years. For corporate bonds we use the RENDIOBB index (the index of Italian corporate bond yields) and assume a duration of three years. For long term bonds we use the estimated term structure of interest rates and determine the holding period returns of an equally weighted portfolio based on two assets with a duration of three years and five years. We checked the quality of this estimation by regressing our monthly returns determined with this procedure on those of the MSCI Italian bond index (that are only available since December 1993) and found that the fit is almost perfect (R^2 is equal to 99.62%).

We express all returns net of withholding tax, on the assumption that for most investors other tax distortions are relatively minor (financial asset income is currently subject to a 12.5% withholding tax. Housing is taxed on the basis of its ratable value, while dividends on stocks directly held and actual rental income is taxed at the marginal income tax rate).

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[6] Further information on the survey is provided in Guiso and Jappelli, (2002) and D'Alessio and Faiella (2000). Information on the Italian pension system and its recent reforms is presented in Brugiavini and Fornero (2001).

[7] We take returns on the Italian stock exchange because, as we shall see later, direct investment in foreign assets is rare in our data. We have checked that our results are robust to assuming that roughly half of

To evaluate the efficiency of households' portfolio we need to determine the expected return and the expected variance covariance matrix of the assets. Given long, stationary series we could simply compute the corresponding sample moments of the assets excess returns. However, this approach is unlikely to work in our case: our sample period is 1989-98 (and cannot be extended because some assets did not exist prior to the mid 1980's), and in the decade we consider we observe a long convergence process of Italian interest rates to German interest rates that accelerated dramatically in the few years before the introduction of the Euro on January 1999.

Estimation error is of particular concern for first moments and calls for use of prior information in estimation (see for instance Merton, 1980, and Jorion, 1985). In our case, we should estimate the first moments by a Bayesian method that exploits prior information on convergence of particularly long-term government bond rates to its German equivalent, and possibly a multivariate GARCH for the second moments. Unfortunately, we do not have enough data points to perform sophisticated estimation exercises. In fact, housing returns are available at a semiannual frequency, and we are therefore forced to use at most twenty-one data points. However, we can exploit prior information on convergence by using a simple Weighted Least Squares procedure, where the raw return series data are down weighted more the farther away they are from December 1998. More precisely, we construct the weights to be a geometrically declining function of the lag operator multiplied by α (where α is set to 0.8). The weights are then multiplied by a constant so that the expected returns on long term

indirect investment in stocks is held in the MSCI world market index, in line with aggregate statistics on Italian mutual funds portfolios in 1998.

government bills are in line with the actual returns of the German Treasury bond in 1998-9. The weighted series are used to compute sample first and second moments[8].

In Table 1 we show the first and second moment of the excess returns data we use. These are expressed as percentage semi-annual rates of return net of the time-varying risk-free rate: for the risk-free rate we report only the January 1999 six month Italian Treasury bill interest rate.

Table 1: Sample first and second moments of asset excess returns (1989-98)

	BOT	BTP	MTG-Bonds	Corporate Bonds	Stocks
Expected return %	1.3169	0.8021	0.427	0.4495	2.2932
Standard Deviation %		1.2223	0.6469	0.7809	7.4875

Note: the risk-free return refers to the second half of 1998

CORRELATION	MTG-Bonds	Corporate Bonds	Stocks
BTP	0.965**	0.842**	0.379
MTG-bonds		0.871**	0.383
Corporate Bonds			0.635**

Note: ** significant at 1% level

We see that stocks have higher expected return and higher variance than all other risky financial assets. Correlation coefficients between bonds are quite high (they range between .84 and .97) – correlation coefficients of stocks and bonds are much lower (between .38 and .64). Most correlation coefficients are significantly different from zero at the 1% level.

This picture is however largely incomplete. We know that two households out of three own real estate, and we argued that this type of investment is highly illiquid. It is therefore of great interest for us to compute first and second moments of the housing

[8] A similar procedure for second-order moments is often used in the financial industry (see RiskMetrics, 1999) and can be shown to be equivalent to particular GARCH models (Phelan, 1995).

stock. To this end we use province-level semiannual price data (source: Consulente Immobiliare[9]) covering the whole 1989-98 period. We compute the return on housing according to the formula:

$$(12) \qquad r_{H,t} = \frac{P_t - P_{t-1}}{P_{t-1}} + \frac{D_t - COM_t}{P_{t-1}} = \frac{P_t - P_{t-1}}{P_{t-1}} + \kappa$$

where *D* denotes rent and *COM* maintenance costs. Given that we lack time series information on these, we set κ=.025 (5% on an annual basis), as in Flavin and Yamashita (2002). It is worth stressing that the choice of κ is immaterial in the analysis of the constrained case, as long as κ is a fixed number (see equation (10)). It becomes important in the case where housing is treated as unconstrained, given that it affects its expected return directly. However, if rental income is time-varying, real estate indices based on observed house prices are flawed, as stressed by De Roon, Eichholtz and Koedijk (2002).

Finally, we aggregate housing returns to the macro-region level (provincial resident population numbers were used to generate weights). This way we generate average return data for the North West, North East, Centre and South (inclusive of the Islands).[10] The first and second moment are then determined using the Weighted Least Squares procedure described above.

Table 2 reveals that expected excess returns on housing are highest in the North East and in the South and lowest in Central Italy (they range between 0.73% and 0.61%

[9] Index constructed using repeat sales of houses.

[10] If we wanted to increase the number of observations, we could generate quarterly price data by using rent price index aggregate data and regress changes in the house price on changes in rent to fill in the gaps in the series. However, the fit of these regressions is relatively low, and this implies that the quarterly return data would be affected by a low signal to noise ratio. Given our interest in second moments this is potentially a serious problem, and we prefer to use semiannual data throughout.

on a semiannual basis). They are close to returns on bonds, but are much lower than returns on stocks. Housing excess return standard deviations range between 0.46% and 0.78%, and are therefore much lower than on stocks, but comparable to Government and Corporate Bonds. Of interest to us is the negative correlation between housing return and most financial asset returns. This is also found in the raw series, that is in the series that are not weighted in the way described above.

Table 2: Expected excess returns and correlation matrix of housing (1989-98)

	NW	NE	CE	SO
Expected excess return %	0.6143	0.7108	0.6517	0.7303
Standard deviation %	0.7816	0.4607	0.5439	0.4986

	NW	NE	CE	SO
BTP	0.018	-0.140	-0.237	-0.274
MTG-bonds	-0.0752	-0.246	-0.355	-0.142
Corporate bonds	-0.150	-0.086	-0.524*	-0.245
Stocks	-0.671**	-0.270	-0.675**	0.031

Note: ** significant at 1% level, * significant at 5% level

The issue arises of whether these correlations are negligible. Some of the simple correlation coefficients are significantly different from zero (for the NW and CE regions). But simple correlation is not the relevant concept for our analysis: partial correlations are important in a multiple asset setting. The simplest way to assess the relevance of partial correlations is to estimate the coefficients of the hedge term in equation (17), that is to estimate the beta hedge ratio $\Sigma^{-1}\Gamma_{bP}$. This can be done by running the regression of housing returns on financial asset returns, as suggested by de Roon, Eichholtz and Koedijk (2002). In our case we use WLS instead of OLS for internal consistency, but stress that OLS point estimates are similar[11]. Parameter estimates and their standard errors are summarized in Table 3.

Table 3: Regression of excess return on housing on financial assets excess returns

Variable	NW	NE	CE	SO
Constant	-0.00141 (.00107)	-0.0003 (.00095)	-0.00041 (.00095)	-0.00037 (.00106)
r_{BTP}	0.928275 (.28242)	0.559817 (.24867)	0.714788 (.25063)	-0.82673 (.27920)
r_{MTG}	-2.38735 (.60929)	-1.88857 (.53646)	-1.29275 (.54069)	2.014983 (.60233)
r_{BONDS}	1.080841 (.3165)	0.848318 (.27867)	-0.11126 (.28087)	-0.73687 (.31289)
r_{STOCKS}	-0.12004 (.01756)	-0.04496 (.015464)	-0.04314 (.01559)	0.0354 (.01736)
p-value	0.000034	0.014788	0.001414	0.02506
R^2	0.784422	0.518884	0.649422	0.482323

Notes: Standard errors in parentheses. Number of observations = 21

We see that in all regions there is at least one non-zero parameter at the 5% significance level and the slope coefficients are jointly significantly different from zero at the 5% level or lower (the p-value of the F-test is reported at the bottom of the table, together with the R^2). The region where this test is least significant is the South (with a p-value of 2.5%).

On the basis of this evidence, we conclude that housing returns present significant correlations with financial asset returns in Italy, and that this provides the basis for introducing a hedge term in household portfolios of house-owners. [12]

[11] Significant coefficients retain their signs, but their magnitude and standard errors are inflated.

De Roon, Eicholtz and Koedijk (2002) find that a similar result is also true for some areas the U.S., but do not analyze the efficiency of U.S. household portfolios. We also find evidence, available upon request, of significant correlations with excess returns on at least some financial assets in other European countries (France, Germany, Spain and the UK). In Tables 4 and 5 we report the percentage participation for each asset and liability recorded in SHIW98 and the corresponding aggregate portfolio share. For instance, we see that almost 75% of the sampled households have a bank current (i.e. checking) account, and that the 27.24% of all financial wealth is held in such accounts.

We also show in the last column of Table 4 where each asset is classified, given that we use asset returns data at a much coarser aggregation level. So the first seven assets (cash, various deposits, and repos) are all classified as risk-free. Of particular interest is the relatively low direct stock market participation (7.42% hold listed shares; 1.58% shares in unlisted companies).

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[12] In our analysis we assume that the relevant stock return is purely domestic. However, according to Bank of Italy aggregate statistics Italian equity mutual funds invested 52% in foreign stocks, 48% in the domestic stock exchange in the fourth quarter of 1998. Unfortunately, we do not know how household indirect equity holdings were split between domestic and foreign stocks, and across countries, but we can run a robustness check. Given that direct stock holdings by SHIW households were mostly domestic, we assume a 50-50 domestic-foreign split in household portfolios, and take as stock return the average of the Italian stock exchange return and the MSCI world stock index return (in Italian Lira). The simple correlation between the domestic return and this mixed stock return is .88; compared to the domestic return, the mixed return has a lower first moment (2.08% rather than 2.29%), but also a lower standard deviation (5.22% rather than 7.49%) resulting in a larger Sharpe ratio. The key regressions of the housing returns on financial asset returns produce results quite similar to the ones shown in Table 3: the coefficients on the stock return lie in the (-.07,-.14) interval for the NW, NE and CE regions, and are all significant. For the South, we find a significant, positive coefficient of .07. The rest of our analysis is largely unaffected. We thank the referee for suggesting this check to us.

Table 4: Participation decision - individual financial and real assets

Asset	**Participation**	**Broad Asset**
Cash	100%	Risk-free
Bank Current Account Deposits	74.94%	Risk-free
Bank Savings Deposits (Registered)	19.31%	Risk-free
Bank Savings Deposits (Bearer)	10.90%	Risk-free
Certificates of deposit	3.68%	Risk-free
Repos	0.94%	Risk-free
Post Office Current Accounts and Deposit Books	11.43%	Risk-free
Post Office Savings Certificates	6.55%	Long-Term
BOT (Italian T-bills)	9.67%	Risk-free
CCT (Italian T-certificates)	4.74%	Risk-free
BTP (Italian T-bonds)	2.70%	Long-Term
CTZ (Italian zero-coupon)	0.78%	Medium-Term
Other Italian Government Debt (CTEs, CTOs, etc.)	0.31%	Medium-Term
Corporate Bonds	5.55%	Bonds
Mutual Funds	10.86%	Bonds (1/2) Stocks (1/2)
Shares of listed companies	7.42%	Stocks
of which: of privatized companies	4.30%	Stocks
Shares of unlisted companies	1.58%	Stocks
Shares of limited liability companies	0.53%	Stocks
Shares of partnerships	0.15%	Stocks
Managed Savings (by banks)	2.03%	Bonds (1/2) Stocks (1/2)
Managed Savings (by other financial intermediaries)	0.5%	Bonds (1/2) Stocks (1/2)
Managed Savings by Trust Companies	0.06%	Bonds (1/2) Stocks (1/2)
Foreign bonds and government securities	0.52%	Bonds (1/2) Stocks (1/2)
Foreign Stocks and Shares	0.46%	Bonds (1/2) Stocks (1/2)
Other foreign assets	0.05%	Bonds (1/2) Stocks (1/2)
Loans to co-operatives	1.67%	Stocks
House	69.76%	House
Mortgage	10.41%	Long Term (neg. position)
Debt	12.33%	Bonds (neg. position)

Table 5: Portfolio share - individual financial and real assets

Asset	**Portfolio share (financial assets)**	**Portfolio share (financial assets + House)**
Cash	2.13%	0.31%
Bank Current Account Deposits	27.24%	2.86%
Bank Savings Deposits (Registered)	4.94%	1.00%
Bank Savings Deposits (Bearer)	2.75%	0.48%
Certificates of deposit	2.52%	0.50%
Repos	1.19%	0.25%
Post Office Current Accounts and Deposit Books	2.54%	0.38%
Post Office Savings Certificates	2.00%	0.31%
BOT (Italian T-bills)	7.64%	1.22%
CCT (Italian T-certificates)	3.92%	0.58%
BTP (Italian T-bonds)	2.14%	0.37%
CTZ (Italian zero-coupon)	0.31%	0.06%
Other Italian Government Debt (CTEs, CTOs, etc.)	0.34%	0.04%
Corporate Bonds	4.92%	0.74%
Mutual Funds	13.99%	2.25%
Shares of listed companies	5.90%	0.99%
of which: of privatized companies	1.86%	0.29%
Shares of unlisted companies	0.77%	0.12%
Shares of limited liability companies	2.19%	0.26%
Shares of partnerships	1.30%	0.16%
Managed Savings (by banks)	6.62%	1.23%
Managed Savings (by other financial intermediaries)	1.53%	0.31%
Managed Savings by Trust Companies	0.04%	0.01%
Foreign bonds and government securities	0.25%	0.08%
Foreign Stocks and Shares	0.14%	0.03%
Other foreign assets	0.00%	0.00%
Loans to co-operatives	0.80%	0.14%
House		85.05%
Mortgage		-2.07%
Debt		-0.54%

However, 10.86% of all households have mutual funds, and these holdings we classify partly as stocks and partly as bonds. Of great interest to us is the high proportion of households who own some housing stock (almost 70%) and the magnitude of this type of investment (that accounts for 85% of total wealth). Liabilities are relatively wide-spread (10.41% households report mortgage; 12.33% other forms of consumer debt), but their quantitative importance is relatively minor.

In Table 6, we treat mortgages as negative holdings of long-term bonds (the only long term bonds available are on government debt, BTP) and other debt as negative holdings of corporate bonds (other debt typically has medium term maturity like corporate bonds). On this basis we re-classify our households in 4 mutually exclusive groups. We then show how this classification changes according to the broad region: we follow standard practice and split the country in North West (that includes the three large industrial cities of Milan, Turin and Genoa), the North East (that includes many middle-sizes cities and towns, such as Bologna, Venice, Verona, Trieste), the Centre (that includes the capital city, Rome, and many medium-sized town such as Florence, Perugia and Ancona) and the South (largely rural, but including Naples and Bari). The two large islands, Sicily and Sardinia, are also counted as South here.

We see that the highest proportion of risk-free asset portfolios (30.1%) is found in the South, SO, the lowest in the Centre, CE (24.2%). The combination of risk-free and housing assets is highest in the SO (49.4%), lowest in the NW (33.8%). The combination of risk-free and risky financial assets (included debts) is most common in the North East, NE, (5.6%), whereas the presence of all three assets is most common in the NE (36.2%) and least common in the SO (only 18.4%).

Table 6 – Classification by Region.

	Total		**NW**		**NE**		**CE**		**SO**	
	n°	**%**	**N°**	**%**	**N°**	**%**	**n°**	**%**	**n°**	**%**
Risk-free asset	1567	26.47%	385	27.13%	217	20.43%	291	24.25%	674	30.10%
Risk-free asset + housing	2499	42.21%	479	33.76%	402	37.85%	511	42.58%	1107	49.44%
Risk-free + risky assets	223	3.77%	78	5.50%	59	5.56%	41	3.42%	45	2.01%
Risk-free + risky assets + housing	1631	27.55%	477	33.62%	384	36.16%	357	29.75%	413	18.45%
Total assets	**5920**	**100%**	**1419**	**100%**	**1062**	**100%**	**1200**	**100%**	**2239**	**100%**

5. Estimation and test results: financial assets portfolios.

First we show mean variance frontier for financial assets alone, using sample averages and variances. We follow the literature and neglect both housing wealth and mortgages and debts. Given that the latter are mostly incurred to purchase housing stock, this is the most natural course of action when analyzing purely financial decisions.

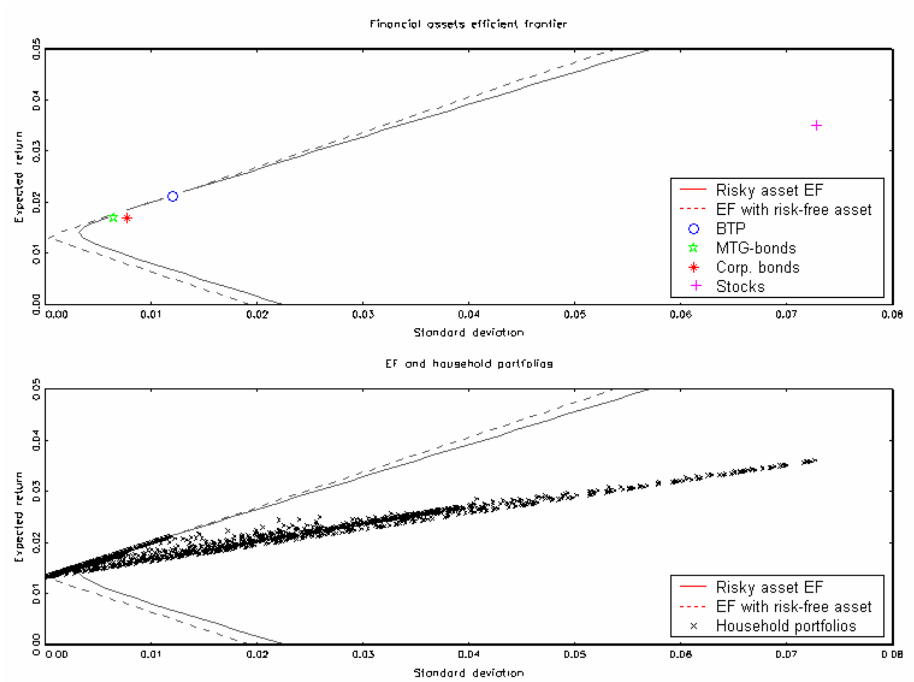

Figure 1 – Risky financial assets efficient frontiers, efficient frontier with the risk-free asset and households portfolios.

In the upper panel of Figure 1 we show the risky financial assets efficient frontier and the efficient frontier with the risk-free asset (this is a broken line). Individual assets are also displayed there: to the far right we have stocks (+ sign), to the extreme left of the risky frontier we find corporate bonds (denoted by a *). In the lower panel we show where individual portfolios lie. Notice that households who have a financial portfolio are 5920 in total: 76.92% of these only have the risk-free asset while 23.08% also have risky assets.

Table 7: Mean-Variance Efficient Portfolio Weights

	Weights
BTP	0.2923
MTG Bonds	0.8932
Corporate bonds	-0.2030
Stocks	0.0175

The tangency of the upper portion of the broken line and the risky financial assets financial frontier defines the market portfolio. In Table 7 we report its weights: the mean-variance efficient portfolio is made of long positions in BTP (long-term government bonds) MTG bonds and stocks, and short position in Corporate bonds[13].

In Table 8 we report the results of a formal efficiency test (described in Section 3) of observed household portfolios. The test statistic is computed for all valid observations (households whose wealth is not entirely in cash) and the percentages of non-rejections are computed at different values of the test size (from 10% to 1%).

Table 8. Efficiency Test – Financial assets only

test size	10%		5%		1%	
	N	%	N	%	N	%
Whole sample	5166	87.26%	5920	100.00%	5920	100.00%
Risk-free only	4554	100.00%	4554	100.00%	4554	100.00%

Note: The table reports the number and % of cases where efficiency is not rejected

[13] In the literature, the short positions have attracted attention (see Jappelli, Julliard and Pagano, 2001), and the argument has been made that one should consider the constrained efficiency frontier where negative holdings are not allowed. We, however, are interested in testing for efficiency, and for this reason consider here the unconstrained frontier. An extension to the case where there are constraints on some assets is discussed in Section 8. A possible way out is to aggregate medium term government bonds and corporate bonds, given their similar duration and the high correlation of their returns. Our key empirical results do not change much when we follow this route, but the tangency portfolio has all positive weights (.1747, .8032, .0221). We do not aggregate these two assets, because in our conditional analysis we treat mortgages as negative corporate bond positions - a negative equilibrium value of corporate bonds is thus possible.

We see from Table 8 that all portfolios containing just the risky assets are (trivially) efficient. Results for those households who hold at least some risky assets are summarized in Table 9: the Table highlights that at most 45% of all risky asset portfolios are efficient (when the test size is set at 10%).

Table 9. Efficiency Test. Diversified Portfolios only

test size	**10%**		**5%**		**1%**	
	N°	%	N°	%	N°	%
Risky financial assets	612	44.80%	1366	100.0%	1366	100.0%

It is perhaps surprising that all portfolios are considered efficient when the test is run at the 5% or 1% levels (the test size is 5% or 1%). This probably reflects three different facts:

a) most households do not invest in stocks, in line with the tangency portfolio;
b) returns on bonds are highly correlated – optimization errors on their shares do not result in major efficiency losses;
c) the efficient frontier is estimated using a limited number of observations, and is therefore estimated with limited precision.

All this suggests that the test may have relatively low power, and the appropriate test size should be chosen at a conservative 10%.

6. Estimation and test results: housing and financial assets.

We show in Figures 2-5 the mean variance frontier for financial assets and housing: given that we know where the households live and house prices differ by region, we compute sample averages and variances for each broad region. We now treat outstanding mortgages as negative holdings of long-term bonds (BTP) and debts as negative holdings of medium term (corporate) bonds. In this Section we disregard transaction costs on housing and therefore treat housing as fully unconstrained.

In the upper panel of Figure 2 we show the risky assets efficient frontier and the efficient frontier with the risk-free asset (this is a broken line) for households living in North Western Italy. Individual assets are also displayed there: to the far right we still have stocks (+ sign), to the extreme left of the risky frontier we find MTG bond (denoted by a star) and corporate bonds (denoted by a *). Just above corporate bonds is housing (denoted by a square). Even though corporate bond seems to be a dominated asset, we know from Tables 1 and 2 that its standard deviation is actually lower than the standard deviation on the house.

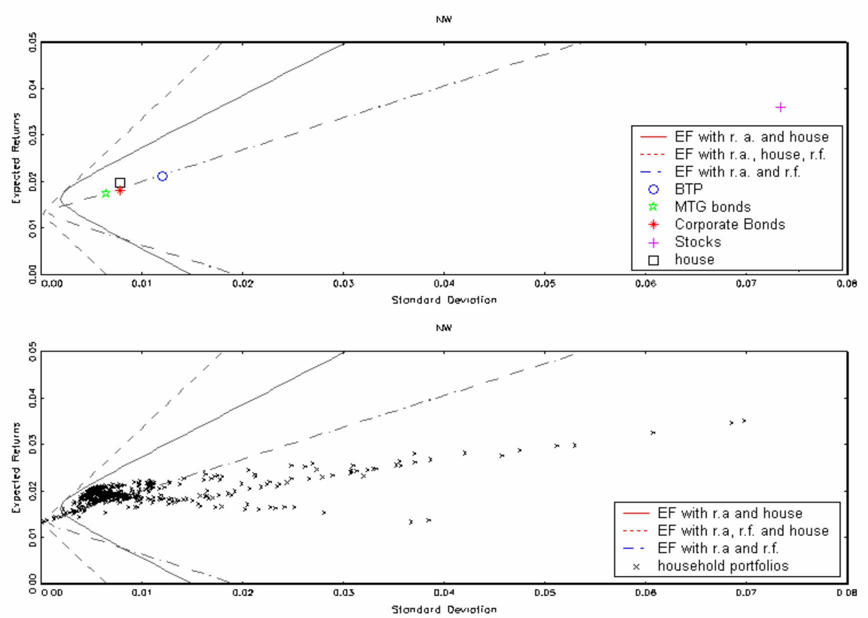

Figure 2 – NW Efficient frontiers with housing

Also, its highly positive correlation with MTG bonds, BTP and stocks gives its short position some insurance value. This is borne out by the mean-variance efficient portfolio weights: as Table 10 shows, the optimal portfolio weight for housing in the NW region is 60% (and the BTP weight falls relatively to the purely financial portfolio shown in the first column). This high wealth percentage in housing is of course largely explained by our assumption that housing rental rate is as high as 5% in real terms.

Table 10: Mean-Variance Efficient Portfolio weights with Housing

	financial assets	NW	NE	CE	SO
BTP	0.2923	-0.5462	-0.3324	-0.3829	1.4222
MTG-bonds	0.8932	1.5346	1.21856	0.7724	-3.099
Corporate bonds	-0.2030	-0.6615	-0.5252	0.0476	1.1269
Stocks	0.0175	0.0735	0.02842	0.0243	-0.052
House		0.5995	0.61071	0.5386	1.6027

In the lower panel we show where individual portfolios lie. A formal efficiency test is discussed later.

Figure 3 displays the efficient frontier for the North East. As we know from Table 2, the figure shows housing investment has higher expected return and lower standard deviation than in the NW. Its optimal portfolio weight is higher (almost 61%).

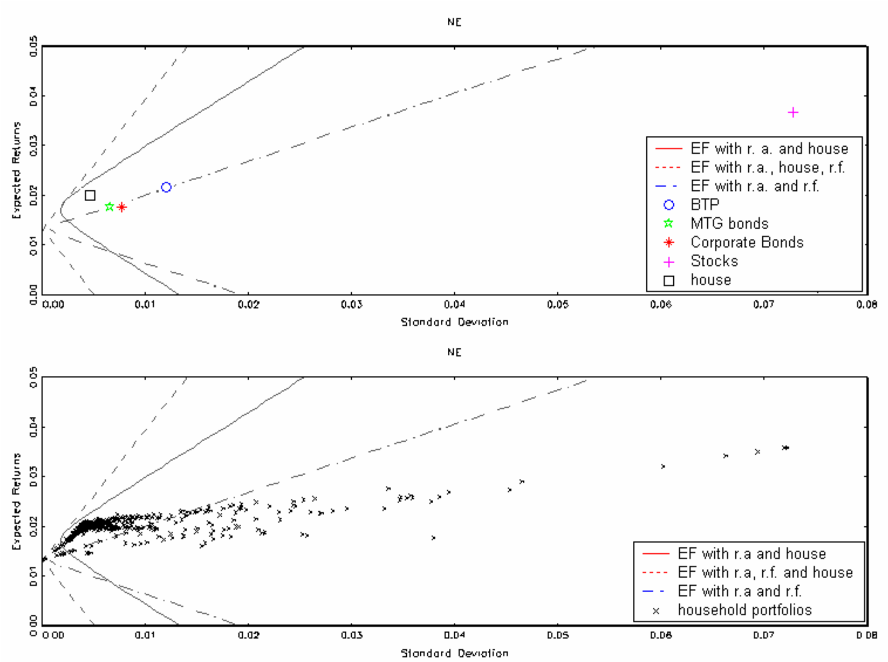

Figure 3 – NE Efficient frontiers with housing

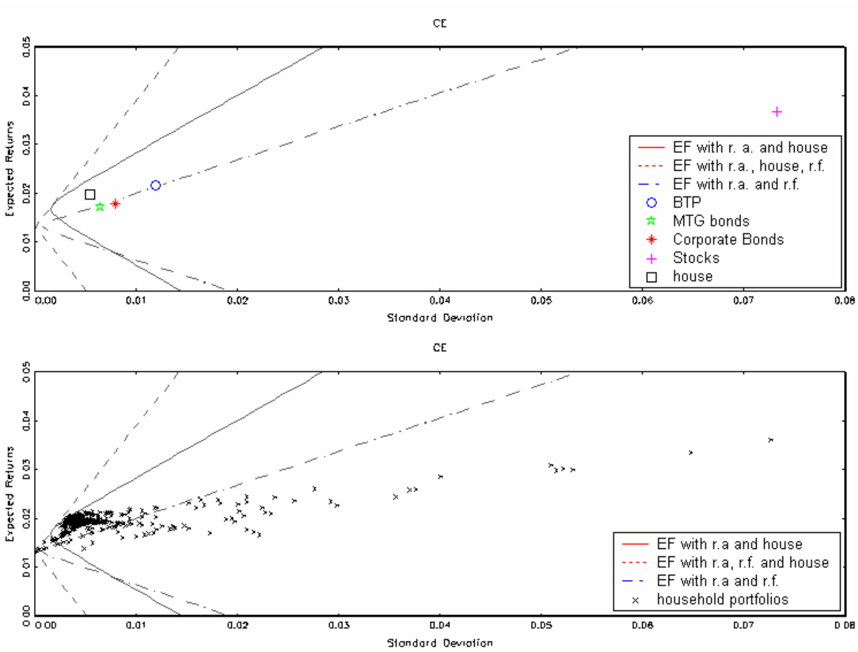

Figure 4 – CE Efficient frontiers with housing

Figure 4 represents the efficient frontier for Central Italy. The house in this case is less attractive as an asset and its optimal portfolio weight is lower than in other macro-areas (54%). For the South the picture is quite different (see Figure 5): the housing expected return is quite large (similarly to NE). As a result the optimal portfolio has an extremely large weight on housing (160%).

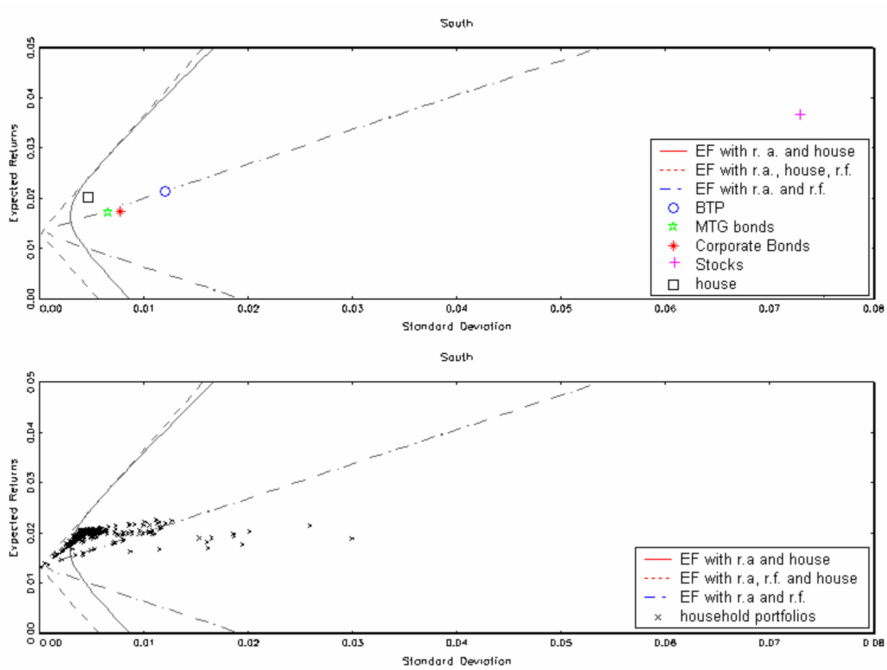

Figure 5 – SO Efficient frontiers with housing

We do not display formal efficiency tests in this case, because the test results are not particularly informative: at any size of the test, the only efficient portfolios are those made just of the risk-free asset (but for a handful of observations: 1 at 10%, 11 at the 5% and 17 at 1% level).

7. Estimation and test results: financial assets conditioning on housing.

Our results so far can be summarized as follows:

- When we consider only financial assets, household portfolios are mostly trivially efficient (because 76% of the sample hold just the risk free asset). Of the diversified portfolios, at most 45% are mean-variance efficient at the 10% level.
- When we take a broader set of assets and liabilities (housing, mortgages and debt) into consideration, many more households hold diversified portfolios (a common combination is the risk-free asset and housing). However, only a few diversified household portfolios are now found to be efficient.

We have already argued (see Section 2) that the illiquid nature of housing should be taken into account. If consumers hold a large fraction of their wealth in housing for reasons other than investment (because rental markets are imperfect, due to information asymmetry, as argued by Henderson and Ioannides, 1983), and do not trade frequently because of high pecuniary and non-pecuniary costs (Flavin and Nakagawa, 2004), then we should investigate their portfolio efficiency conditional on housing. It is in fact possible (and plausible) that their financial decisions are partly dictated by the need to hedge some of the risks connected with their illiquid housing investment.

For each household who has non-zero housing wealth we can compute a specific conditional efficiency frontier that treats housing as constrained (for those without housing the frontier displayed in Section 5 still applies). It's worth stressing that in the constrained case the risk-free portfolio cannot be attained, except trivially (zero housing). This explains why the efficient frontiers we display in Figures 6-9 are not broken lines, contrary to what we have in Sections 5 and 6. In each Figure we display the unconstrained and a few constrained frontiers, corresponding to a random sub-sample of house-owners whose actual portfolio is also shown (marked with a plus sign).

To illustrate, consider Figure 6. This depicts the unconditional frontier with housing for the North West: the presence of a risk-free asset makes it a broken line. In the Figure, we also show two constrained frontiers for the same region, corresponding to two different shares of housing to total wealth (the frontier marked 1 has 18% of wealth into housing; the frontier marked 2 has 47% of total wealth into housing. They correspond to two observed portfolios, displayed as points x_1 and x_2). These frontiers lie entirely to the right of the unconstrained frontier (in general, there could be a tangency point, corresponding to the case where the housing portfolio share is at its optimal value). They do not touch the vertical axis, because a risk free position cannot be achieved with positive housing wealth, given the correlations shown in Figure 2.

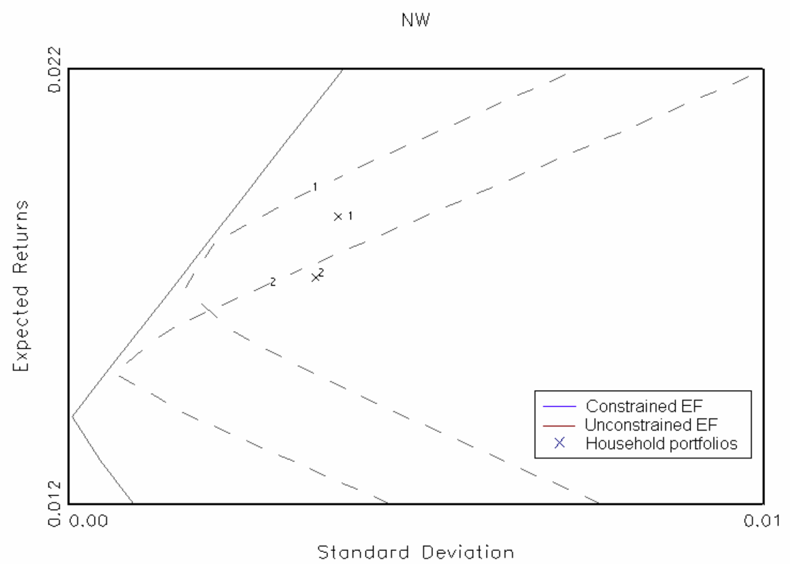

Figure 6 – NW efficient frontiers conditional on housing

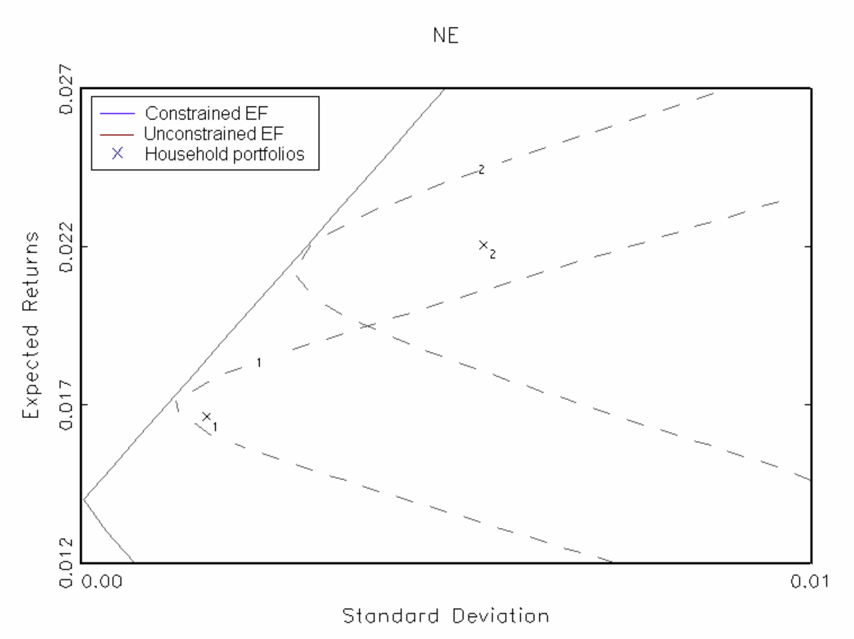

Figure 7 – NE efficient frontiers conditional on housing

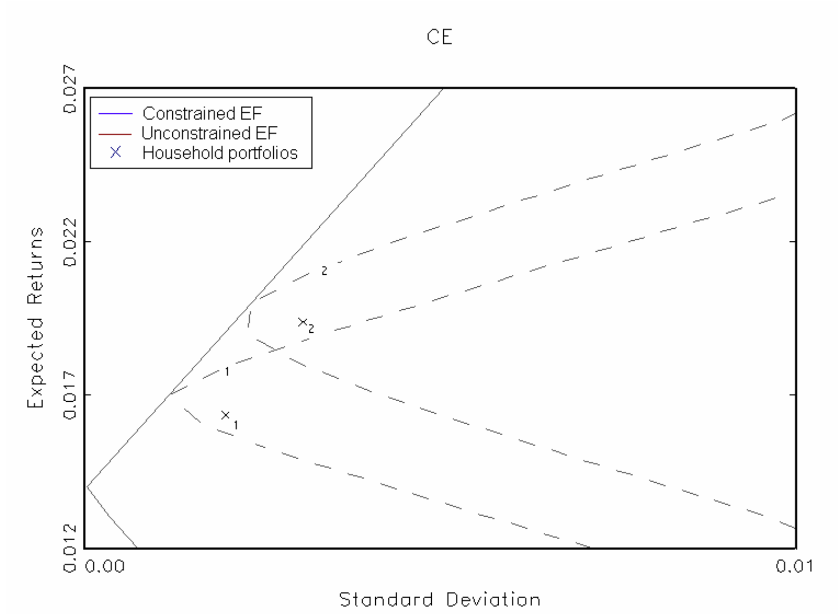

Figure 8 – CE efficient frontiers conditional on housing

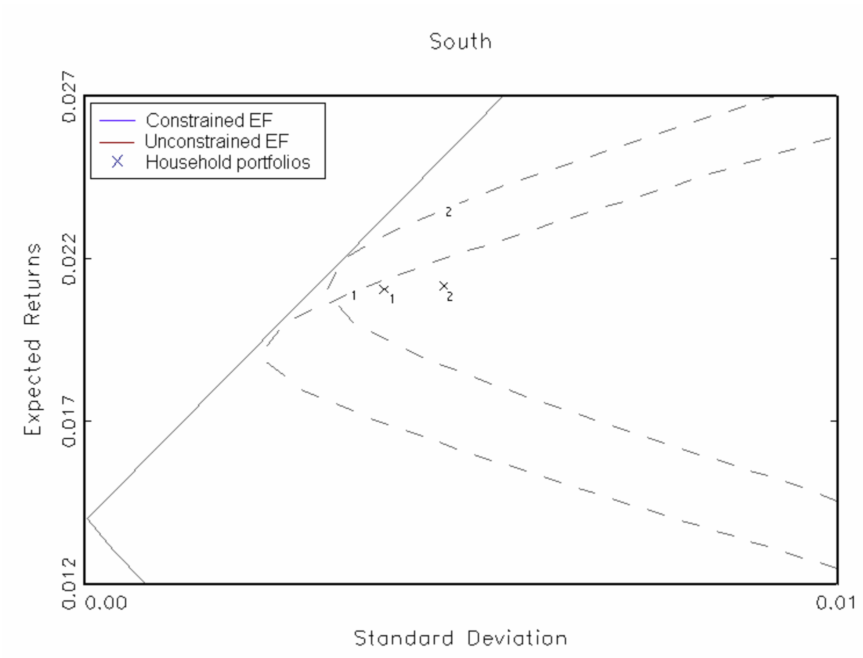

Figure 9 – SO efficient frontiers conditional on housing

We can now compute the test statistic for the conditional portfolios, ξ_I (defined in equation (10), Section 3), and calculate for how many portfolios the test fails to reject the null hypothesis of mean variance efficiency. The test is not defined in the case of portfolios made entirely of risk-free assets (it is a ratio of zero to zero – but in so far as the risk-free asset belongs to the capital market line, we might classify them as all efficient), and is identical to the standard test of Section 6 for portfolios consisting of just financial assets.

In the case of portfolios consisting of risk free plus housing, the test statistic takes the same value within the same region by construction. Also, in our application the expected return for all these portfolios (net of the hedge term) within regions is negative. Therefore, all portfolios with housing but no financial assets are inefficient.

When we consider housing as a constraint, we classify a much smaller number of households in the risk-free portfolios category: 1567 instead of 4554. In fact, of those without risky financial assets, house-owners without a mortgage are now classified in the risk-free + house category (2499 households in all), house-owners with a mortgage or debt (491) could be classified in the last category (risk free+ house+ financial assets), because the mortgage is treated as a negative position on long term government bonds, but we keep them separate in our analysis because all such portfolios turn out to be inefficient for all test sizes. Therefore, the diversified portfolios we consider conditional on house holding are 1140.

Table 11 reports efficiency results for the 1363 households who have diversified portfolios[14] (223 with have a well-diversified financial portfolio, but no housing, and 1140 well-diversified financial portfolio and housing). We see that the test fails to reject efficiency in 261 cases (19%) at the 10% significance level, and this number rises to 595 (44%) at the 5% level and 901 (66%) at the 1% level. Not surprisingly, we find that conditioning on housing many more portfolios are efficient than treating housing as an unconstrained asset (as stressed graphically in Flavin and Yamashita, 2002).

Table 11. Efficient portfolios conditional on housing

Test size		**10%**		**5%**		**1%**	
Portfolios	**Tot. N.**	**N**	**%**	**N**	**%**	**N**	**%**
Risk-free + Risky fin. Ass.	**223**	104	46.64%	223	100.00%	223	100.00%
Risk-free + Risky fin. ass + Housing	**1140**	157	13.78%	372	32.63%	678	59.40%
Total	**1363**	**261**	**19.15%**	**595**	**43.65%**	**901**	**66.10%**

If we look at the group of households who have a well-diversified financial portfolio (but no housing), we find that 46.64% of these portfolios are efficient when the test is conducted at the 10% significance level. It’s worth stressing that the households that fall in this category are just 223.

In the more interesting case, where the household holds both housing and risky financial assets (1140 observations), we find that 157 cases are efficient at the 10% significance levels (14%). When we run the test at the 5% significance level, we find that 33% of these households hold efficient portfolios (372 in all, see Table 11). This number rises to 678 (59% of the group) at the 1% level.

The efficiency test results displayed in Table 11 suggest that a non-negligible proportion of house owners hold portfolios that are not far from their conditional (or constrained) mean variance frontier. This is in stark contrast to the case where housing is treated as a freely-chosen asset (the unconditional test discussed in section 6).

Let us now consider the 1140 fully diversified portfolios (risk free, risky financial assets and housing). In Table 12 we cross tabulate diversified financial portfolios and total conditional portfolios according to the efficiency criterion (at the 10% level for both test statistics):

Table 12. How diversified portfolios are classified: a comparison

Test size = 10%	Efficient (Financial)	Inefficient (Financial)	Total
Efficient (conditional)	**71**	86	157
Inefficient (conditional)	437	**546**	983
Total	508	632	1140

We find that as many as 437 portfolios are classified as efficient when housing is neglected, but inefficient when it is considered. In the next section, we shall argue that this reveals that hedging opportunities are not fully exploited. This is partly compensated by the presence of 86 portfolios for which the reverse holds. This could be evidence that these households use financial assets to hedge housing risk, but could also reveal that housing has diversification properties (for house owners, financial risks are

[14] Compared to Table 9, we have dropped three observations because of a missing value on the house value

relatively small compared to total wealth). Given the high correlations found (see Table 3) and the very large weight attached to housing wealth, the failure to exploit hedging opportunities outweighs the benefits from diversification, and the number of conditionally efficient portfolios (157) is smaller than the number of efficient financial portfolios (508). Similar conclusions can be drawn when the chosen test size for the conditional test is 5% (and kept at 10% for the financial test).

It's worth stressing that the estimated coefficients in Table 3 are the relevant indicators of the way hedging should be performed. For instance, in three regions out of four, more should be invested in MTG bonds compared to the mean-variance efficient portfolio weights displayed in Table 7 (the exception is the SO).

In Table 13 we display efficiency results by region (for two different test sizes: 10% and 5%): we see that the highest proportion of efficient portfolios obtains in the NW. This is particularly true for the conditional analysis – apparently NW households are the best at hedging housing risk. Purely financial portfolios are instead most often efficient in the SO.

Table 13a. Efficient portfolios conditional on housing by region

test size = 10%	**NW**		**NE**		**Centre**		**South**	
Portfolios	**N**	**%**	**N**	**%**	**N**	**%**	**N**	**%**
Risk-free + Risky fin. Ass.	30	38.46%	23	38.98%	17	41.46%	34	75.56%
Risk-free + Risky fin. Ass + house	112	29.71%	11	3.86%	15	5.86%	19	8.56%
Total	**142**	31.21%	**34**	9.88%	**32**	10.77%	**53**	19.85%

Table 13b. Efficient portfolios conditional on housing by region

test size = 5%	**NW**		**NE**		**Centre**		**South**	
Portfolios	**N**	**%**	**N**	**%**	**N**	**%**	**N**	**%**
Risk-free + Risky fin. Ass.	78	100.00%	59	100.00%	41	100.00%	45	100.00%
Risk-free + Risky fin. Ass + house	169	44.83%	83	29.12%	73	28.52%	47	21.17%
Total	**247**	54.29%	**142**	41.28%	**114**	38.38%	**92**	34.46%

The question more generally arises of what makes a household more likely to hold an efficient portfolio. To address this question we run a simple regression of the test statistic (ξ_1) on observable household characteristics such as age, education (secondary junior school degree, high school degree or graduate) and employment position (employee, entrepreneur, retired or unemployed) of the head, region, household income and housing wealth. In those cases where a portfolio has a negative excess return, the test statistic was set equal to the sample maximum (at 9.8).

It is worth recalling that efficient portfolios are either made of the risk-free asset alone, or include housing wealth as well as financial assets. Clearly, these results are highly affected by the wide-spread presence of portfolios characterized only by risk-free assets, and by home-ownership.

As in the last row of Table 11, we focus attention on the more interesting group of house owners with risky financial assets. In Table 14 we report the results of the regression: the sample size is 1138 (two observations were discarded because of a missing value on household income). The dependent variable (ξ_1) takes values in the 0.3-9.8 range: a high value denotes inefficiency.

We see that not all the variables considered are statistically significant: age, education and employment position of the head do not seem to affect efficiency. However, other variables are statistically significant. Household size has a positive effect on inefficiency, other things being equal. The income effect is more complex, because of non-linearity. The impact of income reaches its maximum around its sample mean: for high or low income levels inefficiency is lower. Residence in the NW also has a negative effect on inefficiency, whereas a larger home value has a strong, negative impact.

Table 14. Results of regression of the conditional test statistic on household characteristics

Number of observations	=	1138
F(14, 1123)	=	38.58
R^2	=	0.3248

ξ_1	Coef.	Std. Err.	t	P>|t|
Age-40	-0.00508	0.00511	-0.99	0.32
Household size	0.09419	0.04500	2.09	0.037
Junior_school	-0.05552	0.14433	-0.38	0.701
High_school	-0.01996	0.14169	-0.14	0.888
College	-0.22178	0.17236	-1.29	0.198
Employee	0.29647	0.19721	1.50	0.133
Self_employed	0.17119	0.21128	0.81	0.418
Retired	0.34151	0.19568	1.75	0.081
Ln(income)	7.23564	1.63464	4.43	0
Ln(income)^2	-0.33561	0.07309	-4.59	0
House value	3.29522	0.17983	18.32	0
NW	-0.4206	0.13913	-3.02	0.003
NE	0.06591	0.14496	0.45	0.649
CE	0.17560	0.14354	1.22	0.221
Constant	-33.4717	9.08183	-3.69	0

8. Discussion of empirical results and extensions

The empirical results presented in Section 7 must be interpreted with special care. In fact, the result that the test statistic:

(13) $$\xi_1 = T\frac{\hat{S}_1 - \hat{S}_1(Z)}{1+\hat{S}_1(Z)\dfrac{Z^T\Omega Z}{v_1^T\Sigma v_1}} \xrightarrow{d} \chi^2_{n_1-1}$$

holds independently of the properties of the Ω matrix.

An interesting special case derives when this matrix is block diagonal. In this case, we know from Flavin and Nakagawa's analysis that the optimal portfolio of financial assets is the same as in the standard Markowitz case. This case has attracted much attention in the recent literature, following the seminal paper by Flavin and Yamashita (2002). A number of recent papers have produced micro evidence on the role of housing on portfolio allocations within this framework, in which housing wealth contributes to background risk but also has diversification properties (Kullman and Siegel, 2003, Yamashita, 2003, LeBlanc and Lagarenne, 2004, Cocco, 2005). It is worth checking what our empirical analysis would be if these correlations were indeed zero.

In this case, the conditional test statistic does not simplify to the Sharpe test applied to financial assets alone: the denominator involves the variance of the housing return multiplied by the square of its share in total wealth.

$$(14) \qquad \xi_1^* = T \frac{\left(\hat{S}_1 - \hat{S}_1(Z)\right)}{1 + \hat{S}_1(Z) \frac{1}{\underline{x}_0^T \Sigma \underline{x}_0} \left(\underline{x}_0^T \Sigma \underline{x}_0 + \underline{h}_0^T \sigma_H^2 \underline{h}_0\right)} \xrightarrow{d} \chi^2_{n_1 - 1}$$

By construction, this test statistic is lower than the standard Sharpe statistic for financial assets alone, unless the housing share, h_0, is zero, or housing is risk-less. This is because this statistic tests for the efficiency of the whole portfolio, conditional on one asset being given, and this is conceptually different from testing for efficiency of the allocation of unconstrained assets, even when the optimal financial portfolio does not include a hedge term.

The situation can arise where financial portfolios are inefficient according to a standard Sharpe test, and yet the corresponding overall portfolios (including housing) are found to be efficient even when housing is treated as given and there is zero covariance between housing and all financial assets. This is hardly surprising: if housing wealth is a large fraction of total wealth, the inefficiency of the financial wealth portfolio is relatively minor, compared to the diversification benefits that derive from the existence of this other form of wealth.

We know from Section 3 that for all Italian regions the Ω matrix is not block-diagonal. To assess the relative importance of the diversification effect we have tried a simple exercise: in all four regions we have set the covariance terms to be zero. Test results are reported in Table 15[15].

Of greater interest to us is the fact that more fully diversified portfolios are efficient at all significance levels (for instance: 848 instead of 157 at the 10% level). This suggests that the hedge motive is not widely taken into consideration when households make their portfolio choice. Thus many household portfolios that would be classified as (constrained) efficient if the correlation between housing return and financial assets returns were zero, are instead inefficient.

Table 15. Efficient portfolios conditional on housing (block-diagonal *Ω*)

Test size		**10%**		**5%**		**1%**	
Portfolios	**Tot. N**	**N**	**%**	**N**	**%**	**N**	**%**
Risk-free + Risky fin. Ass.	**223**	104	46.64%	223	100.00%	223	100.00%
Risk-free + Risky fin. ass + House	**1140**	848	74.39%	1092	95.79%	1140	100.00%
Total	**1363**	**952**	**69.85%**	**1315**	**96.48%**	**1363**	**100.00%**

Another issue worth considering is the effect of differential underreporting. We know from D'Alessio and Faiella (2000) that SHIW98 underestimates financial wealth by a wide margin (it accounts only for a third of aggregate household financial wealth), whereas housing wealth is in line with aggregate statistics. The reasons why financial wealth falls short of aggregate statistics can be non-response among the rich and under-reporting among those who do respond. To assess whether the latter has an important

[15] We stress that a straight comparison with Table 8 cannot be made because Table 15 considers mortgages and Table 8 neglects them.

impact on our test, we take the extreme case where differential non-response is not an issue, multiply all financial wealth holdings by a factor of three and re-run the test.

Table 16 displays efficiency test results in this hypothetical case, where all households report the same fraction of their financial wealth. If we compare these results with those in Table 11, we see that more fully diversified portfolios are counted as efficient (for instance: 321 instead of 157 at the 10% level). This increase is in line with expectations (the hedge motive is relatively less important if housing wealth has a lower portfolio weight) and is quite sizeable (now 28.16% of fully diversified portfolios are efficient, rather than 13.78%).

Table 16. Efficient portfolios conditional on housing corrected for under-reporting

Test size		**10%**		**5%**		**1%**	
Portfolios	**Tot. N**	**N**	**%**	**N**	**%**	**N**	**%**
Risk-free + Risky fin. Ass.	**223**	104	46.64%	223	100.00%	223	100.00%
Risk-free + Risky fin. ass + House	**1140**	321	28.16%	682	59.82%	895	78.51%
Total	**1363**	**425**	**31.18%**	**905**	**66.40%**	**1118**	**82.02%**

An important issue that arises when housing is included in the asset mix, is how to account for the liability every household has – to live somewhere. This issue has been stressed in a number of papers (Sinai and Souleles, 2005, Banks et al, 2004, Yao and Zhang, 2005), who point out that housing is a hedge against increases in the price of housing services. It is clear that the risk posed by price increases of housing services is the more important, the less easy it is to substitute out of housing into other goods and services. An extreme example where this substitution cannot take place at all is the case where housing consumption is already at its physical minimum.

In the framework we propose in this paper, we can account for housing needs in a relatively simple way. We define a minimum physical threshold for the main residence as $\overline{H}$ and estimate it in our data. We then take the observed price per squared meter as given and include in wealth only the difference between the current housing wealth and its minimum multiplied by that price. However, when the reported price is much higher than the local average, we replace it with a large, but more sensible value (on the assumption that the household could buy at that lower price if they moved into the smallest possible residence within the same area).

We define $\left(P_H H - \overline{P_H H}\right)$ as net housing wealth where $P_H H$ is the declared house value, $\overline{H}$ is the minimum house size for a given family size and $\overline{P_H}$ is and the relevant alternative house price within the current area of residence. We take the sample first percentile of squared meters for all possible family sizes as the minimum house size (this gives us: 20 square meters for a single, 35 for a couple, 40 for couple with one or two children, 46 for larger families). These values are in line with housing regulations (a single room must be at least 9 square meters in Italy).[16] As for prices, for each household we take the observed price per-square meter, except in those few cases where reported house prices are at the upper end of the distribution (top percentile), where we set them to 6m lire per square meter (roughly 3000 euros).

The resulting net housing wealth variable has an average value of almost 143,000 euros, whereas the original home value is 216,000. Our procedure suggests that about a third of housing wealth should be disregarded when deciding the financial portfolio allocation, because of the housing liability discussed so far.

[16] A square meter is roughly equivalent to 10 square feet.

Table 17. Efficient portfolios conditional on net housing wealth

Test size		10%		5%		1%	
Portfolios	**Tot. N**	**N**	**%**	**N**	**%**	**N**	**%**
Risk-free + Risky fin. Ass.	**223**	104	46.64%	223	100.00%	223	100.00%
Risk-free + Risky fin. ass + House	**1135**	216	19.03%	479	42.20%	756	66.61%
Total	**1358**	**320**	**23.56%**	**702**	**51.69%**	**979**	**72.09%**

When we can compare the results of Table 17 to those in Table 11, we see that taking the housing liability into account makes some 6%-10% more fully diversified portfolios conditionally efficient, depending on the chosen size of the test. [17]

Table 18. How diversified portfolios are classified – net housing wealth

Test size = 10%	**Efficient (Financial)**	**Inefficient (Financial)**	**Total**
Efficient (conditional)	**112**	104	216
Inefficient (conditional)	393	**526**	919
Total	505	630	1135

Table 18, that compares directly to Table 12, shows that the fraction of portfolios that are efficient according to both financial and conditional tests raises substantially (from 6.2% to 9.9%). This is due to a reduction in the number of portfolios that are efficient financially, inefficient conditionally.

The definition of net housing wealth as the difference between the existing main residence and a minimal residence, that meets exogenously defined housing needs, is attractive, but fails to capture preference heterogeneity. Households with a strong preference for housing may consider a much higher minimum threshold for housing

[17] We lose thirty observations because total wealth becomes negative, five of which have fully diversified portfolios. This explains why we have 1135 households in Table 18, as opposed to 1140 in Table 11.

services than households with a weaker preference for housing. A better approximation of housing needs may then be as a given proportion of housing services currently enjoyed.

We thus consider an alternative way to account for the notion that only a part of the main residence is perceived as wealth. We assume that households are not willing to reduce their housing consumption below a given fraction, x, of their existing consumption. This has an effect on the way they consider their main residence, but no effect on other real estate. We thus define net housing wealth, **nhw**, as:

nhw = (1-x) * main residence + other real estate – housing debt.

We see that x times the main residence is the minimum threshold below which a household is not willing to go – for this reason this is not counted as wealth.

Total wealth is the sum of financial wealth and net housing wealth. If x = 0 we are in the standard case considered in Sections 6 and 7, if x = 1 total wealth is financial wealth (as in Section 5), for those households who have neither other real estate, nor housing-related debt. However, even if x=1, total wealth does not coincide with financial wealth for households who have other real estate or housing-related debt. [18]

We then check to what extent conditional efficiency coincides with financial efficiency as a function of x. Figure 10 shows the results for the sample of 1140 households who have both housing and risky financial assets. The lower curve represents the fraction of portfolios that are conditionally efficient out of all portfolios that are financially efficient (508 observations). The upper curve represents the fraction of portfolios that are conditionally inefficient out of all portfolios that are financially inefficient (632 observations). As expected, these two fractions increase in x that is

[18] For all values of x, we make total wealth coincide with financial wealth also for those households whose net housing wealth is negative. In this case in fact we replace **nhw** with zero: this replacement

conditional efficiency tends to coincide with financial efficiency when the less the main residence is counted as wealth. As explained above, we apply this x correction to the main residence only – this explains why these proportions in Figure 10 do not reach unity even when we subtract 100% of the main residence value from housing wealth. In fact, 174 out of 508 (34%) households whose portfolios are financially efficient have other real estate, 244 out of 632 (39%) of households whose portfolios are financially inefficient have other real estate.

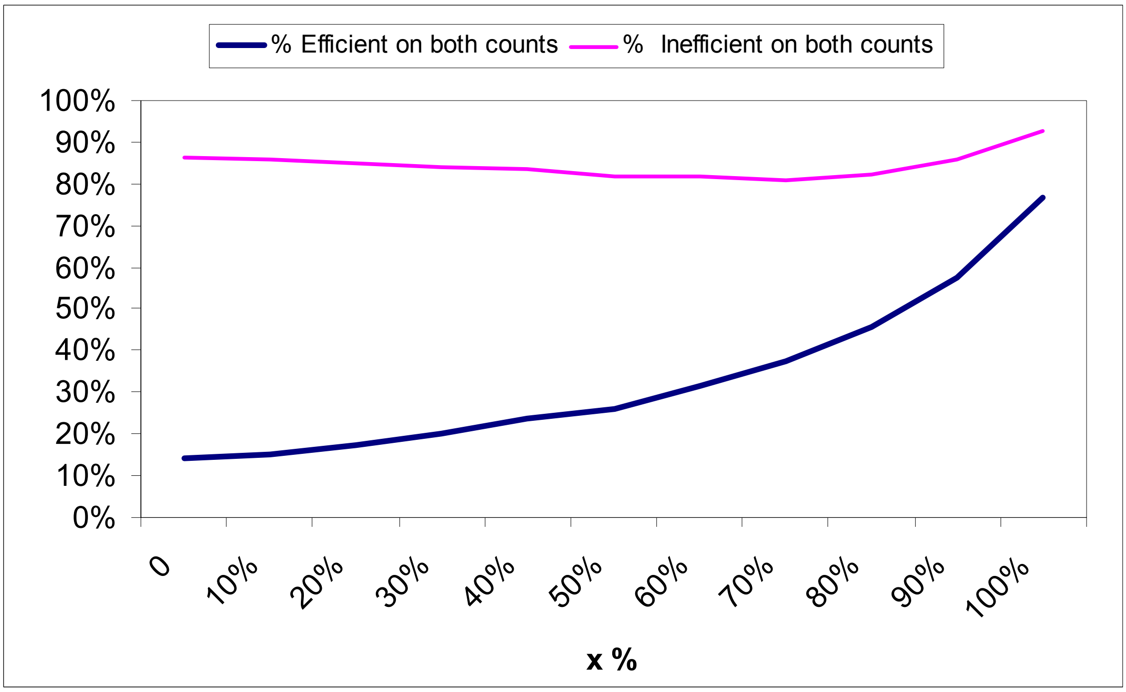

Figure 10 – Sensitivity of efficiency to changes in housing wealth definition

We see that the proportion of portfolios that are efficient on both counts steadily increases: there are only 71 (14%) such portfolios when x=0 (see the first main diagonal entry in Table 12), and as many as 389 (77%) when x=1. This suggests that households who neglect real estate in their portfolio choice (while achieving financial

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occurs in relatively few cases in our sample, because housing debt is a relatively minor item (we set **nhw** efficiency) may do so for good reasons – because they do not consider most of their main residence disposable (part of wealth).

The picture for financially inefficient portfolios is different: the fraction of inefficient portfolios on both counts declines in x from 546 (86%) when x = 0 (see Table 12, second main diagonal entry) down to 515 (81%) when x=70%, then increases to 586 (93%). This suggests that some of these households may hedge housing risk, particularly if they consider about a third of the total main residence value as wealth, but at least four out of five hold inefficient portfolios irrespective of the hedging motive.

One final issue is worth careful consideration. First and second moments of financial asset returns have been estimated using relatively accurate asset price data. Housing returns are instead based on averages of local house price data that are more likely to be affected by sampling variability. This implies that the estimated variance might exceed the actual variance, a potentially relevant issue, as argued in Cauley and Pavlov (2002). On the other hand, it is also possible that the estimated house return variance is an underestimate for each household, because households own a particular property, that is not an average of the local properties that are traded in the market. In either case, the hedge term is unlikely to be affected[19], and its only effect is to produce either an over or an under estimate of the variance of housing return. Then the actual size of the conditional test may differ from the notional size, and this affects the power of the test. The best way to see this is to consider equation (14), corresponding to a block-diagonal covariance matrix. If the estimated variance of housing returns, σ_H^2, is smaller than the true variance, the calculated test statistic is higher, and the test rejects

to zero in less than 50 cases for x<90%, 65 cases when x=90% and 119 cases when x=100%)

[19] The hedge term is given by the regression coefficients of the return on housing on financial returns. Even if the dependent variable is measured with error, as Cauley and Pavlov's analysis implies, these coefficients are consistently estimated by OLS or WLS, as long as measurement error is not correlated with financial returns. We can see no reason why such correlations should be non-zero.

more often than it should under the null for a given size. If it is larger, the conditional test does not reject as often as it should (it lacks power). There is no direct consequence for the financial test, but the comparison across the two is affected. Comparing number of rejections produced by different test statistics for a given test size may not be particularly useful, because one statistic does and the other does not rely on the second moment of housing returns. This analysis carries through to the more general case where the covariance matrix is not block-diagonal (see equation 13).

In the sequel we carry out two exercises that should be immune to this type of concern. In fact, the simplest way to circumvent this problem is to regress the calculated conditional test statistic on the calculated financial test statistic for the sample of households who have fully diversified portfolios (1140 in all). The estimated parameter is negative and significant (its point estimate is -.139 with a standard error of .019), confirming that the two tests strongly diverge in their indication of which portfolios are efficient. Another way to address this issue is to classify as efficient the same proportion of portfolios (30%, say) and check to what extent this group is made by the same households. The results are shown in Table 19, that compares directly to Table 12, but the off-diagonal elements have to be equal by construction. The proportion of efficient portfolios on both counts is even lower than in Table 12 (3.2%).

Table 19. How diversified portfolios are classified: a comparison

Test size = 10%	**Efficient (Financial)**	**Inefficient (Financial)**	**Total**
Efficient (conditional)	**37**	308	345
Inefficient (conditional)	308	**487**	795
Total	345	795	1140

Thus our key result is confirmed: different households appear efficient if we neglect housing.

Finally, in this paper we have ignored non-negativity constraints on asset holdings. In the case of long-term bonds and of corporate bonds this is not a problem for us: we assume negative holdings are allowed in the form of mortgages or other consumer debt. The same applies for the risk free asset, if we are willing to treat informal debt (from friends and relatives) or formal, variable interest debt (as in some mortgage contracts) as negative holdings. [20] However, even in our framework it is hard to explain how consumers can take negative positions in medium term government bonds or stocks.

The efficiency of household portfolios cannot be assessed in this context using the Gourieroux Jouneau test, but a new test has been proposed by Basak, Jagannathan and Sun (2002) that allows for non-negativity constraints (and can be adapted to treat housing as given)[21]. Preliminary results suggest that non-negativity constraints have a minor impact on the empirical analysis. This is not entirely surprising, because short positions on medium term government bonds or stocks do not appear in the Markowtiz optimal portfolio (as shown in Table 7) and these assets have negative partial correlation with housing returns in all regions but the South (see Table 3).

[20] Flavin and Yamashita (2002) analysis hinges heavily on the non-negativity constraint for the least risky asset (the T-bill). This constraint is found to be almost always binding, see Table 3, largely because its expected net return is negative. Our expected return on the risk-free rate is positive, and this makes the constraint much less relevant.

9. Conclusions

In this paper we have argued that standard tests of portfolio efficiency are biased because they neglect the existence of illiquid wealth. In the case of household portfolios, the most important illiquid asset is housing: if housing stock adjustments are costly and therefore infrequent, optimal portfolios in periods of no adjustment are affected by housing price risk.

We have shown that, if financial assets' and housing returns are correlated, the intertemporal expected utility model subject to transaction costs in housing investment implies that financial decisions are affected by the need to hedge some of the risks connected with the existing illiquid housing position. In particular, the investors' optimal strategy is to choose the standard Markowitz portfolio according to their risk aversion and use the risky financial assets to hedge their expositions on the constrained asset (this last decision is independent of their risk aversion). This hedging motive disappears in the case of zero correlation between housing return and financial returns, as pointed out by Flavin and Nakagawa (2004), in which case housing price risk only affects the investor's degree of risk aversion.

We have also shown that the optimal investment in risky financial assets is equal to the one derived in a static mean-variance analysis framework, if the existing housing stock is treated as an additional constraint to the optimization problem. Gourieroux and Jouneau (1999) have proposed an efficiency test for analyzing the performance of a portfolio of risky assets (in a mean-variance framework) when some constraints exist on

[21] However, the statistic proposed by Basak, Jagannathan and Sun tests for differences in portfolios standard deviation for a given expected return. This is conceptually different from the test used in this paper, that allows direct comparison of portfolios that have different expected returns.

a part of the portfolio. We are then able to claim that this test can be applied for a more general test of portfolio efficiency.

In our application, we have used Italian household portfolio data from the 1998 SHIW and time series data on financial asset and housing stock returns to assess whether actual portfolios are efficient. We first consider purely financial portfolios and portfolios that also treat the housing stock as another asset. We then consider the consequences of treating the housing stock as given and test for efficiency in this framework.

Our empirical results support the view that the presence of illiquid wealth plays an important role in determining whether portfolios chosen by home-owners are efficient.

Our results can be summarized as follows:

- When we consider only financial assets, three portfolios out of four are made of just the risk free asset. Of the diversified portfolios, a large fraction (45%) is mean-variance efficient;
- When we take a broader set of assets and liabilities (housing, mortgages and debt) into consideration, many more households hold diversified portfolios (a common combination is the risk-free asset and housing). But very few diversified household portfolios are found to be efficient when housing is treated as unconstrained.
- When we calculate the efficiency test conditional on housing we find that one in seven of fully diversified portfolios (that include the risk-free asset, housing and risky assets) are mean-variance efficient. We also find that these are largely not the same households whose financial portfolios were considered efficient.
- An important issue that arises when housing is included in the asset mix, is how to account for the liability households have to live somewhere. In our robustness analysis, we propose two alternative ways to do this. First, we define net housing

wealth as the difference between the home value and the value of the smallest property a household could move to in the same area. On average, our net housing wealth variable is worth around two thirds of gross housing wealth. We show that taking the housing liability into account this way increases the number of conditionally efficient fully diversified portfolios by a half. Second, we assume that housing needs are a given fraction of the housing services currently enjoyed. Net housing wealth is the difference between total housing wealth and the fraction required to meet housing needs. As this fraction approaches unity, we find that an increasing proportion of financially efficient portfolios is also conditionally efficient. Financially inefficient portfolios, instead, are more often conditionally efficient when this fraction rises to 70%, then more often inefficient.

In summary, compared to the efficiency results relating to portfolios consisting solely of financial assets such as stocks, bonds and a risk-free asset, the introduction of housing and mortgage alters the risk and return trade-off in a direction which pushes very few household portfolios to be efficient. This is not the case, once the illiquid nature of housing investment is taken into account, but there is strong evidence that hedging opportunities are not fully exploited even by those Italian households who hold well-diversified portfolios. This widespread failure to hedge house price risk has important implications for portfolio management.

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APPENDIX – Derivation of equation (7) in a multi-period context.

In this appendix we build on Flavin and Nakagawa's (2004) analysis of the dynamic optimization problem with housing, and use the same notation for comparison's sake. We show that the dynamic optimization problem produces the same asset allocation rule as a static problem that treats housing wealth as given.

Flavin and Nakagawa generalize Grossman and Laroque's model by making current utility a function of both a durable good, a house (H), and a non-durable good (C). The non-durable good is infinitely divisible and costlessly adjustable. As in Grossman and Laroque, the durable good is instead subject to an adjustment cost proportional to its value and is therefore adjusted infrequently. This generalization is of great relevance for the analysis of portfolio choice, because it allows us to consider explicitly the relationship between the real rate of return on housing investment and the real rates of return on financial assets.

The household maximizes expected lifetime utility:

(A1) $$U = E_0 \int_0^{\infty} e^{-\delta t} u(H_t, C_t) dt$$

For analytical simplicity, the house is not subject to physical depreciation.[22] Using the non-durable good as numeraire, define:

(A2) P_t = house price (per square meter) in the household's market[23].

[22] Damgaard, Fuglsbjerg and Munk (2003) have developed a model similar to ours, by deriving the numerical solution for the case with non-zero depreciation. Depreciation implies that the target value for housing is above the mid-point of the (s,S) interval. We prefer to subtract maintenance costs from the return on housing, assuming that maintenance restores the housing stock to its previous state.

[23] Unlike Flavin and Nakagawa, we do not consider a separate price process for the next house to be bought

Assume that wealth is held only in the form of financial assets and housing. The household can invest in a risk-less asset and in any of n risky financial assets. Holdings of the financial assets can be adjusted with zero transaction cost.

Thus wealth is given by:

(A3) $\quad W_t = P_t H_t + B_t + \underline{X}_t \underline{\ell}$

where $\underline{X}_t$ = (1xn) vector of amounts (expressed in terms of the non-durable good) held of the risky assets and $\underline{\ell}$ = (nx1) vector of ones. B_t is the amount held in the form of the riskless asset. All financial assets, including the riskless asset, may be held in positive or negative amounts.[24]

Assuming that dividends or interest payments are reinvested so that all returns are received in the form of appreciation of the value of the asset, let b_{it} = the value (per share) of the i-th risky asset, and assume that asset prices follow an n-dimensional Brownian motion process:

(A4) $\quad db_{it} = b_{it}\left((\mu_i + r_f)dt + d\omega_{it}\right)$

The vector $\underline{\omega}_{Ft} \equiv (\omega_{1t}, \omega_{2t}, ..., \omega_{nt})$ follows an n-dimensional Brownian motion with zero drift and with instantaneous covariance matrix Σ, the corresponding vector (nx1) of expected excess returns on risky financial assets is $\underline{\mu} \equiv (\mu_1, \mu_2, ..., \mu_n)$, and r_f is the risk-less rate. The i-th element of $\underline{X}_t$ in equation (3) is given by $X_{it} \equiv N_{it} b_{it}$ where N_{it} is the number of shares held of asset i. Since asset prices, b_{it}, are taken as exogenous, the household determines X_{it} by its choice of N_{it}[25].

House prices also follow a Brownian motion:

(A5) $\quad dP_t = P_t\left((\mu_H + r_f)dt + d\omega_{Ht}\right)$

[24] This model does not deal with labor income or borrowing restrictions, that are instead considered in the models developed by Cocco (2000) and Cocco (2005).

where ω_{Ht} is a Brownian motion with zero drift and instantaneous variance σ_P^2.

Stacking equations (4) and (5), define the ((*n*+1)x1) vector $d\underline{\omega}_t$:

(A6) $$d\underline{\omega}_t = \begin{bmatrix} d\omega_{1t} \\ \vdots \\ \\ d\omega_{nt} \\ d\omega_{Ht} \end{bmatrix}$$

which has instantaneous ((*n*+1)x(*n*+1)) covariance matrix Ω:

(A7) $$\Omega = \begin{bmatrix} \Sigma & \Gamma_{b_i,P} \\ \Gamma'_{b_i,P} & \sigma_P^2 \end{bmatrix}$$

where:

(A8) $$\Gamma_{bP} = \begin{bmatrix} \sigma_{b1P} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \sigma_{bnP} \end{bmatrix}$$

Note that we depart from Flavin and Nakagawa here, in that we do not assume the covariance matrix Ω to be block diagonal. This is the substantial difference between our models, that generates qualitatively different results.

We shall show that, under the assumptions listed in this Appendix, the optimal holding of risky financial assets, is given by:

(A9) $$\underline{X}_0^T = \begin{bmatrix} -\dfrac{\dfrac{\partial V}{\partial W}}{\dfrac{\partial^2 V}{\partial W^2}} \end{bmatrix} \Sigma^{-1}\underline{\mu} - P_0 H_0 \Sigma^{-1}\Gamma_{bP}$$

---

[25] We follow Flavin, and use X_{it} rather than N_{it} as the choice variable representing the portfolio decision.

In equation (A9), the expression in square brackets is the reciprocal of the coefficient of absolute risk aversion:

$$\text{(A10)} \quad ARA \equiv -\frac{\dfrac{\partial^2 V}{\partial W_t^2}}{\dfrac{\partial V}{\partial W_t}} > 0$$

It is worth pointing out that risk aversion affects the first term on the RHS of equation (A9) but not the second term, that bears the interpretation of a hedge portfolio[26]. In Flavin and Nakagawa's analysis this second term disappears, because they assume $\Gamma_{bp} = 0$, and therefore can prove that the traditional CAPM holds.

Suppose that at time t=0, the household decides that it is not optimal to change the housing stock immediately. During a time interval (0,*s*) when the possibility of such change is negligible, wealth evolves according to:

$$\text{(A11)} \quad dW_t = \left[P_t H_0(\mu_H + r_f) + \underline{X}_t(\underline{\mu} + r_f) + r_f B_t - C_t\right]dt + \underline{X}_t d\underline{\omega}_{Ft} + P_t H_0 d\omega_{Ht}$$

or, rewriting in order to eliminate the term representing risk-free bonds,

$$\text{(A12)} \quad dW_t = \left[r_f W_t + P_t H_0 \mu_H + \underline{X}_t \underline{\mu} - C_t\right]dt + \underline{X}_t d\underline{\omega}_{Ft} + P_t H_0 d\omega_{Ht}$$

Let $V(H,W,P)$ denote the supremum of household expected utility be twice continuously differentiable, conditional on the current values of the state variables (H,W,P). Bellman's principle of optimality can be stated as:

$$\text{(A13)} \quad V(H_0,W_0,P_0) = \sup_{\{\underline{X}_t\},\{C_t\}} E\left[\int_0^s e^{-\delta t} u(H_0,C_t)dt + e^{-\delta s} V(H_0,W_s,P_s)\right]$$

subject to the budget constraint (A12) and the process for house prices (A5). The term inside the brackets intuitively represents the sum of the rewards on the interval (0,*s*) and

[26] This term is different from the classical Merton hedge term that accounts for shifts in the investment opportunity set.

the maximized expected value by proceeding optimally on the interval (s,∞) with the system started at time s in state (H_0, W_s, P_s)[27].

Subtracting $V(H_0, W_0, P_0)$, dividing by s and taking the limit as $s \rightarrow 0$ gives:

$$\text{(A14)}\quad 0 = \lim_{s\rightarrow 0} \sup_{\{\underline{X}_t\}\{C_t\}} E\left[\frac{1}{s}\int_0^s e^{-\delta t} u(H_0, C_t)dt + \frac{1}{s}\left(e^{-\delta s} V(H_0, W_s, P_s) - V(H_0, W_0, P_0)\right)\right]$$

Evaluating the integral and using Ito's lemma, equation (A14) can be rewritten as:

$$\text{(A15)}\quad \begin{aligned} 0 = \sup_{\underline{X}_0, C_0} \{\ & u(H_0, C_0) - \delta V(H_0, W_0, P_0, P_0^{'}) + \frac{\partial V}{\partial W}(r_f W_0 + P_0 H_0 \mu_H + \underline{X}_0 \underline{\mu} - C_0) \\ & + \frac{\partial V}{\partial P} P_0 \mu_H + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}\left(\underline{X}_0 \Sigma \underline{X}_0^T + P_0^2 H_0^2 \sigma_P^2 + 2P_0 H_0 \underline{X}_0 \Gamma_{bP}\right) + \frac{1}{2}\frac{\partial^2 V}{\partial P^2} P_0^2 \sigma_P^2 \\ & + \frac{\partial^2 V}{\partial W \partial P}\left(P_0^2 H_0 \sigma_P^2 + P_0 \underline{X}_0 \Gamma_{bP}\right)\ \} \end{aligned}$$

that is:

$$\text{(A16)}\quad \begin{aligned} 0 = & \sup_{C_0}\left\{u(H_0, C_0) - C_0 \frac{\partial V}{\partial W}\right\} - \delta V(H_0, W_0, P_0, P_0^{'}) + \frac{\partial V}{\partial W}(r_f W_0 + P_0 H_0 \mu_H) \\ & + \frac{\partial V}{\partial P} P_0 \mu_H + \frac{1}{2}\frac{\partial^2 V}{\partial P^2} P_0^2 \sigma_P^2 + \frac{\partial^2 V}{\partial W \partial P}\left(P_0^2 H_0 \sigma_P^2 + P_0 \underline{X}_0 \Gamma_{bP}\right) + \frac{1}{2}\frac{\partial^2 V}{\partial W^2} P_0^2 H_0^2 \sigma_P^2 \\ & + \sup_{\underline{X}_0}\left\{\frac{\partial V}{\partial W}\underline{X}_0 \underline{\mu} + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}\left(\underline{X}_0 \Sigma \underline{X}_0^T + 2P_0 H_0 \underline{X}_0 \Gamma_{bP}\right)\right\} \end{aligned}$$

Non-durable consumption satisfies the standard first order condition:

$$\text{(A17)}\quad \frac{\partial u}{\partial C} = \frac{\partial V}{\partial W}$$

The vector of holdings of risky financial assets, $\underline{X}_0$, is chosen according to:

$$\text{(A18)}\quad 0 = \text{constant} + \frac{\partial V}{\partial W}\left(r_f W_0 + P_0 H_0 \mu_H - C_0\right) + \frac{1}{2}\frac{\partial^2 V}{\partial W^2} P_0^2 H_0^2 \sigma_P^2$$

[27] We assume that the transversality condition holds such that $V(H_0, W_s, P_s)$ is bounded.

$$+\sup_{\underline{X}_0}\left\{ \frac{\partial V}{\partial W}\underline{X}_0\underline{\mu}+\frac{1}{2}\frac{\partial^2 V}{\partial W^2}\left(\underline{X}_0\Sigma\underline{X}_0^T+2P_0\,H_0\,\underline{X}_0\Gamma_{bP}\right)+\frac{\partial^2 V}{\partial W\partial P}\left(P_0\,\underline{X}_0\Gamma_{bP}\right)\right\}$$

Assuming that $\frac{\partial^2 V}{\partial W\partial P}=0$ we can derive the optimal holding of risky financial assets as:

(A19) $$\underline{X}_0^T=\left[\frac{-\frac{\partial V}{\partial W}}{\frac{\partial^2 V}{\partial W^2}}\right]\Sigma^{-1}\underline{\mu}-P_0\,H_0\Sigma^{-1}\Gamma_{bP}$$

and the amount held of the risk-less asset is:

(A20) $B_0 = W_0 - P_0H_0 - \underline{X}_0\underline{\ell}$

Equation (A19) is the same as equation (7), if both members are divided by total initial wealth, W_0.

The assumption that $\frac{\partial^2 V}{\partial W\partial P}=0$ is justified under two sets of circumstances:

a) if the utility function does not depend on housing, as pointed out by Damgaard, Fuglsbjerg and Munk (2003)

b) if the utility function is additive in housing and non-durable consumption.

While condition a) rules out a consumption role for housing, condition b) provides a useful benchmark for the analysis.

Available upon request:

Regressions of housing excess return on financial assets excess returns in some European countries

Variable	France	U.K.	Germany	Spain
Constant	0.0066 (.0127)	0.0122 (.0108)	0.0086 (.0084)	-0.01033 (.0123)
$\mathbf{r_{LT}}$	-2.0432 (.6033)	-.55458 (.4523)	-1.0084 (.3171)	-0.8149 (.5391)
$\mathbf{r_{MT}}$	2.7914 (.5606)	1.1583 (.4602)	1.8139 (.2944)	1.8283 (.5895)
$\mathbf{r_{STOCKS}}$	0.1446 (.0994)	0.1036 (.0819)	0.0132 (.0407)	-0.02736 (.0654)
P-value	***0.0000***	***0.0000***	***0.0000***	***0.0000***
R^2	***0.9468***	***0.8617***	***0.9666***	***0.9581***

Notes: Standard errors in parentheses.

The data are taken from: Iacoviello, Matteo “House Prices and Business Cycles in Europe: a VAR Analysis”, mimeo, Boston College