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THE STATE AID GAME

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# The State Aid Game\*

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## Abstract

We present a model of the impact of state aid on equilibrium market structure and on market performance in an integrating market when the process of integration is driven by consumer inertia. In a partial equilibrium model, it is an equilibrium for governments to grant state aid, even though this reduces common market welfare.

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**Key words:** state aid, exit, market integration.

**JEL codes:** F15, L11, L53

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## 1. Introduction

Despite the prohibition of state aid that distorts competition contained in Article 87(1) of the Treaty establishing the European Community,<sup>1</sup> the mandatory and discretionary exceptions to this prohibition contained in Articles 87(2) and 87(3), and the requirement of Article 88 that the European Commission constantly review systems of state aid, state aid has been an enduring feature of the EC economic landscape. Although state aid has decreased since the end of the 1990s, in 2002 state aid overall in the fifteen member states amounted to around €49 billion, representing about 0.56 per cent of EU GDP. In relative terms, aid ranged from 0.25 per cent of GDP in the United Kingdom to 1.28 per cent in Finland. State aid policy seems certain to remain the subject of controversy as the accession of less economically-developed Member States shifts the standards for permissibility of aid throughout the Community.<sup>2</sup>

The increased competition that accompanies market integration is expected to improve market performance by reducing firms' abilities to hold price above marginal cost and by eliminating waste (reducing X-inefficiency).<sup>3</sup> It is less commonly noted<sup>4</sup> that the increase in rivalry that comes with market integration may, and in general will, result in the exit of less efficient firms. Indeed, such exit, and the concomitant concentration of production in the hands of a smaller number of more efficient firms, is one source of improved performance in the integrated market.

There is a large policy-oriented literature on European Union state aid policy.<sup>5</sup>

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<sup>1</sup>Article 87(1) provides that "any aid granted by a Member State or through State resources in any form whatsoever which distorts or threatens to distort competition by favouring certain undertakings or the production of certain goods shall, in so far as it affects trade between Member States, be incompatible with the common market." Article 88(2) provides that incompatible aid is to be altered or abolished.

<sup>2</sup>See, for example, Ricard (2005). The 10 new members of the European Union devote a larger percentage of their GDP per capita to state subsidies to business than do the 15 older member states (respectively about 1.35% of new member versus 0.45% of the older member states in the period 2002-04). In absolute terms, the new member states granted €6,274 billion aid compared with €42,717 billion for the EU-15 in the period 2002-04 (*EU State Aid Scoreboard*, Spring 2006, p.11).

<sup>3</sup>See, for example, Vickers (1995), Nickell (1996), and Hay and Liu (1997).

<sup>4</sup>See, however, Symeonidis' (2000) discussion of the impact on market structure of an unanticipated toughening of UK competition policy.

<sup>5</sup>For references, see Martin and Evans (1991), Martin and Valbonesi (2000), and Friederiszick *et al.* (2005).

Formal treatments are rare. Collie (2000, 2003), in what is the work most closely related to the present discussion of which we are aware, models of the impact of state aid on market performance in an integrated market. In this paper, in addition to examining the impact of state aid on market performance, we examine the transition period in the run-up to full integration and also consider the impact of state aid on market structure.

The analysis presented in this paper is based on the observation that the economics of equilibrium market structure in an integrating market has elements in common with the analysis of exit from a declining industry.<sup>6</sup> This insight develops from the analysis of the demand curves characterized by consumer inertia — a preference for the product of domestic producers that persists for a limited period even after formal barriers to trade have been eliminated. In such markets, and in absence of government intervention, shifts in the residual demand curves facing individual firms in imperfectly competitive integrating markets dictate a reduction in the equilibrium number of firms. We show that state aid, by frustrating such reductions, neutralizes an efficiency effect of competition in an integrated market, and blocks the way to realization of an efficient specialization of production and division of labor in the common market.

Aid granted by a single country may increase its own net social welfare, although in so doing it reduces common market net social welfare. Aid by several member states can result in a outcome that reduces net social welfare in all member states, and therefore of necessity in the common market.

That market integration may induce exit, absent state aid, is without doubt. An example from the early history of EU market integration is that of the Belgian coal industry in the European Coal and Steel Community. Belgian costs were so high that coal suppliers in the Ruhr would have been able to undersell Belgian mines in Belgium without engaging in freight absorption (Lister, 1960, p. 296; Meade *et al.*, 1962, p. 292). The history of EU competition policy is replete with examples of Member States granting aid to their firms that was generally recognized as contrary to treaty provisions by all parties involved except the legal representatives of the aid-granting member states, who argued in defense of the aid before the European Court of Justice.

In contrast to the general literature on subsidies, which relies mainly on models similar to those found in the strategic trade, tax competition and rent-seeking literatures,<sup>7</sup> our model develops the idea that the incentive to supply state aid is

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<sup>6</sup>See Ghemawat and Nalebuff (1985, 1990) and the literature stemming therefrom.

<sup>7</sup>See Martin and Valbonesi (2006, forthcoming) for a survey.

endogenously created by the very process of market integration. To highlight the issues involved, we first examine (in Section 2) the impact of market integration on equilibrium market structure for the case of integration of two identical markets in which all firms have access to the same technology. In Section 3 we relax the assumption of identical technologies, outline conditions under which market integration implies that less-efficient firms leave the market, and evaluate the welfare impact of market integration. In Section 3.3 we turn to the impact of state aid on market performance and outline the economic case for control of state aid. We conclude and draw policy implications in Section 4. Proofs are given in the Appendix.

## 2. Market Integration and Market Structure

Consider a situation in which two countries, each home to a Cournot oligopoly with inverse demand equation

$$p_i = a - bQ_i, \quad (2.1)$$

for  $i = 1, 2$ , form a common market.<sup>8</sup> Firms in both countries produce with a cost function that exhibits fixed cost and constant marginal cost:

$$c(q) = F + cq. \quad (2.2)$$

In this section, for simplicity, we ignore the fact that the number of firms must be an integer, and ask how market integration affects the equilibrium number of firms.

### 2.1. Pre-integration market structure

We work with a continuous time model. In what is a standard analysis of Cournot oligopoly with linear inverse demand and constant marginal cost, a typical firm (of a total  $n$ ) in one of the pre-integration component markets noncooperatively picks its own output rate to maximize its payoff per unit time,

$$\pi_i = \left[ a - c - b \left( q_i + \sum_{j \neq i}^n q_j \right) \right] q_i - F. \quad (2.3)$$

---

<sup>8</sup>The slope parameter  $b$  can be normalized to some convenient value (usually taken to be 1) by appropriate redefinition of the units in which output is measured. Having normalized  $b$  for a single-country market, the slope of the integrated-market inverse demand curve cannot then be normalized again. With this in mind, we write the slope parameter explicitly in (2.1).

The first-order condition for profit maximization is

$$a - c - b \left( q_i + \sum_{j \neq i}^n q_j \right) = bq_i, \quad (2.4)$$

and this implies that the firm's equilibrium payoff is rate proportional to the square of its equilibrium output,

$$\pi_i = bq_i^2 - F. \quad (2.5)$$

(2.4) is also, implicitly, the equation of firm  $i$ 's best response function. Since the model is symmetric, all firms produce the same equilibrium output rate. (2.4) implies that this equilibrium output rate is

$$q^* = \frac{1}{n+1} \frac{a-c}{b}, \quad (2.6)$$

where the asterisk denotes an equilibrium value for the component market. (2.5) then implies that the equilibrium payoff rate per firm in the component market is

$$\pi^* = b \left( \frac{1}{n+1} \frac{a-c}{b} \right)^2 - F. \quad (2.7)$$

The equilibrium market structure is the number of firms,  $n^*$ , that makes the Nash-Cournot equilibrium payoff rate (2.7) equal to zero:

$$n^* = \frac{\frac{a-c}{b}}{\sqrt{\frac{F}{b}}} - 1. \quad (2.8)$$

The numerator of the fraction on the left,  $\frac{a-c}{b}$ , is the quantity that would be demanded in either of the component markets if price were equal to marginal cost; it is one measure of market size. The denominator is the square root of fixed cost, normalized by the slope of the inverse demand curve. (2.8) therefore says that the equilibrium number of firms in a Cournot market is larger, the larger is the market and the smaller is fixed cost.

## 2.2. Post-integration market structure

In the fully integrated market, firms cannot price discriminate based on nationality. In one perspective, this may be regarded as a definition of market integration.

| $n^*$ | $m^*$ | $[m]$ | $2n^* - [m]$ |
|-------|-------|-------|--------------|
| 1     | 1.83  | 1     | 1            |
| 2     | 3.24  | 3     | 1            |
| 3     | 4.66  | 4     | 2            |
| 4     | 6.07  | 6     | 2            |
| 5     | 7.49  | 7     | 3            |
| 6     | 8.89  | 8     | 4            |
| 7     | 10.31 | 10    | 4            |
| 8     | 11.73 | 11    | 5            |

Table 2.1: Pre-integration equilibrium number of firms per country  $n^*$ , post-integration equilibrium number of firms  $m^*$ , equilibrium integer number of post-integration firms  $[m]$ , and  $2n^* - [m]$ .

Nationality-based price discrimination may also be prohibited by competition policy, as indeed it is in the European Union. The equation of the inverse demand curve in the integrated market is

$$p = a - \frac{1}{2}bQ. \quad (2.9)$$

Going through the same steps as for the single market (alternatively, substituting  $b/2$  for  $b$  in (2.8)), the equilibrium number of firms in the integrated market  $m^*$  satisfies

$$m^* + 1 = \sqrt{2} \frac{\frac{a-c}{b}}{\sqrt{\frac{F}{b}}} = \sqrt{2} (n^* + 1). \quad (2.10)$$

The ratio  $(m^* + 1) / (n^* + 1)$  equals the square root of the number of equally-sized markets that integrate to form a single market — in this case, two.  $m^* > n^*$ , but  $m^* < 2n^*$ . The equilibrium number of firms in the integrated market exceeds the equilibrium number of firms in a single component market, but is less than the total number of firms in all component markets before integration. Table 2.1 shows the relation between  $n^*$  and  $m^*$  for small values of  $n^*$ .

It is worth emphasizing that even though market integration leads to a reduction in the number of firms, it means an improvement in market performance. Pre-integration long-run equilibrium price (by definition, equal to the average cost of producing equilibrium output) is

$$p(n^*) = c + \sqrt{bF}. \quad (2.11)$$



After integration, each surviving firm produces more output, reducing average fixed cost. Post-equilibrium long-run equilibrium price is

$$p(m^*) = c + \frac{1}{\sqrt{2}}\sqrt{bF} < p(n^*). \quad (2.12)$$

Integration not only economizes on fixed cost but also reduces price, thereby increasing consumer surplus.

### 3. Integrated Market Monopoly

#### 3.1. Setup

Our model is of two countries that integrate their markets for a single product. For simplicity, we assume quantity-setting behavior with identical linear demands in each pre-integration market and fixed cost, constant marginal cost technologies, with as well as a lump-sum entry fee. We consider the simplest possible case, with each pre-integration market a monopoly and the post-integration market also a monopoly.<sup>9</sup> Realization of the efficiency results that flow from integration then requires the exit of one of the pre-integration firms. It is this integration-induced change in market structure that creates an incentive for one country to grant its firm state aid, and this, in turn, may make a joint policy to control state aid an attractive proposition.

##### 3.1.1. Demand

(2.1) is the equation of the pre-integration inverse demand curve in country  $i$ ,  $i = 1, 2$ , and (2.9) is the equation of the inverse demand curve in the fully-integrated market. We assume that there is a continuous well-behaved integration function  $\iota(t)$ , with

$$0 \leq \iota(t) \leq 1 \quad (3.1)$$

$$\iota(0) = 0 \quad \iota'(t) > 0 \quad \iota(T) = 1. \quad (3.2)$$

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<sup>9</sup>It is common in both the declining markets literature and the strategic trade policy literature to consider the case of one firm in each country. Rather than simply assume that these are the pre-integration market structures, we explicitly specify the conditions on demand and technology for the configurations we consider to be equilibrium outcomes. The results obtained here generalize to the case of pre- and post-integration oligopoly, at the expense of increasing the complexity of the algebra.

A time period of length  $T$  is required to complete the integration process. At time  $t$  during the integration period, a fraction  $\iota(t)$  of consumers in each market are “in” the integrated market and these consumers consider either supplier a potential source of supply. The complementary fraction  $1 - \iota(t)$  of consumers in each country consider only their domestic supplier a potential source of supply. The higher is  $\iota$ , the smaller is the fraction of consumers still buying only on the pre-integration national market and the more integrated is the common market.

The equation of the inverse demand curve facing each firm is then a weighted average of the national pre-integration demand curve and the full-integration residual demand curve,

$$q_i(t) = [1 - \iota(t)] \frac{a - p_i(t)}{b} + \iota(t) \left[ 2 \frac{a - p_i(t)}{b} - q_j(t) \right], \quad (3.3)$$

for  $i, j = 1, 2$  and  $j \neq i$ . In what follows, we suppress the time argument where this is possible without confusion. It will then be natural simply to write of “integration level  $\iota$ .”<sup>10</sup>

In the “exit from declining markets literature” (Ghemawat and Nalebuff 1985, 1990; Brainard, 1994; others), it is typical to assume that demand declines monotonically to zero over time in a well-behaved way. The assumptions we make about the integration function correspond to such declining demand assumptions, and are rooted in assumptions about consumer behavior. Scitovsky (1950), Waterson (2003), search models of imperfectly competitive markets, and the literature on consumer switching costs all emphasize the importance of consumer behavior for market performance. The European Commission, in its *First Report on Competition Policy*, referred (1972, p. 14) to “differences in the habits of consumers” as one reason for persistent price differences across member states in the Common Market. The formulation given by (3.3) describes a particular kind of demand inertia as a way of modelling the demand side of the integration process.

In a Hotelling model, Schultz (2005) obtains a comparable effect by allowing for two classes of consumers, those who are informed of the prices of both suppliers and those who are informed of the price of only one supplier. Schultz’s transparency parameter, the fraction of consumers informed of both prices, corresponds conceptually to our integration parameter, although the details of the models are quite different. Reinhard Selten has explored the impact of demand

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<sup>10</sup>The formulation adopted here is a general one. If the integration process is linear, we would have  $\iota(t) = t/T$ . Alternatively, the integration process might follow the kind of logistic pattern that is common in diffusion models.

inertia on market performance in experimental markets. Consumer heterogeneity is one of the standard explanations for equilibrium price heterogeneity, and the two groups of consumers in our markets may be thought of as “shoppers” (buying in the integrated market) and “nonshoppers” (buying only in the nonintegrated market).

Inverting (3.3), the inverse demand equation facing firm  $i$  at integration level  $\iota$  is

$$p_i = a - b \frac{q_i + \iota q_j}{1 + \iota}, \quad (3.4)$$

for  $i, j = 1, 2$  and  $i \neq j$ .

(3.4) bears a family resemblance to the Bowley (1924) specification for the inverse demand equation of one variety of a differentiated product group. We ought to expect, therefore, that a partially integrated market for a homogeneous product behaves in some ways like a completely integrated market for a differentiated product.

### 3.1.2. Technology

The firm in country  $i$  produces with constant marginal cost  $c$  per unit and fixed cost  $F_i$  (per unit of time). The cost function of a firm in country  $i$  for a flow of output at rate  $q$  per unit time interval is thus

$$c_i(q_i) = F_i + cq_i, \quad (3.5)$$

for  $i = 1, 2$

Without loss of generality, we assume that the country 2 firm has the lowest fixed cost:

$$F_2 < F_1. \quad (3.6)$$

Empirically, it is known that there are persistent cost differences across plants (Roberts and Supina, 1996, 1997). Country-specific differences in cost (which for simplicity we treat as differences in fixed cost) might reflect locational differences or, for natural resource industries, differences in the quality of mineral deposits.

### 3.1.3. Pre-integration market structure

**Monopoly profit and value** If a single firm supplies country  $i$ , it picks its output to maximize its instantaneous payoff

$$\pi_i = (a - c - bq_i)q_i - F_i. \quad (3.7)$$

In the usual way, monopoly output, the flow rate of profit, and value are

$$q_{im} = \frac{a - c}{2b} \quad \pi_{im} = bq_{im}^2 - F_i \quad V_{im} = \frac{\pi_{im}}{r}. \quad (3.8)$$

**Duopoly profit and value** If a second firm enters, and market  $i$  is a Cournot duopoly, then duopoly outputs, the flow rates of profit, and values are

$$q_{id} = \frac{a - c}{3b} \quad \pi_{id} = bq_{id}^2 - F_i \quad V_{id2} = \frac{\pi_{id}}{r}. \quad (3.9)$$

**Pre-integration natural monopoly** The conditions for the equilibrium number of firms in the pre-integration market to be one are

$$V_{im} \geq 0, V_{id2} < 0. \quad (3.10)$$

These inequalities both hold if

$$\frac{1}{3} < \frac{\sqrt{F_i/b}}{\frac{a-c}{b}} \leq \frac{1}{2}. \quad (3.11)$$

(3.11) characterizes the range of parameter values for which the equilibrium number of firms in each of the pre-integration markets is one. The numerator of the central fraction is the square root of fixed cost, scaled by the slope of the inverse demand curve. The denominator is, as noted in discussion of equation (2.8), a measure of market size. Hence (3.11) can be given the interpretation that the equilibrium number of firms is one if fixed cost is small enough, relative to market size, that it is profitable for one firm to supply the market, but large enough, relative to market size, that it is not profitable for two firms to supply the market.

#### 3.1.4. Post-integration market structure

We assume that (3.11) holds, and thus that before integration, the equilibrium market structure has each country supplied by a single firm. By abuse of notation, we will use the subscript  $i$  to indicate both country  $i$  and the pre-integration home firm that supplied country  $i$ .

It is intuitive that if (3.11) holds, so that each firm makes a profit as a monopolist in its own national market, each firm would also make a profit as a monopolist in the fully integrated market (that is, facing the inverse demand (2.9)).

Firm  $i$ 's payoff function in the fully integrated duopoly market is

$$\pi_i = \left[ a - c - \frac{1}{2}b(q_i + q_j) \right] q_i - F_i, \quad i, j = 1, 2, i \neq j. \quad (3.12)$$

The first-order condition to maximize  $\pi_i$  (this is also, implicitly, the equation of firm  $i$ 's best response function) is

$$a - c - \frac{1}{2}b(2q_i + q_j) \equiv 0, \quad (3.13)$$

from which

$$a - c - \frac{1}{2}b(q_i + q_j) \equiv \frac{1}{2}bq_i, \quad (3.14a)$$

so that when the first-order condition holds, and in particular in Nash-Cournot equilibrium, firm  $i$ 's payoff is

$$\pi_i = \frac{1}{2}bq_i^2 - F_i, \quad i = 1, 2. \quad (3.15)$$

Equilibrium outputs, found by solving the system of equations formed by the two first-order conditions, are identical (given that assumption that firms have identical marginal costs),

$$q_d = 2\frac{a - c}{3b} \quad (3.16)$$

per firm.

Then from (3.15), equilibrium flow payoffs when integration is complete are

$$\pi_i = \frac{1}{2}b \left( 2\frac{a - c}{3b} \right)^2 - F_i = \frac{2}{9}b \left( \frac{a - c}{b} \right)^2 - F_i. \quad (3.17)$$

Rearranging terms,  $\pi_i \geq 0$  and both firms are profitable (at least, do not make losses) in the full-integration market if fixed costs are not too large relative to market size:

$$\frac{\sqrt{F_i/b}}{\frac{a-c}{b}} \leq \frac{\sqrt{2}}{3}, \quad (3.18)$$

for  $i = 1, 2$ .

In contrast, both firms would be unprofitable in a post-integration duopoly if

$$\frac{\sqrt{2}}{3} < \frac{\sqrt{F_i/b}}{\frac{a-c}{b}} \quad (3.19)$$

for  $i = 1, 2$ . Combining (3.11) and (3.19), each pre-integration market is a natural monopoly, as is the post-integration market, if

$$\frac{\sqrt{2}}{3} < \frac{\sqrt{F_i/b}}{\frac{a-c}{b}} \leq \frac{1}{2}, \quad (3.20)$$

for  $i = 1, 2$ .

We assume that demand and technology parameters satisfy (3.20).

### 3.1.5. Integration and payoffs

(3.4) is the inverse demand equation facing firm  $i$  if both firms are in operation at integration level  $\iota$ ,  $0 \leq \iota \leq 1$ . Firm  $i$ 's profit is

$$\pi_i = \left( a - c - b \frac{q_i + \iota q_j}{1 + \iota} \right) q_i - F_i, \quad (3.21)$$

for  $i, j = 1, 2$  and  $j \neq i$ .

Rearranging terms, the first-order condition to maximize (3.21) is

$$2q_i + \iota q_j = (1 + \iota) \frac{a - c}{b} \quad (3.22)$$

and this implies that firm  $i$ 's equilibrium payoff is

$$\pi_i = \frac{b}{1 + \iota} q_i^2 - F_i. \quad (3.23)$$

By symmetry, equilibrium output levels are the same, and at integration level  $\iota$ , equilibrium output per firm is

$$q_d = \frac{1 + \iota}{2 + \iota} \frac{a - c}{b}. \quad (3.24)$$

In the “declining industry” literature, the driving assumption is that the demand curve moves continuously toward the origin. Substituting  $i = 1$ ,  $q_j = q_d$  in (3.4) and rearranging terms shows that firm 1's residual inverse demand equation at integration level  $\iota$  is

$$p_1 = c + \frac{2}{2 + \iota} (a - c) - b \frac{q_1}{1 + \iota}. \quad (3.25)$$

As integration goes forward, the price-axis intercept of firm 1's residual inverse demand curve falls, and the inverse demand curve becomes flatter, with slope changing continuously from  $-b$  for  $\iota = 0$  to  $-\frac{1}{2}b$  for  $\iota = 1$ .

Duopoly output increases as integration goes forward,

$$\frac{\partial q_d}{\partial \iota} = \frac{1}{(2 + \iota)^2} \frac{a - c}{b} > 0, \quad (3.26)$$

as each firm faces a progressively larger number of consumers who are in the integrated market.

From (3.23) and (3.26),

$$\frac{1}{b} \frac{\partial \pi_i^d}{\partial \iota} = -\frac{\iota}{(2 + \iota)^3} \left( \frac{a - c}{b} \right)^2 < 0, \quad (3.27)$$

and instantaneous profit falls as integration goes forward and the pseudo-product differentiation described by the inverse demand equation (3.4) falls.

## 3.2. No State Aid

### 3.2.1. Equilibrium withdrawal integration levels

Let  $\iota_i^d$  be the degree of integration at which firm  $i$ 's duopoly profit just equals zero.  $\iota_i^d$  is implicitly defined by

$$\pi_i^d(\iota) = \frac{b}{1 + \iota} q_d^2(\iota) - F_i = \frac{b}{1 + \iota} \left( \frac{1 + \iota a - c}{2 + \iota} \frac{1}{b} \right)^2 - F_i \equiv 0. \quad (3.28)$$

By our assumptions about the ranking of fixed costs, (3.6), and (3.27), we know that the high-fixed cost firm sees its profit go to zero earlier in the integration process than does the low-fixed cost firm:

$$\iota_1^d < \iota_2^d. \quad (3.29)$$

For  $\iota > \iota_2^d$ , both firms lose money if both are active.

The relationships that determine equilibrium withdrawal levels are illustrated in Figure 3.1.<sup>11</sup> As integration increases, residual demand curves fall and rotate

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<sup>11</sup>To minimize visual clutter, residual marginal revenue curves and the marginal cost curve are omitted from Figure 3.1. However,  $q_1 = 60$  is firm 1's noncooperative duopoly equilibrium output for integration level  $\iota = 0.5$ ;  $q_2 = 65.8$  is firm 2's noncooperative duopoly equilibrium output for integration level  $\iota = 0.925$ .

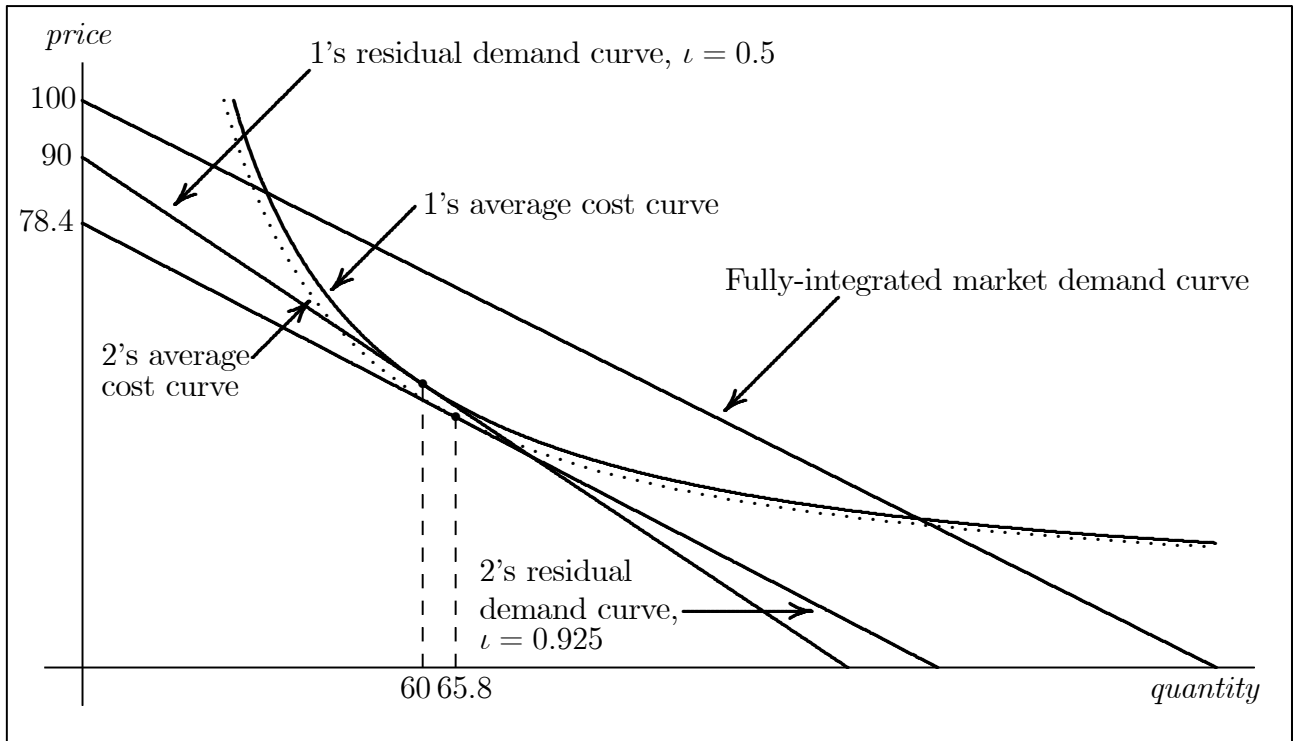


Figure 3.1: Withdrawal integration levels, Cournot duopoly, partially integrated market. Each residual demand curve is drawn for the other firm producing its equilibrium output at the indicated integration level.  $a = 110, c = 10, b = 1, F_1 = 2400, F_2 = 2250$ .

in a counterclockwise direction. At integration level  $\iota = 0.5$ , firm 1's residual demand curve is tangent to its average cost curve. For greater integration levels, firm 1's duopoly payoff is negative. Firm 2 has smaller fixed cost than firm 1; for integration level  $\iota = 0.925$ , firm 2's average cost curve is tangent to its average cost curve, and firm 2 as well has a negative duopoly payoff for greater integration levels.

The subgame perfect equilibrium without state aid is given by Proposition 1, which is proven in the Appendix.

**Proposition 1:** It is a subgame perfect equilibrium for firms to compete as Nash-Cournot duopolists from integration level 0 to integration level  $\iota_1^d$ , for firm 1 to withdraw when integration level  $\iota_1^d$  is reached, and for firm 2 to supply the market as a monopolist thereafter.



As noted in footnote 9, this result generalizes to the case of pre- and post-integration oligopoly.

### 3.2.2. Welfare

We measure welfare as the sum of consumer surplus and producer surplus (economic profit). For cases in which subsidies are granted, we assume that the social cost of a euro of aid is one euro. It may well be — particularly in view of the self-imposed budget constraints of the Stability and Growth Pact — that the opportunity cost of granting aid to a firm is more than one euro. We say more about this below. If the welfare cost of a euro of aid is more than a euro, then it is less likely that the net impact of aid will be beneficial, all else equal.

In equilibrium, from integration level 0 to  $\iota_1^d$ , net social welfare in each country is the sum of the profit of its home firm and domestic consumer surplus. From integration level  $\iota_1^d$  to 1, firm 2 is the single supplier of the partially-integrated market, and from integration level 1 onward, firm 2 is the single supplier of the fully integrated market.

After integration level  $\iota_1^d$ , firm 2 maximizes profit along its partial-integration demand curve (with  $q_1 = 0$ ). Flow welfare in country 1 is the consumer surplus of those of its residents who are in the integrated market and buy from firm 2. Flow welfare in country 2 is the sum of the consumer surplus of its residents and the profit of firm 2 (which includes profit on sales made in country 1).

## 3.3. State Aid

### 3.3.1. By Country 1

In the spirit of the early strategic trade policy literature, suppose country 1 and only country 1 can commit to giving its home firm lump-sum aid in the amount of any losses the home firm might sustain.

The cost to country 1 of this policy is the discounted value of subsidies to firm 1 between integration levels  $\iota_1^d$  and  $\iota_2^d$ .<sup>12</sup> With subsidies, firm 1 is guaranteed at least a normal flow rate of return on investment, and would not exit the market. The optimal action for firm 2 is then to exit at integration level  $\iota_2^d$ .

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<sup>12</sup>In principle, an unrealistically farsighted country 1 government might give firm 1 just enough of a subsidy so that the discounted value of firm 1's losses between integration levels  $\iota_1^d$  and  $\iota_2^d$  and the discounted value of its monopoly profits from integration level  $\iota_2^d$  onward equal zero, so that firm 1 would be willing to stay in the market while receiving a lower subsidy than that discussed in the text.

The benefits to country 1 are the (appropriately discounted) economic profits of firm 1, which includes profit on sales made in country 2, as well as additional consumer surplus to those country 1 consumers who are not in the integrated market during the integration period. Unless discount rates are very high, the net benefit to country 1 will be positive, and granting the subsidy will be privately beneficial for country 1.

A subsidy by country 1 imposes costs on country 2: the profits that firm 2 would otherwise earn are lost after it exits, and some surplus that country 2 consumers would otherwise enjoy is lost in the partially integrated market.

A subsidy also reduces the overall economic benefit from integration. In the case considered here, the fully-integrated market is a monopoly. The globally-efficient outcome is that firm 2, which has lower fixed cost, supply the integrated market. A subsidy granted by country 1 to firm 1 over the interval  $t_1^d$  to  $t_2^d$  imposes higher fixed cost on the integrated market forever.

### 3.3.2. By Both Countries

If country 2 can also commit to loss-neutralizing subsidies for its home firm, it can avoid the losses that would be inflicted by a unilateral country 1 subsidy. Subsidies would continue forever. Consumers would be better off, but net welfare, taking subsidies into account, would be reduced, compared with the no-subsidy case. Further, all potential welfare gains from integration would be lost, since there would be no saving of fixed cost.

### 3.3.3. Welfare flows in the fully-integrated market

Table 3.1 gives the general expressions for per-period welfare outcomes in the fully-integrated market with no subsidies (upper left), with a subsidy by country 1 only (lower left), and with subsidies by both countries (lower right). If the factor used to discount future income flows is sufficiently close to one, the qualitative relationship of these flow values must indicate the qualitative relationship of the corresponding present discounted values. We first discuss the flow values, then turn our attention to numerical evaluation of discounted values for specific parameters.

If country 1 alone grants a subsidy, its welfare must increase, and that of country 2 decrease, compared with the no-subsidy outcome: a subsidy by country 1 alone shifts monopoly profit from country 2 to country 1, and leaves consumer surplus unchanged.

|         | No subsidy  | Subsidy  |
|---------|---|--|
| No      | $0 + \frac{1}{8}b \left(\frac{a-c}{b}\right)^2$   |  |
| Subsidy | $\frac{1}{2}b \left(\frac{a-c}{b}\right)^2 - F_2 + \frac{1}{8}b \left(\frac{a-c}{b}\right)^2$ |  |
| Total   | $\frac{3}{4}b \left(\frac{a-c}{b}\right)^2 - F_2$   |  |
| Subsidy | $\frac{1}{2}b \left(\frac{a-c}{b}\right)^2 - F_1 + \frac{1}{8}b \left(\frac{a-c}{b}\right)^2$ | $0 + \frac{2}{9}b \left(\frac{a-c}{b}\right)^2 + \left[ \frac{2}{9}b \left(\frac{a-c}{b}\right)^2 - F_1 \right]$ |
|         | $0 + \frac{1}{8}b \left(\frac{a-c}{b}\right)^2$   | $0 + \frac{2}{9}b \left(\frac{a-c}{b}\right)^2 + \left[ \frac{2}{9}b \left(\frac{a-c}{b}\right)^2 - F_2 \right]$ |
| Total   | $\frac{3}{4}b \left(\frac{a-c}{b}\right)^2 - F_1$   | $\frac{8}{9}b \left(\frac{a-c}{b}\right)^2 - F_1 - F_2$  |

Table 3.1: Alternative welfare outcomes per period, fully-integrated market: the general case. Rows and upper entries in each cell refer to country 1, columns and lower entry in each cell refer to country 2. Elements in each sum are firm value, discounted consumer surplus, and subsidy (where applicable). See text for definition of parameters.

If country 2 also grants a subsidy, the profit of its firm falls from 0 to less than zero, and the subsidy just matches the amount of the loss. Country 2 consumer surplus increases, as the common market is a duopoly rather than a monopoly. The change in country 2 welfare if it matches country 1's subsidy policy is

$$\frac{2}{9}b \left(\frac{a-c}{b}\right)^2 - F_2 + \frac{2}{9}b \left(\frac{a-c}{b}\right)^2 - \frac{1}{8}b \left(\frac{a-c}{b}\right)^2 = \frac{23}{72}b \left(\frac{a-c}{b}\right)^2 - F_2. \quad (3.30)$$

The first two terms on the left are firm 2's (negative) payoff in the fully-integrated duopoly market, which in absolute value is the flow subsidy paid by country 2. The third term on the left is country 2 resident consumer surplus in the fully-integrated duopoly market. The fourth term is country 2 resident consumer surplus in the fully-integrated monopoly market.

Given our assumptions about the relationship between fixed costs and market size, (3.20), this difference is positive. Thus country 1 increases its welfare if it alone grants a subsidy, and if country 1 grants a subsidy, then country 2 increases its welfare by granting a subsidy to its own firm.

The change in country 1 flow welfare between the two-subsidy and the no-subsidy case is

$$\frac{2}{9}b \left(\frac{a-c}{b}\right)^2 - F_1 + \left[ \frac{2}{9}b \left(\frac{a-c}{b}\right)^2 - \frac{1}{8}b \left(\frac{a-c}{b}\right)^2 \right], \quad (3.31)$$

|         | No subsidy         | Subsidy                     |
|---------|--------------------|-----------------------------|
| No      | 0 + 1250 = 1250    |                             |
| Subsidy | 2750 + 1250 = 4000 |                             |
| Total   | 5250               |                             |
| Subsidy | 2600 + 1250 = 3850 | 0 + 2222.2 - 177.8 = 2044.4 |
|         | 0 + 1250 = 1250    | 0 + 2222.2 - 27.8 = 2194.4  |
| Total   | 5100               | 4238.8                      |

Table 3.2: Alternative welfare outcomes per period, fully-integrated market. Rows and upper entries in each cell refer to country 1, columns and lower entry in each cell refer to country 2. Elements in each sum are firm value, discounted consumer surplus, and subsidies (where applicable).  $a = 110$ ,  $c = 10$ ,  $b = 1$ ,  $F_1 = 2400$ ,  $F_2 = 2250$ .

where the first term (losses of the subsidized firm 1) is negative and the term in braces (change in consumer surplus between duopoly and monopoly) is positive. Combining terms, (3.31) is

$$\frac{23}{72}b \left( \frac{a-c}{b} \right)^2 - F_1, \quad (3.32)$$

and once again this is positive, as for (3.20).

One can also show that “common market” flow welfare falls moving from the no-subsidy to the country 1-subsidy only case, and falls again moving to the two-subsidy case. Thus the noncooperative equilibrium if national governments act to maximize national welfare flows in the fully-integrated market is that both countries grant subsidies. The country that is home to the high-cost firm increases its national welfare by initiating a subsidy game, even if the other country follows suit and even though common market welfare would be greater without any subsidies.

### 3.3.4. Simulations and discounted welfare values

Table 3.2 illustrates the general results of Table 3.1 for specific parameter values. It shows, as does Table 3.1, that considering per-period welfare outcomes in the fully-integrated market, the equilibrium outcome is for both countries to subsidize, even though this reduces common market welfare.

One might wonder, however, if this result could be upset if the appropriately discounted welfare values during the integration period are taken into account.

|         | No subsidy  | Subsidy  |
|---------|---|--|
| No      | $521.59 + 13379.8 = 13901.4$  |  |
| Subsidy | $7933.6 + 14148.3 = 22081.9$  |  |
| Total   | 35983.3   |  |
| Subsidy | $3079.87 + 15475.1 - 112.94 = 18442$<br>$1760.11 + 15466.9 = 17227$ | $521.59 + 16432.9 - 286.56 = 16667.9$<br>$1760.11 + 16432.9 - 25.08 = 18167.9$ |
| Total   | 35669   | 34835.9  |

Table 3.3: Alternative welfare outcomes, linear integration function. Rows and upper entries in each cell refer to country 1, columns and lower entry in each cell refer to country 2. Elements in each sum are firm value, discounted consumer surplus, and (where applicable) discounted subsidy.  $a = 110$ ,  $c = 10$ ,  $b = 1$ ,  $r = 1/10$ ,  $F_1 = 2400$ ,  $F_2 = 2250$ ,  $T = 25$ .

The integral expressions for welfare during the integration period (these are derived in the Appendix) lack closed-form solutions, and must be evaluated numerically. Simulation results suggest that the results of Table 3.1 and Table 3.2 are quite general (as they must be, if the interest rate used to discount future income is sufficiently close to zero).

Table 3.3 gives present-discounted welfare values for the parameterization of Table 3.2 of the model for a 25-year integration period and a linear integration function.<sup>13</sup> The pre-integration rates of profit and consumer surplus are 100 and 1250 per year in country 1, 250 and 1250 per year in country 2, respectively. Integration thus improves welfare in both countries. The gain in country 1 is minimal — 13901 versus a capitalized  $10(1350) = 13500$ . The value of firm 1, which goes out of business, is sharply reduced by integration.<sup>14</sup> In country 1, it is consumers who benefit from integration (in the absence of subsidies).

If country 1 alone grants a subsidy, it improves its own welfare at the expense of country 2. Country 2 consumers, some of whom purchase from a duopoly for a

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<sup>13</sup>The numerical results reported here were generated by Mathematica programs and checked using the version of Maple that is part of Scientific Workplace. The Mathematica programs are available from the authors on request.

<sup>14</sup>In the model considered here, the owners of the firm that goes out of business shift the firm's assets to other markets, where they earn at least a normal rate of return. In practice, appeals for state aid are often rationalized by assertions that capital investments (non-human and human) are partially or wholly sunk, and cannot move to other markets. Analysis of this aspect of the market integration question requires explicit consideration of input choices, and is the subject of ongoing research.

|            | No subsidy  | Subsidy  |
|------------|---|--|
| No Subsidy | 751.43 + 124999 = 125750<br>238585 + 128307 = 366891        |  |
| Total      | 492641  |  |
| Subsidy    | 206936 + 134856 - 640.17 = 341151<br>3208 + 134786 = 137994 | 751.43 + 211965 - 14727.2 = 197989<br>3208 + 211965 - 2183.75 = 212989 |
| Total      | 479146  | 410978   |

Table 3.4: Alternative welfare outcomes, linear integration function. Rows and upper entries in each cell refer to country 1, columns and lower entry in each cell refer to country 2. Elements in each sum are firm value, discounted consumer surplus, and (where applicable) discounted subsidy.  $a = 110$ ,  $c = 10$ ,  $b = 1$ ,  $r = 1/100$ ,  $F_1 = 2400$ ,  $F_2 = 2250$ ,  $T = 25$ .

longer period than without subsidies, are better off if there is a subsidy by country 1. A subsidy by country 1 alone reduces total welfare.

Total welfare is further reduced if both countries grant subsidies. Consumers, in this case able to purchase from a duopoly *ad infinitum*, are even better off than before, but subsidies reduce overall welfare.

But country 1 is unambiguously able to improve its welfare by granting a subsidy to its firm, even if country 2 adopts a retaliatory subsidy policy. Country 2 is worse off if both countries subsidize than if neither do, but country 2 is better off granting its own subsidy if (as one would expect for the parameter values of Table 3.3) country 1 grants a subsidy. It is thus an equilibrium for both countries to subsidize.

The qualitative nature of the results shown in Table 3.3 are robust. The parameter values used for Table 3.4 are, with one exception, identical to those used for Table 3.3; the interest rate used to discount future income is 1/100 for Table 3.4, as opposed to 1/10 for Table 3.3. In Table 3.4 and in Table 3.5 ( $r = 1/100$  with a 100-year integration period), the qualitative relationships of the welfare values for the different policy combinations match those of Table 3.3.

This result may be one explanation for the persistence of EU Member State attempts to grant aid that contravenes the competition policy provisions of the EC Treaty: this is the noncooperative equilibrium outcome.

Two aspects of the situation, both outside our formal model, may explain Member State agreement on those Treaty provisions. First is to argue that, given government budget constraints, the social welfare cost of each euro used to

|         | No subsidy   | Subsidy   |
|---------|--|---|
| No      | 2641.4 + 127322 = 129963                                     |   |
| Subsidy | 159186 + 135399 = 294584                                     |   |
| Total   | 424548   |   |
| Subsidy | 105472 + 150656 - 1428 = 254701<br>10265.5 + 150520 = 160786 | 2641.4 + 189128 - 8437 = 183332<br>10265.5 + 189128 - 1061 = 198332 |
| Total   | 415486   | 381664  |

Table 3.5: Alternative welfare outcomes, linear integration function. Rows and upper entries in each cell refer to country 1, columns and lower entry in each cell refer to country 2. Elements in each sum are firm value, discounted consumer surplus, and (where applicable) discounted subsidy.  $a = 110$ ,  $c = 10$ ,  $b = 1$ ,  $r = 1/100$ ,  $F_1 = 2400$ ,  $F_2 = 2250$ ,  $T = 100$ .

subsidize a private firm is very likely more than one euro (Neary, 1994; Collie, 2003). Then the welfare gains from subsidizing the home country firm are less than suggested by Table 3.3.

Second, one may note that for a common market, it is the welfare effects in all markets, not any one market, that are of interest. In one market, country 1 may better itself at the expense of country 2 by granting aid to home country firms. In another market, it is country 2 that will come out ahead if both countries subsidize. Taking all markets into account restores a situation in which a commitment by both countries not to grant aid, thus maximizing overall welfare, can be a noncooperative equilibrium.

## 4. Conclusion

Paradoxically, market integration, which expands the size of the market available to each firm, has some economic implications in common with those of declining markets. In both cases, the equilibrium number of firms falls over time. The result is that it is in the national interest of component markets to grant state aid, even though such aid reduces or eliminates the economic benefits that flow from integration. This in turn justifies binding international agreements that eliminate the national gains that would flow from the granting of state aid.

The observation that EU member states, having agreed to control of state aid, have a history of granting aid that is regularly found to violate Treaty provisions invites explanation. One part of such an explanation lies in time inconsistency,

as national governments find it convenient to deal with conjunctural crises, especially in the run-up to national elections, with policy choices that will (under the Treaty) be neutralized, but in the future. Another part of the explanation may be that penalties have not always been sufficient (it is only relatively recently that aid found to violate guidelines has been recovered). But another part of the explanation may be that when it is consumer behavior that spreads the integration process over time, if attention is confined to individual product markets, mutual granting of subsidies may be an equilibrium outcome, even though this reduces common market welfare for that product market.



## 5. Appendix

### 5.1. Proof of Proposition 1

The argument of the proof is, with one difference, that of Brainard (1994, Section 2.A). Brainard derives a subgame perfect equilibrium strategy for his declining industry model by working backward from times at which either firm would earn zero profit even as a monopolist. In the model considered here, the analysis works backward from integration levels at which firms would have zero values playing mixed withdrawal strategies.

(A) At any point in time, firms play withdrawal probabilities  $\sigma_1$  and  $\sigma_2$ , respectively, and compete as Cournot duopolists if both are in the market. Equilibrium withdrawal strategies vary with time during the integration period and are constant in the fully-integrated market.

If both firms are in the market at time  $T$  or any time thereafter, it is a subgame perfect equilibrium for the firms to play the mixed withdrawal strategies given by (5.4) below. Equilibrium expected values playing these strategies are zero.

The fully integrated market inverse demand equation is (2.9). If firm  $i$  supplies the integrated market as a monopolist, its profit-maximizing output and payoff rates are

$$q_m^I = \frac{a - c}{b} \quad \pi_i^{mI} = \left( a - c - \frac{1}{2}bq_i \right) q_i - F_i. \quad (5.1)$$

If both firms are active in the fully integrated market, flow duopoly profits are given by (3.23) evaluated for  $q_i = q_d$ .

Let  $\sigma_i$  be firm  $i$ 's probability of withdrawal at time  $t \geq T$ , if both firms are in the market. Withdrawal probabilities are constant for any  $t \geq T$  at which both firms are in the market, since their expected payoffs in different states of the world are the same at all such times. Withdrawal strategies follow an exponential distribution; the probability that firm 1 drops out before time  $t$ , given that firm 2 has not dropped out, is

$$1 - e^{-\sigma_1 t}.$$

With probability density  $e^{-(\sigma_1 + \sigma_2)t} dt$ , neither firm has dropped out at time  $t$ , and firm 1's payoff is  $\pi_1^{dI}$ .

With probability density  $\sigma_1 e^{-(\sigma_1 + \sigma_2)t} dt$ , firm 1 drops out at time  $t$ , firm 2 has not yet dropped out, and firm 1's value from that point onward is 0.

With probability density  $\sigma_2 e^{-(\sigma_1 + \sigma_2)t} dt$ , firm 2 drops out at time  $t$ , firm 1 has not yet dropped out, and firm 1's value from that point onward is  $\pi_1^{mI}/r$ .

Integrating over all future time, firm 1's expected value at time  $t$  if both firms are in the market is

$$V_1^I = \int_0^\infty e^{-(r+\sigma_1+\sigma_2)t} \left[ \sigma_1(0) + \sigma_2 \frac{\pi_1^{mI}}{r} + \pi_1^{dI} \right] dt = \frac{\sigma_2 \frac{\pi_1^{mI}}{r} + \pi_1^{dI}}{r + \sigma_1 + \sigma_2}. \quad (5.2)$$

In the same way, firm 2's value from anytime  $t \geq T$  at which both firms are in the market is

$$V_2^I = \frac{\sigma_1 \frac{\pi_1^{mI}}{r} + \pi_2^{dI}}{r + \sigma_1 + \sigma_2}. \quad (5.3)$$

For firms to be willing to play random strategies, these values must be zero, yielding

$$\sigma_1 = \frac{-\pi_2^{dI}}{\pi_2^{mI}/r} \quad \sigma_2 = \frac{-\pi_1^{dI}}{\pi_1^{mI}/r}. \quad (5.4)$$

Our assumptions mean that  $\pi_j^{dI} < 0$ ,  $j = 1, 2$ , so the numerators in (5.4) are positive. It is reasonable to suppose that duopoly losses are less in magnitude than the capitalized value of monopoly profit (and this is the case for the specifications investigated here). Hence the expressions in (5.4) are less than one, and so are valid withdrawal probabilities.

(B) If both firms are in the market at integration level  $\iota_2^d$  or thereafter, it is a subgame perfect equilibrium for the firms to play the mixed withdrawal strategies given by (5.16) below, and thereafter if both firms are in the market to play the mixed withdrawal strategies given by (5.4). Equilibrium expected values playing these strategies are zero.

Flow duopoly payoffs are

$$\pi_i^d = \frac{b}{1+\iota} \left( \frac{1+\iota a-c}{2+\iota} \frac{1}{b} \right)^2 - F_i, \quad (5.5)$$

and given our assumptions, these payoffs are negative. Note that because  $\iota$  is a function of time, so is  $\pi_i^d$ .

If firm  $j$  drops out, firm  $i$  maximizes profit along the partial-integration demand curve

$$p_i = a - b \frac{q_i + \iota q_j}{1+\iota} \Big|_{q_j=0} = a - b \frac{q_i}{1+\iota}. \quad (5.6)$$

Firm  $i$  maximizes

$$\pi_i^m(\iota) = \left( a - c - b \frac{q_i}{1+\iota} \right) q_i - F_i. \quad (5.7)$$

The first-order condition is

$$a - c - b \frac{2q_i}{1 + \iota} \equiv 0, \quad (5.8)$$

so that

$$a - c - b \frac{q_i}{1 + \iota} \equiv b \frac{q_i}{1 + \iota}, \quad (5.9)$$

$$q^m \equiv (1 + \iota) \frac{a - c}{2b}, \quad (5.10)$$

and the flow monopoly payoff is

$$\pi^m(\iota) = \frac{b}{1 + \iota} \left[ (1 + \iota) \frac{a - c}{2b} \right]^2 - F_i = b(1 + \iota) \left( \frac{a - c}{2b} \right)^2 - F_i. \quad (5.11)$$

If firm 2 exits at time  $t$ , firm 1's value from the moment of firm 2's exit is

$$V_1^m(t) = \int_{\tau=t}^T \pi^m[\iota(\tau)] e^{-r\tau} d\tau + e^{-(T-t)} \frac{\pi_1^{mI}}{r}. \quad (5.12)$$

This is the present discounted value of firm 1's monopoly profit during what is left of the integration period, plus the discounted value of monopoly output in the fully integrated market.

Let  $\sigma_i(t)$  be the probability that firm  $i$  withdraws at time  $t$  during the integration period, conditional on not having withdrawn before.<sup>15</sup> For notational convenience, write

$$\Sigma_i(t) = \int_{\tau=0}^t \sigma_i(\tau) d\tau. \quad (5.13)$$

With probability density  $e^{-(\Sigma_1 + \Sigma_2)} dt$ , neither firm has dropped out at time  $t$ , and firm 1's payoff is  $\pi_1^d$ .

With probability density  $\sigma_1(t) e^{-(\Sigma_1 + \Sigma_2)} dt$ , firm 1 drops out at time  $t$ , firm 2 has not yet dropped out, and firm 1's value from that point onward is 0.

With probability density  $\sigma_2(t) e^{-(\Sigma_1 + \Sigma_2)} dt$ , firm 2 drops out at time  $t$ , firm 1 has not yet dropped out, and firm 1's value from that point onward is  $V_1^m(t)$ .

Firm 1's expected value at time  $t$  during the integration period if both firms are active is

$$V_1^d(t) = \int_t^\infty e^{-[r\tau + \Sigma_1(\tau) + \Sigma_2(\tau)]} [\sigma_1(0) + \sigma_2 V_1^m(\tau) + \pi_1^d(\tau)] d\tau$$

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<sup>15</sup>For a similar formulation in another context, see Fudenberg *et al.* (1983).

$$= \int_t^\infty e^{-[r\tau + \Sigma_1(\tau) + \Sigma_2(\tau)]} [\sigma_2 V_1^m(\tau) + \pi_1^d(\tau)] d\tau. \quad (5.14)$$

In the same way

$$V_2^d(t) = \int_t^\infty e^{-[r\tau + \Sigma_1(\tau) + \Sigma_2(\tau)]} [\sigma_1 V_2^m(\tau) + \pi_2^d(\tau)] d\tau. \quad (5.15)$$

In order for the firms to be willing to play mixed strategies, these expected values must be zero, which requires that

$$\sigma_1(\iota) = \frac{-\pi_2^d(\iota)}{V_2^m(\iota)} \quad \sigma_2(\iota) = \frac{-\pi_1^d(\iota)}{V_1^m(\iota)}. \quad (5.16)$$

Given our assumptions, these are both numbers between zero and one and so are valid withdrawal probabilities.

(C) Now suppose both firms are in the market at any integration level between  $\iota_1^d$  and  $\iota_2^d$ . Firm 2 will stay in: it will make money up to integration level  $\iota_2^d$  even if firm 1 stays in, and at worst break even thereafter. Firm 1 will play out, since it would lose money up to integration level  $\iota_2^d$  and break even thereafter. Suppose then that both firms are in the market at integration level  $\iota_1^d$ . Firm 1 will play out, since it will lose money as long as it stays in over integration levels  $\iota_1^d$  to  $\iota_2^d$  and break even thereafter if both firms are in the market at integration level  $\iota_2^d$ . Firm 2 will stay in, since it will make money over the period  $\iota_1^d$  to  $\iota_2^d$  and break even thereafter if both firms are in the market at integration level  $\iota_2^d$ .

This completes the proof.

## 5.2. Welfare

We write lower-case letters to denote flow values, with upper-case letters reserved for integrals of flow values.

### 5.2.1. Integration levels 0 to $\iota_1^d$

Flow profits per firm  $\pi_i^d$  are given by (5.5). The inverse demand curves (3.4) are implied by an aggregate gross welfare equation

$$a(q_1 + q_2) - \frac{1}{2} \frac{b}{1 + \iota} (q_1^2 + 2\iota q_1 q_2 + q_2^2). \quad (5.17)$$

If both firms produce the output  $q_d$  given by (3.24), (5.17) becomes

$$2aq_d - bq_d^2. \quad (5.18)$$

Taking account of the cost of production, net aggregate welfare under duopoly during the integration period is

$$2(a-c)q_d - bq_d^2 - (F_1 + F_2) = b \frac{(1+\iota)(3+\iota)}{(2+\iota)^2} \left( \frac{a-c}{b} \right)^2 - (F_1 + F_2), \quad (5.19)$$

with flow net aggregate welfare in country  $i$

$$w_i^d(\iota) = \frac{1}{2}b \frac{(1+\iota)(3+\iota)}{(2+\iota)^2} \left( \frac{a-c}{b} \right)^2 - F_i. \quad (5.20)$$

Subtracting  $\pi_i^d$ , from (5.5), from  $w_i^d(\iota)$ , flow consumer surplus in country  $i$  during the integration period if both firms are active is

$$s_i^d(\iota) = \frac{1}{2}b \left( \frac{1+\iota}{2+\iota} \frac{a-c}{b} \right)^2 = \frac{1}{2}bq_d^2. \quad (5.21)$$

### 5.2.2. Integration levels $\iota_1^d$ to 1

In equilibrium, firm 1 drops out at integration level  $\iota_1^d$ . Firm 2 maximizes profit on the inverse demand equation

$$p_2 = a - \frac{b}{1+\iota}q_2. \quad (5.22)$$

Firm 2's profit-maximizing output is

$$q_m(\iota) = (1+\iota) \frac{a-c}{2b} = (1+\iota)q_m, \quad (5.23)$$

where by modest abuse of notation we write

$$q_m = \frac{a-c}{2b} \quad (5.24)$$

for pre-integration monopoly output in the component markets. Firm 2's payoff is

$$\pi_2^m(\iota) = (1+\iota)b \left( \frac{a-c}{2b} \right)^2 - F_2. \quad (5.25)$$

Price is

$$p_2 = c + \frac{1}{2}(a-c).$$

Country 1 consumers who are in the country 1 market only buy nothing, and their surplus welfare is zero.

Country 2 consumers who are in the country 2 market only buy

$$(1 - \iota) \frac{a - p}{b} = (1 - \iota) \frac{1}{2} \frac{a - c}{b}$$

and their surplus welfare is

$$(1 - \iota) \left[ (a - p) \frac{1}{2} \frac{a - c}{b} - \frac{1}{2} b \left( \frac{1}{2} \frac{a - c}{b} \right)^2 \right] = (1 - \iota) \frac{1}{2} b \left( \frac{1}{2} \frac{a - c}{b} \right)^2.$$

Consumers (of both countries) who are in the integrated market buy

$$\iota 2 \frac{a - p}{b} = \iota 2 \left( \frac{a - c}{2b} \right).$$

Their surplus welfare is

$$\iota \left[ (a - p) 2 \left( \frac{a - c}{2b} \right) - \frac{1}{4} b (4) \left( \frac{a - c}{2b} \right)^2 \right] = \iota b \left( \frac{a - c}{2b} \right)^2.$$

Half of this accrues to consumers in each country:

$$\frac{1}{2} \iota b \left( \frac{a - c}{2b} \right)^2.$$

The rate of consumer surplus in country 2 if firm 1 has dropped out of the market:

$$s_2^{m2}(\iota) = (1 - \iota) \frac{1}{2} b \left( \frac{a - c}{2b} \right)^2 + \iota \frac{1}{2} b \left( \frac{a - c}{2b} \right)^2 = \frac{1}{2} b \left( \frac{a - c}{2b} \right)^2 = \frac{1}{2} b q_m^2. \quad (5.26)$$

Firm 2's profit rate if firm 1 has dropped out of the market is given by (5.25).

The rate of net social welfare in country 2 if firm 1 has dropped out of the market is the sum of firm 2's profit and country 2 consumers' surplus:

$$w_2^{m2}(\iota) = (1 + \iota) b \left( \frac{a - c}{2b} \right)^2 - F_2 + \frac{1}{2} b \left( \frac{a - c}{2b} \right)^2 = \frac{b}{2} (3 + 2\iota) q_m^2 - F_2. \quad (5.27)$$

Flow aggregate welfare and consumer surplus in country 1 if firm 1 has dropped out of the market:

$$s_1^{m2}(\iota) = \iota \frac{1}{2} b q_m^2 = \iota \frac{1}{2} b \left( \frac{a-c}{2b} \right)^2. \quad (5.28)$$

If, in contrast, firm 1 should be the surviving supplier, economic profit during the integration period goes to firm 1 and it is all country 1 consumers that are in the market, while a fraction  $1 - \iota$  of country 2 consumers are out of the market. The resulting welfare rates are:

Firm 1's payoff:

$$\pi_1^m(\iota) = (1 + \iota) b \left( \frac{a-c}{2b} \right)^2 - F_1. \quad (5.29)$$

Country 1 consumer surplus:

$$s_1^{m1}(\iota) = \frac{1}{2} b q_m^2. \quad (5.30)$$

Country 1 net aggregate welfare:

$$w_1^{m1}(\iota) = \frac{b}{2} (3 + 2\iota) q_m^2 - F_1 \quad (5.31)$$

Country 2 net aggregate welfare, which is also the rate of country 2 consumer surplus, since firm 2 has exited:

$$w_2^{m1}(\iota) = s_2^{m1}(\iota) = \iota \frac{1}{2} b q_m^2 \quad (5.32)$$

### 5.2.3. Fully integrated market

If one firm supplies the fully integrated market, monopoly output is

$$q_m^I = \frac{a-c}{b} = 2q_m. \quad (5.33)$$

Monopoly profit is

$$\pi_i^m(1) = 2bq_m^2 - F_i = 2bq_m^2 - F_i. \quad (5.34)$$

Consumer surplus in both countries is

$$s^{mi}(1) = bq_m^2, \quad (5.35)$$

and consumer surplus in each country is half this.

If both firms supply the fully integrated market, flow profits per firm (not taking account of any subsidies) are

$$\begin{aligned}\pi_i^d(1) &= \frac{1}{2}b \left( \frac{2a-c}{3} \frac{1}{b} \right)^2 - F_i \\ &= \frac{8}{9}bq_m^2 - F_i.\end{aligned}\tag{5.36}$$

Given the assumptions we have made about demand and technology, (3.11), flow profits are negative.

Consumer surplus in the fully integrated market with both firms active is

$$s^d(1) = b \left( \frac{2a-c}{3} \frac{1}{b} \right)^2 = \frac{16}{9}b(q_m)^2.\tag{5.37}$$

Half of this accrues to consumers in each component market. Net social welfare in the integrated market is the sum of profits and consumer surplus; net social welfare in each component market is the sum of the profit of the home firm and the surplus of domestic consumers.

#### 5.2.4. Discounted values

**No subsidies** Firm 1's equilibrium discounted duopoly payoff from the start of the integration period (suppressing the functional dependence of  $\iota$  on time for notational compactness) is

$$\Pi_1 = \int_0^{t_1^d} e^{-rt} \pi_1^d(\iota) dt = b \left( \frac{a-c}{b} \right)^2 \int_0^{t_1^d} e^{-rt} \frac{1+\iota}{(2+\iota)^2} dt - F_1 \left( 1 - e^{-rt_1^d} \right)\tag{5.38}$$

This is firm 1's discounted profit from time 0 to time  $t_1^d$ , at which time it exits.

Firm 2's equilibrium discounted value is

$$\Pi_2 = \int_0^{t_1^d} e^{-rt} \pi_2^d(\iota) dt + \int_{t_1^d}^T e^{-rt} \pi_2^m(\iota) dt + e^{-rT} \frac{\pi_2^m(1)}{r}.\tag{5.39}$$

This is firm 2's duopoly profit from time 0 to time  $t_1^d$ , its monopoly profit in the partially integrated market from time  $t_1^d$  to time  $T$ , and its monopoly profit in the fully-integrated market, all appropriately discounted.



Equilibrium discounted consumer surplus, by country, is

$$\begin{aligned}
S_1 &= \int_0^{t_1^d} e^{-rt} s_1^d(\iota) dt + \int_{t_1^d}^T e^{-rt} s_1^{m2}(\iota) dt + e^{-rT} \frac{\frac{1}{2} S^{mi}(1)}{r} \\
&= \left[ 4 \int_0^{t_1^d} e^{-rt} \left( \frac{1+\iota}{2+\iota} \right)^2 dt + \int_{t_1^d}^T e^{-rt} \iota dt + \frac{1}{r} e^{-rT} \right] \frac{1}{2} b q_m^2. \tag{5.40}
\end{aligned}$$

$$\begin{aligned}
S_2 &= \int_0^{t_1^d} e^{-rt} s_2^d(\iota) dt + \int_{t_1^d}^T e^{-rt} s_2^{m2}(\iota) dt + \int_T^\infty e^{-rt} s_2^{m2}(1) dt \\
&= \frac{1}{2} b \left( \frac{a-c}{b} \right)^2 \int_0^{t_1^d} e^{-rt} \left( \frac{1+\iota}{2+\iota} \right)^2 dt + \frac{1}{2} b q_m^2 \int_{t_1^d}^\infty e^{-rt} dt \\
&= \left[ 4 \int_0^{t_1^d} e^{-rt} \left( \frac{1+\iota}{2+\iota} \right)^2 dt + e^{-rt_1^d} \frac{1}{r} \right] \frac{1}{2} b q_m^2. \tag{5.41}
\end{aligned}$$

Net social welfare by country is the sum of profit and consumer surplus by country.

**Subsidy by country 1** The present value of the lump-sum subsidy that allows country 1 to break even over integration levels  $t_1^d$  to  $t_2^d$  is the negative of firm 1's losses over that interval,

$$L_1^1 = \int_{t_1^d}^{t_2^d} e^{-rt} \pi_i^d(\iota) dt = \int_{t_1^d}^{t_2^d} e^{-rt} \left[ \frac{b}{1+\iota} \left( \frac{1+\iota}{2+\iota} \frac{a-c}{b} \right)^2 - F_1 \right] dt, \tag{5.42}$$

where the superscript "1" indicates that country 1 grants a subsidy.

If firm 1 receives such a subsidy and firm 2 receives no subsidy, firm 1's discounted payoff is the discounted sum of its duopoly payoffs to time  $t_1^d$ , zero from time  $t_1^d$  to time  $t_2^d$  (when firm 2 exits), and its monopoly profit thereafter,

$$\begin{aligned}
\Pi_1^1 &= \int_0^{t_1^d} e^{-rt} \pi_i^d(\iota) dt + \int_{t_1^d}^{t_2^d} e^{-rt} (0) dt + \int_{t_2^d}^T e^{-rt} \pi_1^m(\iota) dt + e^{-rT} \frac{\pi_1^m(1)}{r} \\
&= \int_0^{t_1^d} e^{-rt} \left[ \frac{b}{1+\iota} \left( \frac{1+\iota}{2+\iota} \frac{a-c}{b} \right)^2 - F_1 \right] dt + \int_{t_2^d}^T e^{-rt} [(1+\iota) b q_m^2 - F_1] dt
\end{aligned}$$

$$+e^{-rT} \frac{2bq_m^2 - F_1}{r}. \quad (5.43)$$

Discounted consumer surplus in country 1 if firm 1 receives the subsidy is

$$S_1^1 = \int_0^{t_2^d} e^{-rt} s_1^d(\iota) dt + e^{-rt_2^d} \frac{\frac{1}{2}bq_m^2}{r}.$$

Firm 2's discounted payoff if firm 1 receives the subsidy is

$$\Pi_2^1 = \int_0^{t_2^d} e^{-rt} \pi_2^d(\iota) dt = b \left( \frac{a-c}{b} \right)^2 \int_0^{t_2^d} e^{-rt} \frac{1+\iota}{(2+\iota)^2} dt - F_2 \left( 1 - e^{-rt_2^d} \right). \quad (5.44)$$

Discounted consumer surplus in country 2 if firm 1 receives the subsidy is

$$S_2^1 = \int_0^{t_2^d} e^{-rt} s_2^d(\iota) dt + \int_{t_2^d}^T e^{-rt} s_2^{m1}(\iota) dt + e^{-rT} \frac{\frac{1}{2}S^{mi}(1)}{r}. \quad (5.45)$$

**Subsidy by both countries** The present value of the subsidy by country  $i$  is the negative of the losses of the home country firm,

$$L_i^{12} = \int_{t_i^d}^1 e^{-rt} \pi_i^d(\iota) dt + e^{-rT} \frac{\pi_i^d(1)}{r}, \quad (5.46)$$

for  $i = 1, 2$ .

The present values of firm payoffs are

$$\Pi_1^{12} = \int_0^{t_1^d} e^{-rt} \pi_1^d(\iota) dt \quad (5.47)$$

and

$$\Pi_2^{12} = \int_0^{t_2^d} e^{-rt} \pi_2^d(\iota) dt, \quad (5.48)$$

respectively.

Discounted consumer surplus in the subsidized duopoly, by country, (see (5.21)) is

$$S_i^{12} = \int_0^1 e^{-rt} s_i^d(\iota) dt + e^{-rT} \frac{s_i^d(1)}{r}, \quad (5.49)$$

for  $i = 1, 2$ .

Net social welfare in each country is the sum of discounted profit and discounted consumer surplus, minus the discounted amount of the subsidy.

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