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EDUCATION AND TRAINING IN A MODEL OF ENDOGENOUS GROWTH WITH CREATIVE WEAR-AND-TEAR

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Education and Training in a Model of Endogenous Growth with Creative Wear-and-Tear*

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Abstract

How does the rate at which firms adopt new technologies affect the level of education and training of a country's workforce? If technological change makes knowledge obsolete and tends to foster general rather than firm-specific skills, what would be the optimum level of education spending in front of a faster arrival of new technologies? This paper tries to answer these questions by developing an endogenous growth model with creative 'wear and tear' in which general education enhances innovation through R&D and lowers adjustment costs to new technologies, while on-the-job training is necessary for firms to realise their profit potentials by implementing the new technologies and reap all the related future quasi-rents. The paper reproduces some stylized facts on the technology-training relationship and shows how the optimum amount of time devoted to education and job training is affected by the rate of technical change itself. In particular, we find that a faster arrival of innovations shifts the private knowledge portfolio towards general human capital. We also find that households tend to under invest in education, thus leading to lower growth rates than technically feasible, and higher training costs than absolutely necessary. This suggests that there is room for education policy reducing private education fees.

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1 Introduction

During the last few decades we have witnessed the rise in the economic relevance of the so called intangible resources, like school education, vocational training, workplace learning and research and development. The process of computerization and the diffusion of ICT in particular, have changed the nature of work and transformed the types of knowledge, skills and attitudes that individuals need for successful employment and work performance (OECD, 2000; Bresnahan, Brynjolfsson, and Hitt, 2002). Under the pressure of ongoing technological progress and the spread of new work practices, firms call upon specialized staff and require individuals to acquire an ever larger set of knowledge and competences. However, these individuals do not necessarily do that at the beginning of their life or at school. With this in mind, policy makers have identified a set of key generic skills that are of particular importance, like motivation, communication skills, analytical reasoning, IT and computer skills, problem-solving and ability to work in teams.

There is ample evidence that generic skills are indeed increasing in importance, both at the workplace and in the wage determination of workers (Juhn, Murphy, and Pierce, 1993; Murnane, Willet, and Levy, 1995; Green, Ashton, and Felstead, 2001; Gould, 2002; Autor, Levy, and Murnane, 2003). At the same time, the relative increase in the supply of more educated workers in the industrialized countries has gone hand in hand with rising returns to education. This puzzling phenomenon has been associated with the idea that technological progress is intrinsically high-skilled biased.

The skill-biased technological change hypothesis has received extensive attention from both economists and policy makers. In particular, since the seminal contribution of Nelson and Phelps (1966), economists have put substantial efforts in establishing that the introduction of a new technology within a firm, or within an industry, is complementary to the employment of a more educated workforce, who enjoys a comparative advantage in adapting to changing environments (Welch, 1970; Bartel and Lichtenberg, 1987).

However, the relationship between the adoption of new technologies and the performance of firms and economic systems is not obvious. Once a new technology enters production, an adjustment process begins that involves both physical capital costs, in terms of equipment set-up, and human capital costs, in terms of workers learning (Jovanovic and Nyarko, 1996;

Bessen, 2002). Moreover, the link between skills and technological change is two-sided. On the one hand, as the economics of technology diffusion has stressed in particular, adjustment costs and the speed of technology adoption are highly affected by the skill distribution of the population (Goldin and Katz, 1998, 2007). On the other hand, technological change does have an influence on the human capital composition of the labour force, since it can stimulate the demand for highly educated workers and/or depress the demand for less qualified workers (Acemoglu, 1998, 2002; Autor, Katz, and Krueger, 1998). But even though there is ample consensus on the impact of technology on the demand for high-skilled workers, there is little evidence of the mechanisms through which technology affects the demand and the supply of different types of human capital, specifically general education and (technology) specific job-training.

As far back as the 1960s, labour economists have investigated the heterogeneous nature of human capital in determining the patterns of skill accumulation within countries. Nonetheless, modern growth models do not seem to take this heterogeneity sufficiently into account. In particular, economic theory does not seem to provide an unambiguous prediction of the sign of the relationship between technological change and aggregate investment in workplace training. The latter is the outcome of an ongoing interaction between employers and workers, in which technological change affects the incentives for both parties (Bartel and Sicherman, 1998). In this respect, one argument is that technological change makes formal education and previously acquired skills obsolete, thus increasing investments in specific training or experience and reducing the investments in general education (Tan, 1989; Weinberg, 2004). An alternative view is that general education enables workers to adjust to and benefit from technological change, so that there are higher incentives to invest in schooling than in specific, on-the-job, training (Welch, 1970; Gould, Moav, and Weinberg, 2001; Gould, 2002).

To sum up then, two main questions still remain unanswered. First, what is the nature of the link between faster arrival of new technologies and the human capital composition, particularly in terms of education and training, of a country's workforce? Second, what are the costs and benefits of such an accumulation of human capital and are they shared in such a way that the most efficient use is made of all (potential) human capital resources?

The present paper tries to shed some light on these questions by developing an endogenous growth model in which the adoption of a new technology is supposed to be costly for firms, since before that technology can be used in production, the associated workers need to learn how to actually handle it. When operating new technologies, we let specific skills, acquired through on-the-job training, be imperfect substitutes for general education at the workplace. We further endogenize the process of general knowledge accumulation by making households decide upon their level of general education in the context of the optimal allocation of their income derived from working and the ownership of assets (firms in this case) over consumption and the accumulation of assets and human capital. However, acquiring education comes at an explicit cost for these private households (i.e. education fees).

There are a number of tradeoffs that are associated with education. First of all, when a higher number of individuals in the economy gets educated, that represents a signal to firms of a higher trainability of individuals, so that firms can expect to benefit from lower training costs and workers can expect to benefit from higher earnings profiles. Moreover, higher education fosters R&D activity, thus increasing the production of new technologies. On the other hand, higher education contributes to an increase in the rate of creative destruction, thus forcing firms to provide more spells of specific training on the job. At the same time, higher education intensities also imply a drop in total working hours available for current production and research activities and higher education expenditures by households at the expense of private consumption expenditures.

The model we develop combines elements from Romer (1990), Aghion and Howitt (1992) and the Ramsey (1928) model, so that productivity growth is the result of firms' 'Love of Variety' and quality improvements in a context of intertemporally optimal consumer decisions. From Romer (1990) we borrow the idea that technological change is of an 'organizational' nature and takes the form of horizontal product differentiation. From Aghion and Howitt (1992), instead, we take the idea that the arrival of new technologies drives the old technologies out of the market, so that technological change takes the form of vertical product differentiation with Schumpeterian creative destruction. However, in our model, the obsolescence of older intermediates is not complete, but gradual, as in a 'creative wear and tear' process where all varieties live forever even if they gradually fade away over time (VanZon and Yetkiner, 2003). To this basic set-up, we add the notion of the principal importance of heterogeneous human capital for economic growth, while we use the Ramsey intertemporal optimisation framework, to endogenize the households decision regarding the allocation of their resources over different activities, among which consumption and the accumulation of (general) human capital through a process of self-financed education.

As opposed to previous work, in our setting it is technological change that opens the possibility to have a mismatch between the skills acquired by individuals and the skills to be used in the workplace when adopting the newest technologies. In other words, technology is always firm specific, and technological change generates new tasks that can only be performed after the acquisition of specific skills on the job has been completed. In addition, technological change affects the rate at which human capital becomes obsolete: as a consequence, workplace training can increase or decrease at higher rates of technological change.

In this model we show that education plays a threefold role in shaping the technology-training relationship: (i) from the point of view of the firm, a higher level of general skills available allows it to reap higher streams of quasi-rents because of lower training costs; (ii) from the R&D-sector's point of view, a higher level of education fosters the invention of new technologies, thus increasing the rate of obsolescence of specific skills at the workplace; (iii) from households' point of view, higher education signals a higher trainability and, thus, a higher expected future earnings capacity once matched to the new task. However, being educated requires time and therefore generates opportunity costs in terms of consumption foregone.

The present paper also differs from previous ones since it considers time as the main unit of measure for describing each activity and each trade-off. Human capital accumulation through school education, job training and R&D, thus, is supposed to be a time-consuming process and our main aim is to show how the sectors of the economic system optimally adjust their allocation of time in response to the arrival of new technologies.

The paper achieves some important results. First, it reproduces the mixed findings regarding the relationship of substitutability/complementarity between general education and specific training. Second, it shows that, in time of rapid accumulation of new technologies, firms change the optimum time 'portfolio-mix' between general education and specific training in favour of the former, since it provides a relatively solid basis for the development of technology-specific skills that are prone to creative destruction.

The paper is further structured as follows. Section 2 surveys endogenous growth models dealing with technology adoption and education/training. Section 3 describes the main features of our model, while section 4 presents some simulation results. Finally section 5 provides the main conclusions and policy implications.

2 Background Literature

Our model links the labour economics literature on education and training with the endogenous growth literature on R&D and human capital formation as the primary cause of productivity growth.

While there is ample theoretical and empirical research on the relationship between technology use and the skill level of workers by education and/or occupational category, the relationship between technological change and the human capital composition of the labour force is not clear-cut. When a new technology is introduced into a production process, the firm may incur large technology-specific adjustment costs because of the need to learn new skills, implementing new organizational forms, or developing complementary capital investments. Bessen (2002), for instance, estimates that capital adoption costs for US manufacturing industries rose by over 4% of GDP in the 1970s and 1980s, this pattern being particularly significant in the period in which information technologies and computers diffused more rapidly. In addition, Greenwood and Yorukoglu (1997) estimate that learning costs as a share of GDP increased by 1.5% in the 1970s, together with a substantial rise in wage inequality. On the third aspect, Brynjolfsson and Hitt (2000) and Bresnahan, Brynjolfsson, and Hitt (2002) find that organizational investments can be considerably larger than the investments in IT equipment itself.

The literature on skill-biased technological change (SBTC) has always pointed to a positive relationship between technological change, i.e. particularly in the form of computerization or ICT diffusion, and the skill upgrading of workers. Sector- or plant-level analyses generally find a robust relationship between the change in the share of skilled workers employed and the use of technology at the level of a single industry (Berndt, Morrison, and Resenblum, 1992; Berman, Bound, and Griliches, 1994; Autor, Katz, and

Krueger, 1998; Haskel and Heden, 1999), or at the level of a single firm (Dunne and Schmitz, 1995; Doms, Dunne, and Troske, 1997; Entorf and Kramarz, 1997; Dunne, Haltiwanger, and Troske, 1997; Dunne and Troske, 2005).

When we look at the close relationship between technology and training, instead, there is only little evidence available. However, it does tell us that, although faster technological change requires more workers to be trained and perhaps more frequently so Lillard and Tan (1986); Gill (1988); Bresnahan, Brynjolfsson, and Hitt (2002); Galia and Legros (2004), the duration of training (which is measured as the average training time required to become fully qualified in the current job) need not be longer (Mincer, 1989). Moreover, little is said on the interaction between general education human capital and specific training human capital in a context characterized by fast technical change. From Nelson and Phelps (1966), Welch (1970) and Bartel and Lichtenberg (1987), we know that highly educated individuals tend to adopt innovations sooner compared to less educated individuals, since the former enjoy a comparative advantage in adapting to change and in implementing new and more complex non-manual tasks (Autor, Levy, and Murnane, 2003; Spitz, 2003). Nevertheless, how technological change ultimately affects training investments for workers with different levels of education is still largely unknown territory.

According to the little evidence available (Tan, 1989; Bartel and Sicherman, 1998): (i) the incidence of training is higher at higher rates of technological change; (ii) the more educated individuals are more likely to receive firm training, but technical change tends to remove this effect as the training gap between highly educated and less educated narrows; (iii) when a new technology is first introduced, the general skills of highly educated workers act like a substitute for firm training. As experience with the new technology is gained, then it is possible to train less educated employees in performing new tasks; (iv) the net effect of technological change on training human capital is obsolescence, whereas the net effect on schooling human capital is an increase in productivity.

At the macroeconomic level, our model is in line with previous work by Acemoglu (1997), Helpman and Rangel (1999), Galor and Moav (2000) and Gould, Moav, and Weinberg (2001). In this respect, a number of contributions study the direct impact of the adoption of a new technology on the human capital composition of workers, even if, generally speaking, the focus of these analyses is on the relationship between technology and the increasing wage inequality that characterizes advanced economies.

After the seminal paper by Grossman and Helpman (1991), works by Greenwood and Yorukoglu (1997), Caselli (1999), Galor and Moav (2000), Aghion, Howitt, and Violante (2002), Violante (2002) and Weinberg (2004), for instance, have focused on the effect that the adoption of a new general purpose technology exerts on the skill premium and on labor productivity. In these models, more educated, and thus more skilled, individuals have a comparative advantage since the time required to learn how to use new technologies diminishes with the level of knowledge that workers have accumulated in the past. The need for a skilled labour force is then particularly perceived at the beginning of the adoption phase, when returns from adoption are more uncertain, whereas, as the technology becomes established, the production process becomes standardised, thus allowing producers to substitute away from expensive high-skilled labour to less expensive unskilled labour.

Helpman and Rangel (1999), instead, show that technological change requiring more education and training, like computerization and ICT diffusion, necessarily produces an initial productivity slowdown, whereas technological change requiring less education and training, like the move from the artisan shop to the factory and the assembly line, can produce either a boom or a bust.

Gould, Moav, and Weinberg (2001) argue that, when technological progress occurs, individuals ask for general education since it guards against the higher depreciation risk associated with technology specific skills. Faster technological change, then, is correlated with higher amounts of investments in technology-specific skills by less educated workers because they suffer from higher rates of obsolescence as compared to highly educated workers.

Our model is also in line with recent work by Krueger and Kumar (2004), who postulate that economic systems that provide a relatively intensive policy support for vocational education, like the European Union, will grow slower than systems favouring general education, like the US, when the arrival rate of new technologies accelerates.

3 The Model

3.1 General Outline

The model that we present here tries to answer two broad questions. The first is how the optimal accumulation of general versus specific skills would change when the rate of technological change does change. The second question is whether the allocation of time over education and other uses of time is the most effective in generating growth.

While trying to answer these questions, the model will also be required to address/reproduce two stylized facts. The first concerns the relative increase in importance of general skills as a source of growth with respect to specific skills in times of fast technological change (Acemoglu and Pischke, 1999; Lindbeck and Snower, 2000b; Green, Ashton, and Felstead, 2001; Stasz, 2001; Gould, 2002; Spitz, 2003). Based on this, we would expect there to be a premium on formal education rather than specific training in times of rapid technological change. However, we would also expect that an increase in the formal level of education could even result in a reduction of output growth because of the increase in 'technology absorption costs' in terms of output foregone during re-training spells that arrive at a faster rate, but may take a shorter time.

The second stylised fact concerns the mixed nature of the relationship between general and specific training by showing conditions under which the two types of human capital act like substitutes and the conditions under which they behave like complements (Gill, 1988; van Smoorenburg and van der Velden, 2000; Allen and der Velden, 2001; Brunello, 2001; Heijke, Meng, and Ris, 2003).

In our set-up, the economy consists of four production sectors and a household sector that interact with each other. The educational sector is responsible for the production of general education, while the R&D sector generates new technologies, and, finally, the final output production sector generates consumption goods using intermediate goods produced by the intermediate goods sector. The model furthermore incorporates three types of temporal trade-offs: (i) human capital production versus immediate participation on the labour market; (ii) technology versus intermediate goods production; (iii) training versus current production. To start with, households should choose between spending time at school for the accumulation of human capital and going to work. The production of human capital further requires people in the economy to allocate time to teaching.

The decision to spend time on non-educational activities is governed by the second trade-off that concerns spending time to develop new technologies through R&D or entering production and spending time to manufacture intermediates, that in turn give rise to the flow of final goods used for consumption purposes.

Finally, the third trade-off occurs at the level of the production process and involves the choice of the firm between accumulating the necessary specific skills through training in order to be able to adopt the latest technology - and reap all the related future earnings - or starting production sooner rather than later by employing low-skilled labour instead of well-trained labour.

To summarise the above, in the model we distinguish between knowledge acquired (by households), knowledge required (for the production of the intermediates incorporating the new technology) and knowledge used (in the actual production of the final good, after the accumulation of technology-specific skills on the job, and in R&D activity). The model starts from the assumption that the introduction of a new technology requires the users to learn how to use it effectively. In order to do that, the workforce associated with a new technology needs to be trained first[1]. During their training period, workers earn a competitive wage, since they are assumed to be indifferent between working (and so 'earning their keep') and training (thus learning to 'earn their keep' later on). However, they do not produce anything, thus giving rise to an opportunity cost for the firm in terms of output forgone. Therefore, the total cost of training includes both the direct wage cost and the opportunity cost of foregone output. The benefits from training, instead, are an increased productivity later on during the produc-

[1]In the model we do not directly focus on mechanisms like 'learning by doing' (Arrow, 1962; Jovanovic and Nyarko, 1996) or 'experience' (Rosen, 1972; Helpman and Rangel, 1999; Weinberg, 2004) for two reasons: first, we want to observe how the adoption of new technologies affects the human capital composition of employees at the beginning of the working process, in order to avoid a situation in which the market activity entails a joint production of learning plus production. Secondly, we are interested in predicting how firms' and households' investments decisions change with the arrival of new intermediates, and both school education and on-the-job training represent two types of investment in human capital in which it is easier to identify monetary (other than opportunity) costs and benefits. For a theoretical synthesis of the discussion around the difference between learning by doing, experience, and training see Killingsworth (1982).

tion phase of a technology, which enables the entrepreneur to economize on wage-costs during that phase. Training, therefore, generates cost-reductions in the future in return for current cost-increases.

Since we assume that there is no production while training, it follows immediately that a model with complete creative destruction of old technologies through the arrival of new ones is less suited for our purposes, since it would generate 'odd' growth patterns that can not be observed in reality. For, in such a creative destruction setting, the arrival of a new superior technology would lead to a zero level of output, while the workforce is retrained to be used with the new technology. Instead, an approach in which new technologies do not fully replace old ones, but gradually drive them out of the market seems to be more suited. This is the 'creative wear and tear effect' also used in VanZon and Yetkiner (2003): in such a 'Love of Variety' set-up, old technologies never die, but simply become less important and therefore less used with the progress of time. This creative 'wear and tear' continuously and gradually releases labour resources that can be re-employed by the new firms created with the aim of using the new technologies that arrive on the market.

In the labour market, general skills play a double role: on the one hand, they increase labour productivity of workers in the production of new technologies, while on the other hand, they are supposed to raise the efficiency of the acquisition of further technology-specific skills at the workplace as they increase 'workers absorptive capacity' (Lloyd-Ellis, 1999) and, thus, reduce technology absorption costs. Particularly when a mismatch between skills required by new tasks and skills acquired by workers arises, an investment in specific training reduces this gap, so that lower levels of formal education can be compensated by higher investments in on the job training and vice versa.

We assume that general skills are acquired by individuals through schooling. While job training is entirely financed by firms, we assume that households pay for acquiring higher levels of general education. The accumulation of human capital through education provides individuals with a higher trainability on the labour market, and therefore higher wages once employed. To obtain these expected returns, households face a double cost: a direct financial cost, given by the fee they need to pay in order to finance educational activities (i.e. both teaching and learning), and an opportunity cost in the form of wage income foregone while being educated.

Technological change, finally, is the result of R&D activity. Following Romer (1990) and Rivera-Batiz and Romer (1991), the knowledge contained in new blueprints is embodied into new intermediate goods that, in turn, give rise to the production of a single final good that serves as the consumption good in our model economy. We further assume that the production and accumulation of blueprints depends positively on the number of hours spent in R&D, and on the productivity of researchers (in part defined by their educational level).

In short, our model therefore contains four production sectors in the model, i.e. the education sector, the intermediate goods sector, the final output sector and the R&D sector. There are two institutional sectors, i.e. private households and firms. Firms have to decide on hiring inputs in order to be able to produce their output, while households have to decide how to spend their time on different activities: learning and the supply of labour to the different sectors. We will first describe how the education sector is supposed to work, before describing how consumers are supposed to allocate their time. In the following subsections we describe how production and training as well as R&D decisions are made.

3.2 Education

We assume that individuals acquire general knowledge by spending time and income in the educational system. By assumption, schooling is a time-consuming activity that is a substitute for working activity and that involves not only pupils but also teachers, so that total time allocated to the education system is split between time spent learning and time spent teaching:

$$H = (H_L + H_T) + H_W \tag{1}$$

where H is the total time available for the entire population, and where H_L is the amount of time that an individual spends at school acquiring general education, while H_T is teaching time; H_W is the total time devoted to working activities, either in direct production or in R&D. $H_L + H_T$ is the total amount of time taken-up by the educational system. Defining $\varphi = H_L/H$, and $\psi = H_T/H$, i.e. φ is the fraction of time spent learning by the population at large, and ψ is the fraction of time devoted by the population to teaching, we further assume that both the schooling intensity φ and the number of teaching hours per pupil hour, i.e. ψ/φ, will contribute positively to the build-up of the level of general human capital per person ϵ:

$$\epsilon = \zeta_0 \cdot \varphi^{\zeta_1} \cdot (H_T/H_L)^{\zeta_2} = \zeta_0 \cdot \varphi^{\zeta_1 - \zeta_2} \cdot \psi^{\zeta_2} \tag{2}$$

where we reasonably assume $\zeta_0 > 0$ and $\zeta_1 > \zeta_2 > 0$ in order for both schooling and teaching to have a positive effect on the level of human capital per person.

As regards the cost-side of education, we assume that households have to devote part of their income to finance the teaching activity that is necessary for increasing ϵ. Households' decisions, therefore, concern the choice between consumption, income from working and the accumulation of human capital in order to benefit from higher future wages on the labour market.

With respect to teaching, we assume that the cost of teaching per unit of pupil time (further called f which is short for teaching 'fee'), depends on the number of teaching hours per pupil hour and the hourly wage of teachers. Assuming that teachers earn a competitive hourly wage, we must have that the teaching cost per pupil hour is given by:

$$f = w \cdot (H_T/H_L) = w \cdot \psi/\varphi. \tag{3}$$

With regard to the productivity effects of education, we simply assume that the production efficiency of labour will depend positively on both human capital per person and the amount of training received per person, as proxied by the length of the training period s.

$$\pi(s, \epsilon) = \gamma_0 \cdot s^{1-\gamma_1} \cdot \epsilon^{\gamma_1}. \tag{4}$$

Equation (4) is a simple linear homogenous power-function in which we assume that $0 < \gamma_1 < 1$, and $\partial\pi/\partial s > 0, \partial\pi/\partial\epsilon > 0$. Equation (4) shows that a constant level of worker efficiency can be attained for different combinations of s and ϵ, where a decreasing value of s must be compensated by an ever increasing value of ϵ in order to keep worker efficiency constant, and vice-versa. Hence, at the micro-level, education and training are assumed to be (imperfect) substitutes.

If we take ω to signal wages per efficiency unit of labour, wages per (physical) unit of labour time are simply the product between efficiency per labourer and ω, giving (after substitution of (2)):

$$w = \pi(s, \epsilon) \cdot \omega = \gamma_0 \cdot \zeta_0^{\gamma_1} \cdot s^{1-\gamma_1} \cdot \varphi^{\gamma_1 \cdot (\zeta_1 - \zeta_2)} \cdot \psi^{\gamma_1 \cdot \zeta_2} \cdot \omega. \tag{5}$$

Equation (5) shows that our assumption that $\zeta_1 > \zeta_2$ implies that, *ceteris paribus*, hourly wages will depend positively on both pupil and teaching time spent in the educational system.

3.3 The Time Allocation Problem of Private Households

The problem of households is to maximize the stream of utility derived from a consumption stream that itself depends on the allocation of time between work on the one hand, and education and teaching on the other. With respect to the latter, we simply assume that consumers are indifferent as to where they work [2].

In order to solve this allocation problem, we use the Ramsey approach, where we accumulate assets (reflecting the ownership of blueprints/firms) rather than physical capital. The stock of assets make up the wealth of households, further called W.

Using c to denote the flow of consumption per capita, the accumulation of wealth is given by [3]:

$$\dot{W} = r \cdot W + w \cdot (H_W + H_T) - f \cdot H_L - c \cdot H. \tag{6}$$

Assuming a standard CIES-utility function, the corresponding Hamiltonian is given by:

$$\begin{aligned} \Omega \ &= e^{-\rho \cdot t} \cdot (c^{1-\theta}/(1-\theta) \cdot H + \lambda \cdot \dot{W} = e^{-\rho \cdot t} \cdot (c^{1-\theta}/(1-\theta) \cdot H + \\ &\quad \lambda \cdot (r \cdot W - c \cdot H + \omega \cdot \pi(s, \epsilon) \cdot (1 - \psi - \varphi) \cdot H) \end{aligned} \tag{7}$$

where ρ represents the constant rate of discount and λ is the co-state variable of this intertemporal utility maximisation problem. Note that we have used (3) and (5) to arrive at (7). In this problem, only c,ψ and φ are variables that are under the control of consumers. Maximising the Hamiltonian with respect to these controls, we find that:

$$\partial\Omega/\partial\varphi \Rightarrow \varphi = \gamma_1 \cdot (\zeta_1 - \zeta_2)/(1 + \gamma_1 \cdot (\zeta_1 - \zeta_2)) \cdot (1 - \psi) \tag{8a}$$

[2] Note that this implies that all production sectors have to offer the same wage rate, otherwise a sector would be offered all labour available or none at all.

[3] A dot over a variable x denotes the time derivative of x, i.e.$\dot{x} = dx/dt$. Furthermore we will be using a hat over a variable to denote the proportional rate of growth of that variable, i.e. $\hat{x} = \dot{x}/x$.

$$\partial\Omega/\partial\psi \Rightarrow \varphi = 1 - (1+\gamma_1\cdot\zeta_2)/(\gamma_1\cdot\zeta_2)\cdot\psi \tag{8b}$$

$$\partial\Omega/\partial c \Rightarrow \hat{c} = (-\hat{\lambda}-\rho)/\theta = (r-\rho)/\theta \tag{8c}$$

$$\partial\Omega/\partial W = -\dot{\lambda} \Rightarrow \hat{\lambda} = -r \tag{8d}$$

It should be noted that (8a) and (8b) both imply a trade-off between learning and teaching time. Simultaneously, however, they fix the amount of learning and teaching time, independently of the other parameters of the model, particularly those associated with the R&D production process[4].

The simultaneous solution for φ and ψ is given by:

$$\varphi^* = \gamma_1\cdot(\zeta_1 \ -\zeta_2)/(1+\gamma_1\cdot\zeta_1) \tag{9a}$$

$$\psi^* = \gamma_1\cdot\zeta_2(1+\gamma_1\cdot\zeta_1) \tag{9b}$$

Equations (9a) and (9b) both imply that teaching time and learning time will depend positively on γ_1 if indeed $\zeta_1 > \zeta_2 > 0$, implying that a higher elasticity of labour efficiency with respect to human capital (i.e. γ_1 in equation (4)) will lead to a rise in the allocation of time to learning and teaching. Yet, the ratio of these optimum values of learning and teaching time is independent of the characteristics of the efficiency function, but instead depends only on those of the human capital accumulation function as given by (2). A higher contribution of teaching hours per pupil hour as given by ζ_2, would indeed increase teaching hours relative to pupil hours, and the other way around. Similarly, if the elasticity of human capital formation with respect to pupil hours (i.e. ζ_1) increases, then φ^* will increase too, while ψ^* will fall. The implication of this is that for groups of countries that would differ with respect to these parameters ζ_1 and ζ_2 learning and teaching hours will be negatively correlated, while for groups of countries that would differ with respect to γ_1, learning and teaching hours will be positively correlated.

[4]Obviously, this is due to our assumption that consumers are well informed about the direct productivity effects of their educational decisions, but ignore the growth effects that these decisions may have through their impact on R&D productivity (we will come back to this in the section covering the R&D sector). A central planner would also take these growth effects into account, leading to a different allocation of time between education and other efforts and to higher growth, as we will show in section 4.

3.4 The Production Set

3.4.1 Preliminaries

The production sector is organised along the lines of Romer (1990), except that it does not use physical capital, but just human capital services to produce intermediates that, together with a freely available fixed factor, go into making the final output. This set-up makes for a production structure that is separable in the intermediates just as in the Romer model. This separability significantly reduces the complexity of the model. We use the assumption of full employment of human capital to obtain the supply of labour available for work in either intermediate goods, final goods or blueprint production as the difference between total time available and time spent in the educational sector. We will go into the supply of labour in somewhat greater detail below.

As regards the decision how much labour to supply for R&D purposes, that depends on the equilibrium wage rate in the R&D sector, that, just like in the Romer (1990) model, depends on discounted profits generated in intermediate goods production. Those profits can be captured by the R&D sector, since it is the ownership of the corresponding blueprint that actually allows the owner to claim the associated profit stream. Transfer of that ownership will take place only if the original owner, i.e. the R&D sector, will be sufficiently compensated for the loss of his claim. In the limit, that compensation will have to be equal to the value of the claim, which is what Romer (1990) and we have assumed. In the remainder of this section, we will describe the features of the production sectors in more detail.

3.4.2 Production Labour Suppy Issues

We assume that the size of the population remains fixed over time. At the same time it constantly needs education, as we would expect it to occur in real life too due to the fact that older (educated) generations die, and the youngest are born with a clean educational slate. We will (somewhat superficially) model this by assuming that at any time just a fraction $0 \leq (1 - \varphi - \psi) \leq 1$ of the population is available for final output production and R&D activities, whereas the complement of that fraction, i.e. $\varphi + \psi$, is tied up in the education process. Given the above, we must have that:

$$H_W = (1 - \varphi - \psi) \cdot H = H_R + H_P \qquad (10)$$

Equation (10) describes the supply of production labour time as the complement of the amount of time spent in the educational system. It also states that the part of the population that is not in the educational system is either engaged in R&D activities (H_R) or in direct production activities (H_P), while H_P itself is in part engaged in training (T) and in production (J):

$$H_P = T + J \tag{11}$$

Equations (10) and (11) reflect the assumption that the labour market is always clearing, and so there is no unemployment pool that absorbs excess supply. Equation (10) also indicates that educated high-skilled R&D labour is a perfect substitute for high-skilled production labour, and the other way around.

3.4.3 Final Output Production and Intermediate Goods Demand

Final output is produced under perfectly competitive circumstances, by assembling the intermediate outputs coming from the intermediate goods sector using only a fixed factor, further called L. To simplify matters, we assume that L is equal to 1 and furthermore that it is freely available to the final output sector. We use an Ethier function to describe this final output production process:

$$Y = L^{1-\alpha} \cdot \int_0^B (x_i)^\alpha di \tag{12}$$

In equation (12), Y is final output, B is the index of the latest technology that has entered the production phase, and x_i is the volume of intermediate output associated with production technology i. $(1-\alpha)$ is the partial output elasticity of the freely available fixed factor, while $\sigma = 1/(1-\alpha) > 1$ is the elasticity of substitution between intermediates.

The demand for each individual intermediate good by the final output sector then follows immediately from the assumption of instantaneous profit maximisation under conditions of perfect competition:

$$p_i = L^{1-\alpha} \cdot \alpha \cdot (x_i)^{\alpha-1}, 0 \leq i \leq B \tag{13}$$

where the price of final output is used as the numeraire. Equation (13) is the inverse demand equation for intermediates: still as in Romer (1990), it will function as a supply constraint for the producers of intermediate goods.

3.4.4 Intermediate Goods Supply and Labour Demand

For reasons of simplicity, we assume a linear production technology so that the production of one unit of each intermediate good requires the use of one efficiency unit of high-skilled labour. Efficiency units of labour used in the production of x_i are called h_i. We therefore have:

$$h_i = x_i, \forall\ 0 \leq i \leq B \tag{14}$$

As already described above, we assume that one physical unit of labour associated with technology i (further called j_i) can generate $\pi(s, \epsilon)$ efficiency units of labour (cf. equation (4)) We therefore have:

$$j_i = \frac{h_i}{\pi(s, \epsilon)} = \frac{x_i}{\pi(s, \epsilon)}, \forall\ 0 \leq i \leq B \tag{15}$$

where the level of intermediate goods production is given by (13).

The corresponding demand for labour in physical units can immediately be obtained by substituting (14) and (15) into (13):

$$j_i = L \cdot (p_i/\alpha)^{-\sigma}/\pi(s, \epsilon) \tag{16}$$

Equation (16) represents the derived demand for labour for each intermediate goods producing firm, given the price it sets for its intermediate.

3.4.5 Quasi-Rents Maximization in Intermediate Goods Production

The price setting behaviour referred to above, together with the behaviour that determines how long the training period s should be, can be formulated as the solution to the problem of maximising the net present value of the flow of quasi-rents that can be obtained by hiring a population of workers, training them for s time-units, and then using these trained workers to generate output that is sold conditional on the inverse demand curve given by (13) and its expected evolution over time. For, as we will show later on, in a situation of steady state growth, wages may be expected to grow equally steadily, and therefore prices (being set using a mark-up over marginal costs) can be expected to grow steadily too.

Because of the symmetry between intermediate goods firms (all firms have the same production technologies and contribute in the same way to final output), the training period s will be the same for all new firms too.

Hence, wages will be the same for each firm as well. We assume furthermore that wages offered to trainees are the same as those for high-skilled workers associated with firms that are in their production phase already. The structure outlined above has been depicted in Figure 1.

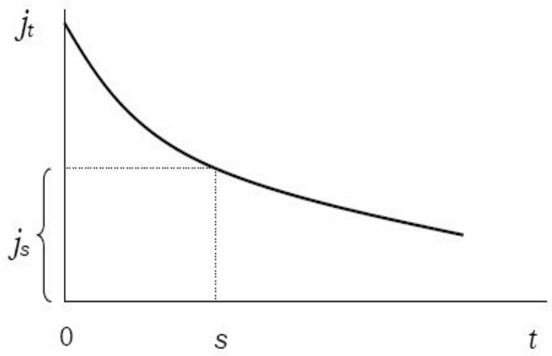

Figure 1: The Evolution over Time of Labour Demand

In Figure 1, the demand for labour to produce an intermediate invented at time $t = 0$ is depicted for all moments in time after $t = 0$, for a given and constant growth rate of the wage rate. An intermediate will be produced after a training phase of duration s. During that phase, the wage rate is assumed to rise at a given proportional rate, and consequently (see equation (17) and (19) further below) the demand for labour at time $t > s$ that is associated with the intermediate invented at time $t = 0$, i.e. j_t , will fall. However, it would be a waste of resources to train more workers than would actually be needed from the start of the production phase at time s, hence a rational entrepreneur would hire only j_s workers to be trained during the training phase of the intermediate.

Figure 1 is formalised as follows. The net present value of the expected flow of quasi-rents (further called Q) for a firm that has bought the latest blueprint at the current time (taken to be time zero), is given by:

$$Q = \int_s^\infty e^{-\rho \cdot T} \left[p_T \cdot x_T - w_T \cdot \frac{x_T}{\pi(s, \epsilon)} \right] - j_s \cdot \int_0^s e^{-\rho \cdot T} w_T dT \qquad (17)$$

where ρ is the rate of discount, and w_T is the expected wage rate per physical unit of labour at time T ($T > s$)[5]. In equation (17), where we have used (14)

[5]Note that we have dropped the technology subscripts, because the symmetry of (4) and

to replace high-skilled labour demand by the demand for intermediates, the first integral describes the flow of quasi-rents, since labour is the only factor of production. That flow starts only after the training-phase has ended and the production-phase has begun. The training-phase takes s units of time. During the training-phase, j_s physical units of labour need to be trained, since that is the amount of labour needed when the production-phase starts. There are training costs involved, and the present value of these costs is given by the second integral in (17).

The intermediate goods producer now has two independent controls to be determined. The first one is the Q-maximising price of intermediates, and the second one is the Q-maximising length of the training period s. It should be noted that equation (17) can be maximised with respect to the price of intermediates only for the future in as far as it refers to the production phase[6]. By differentiating (17) with respect to p_T for $T > s$, we immediately obtain:

$$\frac{\partial Q}{\partial p_T} = x_T \cdot \left[1 + \epsilon \cdot \left(1 - \frac{w_T}{p_T \cdot \pi(s,\epsilon)} \right) \right] = 0 \Leftrightarrow p_T = \frac{w_T}{[\alpha \cdot \pi(s,\epsilon)]} \tag{18}$$

which is the familiar Amoroso-Robinson price-setting rule, where $\epsilon = -\sigma = -1/(1-\alpha)$ is the price-elasticity of the demand for intermediates (see also equation (15)) and where marginal costs are wage costs per efficiency unit of labour[7].

In order to find the Q-maximising value of s, we have to use Leibniz's rule for differentiating integrals, since the bounds of both integrals in (17) depend on s. We therefore get the first-order condition:

$$\begin{aligned} \frac{\partial Q}{\partial s} = 0 = \left[p_s \cdot x_s \cdot - w_s \cdot \frac{x_s}{\pi(s,\epsilon)} \right] \cdot e^{-\rho \cdot s} - \\ - j_s \cdot e^{-\rho \cdot s} \cdot w_s - \frac{\partial j_s}{\partial s} \cdot \int_0^s w_\tau \cdot e^{-\rho \cdot \tau} d\tau - \int_s^\infty e^{-\rho \cdot \tau} \cdot \frac{\partial \left[w_\tau \cdot \frac{x_\tau}{\pi(s,\epsilon)} \right]}{\partial s} d\tau \end{aligned} \tag{19}$$

The first term of (19) within curly brackets is the present value of the loss in quasi-rents that occurs by extending the training-phase by 1 unit of time (*marginal opportunity cost of training*). The second term is the present value of the additional training costs for a given number of trainees associated with a marginal increase in s (*total marginal training costs*). The third

(5) implies that the net present value problem is essentially the same for all intermediate goods producers.

[6]The price of a good that is not produced and therefore not sold can hardly be regarded as a control.

[7]See Aghion and Howitt (1992)

term is the present value of the increase in total training costs for a change in the number of required trainees that could be expected to occur if the production phase starts one unit of time later, i.e. if the training-phase is extended by one unit of time (*marginal direct cost of training*)[8]. The final term represents the present value of the benefits from extra training (*marginal training benefit*), since these would increase labour productivity and hence decrease production costs for the entire duration of the production phase. Equation (19), therefore, represents the equality condition between marginal benefits and marginal costs of additional training.

Using these results, we will now obtain the allocation of time over training and working in intermediate goods production.

3.4.6 Training and Working in Intermediate Goods Production

In order to be able to derive the distribution of time over all its different uses, we focus on the steady state. Assuming steady state growth in A at a proportional rate $\hat{A}$, we should have that the latest intermediate that is actually in the production phase must be the intermediate that has just ended the training phase of length s. Let $B(t)$ be the latest innovation, i.e. the blueprint index of the marginal intermediate just entering the production phase at time t. In that case we must have that:

$$B(t) = A(t-s) = A(t) \cdot e^{\hat{A} \cdot s} \tag{20}$$

Since each intermediate good enters the production function symmetrically, we must have that the demand for labour in the production phase is the same for each intermediate. Hence, the demand for all labour in the production phase must be given by:

$$J(t) = B(t) \cdot \overline{j}(t) = A(t) \cdot e^{(\hat{A} \cdot s)} \cdot L \cdot (w(t)/\pi(s,\epsilon)^{\alpha})^{-\sigma} \cdot \alpha^{2\sigma} \tag{21}$$

where $\overline{j}(t)$ represents the number of production workers active on each intermediate with index less than or equal to $B(t)$ and where $A(t)$ is again the index of the newest technology at time t and $B(t)$ is the index of the

[8]Indeed, as we will show later on, technical change leads to a continuous upward pressure on wages, and hence to a continuous downward adjustment of the demand for labour. So, by extending the training period, one would normally need less people to train because of the anticipated fall in the demand for labour per technology over time in the steady state.

newest intermediate that has just entered the production phase.

Because of technical change, we find that, according to equation (21), the demand for labour would grow as fast as $A(t)$ itself, *ceteris paribus.* For a given level of supply of production labour, this implies that the wage rate is driven up to maintain equilibrium between the demand for production labour and the given supply of production labour. In fact, taking logarithmic time derivatives of (21), while assuming $J(t)$ to be constant, we find that the steady state growth rate of wages must be equal to:

$$\hat{w} = \hat{A}/\sigma = \hat{A} \cdot (1 - \alpha) \tag{22}$$

This positive growth in wages implies that the demand for labour would drop both for intermediates in the production phase, but more importantly, also for labour in the training phase, since we have assumed that workers engaged in technology-specific training must be offered a competitive wage, i.e. the same wage as that of workers in the production phase. However, if the wage rate is expected to rise during the training phase because $A(t)$ grows, then the cost-efficient amount of labour to hire for the newest intermediate with index $A(t)$ at time t would be $j(t + s)$, since that is exactly the amount of production labour that will need to become active after the training phase on intermediate with index has passed.

Let t_C be the moment in time when the intermediate with index C has been invented. Then, in a situation of steady state growth, and assuming $A(0) = 1$, we must have that:

$$C = e^{\hat{A}} \cdot t_C \Leftrightarrow t_C = Log(C)/\hat{A} \tag{23}$$

So, equation (23) defines the steady state arrival time of the newest technology with index C[9]. Furthermore, the demand for labour during the training phase would be equal to the number of workers that would be optimal at the time the production phase of intermediate C commences. This amount of labour for intermediate C is given by (cf. (21)):

$$\bar{j}(t)(t_C + s) = L \cdot [w(t_C + s)/\pi(s, \epsilon)^{\alpha}]^{-\sigma} \cdot \alpha^{2\sigma} \tag{24}$$

where $w(t_C + s)$is the expected wage rate at the time intermediate C would enter the production phase. For a given and constant growth rate of wages

[9]Consequently, since $A(t)$ has been defined as the index of the newest technology at time t, we must have that $t_A = t$. Hence, by definition we must also have that $w_A = w(t)$.

$\hat{w}$, we can write:

$$w_C = w_A \cdot e^{[\hat{w}\cdot(t_C - t_A)]} = w_A \cdot e^{[\hat{w}\cdot Log(C/A)/\hat{A}]} \tag{25}$$

Substituting (25) and (23) into (24), we immediately find the demand for labour for intermediate with index C:

$$\bar{j}(t)(t_C + s) = L \cdot [w(t)/\pi(s,\epsilon)^{\alpha}]^{-\sigma} \cdot \alpha 2\sigma \cdot e^{-\sigma\cdot\hat{w}\cdot[s+Log(C/A)/\hat{A}]} \tag{26}$$

We see from (26) that the demand for labour during the training phase depends negatively on the duration of the training period s. It also depends negatively on the time of arrival of an intermediate (recall from (23) that $t_C = Log(C)/\hat{A}$): the younger it is, the further away in the future the training phase will be, and for growing wages, the lower will be the corresponding demand for labour.

Given (26), the steady-state total demand for labour associated with all intermediates in the training phase can be obtained by direct integration over all intermediates in the training phase, where it should be noted that $\pi(s,\epsilon)$ depends on φ and ψ through (4) and (2):

$$T(t) = \int_{B(t)}^{A(t)} \bar{j}(t)(t_C + s)dt_C = \frac{A(t)\cdot e^{(-\hat{A}\cdot s)}\cdot\hat{A}\cdot L\cdot\pi(s,\epsilon)^{\alpha\sigma}\cdot w(t)^{-\sigma}\cdot\alpha^{2\sigma}\cdot(1-\alpha)\cdot(e^{s\cdot(\hat{A}-\sigma\cdot\hat{w})-1})}{\hat{A}/\sigma - \hat{w}} \tag{27}$$

Both the numerator and the denominator of (27) will approach zero in the steady state because of the presence of the terms $\hat{A}/\sigma - \hat{w}$ and $\hat{A} - \sigma \cdot \hat{w}$ and because (22) should hold in the steady state. Consequently, we need to apply l'Hopital's rule to (27) to find:

$$T(t) = A(t) \cdot e^{-\hat{A}\cdot s} \cdot \hat{A} \cdot L \cdot \pi(s,\epsilon)^{\alpha\sigma} \cdot w(t)^{-\sigma} \cdot \alpha^{2\sigma} \cdot s \tag{28}$$

Using(21) and (28) to obtain the steady state ratio J/T, we find that:

$$J(t)/T(t) = 1/(s \cdot \hat{A}) \tag{29}$$

From (29) it follows immediately that the number of workers in the production phase relative to the number of workers in the training phase falls if the duration of the training period increases, while it also falls if the rate of technical change increases. The latter is easy to understand, since the share of the number of intermediates in the training phase must increase if A increases. Consequently, more people will receive training in times of faster technical change. Substituting (29) into (11), we find for the steady state values of J and T that:

$$J^* = H_p/(1 + s \cdot \hat{A}) \tag{30a}$$

$$T^* = H_p \cdot s \cdot \hat{A}/(1 + s \cdot \hat{A}) \tag{30b}$$

It should be noted from (30a) and (30b) that an increase in the duration of training would increase T^* while decreasing J^*, *ceteris paribus*.

Finally, by adding (21) and (28) together, we obtain the total demand for labour by the intermediate goods sector. We can invert that relation to get the equilibrium wage rate that would clear the market for total production labour both in the training and the production phase:

$$w(t) = \left[\frac{A(t) \cdot e^{-\hat{A} \cdot s} \cdot L \cdot \pi(s, \epsilon)^{\alpha\sigma} \cdot \alpha^{2\sigma} \cdot (1 + s \cdot \hat{A})}{H_P} \right]^{\frac{1}{\sigma}} \tag{31}$$

We see that in the steady state, in which H_P would be constant, the growth rate of wages would still be given by (22). We also see that an increase in the availability of production labour would depress wages, as expected. At this stage a deeper analysis is not possible since s itself, but also $\hat{A}$ itself depends (non-linearly) on wages and wage growth. However, in section 4 we will use (31) as part of a system of simultaneous relations to find out about the connection between education, on-the-job training, technology adoption, production and the rate of innovation at the aggregate level. Before doing that, however, we must describe the demand for R&D labour.

3.4.7 The R&D Sector

As in the Romer (1990) model, we assume that a new intermediate input is produced in accordance with the newest blueprint coming from the R&D sector. As stated before, $A(t)$ is the index of the newest technology available to the market at time t. It therefore also represents the total number of technologies invented up to time t. As in Romer (1990) we have:

$$\hat{A} = \frac{dA}{dt} = \delta \cdot \epsilon \cdot H_R \cdot A \tag{32}$$

where we have included human capital per head ϵ as a determinant of R&D productivity. It follows that the growth rate of A will directly depend on the education decisions taken by households through the presence of ϵ. However, as stated before, in taking these decisions, households do not take this fully into account.

Assuming, as Romer does, that the present value of the expected quasi-rents can be captured by the blueprint producers, and that R&D wages equal the marginal benefits from hiring an additional R&D worker, we find an expression for the R&D workers' wage rate:

$$w_R = \frac{\partial \dot{A}}{dH_R} \cdot Q = Q \cdot A \cdot \delta \cdot \epsilon \tag{33}$$

Substituting (17) for Q as well as (18) for all the intermediates that are in the production phase[10], we find a complicated expression for the wage rate for R&D workers that depends on the wage rate of production workers. Since labour market arbitrage ensures that $w_R = w$ at all times, we can combine (31) and (33) to obtain the equilibrium value of H_P as a function of all the variables and parameters of the model, for which wages for the high-skilled workers earned in both their uses (as R&D and as production workers) are equal. As the expression is still rather complicated, and includes terms containing $\hat{A}$, it is not reproduced here. However, this expression for H_P can now be used to obtain an implicit equation for $\hat{A}$, since we must have from (10), (11) and (32) that:

$$\hat{A} = [(1 - \varphi - \psi) \cdot H - H_P] \cdot \delta \cdot \epsilon \tag{34}$$

Equation (34) provides a compressed description of the supply-side of the growth model. Combining it with the growth demand side as given by (8c) in combination with the steady state requirement that $\hat{c} = \hat{Y}$, we can now obtain an impression of the working of the model by means of some numerical simulations. By numerically solving (34), we can readily 'de-construct' the underlying values of the allocation of time over its various uses. The outcomes of this deconstruction process are described in more detail in the next session.

[10]We assume that there is no output, hence no sales, on intermediates during the training phase.

4 Numerical Results

4.1 Introduction

In this section we present two kinds of outcomes. In section 4.2 we show some of the results that we have generated as part of a sensitivity analysis regarding the calculation of the net present value of the quasi-rents and the corresponding optimum length of the training period.

4.2 Sensitivity Analysis

By substituting all our intermediate results into (17), we can obtain an expression for Q in terms of s, that, together with (19) would enable us to solve both Q and s. Because of the strong non-linearity of this 2x2 equation system, an analytical solution can unfortunately not be found. However, we can get an impression of the sensitivity of the net present value of the quasi-rents, i.e. Q, for changes in s by drawing Q as a function of s, for given values of the parameters. These 'base-run' parameter values are given in Table 1 further below.

It should be noted that equation (17) contains (endogenous) variables that are given from the point of view of the individual entrepreneur, but that will change endogenously over time in the full model that we will simulate later on. Presently, we are only concerned with showing that the specifications we have chosen for the production structure and the productivity function ensure that there is indeed an optimum amount of on the job training that depends in an intuitive way on the parameters listed in the Table below. It should be noted that these parameter values are just some 'fake' numbers not reflecting any empirical regularities, except for their signs.

Table 1: Base-Run Parameter Values

Par	Value	**Par**	Value	**Par**	Value
α	0.500	ζ_1	0.667	$\overline{x}$	1.000
$\hat{w}$	0.025	ζ_2	0.033	w	1.000
ρ	0.075	γ_0	0.500	φ	0.200
ζ_0	2.000	γ_1	0.500	ψ	0.100

In Table 1, $\hat{w}$ is the expected instantaneous growth rate of wages, whereas

ρ is the rate of discount[11]. $\overline{x}$ is a scale factor for the demand for intermediates that will be modelled explicitly later on. For our present illustrative purposes the only thing that is relevant is that it is a constant.

In Figure 2, the downward sloping curve is the one associated with $\partial Q/\partial s$. One observes that for the specific parameter set chosen here, Q itself has a maximum, but Q is relatively unresponsive to changes in s in the neighborhood of that maximum. The maximum value of Q (0.84) is reached for a value of s of 2.07.

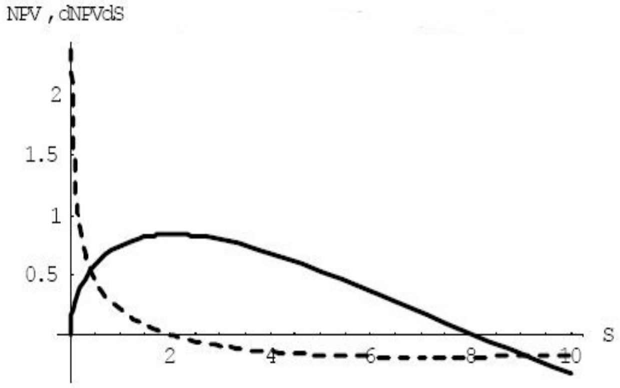

Figure 2: Net Present Value Quasi-Rents

By changing the parameters underlying Q, we can find out more about the qualitative and quantitative behaviour of Q and the consequences for the optimum value of s. In order to do this, we have increased each of the parameters individually by 10% as compared to its base-run value[12], and then obtained the corresponding graph of Q. The shifts in the graph of Q are a direct indication then of the partial derivatives of Q with respect to the parameter under consideration for the entire range of values of s. But rather than showing all the graphs associated with these partial derivatives, we provide information in tabular form below in Table 2[13].

The columns with heading 'Par' hold the names of the parameters we have

[11]Obviously, in the full simultaneous model, the expected growth rate of w will be an endogenous variable. This also goes for φ and ψ.

[12]The sensitivity analysis has been carried out for $\pm 1\%$ and $\pm 10\%$ variations in the parameters' values. In the paper we report only the $+10\%$ case.

[13]In Table 2 a plus, a minus or zero sign implies that the partial derivative of either the net present value of the quasi-rents or the length of the training period s is positive, negative or zero.

Table 2: Parameters Sensitivity

Par	$\partial Q/\partial s$	$\partial s/\partial Par$	**Par**	$\partial Q/\partial s$	$\partial s/\partial Par$
α	-	-	ζ_2	-	0
$\hat{w}$	-	-	γ_0	+	0
ρ	-	-	γ_1	-	-
ζ_0	+	0	$\overline{x}$	+	0
ζ_1	-	0	w	-	0

changed, while holding the other parameters constant. The other columns hold information regarding the sign of the partial derivatives of Q and the optimum value of s (hence the shift in the $\partial Q/\partial s$ curve) with respect to the parameter under consideration.

Looking at the effects of individual parameter changes, we see that an increase in α implies an increase of the price-elasticity of demand. This in turn lowers the profit margin, hence profits themselves, hence the present value of the quasi-rents Q too. This will lower the optimum amount of training, as one would expect, *ceteris paribus*. An increase in the rate of discount ρ has qualitatively the same effects. The present value of the quasi-rents is negatively affected, and so is the optimum amount of training. A rise in ζ_0 has a positive effect on Q, but no effect on s. The latter also goes for changes in ζ_1 and ζ_2 which is a result that can be analytically derived from setting (17) equal to zero. A rise in γ_1, i.e. the human capital elasticity of labour efficiency, also reduces the training elasticity of labour efficiency. We observe a net negative effect, both on Q and on s, again as expected. A change in the amount of formal education φ has a positive effect on productivity, hence on Q. This also goes for a rise in ψ. However, in both cases s is not directly affected. Nonetheless, since the latter 'parameters' would also influence the growth rate of the economy, *ceteris paribus*, it is to be expected that the general equilibrium effect will be non-zero, as we will show in the next section. A change in the growth rate of wages reduces Q. It also reduces the optimum amount of training, since not only training costs rise during the training phase, but in addition to this, quasi-rents during the production phase will fall. Similar results can be observed for a rise in the initial wage rate, except that a change in the initial wage-level does not have an effect on s. Finally, a rise in the autonomous demand for the intermediate under consideration would raise Q but not s.

Summarising the above, we have to make a distinction between parameter changes that have a direct growth effect, and so affect the intertemporal balancing of costs against benefits. Changes in wage levels or the level of autonomous demand do indeed have an impact on Q but not on s, whereas this is distinctly different for the growth parameters, like the growth rate of wages or the rate of discount. A striking result is the insensitivity of s for changes in the parameters associated with the education system. However, these would directly affect φ and ψ, and hence the growth rate of the economy, which in turn is an important determinant of Q and s again. Thus, there will be a general equilibrium effect that is not accounted for in this sensitivity analysis.

4.3 Steady State General Equilibrium Results

The simultaneous model that we are going to solve numerically for the variables $s, \hat{A}, H_R, J, T$ and φ consists of the following equations:

1. equation (22) that describes the link between technical change and wage growth;
2. the implicit equation for production labour that we obtain from combining (31) and (33);
3. equation (34) that describes the rate of technical change in function of the level of production labour;
4. equations (30a) and (30b) that describe the shares of labour in the production phase and labour in the training phase in total non R&D production labour;
5. equation (19) that (implicitly) describes the optimum value of s;
6. equations (9a) and (9b) that describe the optimum allocation of time to education and teaching;
7. the growth demand side of the model, linking the interest rate to the growth rate, as given by (8c).

We now put the productivity parameter associated with the R&D production function (32), i.e. δ, equal to 0.6, and the intertemporal elasticity of substitution $1/\delta$ equal to 1.5. We then solve the model (excluding the equation for φ, i.e. (9.A)) for varying values of φ. We still have the same values for the other structural parameters as given by Table 1, except for the rate of discount that we have reduced to $\rho = 0.03$. However, the growth rate of wages is now linked to the rate of technical change, while $\overline{x}$, is given by the parameter combination $\overline{x} = L \cdot \alpha^{\sigma}$with $L = 1$ (cf. equation (13)).

The way in which the equilibrium steady state growth rate $\hat{A}$depends on the amount of education can now readily be observed by solving the simultaneous system outlined above, for all values $0 \leq \varphi \leq 1$. The result is presented in Figure 3. In this figure, we find two curves with an 'inverted' U-shape. The solid curve is the one associated with a value of $\delta = 0.6$, whereas the dotted curve is associated with a value of $\delta = 0.7$. The vertical axis of Figure 3 below is associated with the growth rate of A, indicated by GA. There is a solid vertical that represents the optimum value of φ as given by (9.A). The point of intersection with the inverted U-shaped curves defines the general equilibrium.

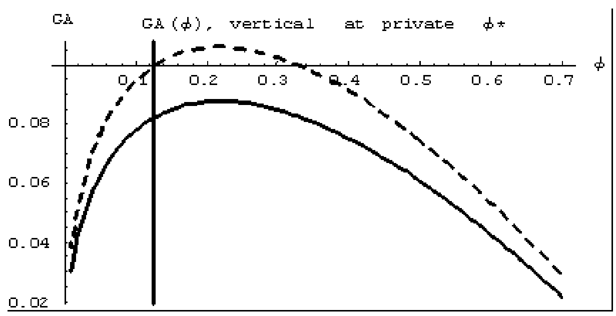

Figure 3: Growth as a function of education efforts

Looking at this Figure, there are two main conclusions to be drawn. First of all, the rate of technical change depends positively on the value of the productivity parameter δ, since the dotted curve lies entirely above the solid curve. Secondly, from a growth perspective, there seems to be an optimum value of the level of education, since the growth curve reaches a definite maximum for $\varphi \approx 0.22$. The reason why the curve has an inverted U-shape is the following: if an individual spends all his/her time at school, he can not produce any output. If, on the contrary, an individual spends no time at all at school, he does not get trained enough to produce; this follows immediately from (34). We also conclude that the general equilibrium solution falls short of the maximum growth solution, suggesting indeed that the amount of general education that people obtain for themselves is too low.

In Figure 4, we have also plotted the corresponding graphs for the duration of training s.

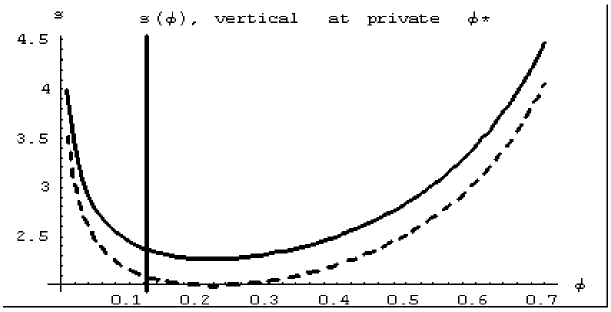

Figure 4: The duration of training

This graph shows that for low levels of education, an increase in the level of φ will be associated with a lower level of training, suggesting that training and education act like substitutes. For high levels of φ, however, the amount of training rises with the level of education itself, suggesting that education and training act like complements. This result is consistent with the mixed results we have found in the introductory part regarding the complementarity/substitutability of training and education in function of the level of education. The substitutability of the two derives from the fact that on the downward sloping part of the curve in Figure 4, the rate of technical change is rising, as we can see from Figure 3, and therefore also the growth rate of wages (cf. equation (22)).

Table 2, that contains our partial sensitivity results, shows that an increase in the growth rate of wages reduces the optimum amount of training, because profit flows are strongly eroded. On the downward sloping part of the training duration curves above, the rise in the growth rate of wages causes training levels to fall by more than the optimum amount of training expands with an increase in the level of education. When the rate of technical change starts falling again after starts exceeding its growth maximising value, the growth rate of the wage rate is reduced as well, and the partial complementarity results become stronger again, resulting in the observed positive correlation between education and training for higher levels of education. Interestingly, this downward shift of the training duration curve signals that in times of strong technical change, general education becomes a relatively more important determinant of productivity than technology specific training, since for every level of general education, the optimum level of training falls.

With respect to Figure 4, it should furthermore be noted that the general equilibrium value of s is higher than the growth maximising value of s, which is reached for exactly the same value of φ as the growth maximum itself. So, not only does the general equilibrium generate a shortage of education, but it causes overinvestment in training as well, again emphasising the substitutability of education and training.

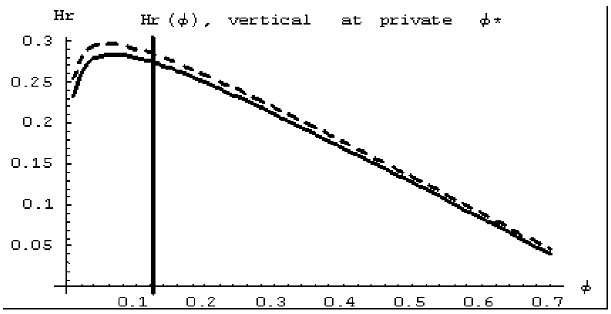

Figure 5: R&D employment

Figure 5 shows how the number of R&D workers changes with the level of education. Again, this is as expected: at first the number of R&D workers rises for an increase in the level of education, and then it falls when the reduction in available hours per R&D worker falls below the increase in the absolute productivity of that worker.

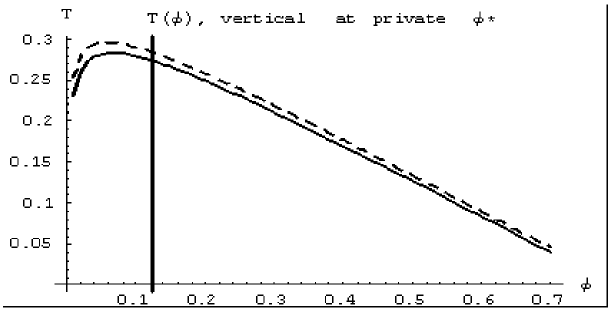

Figure 6: Labour in the training phase

In Figure 6, we depict what happens to the number of workers in the training phase. The results are very interesting on two accounts. First, an increase in the productivity of R&D workers raises the number of production workers in the training phase. This is consistent with the idea that increased technical progress brings about stronger "creative wear and tear" of technology-specific knowledge, requiring more people to be retrained. Secondly, the number of people in the training phase peaks at levels of education that are somewhat lower than in the low productivity/growth case. This is due to the fact that the strong upward pressure on wage growth for low but growing levels of education have a strongly negative effect on the duration of training (see Figure 4) that leads to a fall in the number of people in the training phase even before the peak in the rate of technical change itself has been reached.

In Figure 7, we show that the number of people in the production phase falls with a rise in the productivity of R&D labour. This is the corollary of the rise of the number of people in the training phase. However, it is also caused by the rise in the number of R&D workers.

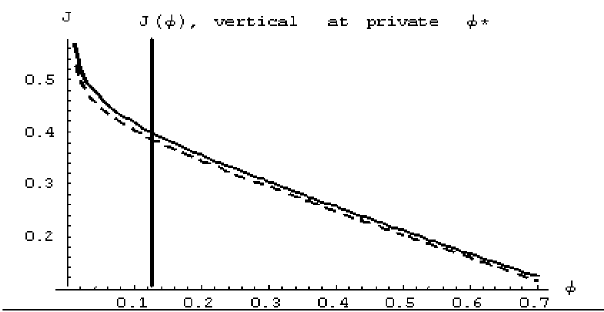

Figure 7: Labour in the production phase

To sum up then, faster technological change leads to higher growth indeed, but also to more people being (re-) trained for shorter periods of time. Nonetheless, growth opportunities are left unused as private households fail to take account of the growth impact of their education decisions.

5 Summary and conclusions

In this paper we develop an endogenous growth model that combines private household investment in education with investment by firms in workplace training in a context of costly technology adoption and 'wear-and-tear' creative destruction. Its aim is to answer the question how technological change affects the composition of human capital in terms of education and training.

In our set-up, time is the unit of measure of each activity. Its total amount available can be spent for three different purposes, i.e. the accumulation of human capital, the production of goods and the production of new technologies.

The accumulation of general knowledge occurs through schooling and teaching, while its utilization occurs in goods production and in R&D. The labour time available for the production of goods can be split in time-consuming training activities, necessary for adopting the newest technologies, and current manufacturing of (intermediate) goods.

General knowledge is acquired within the educational system, where people spend their time as pupils or in teaching activities. We assume that teachers earn a competitive wage, covered from fees paid by private households. The latter are engaged in education with the aim of obtaining higher future wages in the labour market through higher labour efficiency arising from human capital accumulation. Technological knowledge is produced through R&D and is embodied in new intermediates that are patented and sold to final good producers.

In line with R&D-based endogenous growth models, we assume that new designs are the outcome of the time spent in research activities and the amount of education acquired by R&D workers. Finally, knowledge is used at the plant level for the production of intermediate goods: however, the arrival of a new intermediate requires workers to learn how to produce that intermediate effectively, and this occurs through a time-consuming on-the-job training activity that is entirely financed by the firm.

In our model, education pervades the entire economy and plays three roles: from the consumers' point of view, more education means higher future labour earnings due to the higher trainability and labour efficiency of individuals; from the R&D point of view, more education means higher numbers of new technologies produced in research labs, and then faster technological change; finally, from the point of view of the firm, more education means lower training costs, and thus the possibility to adopt new technologies sooner in time, thus bringing sales forward in time.

Our partial equilibrium simulation results show that, generally speaking, both the net present value of the quasi-rents themselves, and the optimum duration of the training phase react to parameter changes that would affect the intertemporal distribution of costs and benefits. We also find that the parameters of the educational system do not affect optimum training period choices, even though they do affect the net present value of the quasi-rents. However, in a general equilibrium situation this is completely different, since in that case such parameter changes would have a direct impact on the productivity of R&D workers, hence on growth and therefore on the growth of wages too, and hence on training decisions again. In that case we find that more technical change would lead to less training.

We extend the analysis to a general equilibrium setting, in which we introduce both the households' educational choices - that are merely driven by the possibility to reap higher future wage rates - and the level of education as a factor that determines the productivity of R&D workers as well as that of high-skilled production workers. Thus we obtain a bell-shaped relationship between the rate of technical change and the level of education, since time spent in the education system cannot be utilized either for final good production or for new technology production. This means that, on the one hand, it is possible to derive an optimum amount of education that maximizes the rate of technological change, while on the other hand, too much time spent in acquiring knowledge, or teaching, at school reduces the time available for production activities, thereby reducing the time and financial resources available for the production of future innovations. Moreover, an increase in the rate of technological change shifts the curve upwards while leaving the growth maximizing level of education unchanged.

Nonetheless, the duration of training decreases, whereas the number of people in the training phase increases. So the effect of a rise in research labour productivity is to (re-) train more people for shorter periods of time. This finding does confirm the available evidence that emphasizes the training-enhancing nature of technology, even though it is unclear *a priori* whether and how the total volume of training is affected.

Interestingly, when we include households education decisions in the analysis, we see that the optimum private level of educational time, which is driven by their wish to earn higher future wages, is strictly below the growth maximising amount of education time: from a growth perspective, therefore, households tend to under-invest in education. Similar results emerge when we look at the general equilibrium relationship between education and workplace training. In this case we find that the observational complementarity between the duration of education and training turns into a U-shaped relationship, indicating observational substitutability for low levels of education-time and complementarity for higher levels of education-time. This observational substitutability arises from a general equilibrium effect that pushes up wages as the rise in the time devoted to education raises the growth rate of the economy and hence the demand for production labour. This reduces the optimum duration of training, as was shown in the parameter sensitivity results.

We also observe that this U-shaped curve shifts downward in the training and education-plane when technological change speeds up, thus changing the human capital composition of the workforce in favour of general education. Because a higher rate of innovation increases the number of people being (re-) trained, the number of workers available for direct production decreases, since the number of R&D workers increases. Nonetheless, output grows faster than before, because of the 'Love of Variety' effect.

With respect to households' education decisions, we find that the privately optimal level of training is higher than the growth maximising level of training, suggesting that households do not invest enough to minimize firms' training costs, thus forcing producers to provide more training than it would be necessary for maximizing growth. A change in education fees may change the level of education such that training costs are reduced by more than training fees are lowered for households. At the moment, this is left for future research.

Rounding up, our model shows that in times of increasing technical change, the optimum 'portfolio-mix' between education and training changes in favour of the former, since that provides a relatively solid basis for the development of technology- specific skills that are prone to creative destruction. However, when we endogenize education costs, we see that private households' decisions regarding education seem to leave growth opportunities and training cost reductions unexploited, thus 'calling out' for public policy intervention.

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