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THE DYNAMIC GENERAL EQUILIBRIUM IN THE ITALIAN PARETIAN SCHOOL

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The dynamic general equilibrium in the Italian Paretian School*

Abstract

The paper examines the theoretical contributions of two eminent representatives of the Italian Paretian School, namely Luigi Amoroso and Giulio La Volpe. Both contributed to the most ambitious project undertaken by the Paretian School, that is, dynamization of the general economic equilibrium. This research programme was grounded on two pillars. The first concerned the use of then sophisticated mathematical instruments, such as functional, differential equations, and the calculus of variations. The second pillar involved maintenance of a strict analogy with physics and mechanics. But whilst Amoroso mainly used a theory based on the past, La Volpe founded his proposal on the future.

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Introduction

The Paretian School was one of the most important research strands in the interwar period in Italy. It included a small but combative group of economists, among whom the names of Luigi Amoroso and Giulio La Volpe stand out.[1] The most ambitious project undertaken by the Paretian School was dynamization of the theory of general equilibrium by grounding it on two pillars: first, a common effort to build 'economic equilibrium' as well as 'social equilibrium' according to the idea of 'continuous movement'; second, the use of then sophisticated mathematical instruments, such as functional and differential equations, drawn by analogy from mechanics. Luigi Amoroso, following his scientific background and emphasizing analogies with the physical sciences, in particular with rational mechanics, proposed a view of economic dynamics based on the past; La Volpe took a totally different route by concentrating on choices grounded on plans and predictions, that is, on the future. The Paretian approach to dynamics was one of the most original products of the Italian tradition during that period, at least in the field of pure theorizing; but, for the reasons that we will attempt to explore, it did not have a significant impact in the working out of economic dynamics in the post-war period.

In section 1 we scrutinize Pareto's legacy as regards economic dynamics. Section 2 is devoted to Amoroso's proposal, which is presented according to its different phases. Section 3 focuses on La Volpe's research on general dynamic economic equilibrium. Section 4 makes some final remarks.

1. The difficult Paretian legacy in dynamic theory

It is in the *Cours d'économie politique*, Pareto's first important book in 1896-97, that we can detect the signs of what was to become the Paretian legacy, particularly where he stated that the study of 'continuous movement' constitutes a problem of economic dynamics. This was a crucial point, because even if in 1895 Pareto had recognized that economic science had still not found dynamic general equations (1966b: 67), he opened the Chapter 4 of the *Cours* with the blunt statement that the "economic phenomenon is not static, but it is dynamic" (1964: §925). A sentence that represented a challenge to himself and to his many followers who in the subsequent years attempted to single out an approach allowing the analysis of continuous and, generally, of dynamic movements.

[1] Besides Amoroso and La Volpe, the Paretian School comprised, among others, Arrigo Bordin, Alfonso De Pietri Tonelli, Valentino Dominedò, Eraldo Fossati, Giuseppe Palomba, and Guido Sensini.

In the §587 of the *Cours*, Pareto hypothesized the existence of, first, subsequent equilibria, which could be investigated by means of comparative statics. This type of equilibrium referred to Walrasian *tâtonnement*, that is, the problem of the adjustment between an equilibrium and the subsequent one that Pareto postulated without analysing it, perhaps also because he realized that such an adjustment led to several equilibria and not to a unique one. Then, he shifted his perspective on the whole of equilibria sketching the 'continuous or evolutionary movement', which ought to correspond to the true object of economic dynamics. Once picked out this level of analysis, he hypothesized that the equilibrium of an economic system might be depicted as something analogous to converging forces, constraints, and inertia, that is to say, to factors warranting the mechanical equilibrium.

As it has been defined, the notion of 'continuous movement' comprised another important dynamic concept, that of 'moving equilibrium', then understood as social movement. This happened because, as pointed out by Samuels (1974: 10), in Pareto "social equilibrium was a function of a continuous changing array of forces",[2] which, at the same time, could be analyzed by making use of analogies and concepts drawn from rational mechanics.[3] The 'continuous movement' represented the Paretian challenge, because despite its social content, originally it had to be rigorously investigated by means of the logic of physics and mathematical tools. Pareto was immediately aware that the analogy between economics and rational mechanics would be fully accepted only if inertia found an economic analogue. In fact, if utilities and budgets could exemplify forces and constraints respectively, at first glance nothing was adapted to express the forces of inertia. Defining economic inertia, Pareto would be able to interpret continuous or dynamic equilibrium as the result of economic factors only, without resorting to exogenous forces.

Pareto gave a second outline of an approach to economic dynamics in the short appendix to his 1901 article, *Le nuove teorie dell'equilibrio economico.* In this case Pareto adopted a more traditional strategy, since he considered economic variables in a generic short period, *dt*. In comparison to a static scheme, the transformation had a purely formal character since the equilibrium relations were related to a single period. To pass from static to dynamic economics, the same variables must be referred to different periods, and the functional relations between periods must be made plain, but this analysis was totally absent in Pareto's note. Moreover, a point that has to be stressed concerns the reference to the coefficients of production, which are finite (discrete)

[2] Although Samuels's quotation referred to the Pareto's *Treatise on General Sociology*, it also fits well with the *Cours* since the conception of social equilibrium did not change from the *Cours* to the *Treatise*. See also Boianovsky and Tarascio (1998).

[3] See Pareto 1964: §592.

magnitudes, even if *dt* is an infinitesimal period (1901: 254). This involved time and the consequent admission that innovations and changes might modify the productivity of economic process. Thus, dynamics implied changes having an exogenous source not envisaged by mechanical schemes.

Accordingly with results until then judged formally inadequate, the 1906 *Manual of Political Economy* (1971) contained few references to dynamic analysis, with the exception of the concluding paragraphs on economic crises included in the sections devoted to “concrete economic phenomena” and not to pure economics. This abandonment of dynamic inquiry can be explained by analyzing the difficulties that Pareto met in the attempt of applying into economic dynamics the paradigm of rational mechanics already employed in static analysis (Donzelli 1997). The problems arose because the analogy involved adoption of the d’Alembert principle, that is, analysis of the difference between the “living forces” and the “inertial forces” expressed by the presence of a tie which has no analogies in the economic field. Mechanical inertia, expressing resistance to the change of velocity of a physical body, has no equivalent in economics where the relation between ‘tastes’ and ‘obstacles,’ on which Pareto based his idea of equilibrium, has not dynamic features. Step by step economic dynamics revealed different sources.

Pareto stressed this insurmountable theoretical difficulty in a letter to the young Amoroso, in 1907:

> The trouble is not to recognize that habit tallies to inertia, as it seems probable, but to find something analogous from an economic viewpoint to the mechanical mass, and to what the mechanical acceleration multiplied to mass corresponds to. If we do not know the analogue of the relation between forces and acceleration, we cannot write the equations of economic dynamics (Pareto 1973: 594).

In Pareto’s eyes, economic phenomena seemed too complex to be investigated by means of differential equations drawn from rational mechanics. The old problem of economic dynamics continued to torment him, as he stated in 1911 *Economie mathématique* (1911):

> in facing a dynamical problem, it is necessary to know not only the direction in which an individual is moving, but also the movement intensity […] Moreover, in treating equilibrium questions, we must know the values of variables that offset such variations (1966a: 326).

This statement represented the true ending point in his research on the rational or economic formulation of the general dynamic equilibrium. As Guido Sensini (1932: 100) later wrote, once Pareto had established that changes of economic variables “seem[ed] largely due to circumstances that [were] quite unlike strictly economic ones”, he steered his research on dynamics along a non-mathematical or sociological path. According to Sensini, in a private meeting in 1914 Pareto said:

> It is vain attempting to build mathematical economic dynamics, since, in the state of our studies, it is not possible. I have made dozens of attempts to find a principle that might be analogous to that of d’Alembert, but I

> have not succeeded […] Only Sociology might give us such a principle, or such a series of principles on which we may found mathematical economic dynamics (Sensini 1960: 553).

These doubts re-emerged in 1916 *Treatise on General Sociology,* where the furrow between economic equilibrium defined in static terms and continuous movement involving dynamic equilibrium seemed to deepen. The latter definitively took the form of 'social equilibrium', which, as the 'continuous movement' depicted in the *Cours*, was characterised by uninterrupted change (1963: §2063) and by subsequent equilibria alternating to disequilibria. But differently, Pareto seemed to recognize that pure economics could investigate only the equilibrium positions (1963: §2069[1]) and moreover could explain them partially. In fact, on the one hand, every departure from the line of the continuous equilibrium is artificially made, that is, it has a non-economic cause; on the other, the system proceeds along this line of continuous equilibrium because of a "numberless imperceptible mutations" (1963: §2077), which may have both logical and non-logical origins rooted in the whole organization of the social system (1963: §2079).

Summing up, the Paretian legacy mattered more in terms of the problems it raised than the result it obtained. Firstly, the content of economic analysis was defined by the fact that the unknowns are determined by differential equations and not by an adjustment process around stationary values (as in comparative statics). Secondly, it became clear that the rational mechanics model, so rich in results when applied to the static scheme, had inconsistencies when it dealt with economic dynamic phenomena. Thirdly, recognizing that the relation between 'tastes' and 'obstacles' has a mainly static character, he implicitly admitted that the factors on which economic equilibrium is grounded are not useful to understand economic dynamics, which, at that point, required a totally different approach, non necessarily anchored to the equilibrium idea. Obliged to be content with a partial outcome, that of successive equilibria, Pareto left uncompleted the more arduous task: the dynamization of general economic equilibrium along the line of evolutionary or continuous movements. The latter was to be the main objective of the Italian Pareto's disciples, beginning with Amoroso and then La Volpe, who flattered themselves to win the Pareto's challenge by resorting to more sophisticated mathematical tools.

2. Towards a pure dynamic theory: Amoroso's research program

Notwithstanding his mathematical education,[4] while still a student Luigi Amoroso turned to economics when, on attending Pantaleoni's course in Rome, he discovered 'the economist of

[4] Luigi Amoroso (1886-1965) has been the most important representative of the Italian school of mathematical economics. He began his studies in mathematics at the *Scuola Normale* of Pisa and continued them at the University of Rome, where he graduated in 1907. His brilliant academic career began as a teaching assistant on courses in geometry

Céligny,' and became Pareto's assistant at the beginnings of his academic career. This experience was decisive in Amoroso's life, since he has been considered the most important exponent of the Paretian School and one of the most influential interpreters of the dynamic general equilibrium theory. In particular, he focused his scientific research on the mathematical features of the Paretian theory, undertaking the task of promoting mathematical economics in Italy as an autonomous discipline different from political economy, a discipline that ought to be characterized by its own content and by a rigorous method drawn from rational mechanics. Amoroso's mission led to the publication in 1921 of the *Lezioni di economia matematica*, which for many decades was one of the most authoritative texts on the subject.

2.1 *The problem of the «existence and uniqueness» of the equilibrium solution*

In the wake of Pareto, Amoroso's chief concern was to demonstrate mathematically the interdependence among all the economic magnitudes, thereby superseding the ingenuous causal vision typical of the classical school and of part of marginalist theory. However, Amoroso himself acknowledged that the method of equality between the number of equations and that of the unknowns was rather elementary and inadequate in mathematical terms. He addressed this problem in his important article of 1928 entitled *Discussione del sistema di equazioni che definiscono l'equilibrio del consumatore* (1928), unfortunately published only in Italian but cited by Schumpeter in his *History of Economic Analysis*. In the introduction, Amoroso stated that:

> The classical theory of consumer behaviour, in the well-known formulation by Walras, Edgeworth and Pareto, and given in my lessons for *Mathematical Economics*, is not complete. Indeed, in reducing the problem to one of the maximum, it restricts the given quantities to satisfy certain equations which are necessary, but not *necessary and sufficient* for a maximum. An analysis of the system is required, showing that solutions do exist, the number of solutions, etc. The present note intends to provide such an analysis (1992: 99).

Amoroso examined the existence and uniqueness of the equilibrium position of the individual consumer. This was not yet analysis of general equilibrium. Amoroso showed in rigorous manner, by introducing numerous lemmas in mathematical style, that, given particular assumptions, the solution which guarantees maximum satisfaction for the consumer exists and is unique. Curiously, he based his argument on proof by contradiction, the demonstrative technique typically used by the

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and rational mechanics. Already by the 1913-1914 academic year he was teaching a course on 'higher analysis.' He subsequently became a lecturer in political economy and mathematical physics. In 1914 he was appointed to the chair of financial mathematics at Bari; he then moved to Naples and finally to Rome, where he was professor of political economy until 1956.

proponents of the axiomatic method. This has prompted Guccione and Minelli (1999) to observe that Amoroso's 1928 essay can be considered a first example of the axiomatic method employed in economics.

The conditions that guaranteed what Amoroso called the 'fundamental theorem' of consumer equilibrium were the traditional ones applied to the structure of the utility function. He argued that it is necessary to start from the experimental truths represented by the fact that the first derivative of this function is positive and the Hessian matrix is negative. If the utility function possesses these characteristics, the existence and uniqueness of the consumer equilibrium can be demonstrated in rigorous manner. Though significant, this result was only partial, because it concerned the optimal choice of an individual consumer, and Amoroso did not develop it any further: in the 1930s, as we shall see, he concentrated mainly on dynamic analysis. However, it was reaffirmed in his 1942 book gathering his lectures to the Istituto di Alta Matematica of Rome, which contains the following passage:

> It is proven that under certain conditions, which substantially reproduce those stated in lecture II regarding the curvature of the ophelimity slope, the system formed by equations [$p_1 x_1 + p_2 x_2 + ... + p_n x_n = R$, i.e., the traditional balance constraint where R is the budget or income] and [$\frac{1}{p_1}\frac{\partial \Phi}{\partial x_1} = \frac{1}{p_2}\frac{\partial \Phi}{\partial x_2} = ... = \frac{1}{p_n}\frac{\partial \Phi}{\partial x_n}$] admits in general to one and only one solution. It therefore univocally determines the unknowns $x_1, x_2, ..., x_n$. Without dwelling on proof of this general theorem, we show its validity in the particular case in which the utility function assumes the following form:
>
> $$\Phi = A x_1^a x_2^b ... x_n^k$$
>
> where *A* is a positive constant; and *a*, *b*, ..., *k* are also positive constants whose sum is less than unity (Amoroso 1942: 22-23).

Doubts may arise if the set of agents is considered rather than the consumer. But Amoroso pointed out:

> [Notwithstanding] the conditions postulated previously [in the 1928 essay] for the index functions of the ophelimities [according to which is] sufficient to ensure that the system always admits in general one and only one solution, the matter is still unresolved. [However] I do not intend to address this question here and shall merely show that the theorem is effectively true in the particular case that the index functions of ophelimity assume the simple form $\Phi = A x_1^a x_2^b ... x_n^k$, where *A* and *a,b, ...,k,* are positive constants (Amoroso 1942: 38).

These two quotations aid understanding Amoroso's position, and in general of the Paretians, on the twofold problem of existence and uniqueness. Amoroso opted for constructive proof: that is, he showed that, under certain realistic hypotheses, the analytical solution has the properties desired. He was not in search like a mathematician for the minimum conditions that guarantee the most general result; rather, he behaved like an engineer who must determine the unknown magnitudes. Again in his 1942 book, Amoroso showed that he was not interested in proof of theorems that guarantee particular properties, but rather in the characteristics of the utility function dictated mainly by experience. Following Pareto, Amoroso believed that the theory of general economic equilibrium, despite its complexity, should be empirical in nature. He therefore gave scant importance to purely mathematical questions that could add little to the real explanation of economic phenomena.

2.2 *The elements of business cycle theory*

The recognized homogeneous character of Amoroso's scientific production rests on the dynamic perspective that he uninterruptedly maintained on economic phenomena. Already in his first 1912 article *Contributo alla teoria matematica della dinamica economica*, Amoroso seemed persuaded that:

> Like rational mechanics, mathematical economics has two parts: a static part and a dynamic part. The mathematical formulation of the principles of economic statics is defined in the work of Vilfredo Pareto. Economic dynamics lags far behind (see Guerraggio 1998: 739).

However, notwithstanding his intellectual devotion, Amoroso assumed a starting point that differed from Pareto's in one respect at least. Whilst Pareto conceived static analysis as analysis concerned with virtual time, Amoroso interpreted static analysis as the study of the stationary state.

The epistemological model upon which Amoroso relied to build his dynamic economic equilibrium was the rational mechanics inherited from Pareto and adapted to the economic reasoning by extensive recourse to the economic analogy of the inertia principle. For Amoroso, the latter concept not only offered a further occasion to reaffirm the semantic and syntactic similarities between the two disciplines, as proved by the concluding paragraphs and the 1921 book, where the basic mechanical concepts of space, force and constraint became wealth, utility and income; it also opened up his economic analysis to new sophisticated tools such as the calculus of variations, more suitable to express time change in the economic variables. On celebrating the centenary of Pareto's birth, Amoroso wrote:

> The passage from Statics to Dynamics is very arduous, involving the setting of a theoretical process similar to the one developed by Galileo and d'Alembert in Mechanics, anchored to the inertia of the system. The economic analogue of what the inertia forces represented in Mechanics must be retraced. These resistances must be quantified and represented by means of logarithms. And this has been recently done according to Pareto's work (Amoroso 1948: 13-14).

Amoroso developed his research on the continuous equilibrium in two stages. In the first part of the 1930s he concentrated, like many other Italian economists, on the driving forces of the business cycle, turning at the end of the decade to the theory of the general equilibrium. We consider these phases one at a time, finally dwelling upon his interest in a mathematical theory of corporatism.

In *Contributo alla teoria matematica della dinamica economica* (1932) and in the subsequent *La dinamica della circolazione* (1934), Amoroso assumed a very simple market characterized by generic goods and postulated that agents decide to buy or to sell according to the attitude of prices themselves to change. This attitude is the inertia of the system, defining which Amoroso made dynamic the economic system. The latter comprises three sectors – production, consumption and banking – all decisions concerning the present and future are a function of current and past rates of changes of such values. That is to say, consumers decide also according to price trends determined by past decisions. In this way habits enter into the scheme. In those years, Griffith C. Evans (1924; 1930) hypothesized a “weight” of history which decreases with the distance of the fact from the present, expressing it with higher order derivatives. In Amoroso’s theory, Evans’s reasoning appears suitable for those reactions of inertia which are of a mechanical type, for example, reactions of production to past investments.

It is evident that the use of time derivatives raises the question of the persistence of effects of past trends and the influence of expectations. In other words, we have to establish up until what time past actions can be considered as important for current choices, and to what extent forecasts determine current choices. This shows that Amoroso, despite his mechanical faith, opened his theory to mental variables. Better, he was forced to introduce subjective variables, because, as seen by Pareto, only this kind of economic variables involved time.

Amoroso also introduced expected values: prices of goods and rates of interest depend also on expected production, but he implicitly postulated that these predictions were always exact, i.e. perfect foresights. Obviously, these forecasts (*guiding* or *speculative forces* in the author’s language) were as the weaker aspect in Amoroso’s theoretical scheme since expectations were not subject to the analogy with classical mechanics.

Subsequently, Amoroso shifted from a microeconomic perspective to a macroeconomic one, applying the action-reaction principle to mass phenomena. In *La dinamica della circolazione* (1934), he rewrote the system including three equations on trade, production and banking, attributing to the latter, the banking and credit system, the role of dampening price oscillations.

The model comprised three equations (here modified):

the trade equation $P_t = m - a\dot{P}_t - a\dot{r}_{t+\omega} - a\dot{Q}_{t+\omega}$; [1]

the banking equation $r_t = m + a\dot{P}_{t-\omega} - a\dot{r}_t + a\dot{Q}_{t+\omega}$; [2]

and the production or supply equation $Q_t = m + a\dot{P}_{t-\omega} - a\dot{r}_{t-\omega} - a\dot{Q}_t$; [3]

where m is the stock of money, P is a price index of a good or of a bundle of goods at time t and $\dot{P}$ is the derivative of the index price; r is the interest rate and $\dot{r}$ is its time derivative; Q is the production and $\dot{Q}$ is its time derivative; a is a positive constant (Amoroso 1934: 832-3).[5]

The economic system displays a cyclical motion as a result of the interaction among different types of reaction to given changes. Firstly, the reactions of inertia (the derivative marked by t) meaning the tendency of a system to react to a given change. Secondly, the mechanical reactions (marked by $t - \omega$) showing the response to past variations, as those characterising the supply sector. Thirdly, there may also be actions grounded on forecasts (marked by $t + \omega$), such as those characterising the business sector and partially the banking one. Moreover, the time lags are different according to the sector. The cycle is fully endogenous because it is not the product of variations of prices, interest rate and supply due to exogenous factors, but because variables' changes bring about different effects on the current variable, i.e. current choices. So the past growth of supply determines the fall in current prices and the rise in current interest rate; increase in foreseen production cuts prices and boosts interest rate. Briefly, Amoroso applied the action/reaction principle to whole economic system.

Money influences the level of prices but not its rate of change. Because it is a stock, the amount of money cannot present acceleration and it does not condition individual choices. This entails the ineffectiveness of any type of monetary measure. Amoroso was also rather mysterious regarding the role of the banking sector. Even if savings were not included in the equations, Amoroso evoked an embryonic theory of loanable funds when he attributed to the bank the function of equalizing savings to new investments, by means of the discount rate (1934: 835-6). In this way, the banking system mitigates economic fluctuations.

It must be emphasized that the cycle resulting from the meeting of fundamental actions and reactions in the three sectors, that is industrial, business and banking, rules out every kind of political intervention such as monetary policies.

A key theoretical point that Amoroso did not clarify satisfactorily concerns precisely the passage from micro- to macro-economic analysis. Individual behaviours, when they are aggregated or considered as mass phenomena, can generate crises or perturbations. It is not clear whether Amoroso was thinking of a simple addition process or if he meant something else. Subsequently, Amoroso (1943: 428 ff.) introduced a distinction between two types of inertia: first, the known individual inertia and, second, the inertia of the market. Both types of resistance concern market

[5] By including the rate of change of the price index, Amoroso followed Fisher (1925a).

choices, but only the aggregate inertia can generate the rigidity of the market or the stickiness of variables. Amoroso stated that market failures are generated by aggregate behaviours. We presume that he thought of mass phenomena as imitation and tradition which, following Pareto (1963: §349) are non-logical by definition: “habits and worries weigh directly upon our sensitiveness, independently and sometimes contrasting with the satisfaction of temporary needs” (1943: 431).

2.3 *The general economic equilibrium*

Having completed his cyclical scheme in 1934, Amoroso returned to his original project of constructing a micro-theory of general dynamic economic equilibrium.

It was during the early forties that he developed his ideas on general equilibrium, partially anticipated in the article *The Transformation of Value in the Productive Process*, published by *Econometrica* in 1940, were extensively illustrated in the 1942 book *Lezioni di Meccanica Economica*. Amoroso’s starting point was his dissatisfaction with static general equilibrium theory. The greatest need for a dynamic vision was in the savings sector or in the accumulation of capital as we would define it today. In his words:

> If there is one thing that characterizes economic life and modern industrial society in particular, it is the continuous growth of plant, and of capital in general, the tool which enables man to prevail over nature. But new capital is generated by savings: hence the importance attributed to the process of saving in economic theory. This representation, however, appears mutilated, because, if such a process is treated as stationary […] savings is considered unvaried, either the aggregate stock or the individual shares of savings […] This mutilation is grave, and authorizes us to conclude that equilibrium theory is, by its very nature, incapable of representing the core of this concrete phenomenon (Amoroso 1942: 119-20).

Such incongruence emerged not only in the analysis of savings, but also in trade and distribution: here the assumption of constant prices, a requisite of stationary equilibrium, clearly conflicted with reality.

The extension of the time horizon to several periods requires new analytical tools and, according to Amoroso, the substitution of differential with functional calculus. The calculus of variations seemed to be the tool that could best lead to dynamic optimization. In the 1942 book’s sections devoted to both the consumer and the entrepreneur, assuming that both the initial and the final conditions are given, Amoroso argued that the conditions of consistency are verified, the number of equations being equal to the unknowns.

The introduction of a principle analogous to that of mechanical inertia was a new facet to consumption theory. More generally, Amoroso stated that the quantity consumed in a time, dt, is a function of time and represents the velocity of the consumption flow. If we take into account the weight of habits in slowing down or accelerating of consumption, we must express the utility as functions of both consumption and the change of consumption, c and $\dot{c}$. In symbols:

$$U = U(c, \dot{c}) \qquad [4]$$

Amoroso labelled this equation [4] Lagrangian ophelimity (Amoroso 1942: 124), evoking both the rational mechanics and Pareto himself. In practice, he intended to extend the Paretian ophelimity conceived for static analysis ($\dot{c} = 0$) to dynamic analysis ($\dot{c} \neq 0$).

[4] symbolizes a consumption trajectory that the consumer tries to determine according to a given time. From a mathematical viewpoint, the problem is well determined since the latter consists in identifying the optimal trajectory by means of the following functional:

$$\int_{t_0}^{t_1} U(c, \dot{c})dt \qquad [5]$$

This is a typical calculus of variations problem, which presented the following Euler equation as a necessary and here also sufficient condition:[6]

$$\frac{\partial U}{\partial c} - \frac{d}{dt}\left(\frac{\partial U}{\partial \dot{c}}\right) = 0 \qquad [6]$$

This was the Lagrangian marginal utility, where the first term represents the traditional marginal utility while the second is the dynamic part, the loss of utility being due to habits (the inertia of the system) and, finally, to the costs associated with change. If the consumer has to distribute his income among many goods, this equilibrium condition involves maximum utility when the weighted Lagrangian marginal utilities are levelled, exactly as in the static analysis.

The textual reference to inertia is an evident reference to the mechanical analogy, which is strengthened by the second order differential equation [6], just as in the dynamics of bodies. In practice, some problems may emerge from a mathematical viewpoint, since the system is composed of a large number of differential equations, the squaring of which is possible in only a few cases, even if the demand functions could be theoretically deduced from the system of prices and the time assumed as exogenous variable.

This epistemological project was most fully manifest in the energetic interpretation of the productive process,[7] as set out in 1942 in *Meccanica economica*.[8] On this view, the similarity

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[6] As an example of a function with this objective, Amoroso (1942: 134) pointed out the following $U = \log x - e^{-x'}$, from which he drew the Euler equation $xx'' + e^{x'} = 0$

[7] An in-depth discussion on this topic is included in Mirowski (1989: Chap. 5).

[8] The question at the basis of Amoroso's research was the very same one that Fisher (1925b) posed in his doctoral thesis in 1895: in economic sectors, to what extent did equivalent concepts to the concept of energy exist? Fisher limited his discussion to the static case in consumption, and concluded that utility could be interpreted as potential energy; total expenditure, assuming fixed prices, could be interpreted as a form of kinetic energy. In the process of rational choice,

between mechanical and economic phenomena is entirely evident: in the production sphere there operates the principle of energy conservation, one of the fundamental laws of classical dynamics; but with the non-negligible difference that, whilst in the natural sciences the principle of minimum action is built on experimental practice without explanation of its origin, in economics it is the intentional outcome of rational behaviour. We will reconstruct Amoroso's reasoning starting with the sections devoted to the firm.

The mathematical nature of the entrepreneur choice problem is analogous to that of the consumer when it depends on the rate of change of productive factors and not only on the absolute values (Amoroso 1942: *Lezione XIV*). The total revenue (T) and the total cost (Θ) are presented as follows:

$$T = pz(x, \dot{x})e^{-it} \qquad\qquad \Theta = qxe^{-it} \tag{7}$$

where p is the product price, q is the factor price, x is the flow of productive investment, and z is the quantity of product obtained in a given time. It must be stressed that this amount depends on both the amount of investments and the rate of change of such investments, ($\dot{x}$), as in the case of the dynamic choice of the consumer. The entrepreneur tries to maximize the integral *I*, that is, the profits during the time interval $t_0 \le t \le t_1$:

$$I = \int_{t_0}^{t_1} (T - \Theta)dt \tag{8}$$

The application of the Euler equation leads to the following Lagrangian production equation, according to Amoroso's terminology (1942: 150):

$$\frac{\partial T}{\partial x} - \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = \frac{\partial \Theta}{\partial x} \tag{9}$$

The left side of [9] represents the marginal revenue, the right one is the marginal cost. At the optimum point, the marginal revenue depends here also on the factor rates of change and must always be equal to the marginal cost. In the stationary state case the derivative of $\dot{x}$ is zero and [9] becomes the known levelling between marginal productivities and factor prices.

The energetic view of the productive process springs from a specific interpretation of [9]. According to Amoroso, the left side of the equation can be considered as a sort of kinetic energy, even if this is only a formal analogy since the function is neither quadratic nor homogeneous. On

the principle of energy conservation assured that their sum was constant. In other words economic action was subject to the action of conservative forces (marginal utilities) the final result of which was independent of time. Amoroso extended this approach to production: "Is it possible to combine the economic concept of value and the mechanical concept of energy in such way as to link the transformation (of value) which is effected in the productive process to the transformation (of energy) which is displayed in the mechanical process?" (Amoroso 1940: 6) the contrary, the right side can be treated as a drop in potential energy (*dV*) consequent upon the consumption of resources ($d\Theta = -dV$). When the production is nil, the potential energy is at maximum level; it then decreases with the beginning of the production process. If the productive system is stationary, $(\dot{x} = 0)$, equation [9] can be written in a form analogous to the principle of conservation of energy, $dT + dV = 0$, which becomes the following one when expressed in general terms:

$$U + V = Cost. \qquad [10]$$

where *U* symbolizes the kinetic energy and *V* is the potential energy of the system. The latter equation means that in competitive conditions determining equality between price and marginal cost, the theorem of conservation of energy holds, in the sense that the sum of the value of total production and costs is equal to a constant:

> If the prices are supposed to be invariable and if the effect of interest is neglected, the transformation in value which is effected in the productive cycle may be interpreted in the form of energy in the sense that the sum of the values expressed by the plant's potential energy, and the kinetic energy is constant. This means that every upward or downward variation in cost must find its exact equivalent in the increase or decrease of the latent product (Amoroso 1940: 9).

On Amoroso's positivist view, the rule of parity between marginal cost and marginal revenue is not only the consequence of a rational behaviour; it also signifies transformation of one type of energy into another. Nature behaves like an entrepreneur seeking to obtain the maximum output in energetic terms from a stationary plant. Then the principle of minimum action is evidenced in how the natural transformation of energy occurs at the minimum cost.

But, he added, "the transformation of value which is effected in the productive process represents a rule of conduct" and not a natural law (1940: 11). This means that a different kind of energy exists, that is, a monetary energy analogous to thermal energy, Ω, which expresses the value due to the fluctuations of prices and to the accumulation of interest. Ω represents a dispersion of energy – a *monetary energy* according to Amoroso – occurring when the productive process takes time and interests must be paid. Moreover, dispersion or concentration of energy may be due to the fluctuation of prices. So, the [10] must be rewritten as follows (1940: 10):

$$U + V + \Omega = Cost. \qquad [11]$$

In Amoroso's words:

> The employment of the factors of production is extended in every instant up to the point where the marginal cost is brought down to the value of marginal latent production, to the net amount of the price fluctuations and of the accumulation of interest to be paid (1940:11).

We conclude by noting that the principle of conservation of energy has been applied by Amoroso grounding his argument on proof by contradiction (as above), in order to show the differences between the economic processes and the natural processes.

2.4 *A mathematical theory of corporatism*

Amoroso's inquiry into general economic equilibrium moved him towards an economic theory of corporatism.[9] A step in this direction was the 1939 essay, *La teoria matematica del programma economico,* where Amoroso pointed out the analytical conditions which must be respected when planning both consumption and production and, finally, dwelt on the specific form of forecasts which characterize the "corporative" economics. Moving from economic liberalism to economic planning, Amoroso replaced the pivotal role attributed to individual economic habits and forecasts with determination of governmental economic targets. Here, the calculus of variations was applied to a governmental program with a known target.

Amoroso conceived his theory in order to legitimize the economic policy of the then Fascist regime, even if economic science was never subordinate to the regime, contrary to the case of Italian statistics (Guerraggio and Nastasi 2005: 137). However, the analytical tools proposed by Amoroso, La Volpe, and Bordin fitted well with the programmatic needs of a corporative economy. In effect, the problem of planning involved an instrumental innovation that Amoroso drew from rational mechanics and that enabled him to look at economic dynamics in a way fundamentally different from Pareto's theory of subsequent equilibria (Tusset 2004: Chap. 1). Rather than considering causal relations, one can compare the movement of a system in a given time with other possible trajectories or movements in the same period. Whilst in mechanics the calculus of variations enables comparison among different trajectories, in economics it enables comparison among different plans. Consequently, the optimal path ought to be singled out, obviously once the target is given. The introduction of these analytical tools induced Amoroso explicitly to propose a *mathematical theory of corporatism* (Amoroso 1938b). He wrote:

> Economic action is implemented by choosing the target and directing to it the existing resources. The aim may be both the industrial or trade program of a private board and the regulatory plan of national economy, established in the upper echelons system and here assumed as given (1938b: 80).

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[9] In his writings on corporatism Amoroso paid no attention to the contemporary international literature on centralized economies; nor did he refer to other economists who wrote on economic planning. We identify two reasons for this omission. First, Amoroso's interest in Italian corporative economy originated not from theoretical or scientific concerns but from his adherence to Fascist ideology, as proved by his political writings (see Amoroso and De Stefani 1933). Second, he treated the corporative economy as a national experience with no immediate analogue in other national economies.

On hypothesizing an economic plan for the corporative economy, Amoroso affirmed that the centralization of decisions was compatible with individual plans. This was not an isolated position, for several economists envisaged compatibility between national economic aims and individual decisions.

In fact, the idea of a national economic plan did not call the individual hedonistic principle into question, since the latter was in any case considered the true engine of economic activity. From this perspective, the forces of inertia may be authentic obstacles to the achievement of targets, when they express habits not suited to the aims established. Habits conflict by definition with programs, which because they are "revolutionary acts" (Amoroso 1942: 152) always involve a discontinuity with the past. Hence, in singling out the path for the national economy judged optimal in relation to the targets, the corporative plan must take account of the contrary forces springing from inertia. This requires actions on habits, the interest rate and other factors influencing individual forecasts and choices. The result is a paternalist state which acts on individual behaviors. Amoroso never argued for a centralized or planning government, but implicitly affirmed that the latter could establish the economic trends of the entire economy. As a mathematical economist, Amoroso limited himself "to shedding light on the path towards the given aim", since "economics is logics and not politics" (Amoroso 1939: 144).

3. The 'moving equilibrium' of Giulio La Volpe

For Amoroso, his approach to the differential equations of general dynamic economic equilibrium marked the completion of a long and unremitting inquiry. In the case of La Volpe, by contrast, his main results were obtained in the first part of his long scientific activity. Having graduated in 1930 from the Regio Istituto Superiore di Scienze Economiche of Naples, when only thirty years old in 1936 he published probably the most important Italian contribution on this topic, *Studies on the theory of general dynamic economic equilibrium* (1993), a long and technical essay in which La Volpe tried to translate the Paretian framework into dynamic terms. In the post-war period, La Volpe was attracted by empirical research which he published in *Ricerche Economiche*, the journal of which he had been managing editor for many years. His interest in the relation between static and dynamic economics never ceased, even though he abandoned the theoretical perspective shown in the *Studies* for more empirical analysis. Here we will limit our attention to his 1936 most important book, translated into English in 1993.

The book's treatment follows the traditional Paretian scheme: the first part of the first chapter deals with the dynamic consumption problem, the second is dedicated to production analysis and the third is devoted to the compatibility between agents' choices, that is, to the equality between the number of equations and the number of unknowns. In the second chapter La Volpe addresses special aspects, such as the presence of durable goods, the legacies of consumers, the financing and the number of firms, and monopoly.

Certainly La Volpe fostered the Paretian interest in the continuous equilibrium, but adopting a really different perspective. In fact, he overturned Amoroso's framework grounded on the past, adopting a view that put future and forecasts at the centre of his analysis. He wrote:

> I have tried to construct a micro-dynamic theory of general equilibrium, showing how at every moment there is market equilibrium as a result of the behaviour of economic agents, consumers and firms, founded on expectations and plans for the future; showing how this equilibrium shifts continuously in time, through the modification of individual plans. This is the salient point of the theory. While it is true that it is difficult to predict the future, impossible to avoid errors when projecting present market trends into the future and easy to exaggerate one's predictions in one direction or the other, it is no less true that this is how economic activity is regulated at any moment (La Volpe 1993: 2).

This obliged him to abandon also the rational mechanics and adaptive and inertial behaviours as starting points, preferring to consider an agent that plans his choices along his lifetime. According to the latter, economic dynamics must be grounded on the foreseen value of the decision variables. This differentiated the La Volpe's economic dynamics from both Pareto's and Amoroso's. However La Volpe followed Pareto conceiving equilibrium not as the "state to which the economic tends over a shorter or longer period of time," but as the "effective state attained at a given moment on the basis of the simple hypotheses that have been assumed" (1998: 254). As in the case of Pareto's continuous equilibrium, "*in the historical* [evolutionary] *motion of a given economy the various forces are, given the constraints, in equilibrium at every instant of time*" (1998: 254, *italics* in the original). This also means that the stability of equilibrium was not at all a concern in La Volpe's view.

As usual in those years, the main theoretical outcomes concerned consumer behaviour then extended to firm's behaviour. Accordingly, we may limit our analysis to the consumer theory.

In order to study consumption, La Volpe stated that the current consumer's choices were based on all the lifetime foreseen consumptions, assuming the current constraints. He wrote:

> In any infinitesimal interval of time, the consumer decides his market conduct with an eye to the future and the aim of maximum satisfaction over the (expected) period of his life. It is a plan for the allotment of all resources (present and future) held and will certainly determine his present conduct and also his future conduct if his expectations and other factors do not change. Otherwise he will not follow any predetermined plan and will re-examine his choices at every moment (La Volpe 1993: 9):

According to La Volpe, this makes the problem of the consumer one of the most complex to resolve.

> [The consumer] has to take into account the trend of prices and the rate of interest, as he forecasts them at the time. Together with his consumption and savings, he has to decide how much of his wealth he wants to invest and how much he needs for the maintenance of consumer durables; at the same time he has to decide how to distribute the former between different types of investments having different terms, and the latter between different consumer goods. He has to decide the cash balance necessary for everyday transactions, how much of his labour to supply, the distribution of expenditure between different consumer items: a whole series of simultaneous, closely related choices, very difficult to analyze rigorously (La Volpe 1993: 9-10).

From an analytical view, an inter-temporal utility constituted the starting point. The distinctive feature of La Volpe's scheme is that it also depends on the expectations formed at time t_0 on the foreseen lifetime, ν. For this reason he speaks of a *future utility evaluation function*. The total utility achievable in a given interval of time (t_0, ν) is expressed as follows:

$$\int_{t_0}^{\nu} U_i(t,\tau,C_0(t_0,\tau),...,C_{h+m}(t_0,\tau))d\tau \qquad [12]$$

which refers to a given basket of goods and services, C_{h+m}, given future expectations established at time t_0 concerning a certain future time $\tau \geq t_0$ (1993: 11). In each instant, the consumer must decide how to determine his optimal consumption pattern, taking account of the dynamic budget constraint. The latter consists of predictions concerning the quantity of labour and other services that the consumer may receive in the future, of the amount of profits that he will obtain, his financial choices, and the planned flow of consumption. Using La Volpe's notation, the budget constraint becomes the following expression (the time suffixes have been omitted in order not to encumber the notation):

$$\sum_{j=0}^{h} p_j(H_j - C_j) + rF + R - \dot{F} - \sum_{j=h+1}^{h+m} p_j C_j = 0 \qquad [13]$$

where the first term in [13] represents the balance of services purchased or supplied in the time interval (which are between 0 and h in number), including labour (1993: 14). The second term, rF, denotes interest on accumulated savings, while the third, R, is the proportion of income from shareholding. There are two further terms: the variation in savings by unit of time, $\dot{F}$, and the final summation, which indicates spending on consumption (consumption goods range from h_{t+1} to h_{t+m}). La Volpe hypothesises that, at this stage of the analysis, both current and expected prices, as well as both current and expected interest rate, are constant, so that the unknowns are reduced to the determination of current and future consumption, and of planned financing.

The maximization of [12] under the inter-temporal budget constraint is a particular problem resolved through the calculus of variations. La Volpe obtains the following equation (1993: 16), which describes the optimal path for each good:

$$\frac{\dot{U}_{C_j}}{p_j} = Ae^{-rt} \qquad [14]$$

where the parameter A is a constant of integration that must be determined.

In practice, [14] defines a differential equation system, one equation for each good, which together with budget constraint, initial condition, and transversality condition permit the determination of the consumer's problem of optimal choice.

All these analytical conditions allowed La Volpe to argue the so-called two laws of consumption dynamic equilibrium. The first law affirms that the consumer maximizes its well-being when the weighted marginal utilities referred to the different goods in different times are equalized. The true dynamic facet is introduced, however, with the second law, according to which the distribution of the maximum satisfaction in time requires the rate of the weighted marginal utility to be a decreasing function of the rate of interest, and assuming that the constant A depends on the initial parameters. The larger the value of this constant, the smaller the consumption and, consequently, the larger the individual savings.

On inspection of this (at that time) complex analytical construction, we can conclude that La Volpe, like Amoroso, re-defined the equilibrium of the consumer in dynamic terms. Theoretically, this enabled the demand function to be mapped for each good with prices and time as arguments. Obviously, once the time is given, the dynamic analysis changes into a static analysis.

There is an aspect to La Volpe's account of dynamic equilibrium that should be stressed in particular – both from the mathematical viewpoint and from that of economic interpretation – because it is an indubitable advance on Amoroso's position and highlights its originality. Amoroso had shown that the application of the calculus of variations to problems of dynamic optimization required the specification of two conditions: the initial stock of factors, and the initial pace (the initial velocity) of their use. Whilst the former condition has an immediate economic meaning, the latter is difficult to interpret. When faced with the same analytical problem, La Volpe adopted the first condition and introduced a second one, thereby anticipating what in the subsequent literature became the (necessary) transversality condition. In order to solve the consumer's problem, La Volpe assumed that the following condition holds:

$$F(t,\tau) = 0 \quad if \quad t = \tau \qquad [15]$$

This states that in the final instant of the period considered savings must be nil, so that the individual must have used all the financial resources at his disposal. As a Paretian, La Volpe was concerned about the scant realism of this hypothesis, and to remedy this limitation of the model, he also considered the case of legacies, the 'need to bestow', as he put it (1993: 33 ff.).

It must be stressed that La Volpe was aware that his proposal allowed the horizon to be extended from the single period to the entire life-cycle of the individual. Resuming [14], integrating it, and taking account of the transversality condition, he obtained the following expression :

$$\int_{t_0}^{v} \sum_{j=0}^{h+m} p_j C_j e^{-rt} = F(t_0) + \int_{t_0}^{v} (\sum_{j=0}^{h} p_j H_j + R) e^{-rt} \qquad [16]$$

which, to simplify the notation, considers as constant the interest rate. In [16] the left-hand term represents the current value of consumption, calculated at time t, while the right-hand term is the current value of the individual's total wealth (1993: 19).[10] La Volpe's conclusion is that the current value of expenditure on all consumption envisaged, at a given instant, by individuals for their entire expected lifetimes must be equal to the wealth possessed at that instant. With relation [16] he anticipated the core theoretical principle of the theory of consumption based on the life cycle and in the 1950s in order to overcome the limitations of the Keynesian consumption function. He wrote: "The present value of expenditure for all the consumption items that individuals plan, at a given time, to make in the course of their expected lifetimes equals their wealth at that moment" (1993: 19). This leads to a clear conclusion: if an individual gets into debt in a given interval of time, during the subsequent one he must save because it is not possible to dissipate more than one's own wealth. Obviously, this does not rule out that some individuals may not accumulate wealth in order to pass it on to their descendants.

Finally, by postponing the consumption of current resources, or by anticipating the use of future resources, the individual achieves his maximum well-being. However, once La Volpe had clarified the analytical aspects of the dynamic equilibrium, he seemed almost pessimistic about its heuristic use:

> In individual dynamic equilibrium, the problems are all solved simultaneously by a calculation of overall economic utility. Consumption and savings, supply and demand of goods, services and financial means, and expected wealth are all related quantities which are not determined separately but result from the judicious seeking of maximum welfare which links them all. Hence each of them depends on all the data of the problem: on the valuation of the utility of present and future consumption, on present and expected resources as well as on present market prices and rate of interest, and on the functions of their expected values which in turn depend on relative present values and their trends. If even a single one of these elements changed, individual behaviour would be completely modified (1993: 20).

[10] It should be pointed out that La Volpe cited Ramsey's 1928 article, *A Mathematical Theory of Saving*, as among the works to which he had made "particular reference" when studying the consumer economy (1993: 101, note 2).

The analysis of the firm's dynamic equilibrium does not conceptually differ from that of the consumer. On the contrary, it is facilitated by the dynamic character of investments. Every investment is always carried out with a certain delay in comparison to the moment when it is decided, so the marginal cost must increase according to the length of the productive process. In this way, if we know prices and rate of interest, we obtain as many supply functions as the number of firms.

It may be of some interest to point out that La Volpe applied the same idea of lifecycle also to the firm stating that:

> In plans formulated at any time, producers decide to buy a quantity of each type of limited durable, at any moment in the (expected) lifetime of the firm, such that the expected value of the marginal productivity is equal to the (expected) market price at that moment (1993: 61).

The entrepreneur formulates a detailed investments plan for the entire life of the firm, even if, La Volpe admitted, such a plan "becomes vaguer and vaguer the further into the future it looks" and this because of the difficulties of predicting "industrial yields and especially purchase prices of means of production, rates of interest and sales prices of products" (1993: 66).

Once the consumers' demands and the firms' supplies have been determined, the consistency of the equation system must be verified. La Volpe established that the number of unknowns was equal to the functional relations, that is, that the system was determined. By integrating the system referring the variables to a given time *t*, we obtain the solution referring to that time and to the historical motion that begins at that moment. The result is a 'big picture' of competitive markets:

> In conformity with the fundamental principle of dynamics […] the market attains a position of equilibrium at every moment. The consumers are in equilibrium because the plans which regulate their behaviour imply maximum present and future satisfaction. Firms are in equilibrium because they regulate investments and thus finance on the basis of long- or short-term plans which aim for the greatest possible difference between sales and production costs. The whole market is in equilibrium because prices and interest rates settle so as to balance simultaneously the total supply and demand of goods, services and financial means (1993: 76).

In La Volpe's opinion, this moving (temporary) equilibrium grounded on future forecasts is always a changing and fragile edifice. As time passes, consumer tastes alter, production techniques change because of technical progress, and finally individual forecasts are not always correct. Consequently, this complex analytical representation hardly provides exact indications on the real economic working of the system.

La Volpe's dynamic approach can be judged one of the most interesting products of the Paretian School. At the analytical level, it singled out the transversality conditions and at the economic level, it showed an articulated approach to expectations. Notwithstanding these aspects, it was completely ignored until the later tribute by Morishima stressing the analogies of this work with Hicks's celebrated *Value and Capital*.

In 1938 and subsequently in 1965, 1967 and 1977, La Volpe returned to the general dynamic economic equilibrium with a proposal that he defined "variational dynamic analysis". He tried to overcome the limits of realism and of dynamic consistency he envisaged in the 1936 theory, by introducing the time gap or the "delayed variables" as the key to dynamic economics. Moreover, he abandoned the logic of interdependence, preferring causality. However, this project remained only a collection of premises and methodological features, without a true analytical scheme.

4. Concluding remarks

In accordance with the main international research programs, also in Italy the 1930s were years of economic dynamics. The attempts to establish a true dynamic theory in economics followed different paths, also in the mechanistic legacy of Pareto. This is proof of the traditional pluralism characterizing the Italian sciences, including economics. Amoroso considered rational mechanics in an attempt to find the economic analogy of the inertia principle. The twin notions of Lagrangian utility and productivity represented a theoretical enrichment permitting inclusion of the time factor in micro-economic decisions. Moreover, the systematic use of functional calculus instead of differential calculus put Amoroso on the frontiers of research into dynamic economics. La Volpe acknowledged the importance of expectations, as suggested by the then international debate on this topic, but asserted the perfect rationality of the foresights. Nevertheless his anticipation of the lifetime cycle made his youthful work interesting.

The proposals of the Paretian School let some aspects unresolved. A first limit pertains the vagueness surrounding the links between different periods or plans to which dynamic analysis ought to be applied. In both Amoroso's and La Volpe's theories, the present is conditioned by the past and in its turn conditions the future, but the analytical problem of the variables connecting different periods remained seemingly unaddressed. However, it is true that Amoroso developed a cycle theory involving periodical changing of variables and that La Volpe's consumer and firm theory may be read from a different perspective. In fact, the insertion of a lifetime constraint conditioning all agents' choices can be interpreted as the insertion of a factor linking all the temporary plans implemented by consumers and firms. The lifetime plan includes the shorter time plans virtually linking them together.

This lack finds another interpretation. Both Amoroso and La Volpe remained loyal to the Pareto's idea of continuous equilibrium and this limited the development of a dynamic theory built on the sequence of different periods.

Furthermore, this means that we should not consider these theories as fully grounded on a deterministic conception. La Volpe postulated perfect foresights in principle, but they may be denied at any time. Both consumers' and firms' plans are subject to unforeseeable changes in technology and tastes must be continually revised: "What is new comes above all from the hedonistic estimates of the consumers, from the law of technology or from the expected prices" (1998: 259).

A further limitation that may be ascribed to all the Paretians is that they did not elaborate a complete macro-dynamic theory. With the partial exception of Amoroso for a few years, they never abandoned the micro-economic scheme, remaining loyal to the Paretian view affirming market interdependence, and preventing the causal analysis of economic dynamism. But it must be remembered that the whole of Italian economic culture found it difficult to accept the passage to aggregate or macroeconomic scheme.

Finally, given these characteristics, it is no surprise that the Paretians remained indifferent, if not critical, toward the axiomatization of general equilibrium that began in the 1940s. On the one hand, their reference model remained physics, with its concrete magnitudes, and not mathematics with its formal elegance. On the other, the new frontiers of topology applied to economics had no contribution to make to the Paretians' privileged field of research, i.e. economic dynamics.

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