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TRILATERAL CONTRACT AND THE HOLD-UP PROBLEM

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Trilateral Contract and the Hold-up Problem

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Abstract

We present a novel solution for the hold up problem, when more than two parties are involved. The case we consider is a company selling identical products to two buyers that have a common interest in inducing the seller to make a quality enhancing investment. We show that a trilateral contract may provide the correct incentives to restore optimal efficiency. The contract induces a coalition proof Nash equilibrium and holds under complete as well as incomplete information. The extension to more than two buyers is straightforward.

JEL: D82, L14
Keywords: multilateral contract, trilateral contract, hold-up problem

1 Introduction

The hold-up problem has been extensively analyzed by the economic literature in the last decades. In its classical version, this problem applies when two parties, for instance a manufacturer and a customer, or, more generally, a seller and a buyer, are unable to extract all the surplus from their interaction. Typically, the party that should make a quality enhancing relation-specific investment is unable to receive all the benefits that accrue from this investment (Klein et al., 1978, Williamson, 1985), as future (re)negotiation may confer parts of the benefit from the customized investment to the party with higher bargaining power. When neither the investment nor the induced quality can be verified by a third party, the contract cannot be contingent on them. Therefore, a contract with a fixed price would give the seller no incentive to invest. Alternatively, a contract where the price is fixed but the buyer has the option to buy gives no incentive to invest: the buyer may renegotiate the terms of the contract once the investment is sunk (Hart and Moore, 1999).

In the following, we present a novel solution for the hold-up problem, in case there is more than one buyer involved in the transaction. We consider the setting in which a seller produces identical products for two noncompeting buyers that have a common interest in inducing the seller to make a quality enhancing investment. We show that a trilateral contract may provide the correct incentives to lower the hold-up problem and restore optimal efficiency.

The intuition on why the contract may solve the problem is simple. If trade between the seller and each of the buyers is sequentially, then it is possible to make payments contingent on exchanged (and verifiable) payments. More specifically, the first buyer purchases the product in case the quality is high, paying a price equal to his outside option, the market price. Yet, the contract obliges the second buyer to pay an extra transfer contingent on the first buyer exchanging the payment with the seller; thereafter, he also may buy the product paying the market price. The strictly positive extra transfer received by the seller induces a higher level of investment than the one induced by the market prices. This way, even though prices are fixed and there is a trilateral option contract, the seller has incentive to invest sufficiently into quality enhancement. We show that the overall induced investment may be as high as the optimal level of investment. The contract is self-enforcing, which means that neither party has incentive to renege (Baker, Gibbons, and Murphy, 2002b), and it is coalition renegotiation proof: no subgroup of agents has incentive to renege

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(Bernheim, Peleg, and Whinston, 1987). By specifying payments among the firms upon contracting, we ensure that all parties have incentive to participate in the contract.

There are numerous examples of situations in which this kind of contract could be of use. Since we consider no competition among the buyers, the buyers might either be end-consumers, or firms operating in different markets. Examples include software or operating system provision for firms in different industries. Also research collaborations may benefit: pharmaceutical companies may want to increase investment in gene therapy, to further developing different medicines.

Our trilateral constitutes a short term cooperative project, in which investment is incurred one single time. In this sense, it differs to solving the problem by vertically integrating or restructuring firm boundaries and asset ownership, as suggested by Baker, Gibbons, and Murphy (2002a) and Grossman and Hart (1986). Neither does the contract rely on repeated interaction with the same agent or within a group, where incentives arise based on reputational effects (Dixit, 2003, Kandori, 1992, Radner, 1981). Also, it does not require any additional agent like an an intermediary or arbitrary, cases considered in Dixit (2004) and Laffont and Martimort (1997). In our contract, the interaction among the agents involved in the transactions suffices to induce the right incentives.

The remaining part of the paper is organized as follows: section 2 introduces the basic model of a trilateral contract, concentrating on the case with only two buyers. After presenting the benchmark and several verifiability issues in section 2.1, section 2.2 presents the model under complete information, taking into account the fact that the two buyers might have different valuations of the product. We show that the optimal efficiency can be restored. In section 3, we comment on joint renegotiation. Section 4 considers the case of asymmetric information, still concentrating on two downstream firms: also when the valuation of each buyer is private information, we show that there exists a modification of the multilateral contract that induces truthful revelation and restores the optimal level of investment. The extension of the model to more than two buyers is exposed in section 5; section 6 concludes.

2 Model

Consider three players: One upstream firm A and two downstream firms P_i, $i \in \{1, 2\}$. A produces two goods with zero marginal cost. The two downstream firms are not competing with each other. A chooses the level of effort $e \geq 0$, to maximize its profit. Effort is costly, with $c(e)$ being an increasing convex function. The probability that the good is of high quality is $\pi(e) \in [0, 1]$, which is an increasing quasiconcave function. The high quality product generates a monetary value $v_i = \beta_i^k m$, $\forall i \in \{1, 2\}$ for the downstream firms, with $k \in \{H, L\}$ and $\beta^H \geq \beta^L \geq 1$. Assume for now that the valuations of the downstream firms β_i^k are common knowledge. The low quality good generates a monetary value of zero. The good can be sold to the market, in which case it generates a monetary value of $m > 0$ if the quality is high and a value of zero if the quality is low[1].

In order to derive a closed-form solution for the model, we employ specific functional forms for the probability and cost functions. Namely, we assume that $\pi(e) = min\{\eta e, 1\}$, with $\eta > 0$, and $c(e) = \frac{\alpha e^2}{2}$, with $\alpha > 0$. Moreover, we assume that α is sufficiently high and η sufficiently small to prevent A to choose such a large investment level as to induce $\pi(e) = 1$. All players are risk neutral with standard utility functions.

Timing is as follows: at time $t = 0$, partners decide upon the terms of the contract. After agreeing on the contract, A chooses the level of its investment, and production occurs. At $t = 1$, P_1 and P_2 decide whether to buy or not, after having observed the quality of the good.

[1] The contract works also for m = 0, setting the valuations of the downstream firms equal to $v_i = \beta_i^k$.

2.1 Benchmark and Verifiability

To identify the level of investment that is socially optimal, we consider the case of a social planner that chooses the amount of investment e to maximize social welfare:

$$\begin{aligned} & \max_e \pi(e)(\beta_1+\beta_2)m - c(e) \\ = \quad & \max_e \eta e(\beta_1+\beta_2)m - \frac{\alpha e^2}{2}. \end{aligned} \tag{1}$$

From the first order condition one can easily derive the optimal investment level

$$e^{FB} \equiv \frac{\eta}{\alpha}(\beta_1+\beta_2)m.$$

This is also the effort level the upstream firm A would choose if it were sure to get a payment of $\beta_i m$ of each downstream firm for a high quality product.

If the investment is verifiable, but quality is not, still the first best is easily obtainable. A contract which specifies $e = e^{FB}$ and a fixed price $p_i \in [\frac{1}{2}c(e^{FB}), \beta_i m\pi(e^{FB})]$ for the product (independently of the realized quality) induces an efficient outcome and guarantees to each party profits at least as big as no trading.

If investment is not verifiable, but quality is, there still exist incentive compatible contracts which induce efficient outcomes. In fact, any contract which specifies a pair of prices (p_{h_i}, p_{l_i}) such that $p_{h_i} - p_{l_i} = \beta_i m$ and lump sum transfers $\tau \in R_+$ to distribute profits induces an efficient outcome: the most intuitive case is $p_i = \beta_i m$ for the high quality product and $p_i = 0$ for the low quality product, with $\tau = 0$.

Now suppose that neither quality nor investment is verifiable. On the one hand, any contract that specifies one fixed price for the good does not provide incentive to invest sufficiently to the upstream firm A. On the other hand, any option contract where each downstream firm has the option to buy the good at time $t = 1$ at a fixed price may be subject to renegotiation. In fact, any contract which fixes a price $p_i > m$ may be easily renegotiated at time $t = 1$: if the downstream firm refuses to buy then, in case the quality is high, the upstream firm can sell it to the market at a price m. Following Hart and Moore (1999), we assume that in the renegotiation stage the downstream firms have all bargaining power. Hence, anticipating the renegotiation, A invests

$$\begin{aligned} & \max_{e|p_i=m} U_A \\ = \quad & \max_{e|p_i=m} \pi(e)(2m) - c(e) \\ = \quad & \max_{e|p_i=m} \eta e 2m - \frac{\alpha e^2}{2}. \end{aligned} \tag{2}$$

From the first order conditions one can easily derive the induced investment level

$$e^{IC} \equiv \frac{\eta}{\alpha}2m,$$

which henceforth is called the incentive compatible investment level. This is the highest level of investment that can be induced when neither quality nor investment is verifiable.

2.2 Multilateral contract

The case of non verifiable quality and non verifiable investment is considered in the following. We show that there is scope for improvement, by signing a more complex contract instead of signing two independent bilateral option contracts. Consider the following contract. The downstream firms $P_{1,2}$ sign an option contract at time $t = 0$: P_1 pays a price $p_1 = m$ to A in case it buys a high quality good at time $t = 2$. P_2 pays $p_2 = m$ to A in case it buys at time $t = 3$. A chooses the level of investment at $t = 1$. Moreover, at time $t = 0$, P_1 pays x_1 to P_2, and A pays x_0 to P_2. Finally, P_2 pays an amount equal to ρ to A at time $t = 2$, conditionally on the fact that P_1 buys the good at time $t = 2$; this "extra-payment" ρ is specified in what follows. The payments are summarized in figure 2. While it may seem that some payments could "cancel out" - A pays x_0 to P_2, and P_2 pays ρ and m to A - this is not the case, since the payments are conditional upon different events. The timing of the game is crucial: only by making payments conditional on previous payments, incentive compatibility is given.

Proposition 1 *There exists a self-enforcing trilateral contract c that induces the optimal level of investment, and therefore increases overall welfare.*

In the following, we show which is the induced level of investment, assuming that the contract is renegotiation proof. Thereafter we show that it is indeed renegotiation proof, and that it satisfies individual rationality.

Proof Suppose the contract has been signed. Knowing that it gets the payments $p_1 = m$, $p_2 = m$ and ρ, in case the product is of high quality, and since the payment x_0 is paid *before* the level of investment is chosen[2], A maximizes

$$\begin{aligned} & \max_{e \mid \left\{ \substack{p_i = m, \forall i \in \{1,2\} \\ \rho} \right\}} U_A \qquad (3) \\ = & \max_{e \mid \left\{ \substack{p_i = m, \forall i \in \{1,2\} \\ \rho} \right\}} pi(e)(\sum_{i=1}^{2} p_i + \rho) - c(e) \\ = & \max_{e \mid \left\{ \substack{p_i = m, \forall i \in \{1,2\} \\ \rho} \right\}} \eta e(2m + \rho) - \frac{\alpha e^2}{2}. \end{aligned}$$

From the first order conditions follows that

$$\tilde{e} \equiv \frac{\eta}{\alpha}(2m + \rho)$$

[2] In this particular case, even if x_0 was paid *after* choosing the level of investment, A would incur the same level of investment as specified below, as long as x_0 and ρ are specified as in the following.

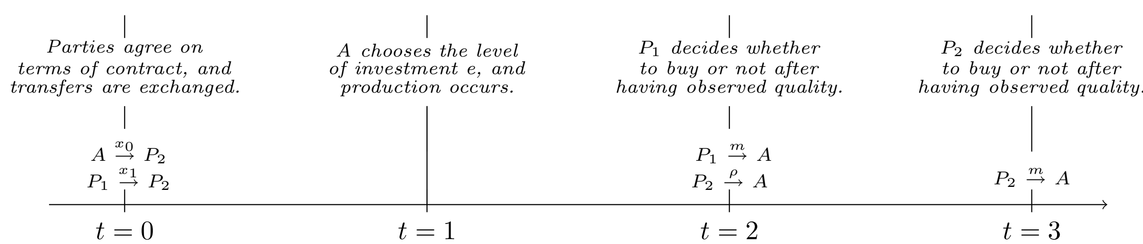

Figure 1: Timeline Trilateral Contract

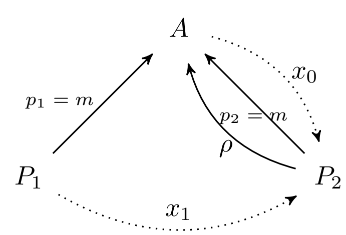

Figure 2: Trilateral contract

is the optimal level of investment for A, given the contract has been signed. It is strictly increasing in ρ. For any $\rho > 0$, $\tilde{e} > e^{ic}$; for $\rho = (\beta_1 + \beta_2 - 2)m$, $\tilde{e}$ equals the optimal level of investment e^{FB}.

We now study the conditions under which this contract is renegotiation-proof. Suppose the quality of the good is high. Both downstream firms have the possibility to refuse buying from the upstream firm, which can sell to the market at a price m. This leads to the outside option for P_1 and P_2 being to buy at a price m. Therefore, P_i does not renegotiate the contract if

$$\beta_i m - p_i \geq \beta_i 1m - m \quad \forall i \in \{1, 2\}. \tag{4}$$

It does not buy a good of low quality if

$$- p_i \leq 0 \quad \forall i \in \{1, 2\}. \tag{5}$$

Both inequalities are clearly satisfied. We still need to show that the contract is individual rational and that it induces a higher than the incentive compatible investment level. P_1 signs the contract if

$$- x_1 + \pi(\tilde{e})(\beta_1 m - p_1) \geq \pi(e^{ic})(\beta_1 m - m). \tag{6}$$

Setting $p_1 = m$, the previous can be written

$$x_1 \leq [\pi(\tilde{e}) - \pi(e^{ic})](\beta_1 m - m).$$

P_2 signs the contract if

$$x_0 + x_1 + \pi(\tilde{e})(\beta_2 m - p_2) - \pi(\tilde{e})\rho \geq \pi(e^{ic})(\beta_2 m - m), \tag{7}$$

which, for $p_2 = m$, is

$$x_0 + x_1 \geq \pi(e^{ic})(\beta_2 m - m) - \pi(\tilde{e})(\beta_2 m - m - \rho).$$

Assume the participation constraints (6) and (7) of the downstream firms to be binding. Then A has to provide x_0:

$$x_0 = \pi(e^{ic})(\beta_1 + \beta_2 - 2)m - \pi(\tilde{e})(\beta_1 m + \beta_2 m - 2m - \rho).$$

To see if this is implementable, we need to check the participation constraint of A. A signs the contract if

$$\pi(\tilde{e})(2m + \rho) - c(\tilde{e}) - x_0 \geq \pi(e^{ic})(2m) - c(e^{ic}), \tag{8}$$

which results in

$$x_0 \leq \pi(\tilde{e})(2m + \rho) - c(\tilde{e}) - \pi(e^{ic})(2m) + c(e^{ic}).$$

Replacing x_0, $\pi(\cdot)$, $c(\cdot)$ and the levels of effort e^{ic} and $\tilde{e}$, equation (8) results in

$$-\rho(\beta_1 m + \beta_2 m - 2m - \frac{1}{2}\rho) \leq 0.$$

Choosing $\rho = (\beta_1 + \beta_2 - 2)m$, the participation constraints of A, P_1 and P_2 are fulfilled. Since $\rho > 0$, the induced level of investment is higher than the incentive compatible one; in fact, it is what is required to induce the optimal level of investment. The resulting x's are:

$$\begin{aligned} x_0 &= \frac{\eta^2}{\alpha} 2m^2(\beta_1 + \beta_2 - 2), \text{ and} \\ x_1 &= \frac{\eta^2}{\alpha} m^2(\beta_1 - 1)(\beta_1 + \beta_2 - 2). \square \end{aligned}$$

With the x's specified as above, the extra benefit resulting from the trilateral contract (compared to the incentive compatible case), is completely skimmed by the upstream firm A: while each downstream firm makes a profit equal to the incentive compatible one, A's profit is higher than the incentive compatible[3].

Which of the two downstream firms P_1 and P_2 buys first is decided randomly. Since the expected payoff of the respective downstream firm is, as stated previously, equal to the incentive compatible payoff - in case the firm is of the high type, this is $\pi(e^{ic})(\beta^H - 1)m$, in case the firm is of the low type, $\pi(e^{ic})(\beta^L - 1)m$ - they are indifferent on being the first or the second buyer. That makes sense as long as we assume the whole setting happening in a very short period of time. However, one could also think about what would happen in case there actually are time frictions. Then, the downstream firm P_2 must have an incentive to buy in the 3^{rd} period, knowing about the existence of the product already in period $t = 2$. The existence of a temporary credit constraint might intuitively explain the delay in purchasing, as P_2 first receives x_0 and x_1 in period $t = 0$, and later on pays "extra" ρ in period $t = 2$. Up to now, we assumed all players to be risk neutral; yet, another intuitive explanation for why one buyer would choose to buy second could be risk aversion. While the expected profit of each downstream firm is equal, if the product is of low quality, the downstream firm P_1 has lost x_1, while the downstream firm P_2 has still gained $x_0 + x_1$[4].

3 Joint deviation under complete information

In the previous section we have shown that the downstream firms, each by itself, do not have incentive to renegotiate the contract. Yet, it seems plausible to assume that also subsets of the participants could try to coordinate their actions in a mutually beneficial way; especially since all participants can communicate at any stage of the contract. Therefore, we need to investigate the possibility of joint renegotiation.

Define a contract to be coalition proof if it induces a Coalition-Proof Nash equilibrium. Being a Coalition-Proof Nash equilibrium means that no subcoalition of the agents taking part in the contract has incentive to deviate from the specified equilibrium (Bernheim, Peleg, and Whinston, 1987). There exists a modification of our trilateral contract, $\bar{c}$, that is robust to coalition deviation, making use of the fact that the deviation is not self-enforcing. Hence, we show that

Proposition 2 *The contract $\bar{c}$ is coalition deviation proof.*

The contract $\bar{c}$ includes, additionally to what c specifies, a clause inhibiting all participating firms to make legally enforceable side contracts *conditional on the asserted quality*. Furthermore, it contains a clause specifying the exchange of a payment $0 < m_c < m$, payed from the first downstream firm P_1 to the second downstream firm P_2, in case P_1 claims low quality and P_2 claims high quality[5]. Previously we have shown that no single individual can profitable misreport the level of realized quality; hence, it remains to consider the three possible coalitions of subsets of the players: $\{A, P_1\}$, $\{A, P_2\}$, and $\{P_1, P_2\}$. The coalition $\{A, P_2\}$ cannot gain anything by jointly deviating. Since the exchange of the payment ρ depends only on what the first downstream firm, P_1, reports, any possible joint deviation wanting to extract this amount has to include P_1. However, we show that neither of the two possible coalitions has incentive to deviate.

Proof See section A.1 in the appendix. □

The intuition behind the proof is the following. The coalition $\{A, P_1\}$ cannot gain anything in case the good is of high quality, since P_1, when claiming would reduce the overall amount their coalition receives by ρ. On the other hand, if the good is of low quality, then by falsely claiming

[3] Yet, also a contract in which the two downstream firms receive part or all extra generated profit is feasible. Then the x's have to be specified differently.

[4] In any case, it is also possible to establish a symmetric case, in which both firms are paying and receiving the exact same transfers.

[5] Also with this clause the incentive compatibility constraint for P_2 when the good is of low quality is still satisfied: P_2 does not have incentive to buy a low quality product (claiming it to be high quality), since$-m + m_c < 0$.

high quality, P_1 can increase the overall amount their coalition receives by ρ. However, whichever payment A and P_1 agree upon as compensation for P_1 reporting falsely, since the two parties are not allowed to make legally enforceable side contracts, the exchange of this payment is not incentive compatible.

The coalition $\{P_1, P_2\}$ cannot gain anything when the good is of low quality, since by falsely claiming high quality they have to pay for products their valuation is actually 0 - remember that the payments x_0 and x_1 are exchanged already before production incurs, so they do not depend on the reported quality of the good. If however the good is of high quality, it is again the clause inhibiting to make legally enforceable side contracts, combined with the clause of P_2 getting a payment $0 < m_c < m$ in case P_1 claims low quality and P_2 claims high quality that inhibits a profitable joint deviation. Hence, reporting falsely the own valuation is not incentive compatible even when the downstream firms cooperatively try to deviate.

4 Asymmetric Information

Up to now we considered the valuations of the downstream firms to be common knowledge. But does the contract hold as well under information asymmetries? In this section show that the solution proposed to alleviate the hold up problem by means of a multilateral contract may be extended to situations in which the valuation of the good to each buyer is private information. This is an important case to consider, since obviously valuations which differ may not be observed, neither by other firms working in the same sector (i.e., the other downstream firm), nor by a possible supplier (i.e., the upstream firm). Again, we consider the case of two downstream firms. Assume that, before the contract is made, each downstream firm privately observes its type $\beta^k, k \in \{H, L\}$, where, as before, $\beta^H \geq \beta^L \geq 1$. The types β^H and β^L are identically and independently distributed, with Pr $\{\beta_i = \beta^H\} = p \in [0, 1]$, the distribution being common knowledge. Let $\hat{\beta}_i$ be the reported type.

Proposition 3 *By specifying the extra-payment ρ and the transfers conditional on the reported values $\hat{\beta}_i$, there exists a contract $\hat{c}$ that induces truthful revelation. The optimal level of investment is reached.*

Proof See section A.2 in the appendix. □

The intuition on why the contract $\hat{c}$ works is the following. The three parameters $\{\rho(\hat{\beta}_i), x_0(\hat{\beta}_i), x_1(\hat{\beta}_i)\}$ can be specified such that they induce truthful revelation is given and simultaneously fulfill the participation constraints for the downstream firms as well as of the upstream firm. Hence, we need to specify a different set of parameters for each possible state - both firms reporting high, both firms reporting low, as well as the two cases when they report differently. With this set we can show that truthful revelation holds in dominant strategies: neither of the downstream firms has incentive to misreport its type, independently of being of high or low type, independently on what the other downstream firm reports.

Assuming that each firm can decide whether to participate or not *after* each downstream firm has revealed its type, we show that the ex-post participation constraints are fulfilled[6]: all agents have incentive to participate in the contract.

Different to the case of complete information, now the upstream firm still capture some of the payoff when being of high type, even in cases in which the $x's$ are specified as to maximize the upstream firm A's payoff. This can be explained by the existence of informational rents for the downstream firms: when being of low type, the participation constraint of the respective downstream is binding, while when it is of high type, the participation constraint is not binding, but the constraints on truthful reporting are. Since the x_1 transferred in the case of reporting (β^L, β^H) is already the biggest P_1 can provide (the participation constraint is binding), to keep truthful reporting a (weakly) dominant strategy, also the x_1 exchanged in case of reporting (β^H, β^H) cannot be decreased. This results in the downstream firms, when being of the high type, capturing some

[6]Since ex-post are the strongest kinds of constraints, also the interim and ex-ante participation constraints are fulfilled.

of the benefit generated. A similar reasoning holds for the x's specified when the other downstream firm is of low type (i.e., for $x(\beta^L, \beta^L), x(\beta^H, \beta^L)$), and also for the x_2's received by P_2.

Again, as in the case of complete information, the payoffs for the downstream firms do not depend on the "position" in which the respective firm is placed: independently on being the first or second downstream firm, each firm receives the same amount when being of high than when being of low type.

5 More than two downstream firms

When extending the model to more than two downstream firms, several modifications of the trilateral contract come to mind. In the following, we show one possibility of a non-symmetric case with the number of downstream firms greater than two, assuming complete information. We show that the optimal level of investment can be induced.

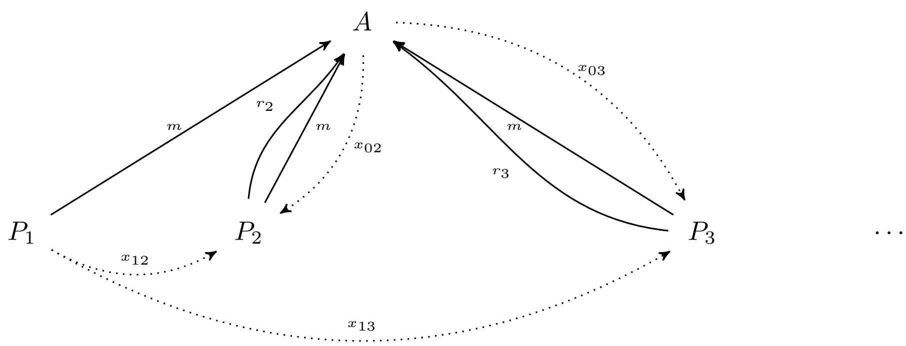

Figure 3: Multilateral contract with more than two downstream firms, non-symmetric case

The setting is very similar to the one considered in the previous sections. There are n downstream firms, each buying the product at a price equal to the market price $p_i = m$ in case the quality is high. Conditional on the first downstream firm P_1 buying, the other downstream firms P_i, $i \in \{2, \ldots, k\}$ pay a share $r_i = \beta_i \frac{\sum_{i=1}^k \beta_i m - km}{\sum_{i=1}^k \beta_i}(1 + \frac{\beta_1}{\sum_{i=2}^k \beta_i})$ to the upstream firm.

A maximizes again

$$\begin{aligned} & \max_{e|\left\{\substack{p_i = m, \forall i \in \{1, \ldots k\} \\ \rho}\right\}} U_A \\ = & \max_{e|\left\{\substack{p_i = m, \forall i \in \{1, \ldots k\} \\ \rho}\right\}} \pi(e)(\sum_{i=1}^k p_i + \sum_{i=2}^k r_i) - c(e) \\ = & \max_{e|\left\{\substack{p_i = m, \forall i \in \{1, \ldots k\} \\ \rho}\right\}} \eta e(km + \rho) - \frac{\alpha e^2}{2}, \end{aligned}$$

which results in

$$\bar{e} = \frac{\eta}{\alpha}(km + \rho), \text{ with} \rho = \sum_{i=2}^k r_i.$$

Summing over the payments r_i of the $(k-1)$ downstream firms, $\sum_{i=2}^k r_i = \rho = \sum_{i=1}^k \beta_i m - km$, it can be seen that ρ is equal to the amount required to induce the optimal level of investment. The incentive compatible level of investment is still $e^{ic} = \frac{\eta}{\alpha}(km)$. The reasoning for renegotiating and buying a product of low quality is the same as in section 2, and also the participation constraints are similar. For the first downstream firm, it is

$$\pi(\bar{e})(\beta_1 m - m) - \sum_{i=2}^{k} x_{1i} \geq \pi(e^{ic})(\beta_1 m - m).$$

For the other downstream firms $P_i, \ \forall i \in \{2, \ldots, k\}$, it is

$$\pi(\bar{e})(\beta_i m - m - r_i) + x_{0i} + x_{1i} \geq \pi(e^{ic})(\beta_i m - m).$$

Summing up over all downstream firms, this results in

$$\sum_{i=2}^{k} x_{0i} + \sum_{i=2}^{k} x_{1i} \geq \pi(e^{ic})(\sum_{i=2}^{k} \beta_i m - (k-1)m) - \pi(\bar{e})(\sum_{i=2}^{k} \beta_i m - (k-1)m - \rho).$$

The participation constraint of the upstream firm is

$$\pi(\bar{e})(km + \rho) - c(\bar{e}) - \sum_{i=2}^{k} x_{0i} \geq \pi(e^{ic})(km) - c(e^{ic}). \tag{9}$$

Taking the participation constraints of all downstream firms as binding and inserting the respective values for $\sum_{i=2}^{k} x_{0i}$ and $\sum_{i=2}^{k} x_{1i}$, equation (9) becomes $\sum_{i=1}^{k} \beta_i \geq k$. For $\beta_i \geq 1, \forall i \in \{1, \ldots, k\}$, this condition is fulfilled. The contract is implementable. □

6 Conclusion

Extending literature on the hold-up problem, we have shown that such dilemma can be solved in case there are more than two parties involved in the transaction. Making transfers conditional upon verifiable payments, our trilateral contract restores first best efficiency. This result holds both under complete and under asymmetric information, and in the latter case induces truthful revelation of types. The contract is coalition renegotiation proof and extendable to more than two downstream firms.

As specified up to now, our contract induces the first best level of investment, and satisfies the participation constraints of all agents. Yet, it is not the unique possible implementation of the contract. What is crucial is the exchange of payments *conditional* on another partner buying the product.

Since it might seem counterintuitive that even though the upstream firm has to incur a costly investment at the beginning, in addition it has to pay a transfer to the second downstream firm, it can also be shown that the contract works without monetary transfers from the upstream firm A, as well as without any transfers at all exchanged unconditional on the quality of the product. Yet, the range of valuations of the downstream firms that lead to a higher than the incentive compatible level of investment is smaller than in the model presented previously. The reason is that the unconditional transfers serve to relax the participation constraint of the downstream firm P_2. If A does not provide any transfer, the amount x_1 that P_2 receives is limited by the participation constraint of P_1. If instead there are no unconditional transfers exchanged at all, the participation constraint of P_2 is even more binding.

Depending on how the bargaining power is distributed, the surplus generated by the trilateral contract may be divided differently among the upstream firm and the downstream firms. While in the base-model we assumed the upstream firm to capture all the extra surplus, it is also possible to specify the transfers such that the surplus is divided differently.

In a similar sense, it is not necessarily obvious why the two downstream firms might have a different role in the contract. While the payments the second downstream firm receives upon signing the contract could be seen as temporary compensation for having to wait until the second period for buying the product, it can be shown that the contract also works in a symmetric setting:

also the first downstream firm may be induced to pay an extra payment to the upstream firm, conditional on the second downstream firm buying the high quality product. Also this case can easily be extended to more than two downstream firms.

While we are not aware of a contract like this being already applied in the real world, we consider our research as illustrating a suggestion for research collaborations and other companies mentioned in the introduction; also because, depending on the real cost-function of the investor, the gains from such a contract may be considerable, as seen in our numerical example. Since our contract is easily extendable to more than two downstream firms, it might be thought of the biggest players in the pharmaceutical industry to jointly invest into base-research projects. Yet, it remains considering the case when we introduce competition among the downstream firms, to apply the setting to a broader field of applications.

A Appendix

A.1 Proof of Proposition 2

Proof In the following, we will consider the possible deviations in turn.

a) Coalition $\{A, P_1\}$

Suppose first that the good is of high quality. A and P_1 cannot benefit when P_1 reports falsely low quality, since this reduces the amount the upstream firm A receives by ρ: hence, even with specifying any exchange of payments among the two firms, at least one of the two is strictly worse of, when misreporting than when telling the truth.

Now assume that good being of low quality. In this case, A and P_1 may benefit when P_1 reports falsely the good being of high quality. While P_1 then pays $p_1 = m$ for the product that has a value of 0 for it, A receives $\rho = (\beta_1 + \beta_2 - 2)m$ from P_2. In case $\beta_1 m + \beta_2 m \geq 3m$, A may promise P_1 a payment of $\epsilon_{01} \in (m, \rho)$ for claiming the good to be of high quality, when instead it is low quality. This payment ϵ_{01} may be agreed upon after the quality is realized[7]. ϵ_{01} may be exchanged either before or after the downstream firm P_1 claims high quality. If A and P_1 agree upon exchanging it *after* P_1 has claimed high quality, since the payment is not legally enforceable, A is strictly better of by not paying ϵ_{01}, since

$$p_1 + \rho > p_1 + \rho - \epsilon_{01}.$$

Hence, P_1 wants to receive ϵ_{01} *before* claiming high quality. However, once P_1 has received ϵ_{01}, P_1 is strictly better of by not respecting the side-agreement with A, since

$$\epsilon_{01} - p_1 < \epsilon_{01}.$$

Therefore, this side-agreement is not deviation-proof, and hence not self-enforcing. It remains to check if the downstream firms have incentive to jointly deviating.

b) Coalition of $\{P_2, P_1\}$

Assume first the good is of low quality. Hence, the good has a value of 0 for each of the two downstream firms, which is the realized profit when reporting truthfully the quality of the product. Therefore, they do not have incentive to jointly claim high quality, as they had to pay in any case an amount greater zero for receiving the good, and hence at least one of the two firms is strictly worse of compared to reporting truthfully[8].

Now suppose the product is of high quality. When telling the truth, the downstream firms pay $(2m + \rho)$ for two high quality products. If instead they deviate by agreeing upon the fact that P_1 claims low quality in case quality is high, they only pay $2m$ to the upstream firm, making a "profit" of ρ. ρ can be shared in a way such that both parties are strictly better of, specifying shares $\{\epsilon_{21}, \rho - \epsilon_{21}\}$ for P_1 and P_2 respectively, with $\epsilon_{21} \in (0, \rho)$. Again, ϵ_{21} may be exchanged before or after P_1 claims low quality. If ϵ_{21} is exchanged *after* P_1 has claimed low quality, since the side-agreement is not legally enforceable, P_2 is strictly better of by deviating and not paying ϵ_{21}, since

$$\beta_2 m - p_2 + m_c > \beta_2 m - p_2 + m_c - \epsilon_{21}.$$

[7]Before the quality is realized, the upstream firm prefers to not agree upon such a side-payment, since it strictly gains less when the quality is indeed high. In any case, the following reasoning also holds if the payment is agreed upon before the quality is realized.

[8]The two downstream firms can both claim the good to be high quality, in which case the first firm pays $-p_1 < 0$ and the second firm pays overall $-\rho - p_2 < 0$. Alternatively, they could agree upon P_1 claiming low and P_2 claiming high quality, in which case P_2 is strictly worse of $(-p_2 < 0)$ and P_1 is indifferent; or they can agree upon P_1 claiming high and P_2 claiming low quality, in which both P_1 and P_2 are worse of, with $-p_1 < 0$ and $-\rho < 0$ respectively

Now assume ϵ_{21} to be exchanged *before* P_1 reports low quality[9]. Recall that m_c is a payment specified as to be paid from P_1 to P_2 in case P_2 reports high quality *after* P_1 *has reported low quality*. Then, if P_1 has claimed the good to be of low quality, given that the quality is high, P_2 strictly prefers to claim high quality:

$$\beta_2 m - p_1 - \epsilon_{21} + m_c > \beta_2 m - m - \epsilon_{21}.$$

Hence, P_1 knows that reporting truthfully high quality, makes him better of, since

$$\beta_1 m - p_2 + \epsilon_{21} > \beta_1 m - m + \epsilon_{21} - m_c.$$

Therefore, also this deviation is not self-enforcing. It follows that our contract is coalition deviation proof.□

A.2 Proof of Proposition 3

Proof Incentive compatibility in (weakly) dominant strategies requires that there exists a strategy $\hat{\beta}_i = \beta_i^k$, $\forall\, i \in \{1, 2\}$, such that

$$U_i(\hat{\beta}_i, \hat{\beta}_{-i}|\beta_i) \ \geq U_i(\hat{\beta}'_i, \beta_{-i}|\beta_i)\ ,\ \forall \hat{\beta}_i \text{ and all } \hat{\beta}'_i. \quad (10)$$

To find an equilibrium in dominant strategies, we need to specify the three parameters $\{\rho(\hat{\beta}_i), x_0(\hat{\beta}_i), x_1(\hat{\beta}_i)\}$ such that condition (10) is fulfilled. In addition, we want to satisfy the participation constraints of the downstream firms (equations (25) - (32)) and of the upstream firm (equations (33) - (36)). Define $x_2(\hat{\beta}_i) \equiv x_0(\hat{\beta}_i) + x_1(\hat{\beta}_i)$. In addition, to simplify notation, we write in superscript the respectively reported type of each downstream firm (e^{12} instead of $e(\hat{\beta}_1, \hat{\beta}_2)$). Specify the x's as follows[10]:

$$\begin{aligned}
\tilde{x}_1(\hat{\beta}_1^H, \hat{\beta}_2^H) &= \pi(e^{HH})(\beta^H m - m) - \pi(e^{LH})(\beta^H m - \beta^L m) - \pi(e^{ic})(\beta^L m - m), && (11)\\
\tilde{x}_2(\hat{\beta}_1^H, \hat{\beta}_2^H) &= \pi(e^{ic})(\beta^L m - m) - \pi(e^{HH})(\beta^H m - m - \rho^{HH}) + \pi(e^{HL})(\beta^H m - \beta^L m), && (12)\\
\tilde{x}_1(\hat{\beta}_1^L, \hat{\beta}_2^H) &= [\pi(e^{LH}) - \pi(e^{ic})](\beta^L m - m), && (13)\\
\tilde{x}_2(\hat{\beta}_1^L, \hat{\beta}_2^H) &= \pi(e^{ic})(\beta^L m - m) - \pi(e^{LH})(\beta^H m - m - \rho^{LH}) + \pi(e^{LL})(\beta^H m - \beta^L m), && (14)\\
\tilde{x}_1(\hat{\beta}_1^H, \hat{\beta}_2^L) &= \pi(e^{HL})(\beta^H m - m) - \pi(e^{LL})(\beta^H m - \beta^L m) - \pi(e^{ic})(\beta^L m - m), && (15)\\
\tilde{x}_2(\hat{\beta}_1^H, \hat{\beta}_2^L) &= \pi(e^{ic})(\beta^L m - m) - \pi(e^{HL})(\beta^L m - m - \rho^{HL}), && (16)\\
\tilde{x}_1(\hat{\beta}_1^L, \hat{\beta}_2^L) &= [\pi(e^{LL}) - \pi(e^{ic})](\beta^L m - m), && (17)\\
\tilde{x}_2(\hat{\beta}_1^L, \hat{\beta}_2^L) &= \pi(e^{ic})(\beta^L m - m) - \pi(e^{LL})(\beta^L m - m - \rho^{LL}). && (18)
\end{aligned}$$

First, we show that truthful revelation is given. Thereafter we show that the participation constraints of the downstream firms are satisfied, followed by demonstrating that also the participation constraint of the upstream firm is fulfilled.

1. Truthful Revelation

The payoffs of the downstream firms are, as before, respectively:

$$\begin{aligned}
U_1(\hat{\beta}_1, \hat{\beta}_1|\beta_1) &= \pi(\tilde{e}(\hat{\beta}_1, \hat{\beta}_2))\,[\beta_1 m - p_1] - x_1(\hat{\beta}_1, \hat{\beta}_2), && (19)\\
U_2(\hat{\beta}_2, \hat{\beta}_1|\beta_2) &= \pi(\tilde{e}(\hat{\beta}_1, \hat{\beta}_2))\left[\beta_2 m - p_2 - \rho(\hat{\beta}_1, \hat{\beta}_2)\right] + x_0(\hat{\beta}_1, \hat{\beta}_2) + x_1(\hat{\beta}_1, \hat{\beta}_2). && (20)
\end{aligned}$$

[9]Again, the same reasoning holds as with the previous coalition w.r.t. specifying the exchanged payment before or after quality is realized: P_2 prefers to specify the payment *after* quality is realized, since this way he is strictly better of in case the good is of low quality. However, the following reasoning also holds in case they specify ϵ_{21} to be exchanged before quality is observed.

[10]The $x's$ are chosen according to the following strategy: to minimize the amount A has to provide, we choose x_1 such that P_1 pays the biggest amount possible satisfying its incentive compatibility and participation constraints. Similarly, x_2 is chosen such that P_2 receives the smallest amount possible such that its incentive compatibility and participation constraints are fulfilled.

Replace $p_1 = p_2 = m$, and consider first the downstream firm P_1.

a) Downstream firm P_1

We show that it has no incentive to misreport its type, independently of being of high or low type, and independently on what the second downstream firm P_2 reports.

Suppose P_2 reports being of type β^H, and suppose P_1 *is* of type β^H. Having specified the x's as above, when truthfully *reporting* being of type β^H, P_1 receives a payoff of

$$\pi(e^{LH})\left[\beta^H m - \beta^L m\right] + \pi(e^{ic})\left[\beta^L m - m\right];$$

when reporting to be of type β^L, it receives

$$\pi(e^{LH})\left[\beta^H m - \beta^L m\right] + \pi(e^{ic})\left[\beta^L m - m\right].$$

As can be seen, the two payoffs are equal: reporting the true type weakly dominates non-truthful reporting. Now suppose the dowstream firm P_1 is of type β^L, while the downstream firm P_2 still reports being of high type. When P_1 reports to be of type β^H, it gets

$$\left[\pi(e^{LH}) - \pi(e^{HH})\right]\left[\beta^H m - \beta^L m\right] + \pi(e^{ic})\left[\beta^L m - m\right], \tag{21}$$

while when truthfully reporting β^L, it receives

$$\pi(e^{ic})\left[\beta^L m - m\right]. \tag{22}$$

x_0 is exchanged *before* the level of investment is incurred, hence, it can be shown that the payoff in (21) is smaller than the payoff in (22), since

$$\left[\pi(e^{LH}) - \pi(e^{HH})\right]\left[\beta^H m - \beta^L m\right] \leq 0;$$

A maximizes again

$$\begin{aligned}
&\max_{e|\left\{\substack{p_1 = m + \rho(\hat{\beta}_1, \hat{\beta}_2),\\ p_2 = m}\right\}} U_A \\
= &\max_{e|\left\{\substack{p_1 = m + \rho(\hat{\beta}_1, \hat{\beta}_2),\\ p_2 = m}\right\}} \pi(e(\hat{\beta}_1, \hat{\beta}_2))[\sum_{i=1}^{2} p_i + \rho(\hat{\beta}_1, \hat{\beta}_2)] - c(e(\hat{\beta}_1, \hat{\beta}_2)) \\
= &\max_{e|\left\{\substack{p_1 = m + \rho(\hat{\beta}_1, \hat{\beta}_2),\\ p_2 = m}\right\}} \eta e(\hat{\beta}_1, \hat{\beta}_2)[2m + \rho(\hat{\beta}_1, \hat{\beta}_2)] - \frac{\alpha e(\hat{\beta}_1, \hat{\beta}_2)^2}{2}.
\end{aligned}$$

This results in $e(\hat{\beta}_1, \hat{\beta}_2) = \frac{\eta}{\alpha}(2m + \rho(\hat{\beta}_1, \hat{\beta}_2))$. Set $\rho(\hat{\beta}_1, \hat{\beta}_2) = (\hat{\beta}_1 + \hat{\beta}_2 - 2)m$, to induce again the optimal level of investment. Then, $e(\hat{\beta}_1, \hat{\beta}_2) = \frac{\eta}{\alpha}[\hat{\beta}_1 m + \hat{\beta}_2 m]$, and therefore, for $\beta^H \geq \beta^L \geq 1$, $e^{HH} \geq e^{HL}$. Hence, $\left[\pi(e^{LH}) - \pi(e^{HH})\right]\left[\beta^H m - \beta^L m\right] \leq 0$.
Now suppose the downstream firm P_2 reports being of type β^L. Suppose P_1 is of type β^H. When reporting being of type β^H, it receives a payoff of

$$\pi(e^{LL})\left[\beta^H m - \beta^L m\right] + \pi(e^{ic})\left[\beta^L m - m\right],$$

while when reporting being of type β^L, it receives

$$\pi(e^{LL})\left[\beta^H m - \beta^L m\right] + \pi(e^{ic})\left[\beta^L m - m\right].$$

Again, the two payoffs are equal; again, reporting the true type weakly dominates non-truthfully reporting. Now suppose P_1 is of type β^L, with P_2 still reporting being of type β^L. When reporting being of type β^H, P_1 receives a payoff of

$$\left[\pi(e^{LL}) - \pi(e^{HL})\right]\left[\beta^H m - \beta^L m\right] + \pi(e^{ic})\left[\beta^L m - m\right],$$

while when truthfully reporting β^L, it receives

$$\pi(e^{ic})\left[\beta^L m - m\right].$$

Following the reasoning above, since $e^{HH} \geq e^{LH}$, reporting the truth dominates non-truthful reporting. Since, in expected terms, final payoffs of the downstream firms are equal - given that they are of the same type - a symmetric reasoning holds for the truthful reporting of P_2.

b) Downstream firm P_2

Suppose the first downstream firm P_1 reports being of type β^H. Suppose P_2 *is* of type β^H. When *reporting* being of type β^H, it receives a payoff of

$$\pi(e^{HL})\left[\beta^H m - \beta^L m\right] + \pi(e^{ic})\left[\beta^L m - m\right],$$

while when reporting being of type β^L, it receives a payoff of

$$\pi(e^{HL})\left[\beta^H m - \beta^L m\right] + \pi(e^{ic})\left[\beta^L m - m\right].$$

Again, the two payoffs are equal; Reporting the true type weakly dominates non-truthfully reporting. Now suppose P_2 is of type β^L. When reporting being of type β^H, it receives an overall payoff of

$$\left[\pi(e^{HL}) - \pi(e^{HH})\right]\left[\beta^H m - \beta^L m\right] + \pi(e^{ic})\left[\beta^L m - m\right],$$

while when reporting being of type β^L, he receives an overall payoff of

$$\pi(e^{ic})\left[\beta^L m - m\right].$$

Following the reasoning for the downstream firm P_1, reporting the truth dominates non-truthful reporting.

Now suppose P_1 reports being of type β^L. Suppose P_2 is of type β^H. When reporting type β^H, it receives a payoff of

$$\pi(e^{LL})\left[\beta^H m - \beta^L m\right] + \pi(e^{ic})\left[\beta^L m - m\right],$$

while when reporting type β^L, it receives a payoff of

$$\pi(e^{LL})\left[\beta^H m - \beta^L m\right] + \pi(e^{ic})\left[\beta^L m - m\right].$$

Again, the two payoffs are equal; Reporting the true type weakly dominates non-truthfully reporting. Now suppose P_2 is of type β^L. When reporting type β^H, it receives a payoff of

$$\left[\pi(e^{LL}) - \pi(e^{HL})\right]\left[\beta^H m - \beta^L m\right] + \pi(e^{ic})\left[\beta^L m - m\right],$$

while when reporting β^L, it receives a payoff of

$$\pi(e^{ic})\left[\beta^L m - m\right].$$

Following the reasoning for the downstream firm P_1, again reporting the truth dominates non-truthful reporting.

So, with the $x's$ specified as in (11) - (18), the downstream firms have incentive to truthfully reveal their types. But do they also want to join the trilateral contract? In the following, we show that the participation constraints are fulfilled.

2. Participation Constraints

a) Participation Constraints of Downstream Firms

The participation constraints of P_1 and P_2 are, as before, respectively:

$$\begin{aligned}\pi(\tilde{e}(\hat{\beta}_1,\hat{\beta}_2))(\beta_1 m - p_1) - x_1(\hat{\beta}_1,\hat{\beta}_2) &\geq \pi(e^{ic})(\beta_1 m - m),\\ \pi(\tilde{e}(\hat{\beta}_1,\hat{\beta}_2))[\beta_2 m - p_2 - \rho(\hat{\beta}_1,\hat{\beta}_2)] + x_2(\hat{\beta}_1,\hat{\beta}_2) &\geq \pi(e^{ic})(\beta_2 m - m),\end{aligned}$$

which is

$$x_1(\hat{\beta}_1,\hat{\beta}_2) \leq \pi(\tilde{e}(\hat{\beta}_1,\hat{\beta}_2))(\beta_1 m - m) - \pi(e^{ic})(\beta_1 m - m), \tag{23}$$

$$x_2(\hat{\beta}_1,\hat{\beta}_2) \geq \pi(e^{ic})(\beta_2 m - m) - \pi(\tilde{e}(\hat{\beta}_1,\hat{\beta}_2))[\beta_2 m - m - \rho(\hat{\beta}_1,\hat{\beta}_2)]. \tag{24}$$

Assuming truthful reporting, for the respective values of $\hat{\beta}_i$ and β_i, 23 and 24 become

$$x_1(\hat{\beta}_1^H,\hat{\beta}_2^H) \leq [\pi(e^{HH}) - \pi(e^{ic})](\beta^H m - m), \tag{25}$$

$$x_2(\hat{\beta}_1^H,\hat{\beta}_2^H) \geq \pi(e^{ic})(\beta^H m - m) - \pi(e^{HH})(\beta^H m - m - \rho^{HH}), \tag{26}$$

$$x_1(\hat{\beta}_1^L,\hat{\beta}_2^H) \leq [\pi(e^{LH}) - \pi(e^{ic})](\beta^L m - m), \tag{27}$$

$$x_2(\hat{\beta}_1^L,\hat{\beta}_2^H) \geq \pi(e^{ic})(\beta^H m - m) - \pi(e^{LH})(\beta^H m - m - \rho^{LH}), \tag{28}$$

$$x_1(\hat{\beta}_1^H,\hat{\beta}_2^L) \leq [\pi(e^{HL}) - \pi(e^{ic})](\beta^H m - m), \tag{29}$$

$$x_2(\hat{\beta}_1^H,\hat{\beta}_2^L) \geq \pi(e^{ic})(\beta^L m - m) - \pi(e^{HL})(\beta^L m - m - \rho^{HL}), \tag{30}$$

$$x_1(\hat{\beta}_1^L,\hat{\beta}_2^L) \leq [\pi(e^{LL}) - \pi(e^{ic})](\beta^L m - m), \tag{31}$$

$$x_2(\hat{\beta}_1^L,\hat{\beta}_2^L) \geq \pi(e^{ic})(\beta^L m - m) - \pi(e^{LL})(\beta^L m - m - \rho^{LL}). \tag{32}$$

Replacing $\tilde{x}_1(\hat{\beta}_1,\hat{\beta}_2)$ and $\tilde{x}_2(\hat{\beta}_1,\hat{\beta}_2)$ into the participation constraints (25) - (32), it can be seen that equations (27), (30), (31), and (32) are binding. Equations (25), (26), (28), and (29) can be simplified to, respectively

$$\begin{aligned}(\pi(e^{LH}) - \pi(e^{ic}))(\beta^H m - \beta^L m) &\geq 0,\\ (\pi(e^{HL}) - \pi(e^{ic}))(\beta^H m - \beta^L m) &\geq 0,\\ (\pi(e^{LL}) - \pi(e^{ic}))(\beta^H m - \beta^L m) &\geq 0,\\ (\pi(e^{LL}) - \pi(e^{ic}))(\beta^H m - \beta^L m) &\geq 0,\end{aligned}$$

which are all clearly satisfied for $\beta^H \geq \beta^L \geq 1$. So, the specified $\tilde{x}$'s also satisfy the participation constraints of P_1 and P_2 .

It remains to check if A can provide $x_0(\hat{\beta}_1,\hat{\beta}_2) = x_2(\hat{\beta}_1,\hat{\beta}_2) - x_1(\hat{\beta}_1,\hat{\beta}_2)$, which is, in each case:

$$\begin{aligned}\tilde{x}_0(\hat{\beta}_1^H,\hat{\beta}_2^H) &= \pi(e^{ic})(2\beta^L - 2)m\\ &\quad -\pi(e^{HH})(2\beta^H m - 2m - \rho^{HH}) + [\pi(e^{HL}) + \pi(e^{LH})](\beta^H - \beta^L)m\\ \tilde{x}_0(\hat{\beta}_1^L,\hat{\beta}_2^H) &= \pi(e^{ic})(2\beta^L - 2)m\\ &\quad -\pi(e^{LH})(\beta^H m + \beta^L m - 2m - \rho^{LH}) + \pi(e^{LL})(\beta^H - \beta^L)m\\ \tilde{x}_0(\hat{\beta}_1^H,\hat{\beta}_2^L) &= \pi(e^{ic})(2\beta^L - 2)m\\ &\quad -\pi(e^{HL})(\beta^H m + \beta^L m - 2m - \rho^{HL}) + \pi(e^{LL})(\beta^H - \beta^L)m\\ \tilde{x}_0(\hat{\beta}_1^L,\hat{\beta}_2^L) &= \pi(e^{ic})(2\beta^L - 2)m\\ &\quad -\pi(e^{LL})(2\beta^L m - 2m - \rho^{LL}).\end{aligned}$$

b) Participation Constraint of Upstream Firm

The ex-post participation constraint of A is

$$\pi(e(\hat{\beta}_1,\hat{\beta}_2))[2m + \rho(\hat{\beta}_1,\hat{\beta}_2)] - c(e(\hat{\beta}_1,\hat{\beta}_2)) - x_0(\hat{\beta}_1,\hat{\beta}_2) \geq \pi(e^{ic})(2m) - c(e^{ic}),$$

which, for each case, results in:

$$\begin{aligned}
x_0(\hat{\beta}_1^H, \hat{\beta}_2^H) &\leq \pi(e^{HH})(2m + \rho^{HH}) - \pi(e^{ic})(2m) - c(e^{HH}) + c(e^{ic}) && (33)\\
x_0(\hat{\beta}_1^L, \hat{\beta}_2^H) &\leq \pi(e^{LH})(2m + \rho^{LH}) - \pi(e^{ic})(2m) - c(e^{LH}) + c(e^{ic}) && (34)\\
x_0(\hat{\beta}_1^H, \hat{\beta}_2^L) &\leq \pi(e^{HL})(2m + \rho^{HL}) - \pi(e^{ic})(2m) - c(e^{HL}) + c(e^{ic}) && (35)\\
x_0(\hat{\beta}_1^L, \hat{\beta}_2^L) &\leq \pi(e^{LL})(2m + \rho^{LL}) - \pi(e^{ic})(2m) - c(e^{LL}) + c(e^{ic}) && (36)
\end{aligned}$$

Inserting the respective values for $\rho(\cdot)$, $e(\cdot)$, $\pi(\cdot)$, $c(\cdot)$, and $\tilde{x}_0(\cdot)$, equations (33)- (36) become

$$\begin{aligned}
2(\beta^L)^2 - 4\beta^L + 2 &\geq 0,\\
\frac{1}{2}(\beta^H)^2 + \frac{5}{2}(\beta^L)^2 - \beta^H\beta^L - 4\beta^L + 2 &\geq 0,\\
\frac{1}{2}(\beta^H)^2 + \frac{5}{2}(\beta^L)^2 - \beta^H\beta^L - 4\beta^L + 2 &\geq 0,\\
2(\beta^L)^2 - 4\beta^L + 2 &\geq 0.
\end{aligned}$$

It can be checked that all four equations hold for $\beta^H \geq \beta^L \geq 1$. So we have shown that there exist $\tilde{x}$'s that induce truthful revelation and fulfill the participation constraints of each firm. □

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