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MODELING AND FORECASTING REALIZED RANGE VOLATILITY

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Abstract In this paper, we estimate, model and forecast Realized Range Volatility, a new realized measure and estimator of the quadratic variation of financial prices. This estimator was early introduced in the literature and it is based on the high-low range observed at high frequency during the day. We consider the impact of the microstructure noise in high frequency data and correct our estimations, following a known procedure. Then, we model the Realized Range accounting for the well-known stylized effects present in financial data. We consider an HAR model with asymmetric effects with respect to the volatility and the return, and GARCH and GJR-GARCH specifications for the variance equation. Moreover, we also consider a non Gaussian distribution for the innovations. The analysis of the forecast performance during the different periods suggests that including the HAR components in the model improve the point forecasting accuracy while the introduction of asymmetric effects only leads to minor improvements.

Key words: Statistical analysis of financial data, Econometrics, Forecasting methods, Time series analysis, Realized Range Volatility, Realized Volatility, Long-memory, Volatility forecasting.

JEL codes: C22, C52, C53, C58

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1 Introduction

In the last years, realized volatility measures, constructed from high frequency financial data and modeled with standard time series techniques, have shown to perform much better than traditional generalized autoregressive conditional heteroskedasticity (GARCH) and stochastic volatility models, when forecasting conditional second order moments. Most of the works that forecast volatility through realized measure, have concentrated on the *Realized Variance* (*RV*) introduced by Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002). The *RV* is based on the continuous time price theory and it is defined as a function of the sum of squared intraday returns. Basically, the *RV* is a highly efficient and unbiased estimator of the quadratic variation and converges to it when the intraday period goes to zero. Later on, Martens and van Dijk (2007) and Christensen and Podolskij (2007) introduced the *Realized Range Volatility* (*RRV*), another realized estimator consistent for the quadratic variation. The *RRV* is based on the difference between the minimum and maximum prices observed during a certain time interval. This new estimator tries to exploit the higher efficiency of the range relatively to that of the squared daily close-to-close return in the estimation of quadratic variation.

When dealing with high frequency financial market data, the asymptotic properties of the simple estimators are highly affected by the microstructure noise (non continuous trading, infrequent trade, bid ask bounce). As a result, an important part of the literature has presented different corrections to restore the efficiency of realized estimators for the volatility. These studies aimed at improving over the first generation of models, whose purpose was to construct estimates of realized variances by using series at a moderate frequency (see Andersen et al. (2003)). Some of the corrections presented to the *RV* are the Two Time Scale Estimator (TTSE), the sub-sampling method of Zhang et al. (2005), the generalization introduced by Zhang (2006). We also mention the approach for identifying the optimal sampling frequency by Bandi and Russell (2008), through a minimization of the MSE. Furthermore, kernel estimation was introduced by Hansen and Lunde (2006), while Barndorff-Nielsen et al. (2008) provide a generalization of this approach. Differently, Martens and van Dijk (2007) proposed a correction for the *RRV* based on scaling the range with the daily range and Christensen et al. (2009) presented another approach based on an adjustment by a constant which has to be estimated by simulation methods.

With the availability of new observable series for the volatility, many authors have applied traditional discrete time series models for their forecast (and implicitly for the forecast of returns volatility). Financial data are characterized by a series of well-known stylized facts. Being able to capture them, will result in a more accurate prevision of our variable of interest. These stylized facts are also observable over realized variance series and require appropriate modeling strategies. The presence of long-memory in volatility, documented in several studies, has been modeled through different specification: Andersen et al. (2003) introduced an ARFIMA model, and their forecasts for the RV generally dominate those obtained through GARCH models; Corsi (2009) presented the Heterogenous autoregressive (HAR) model, that reproduces the hyperbolic decay of the autocorrelation function by including the sums of *RV* over different horizons in order to capture the time strategies of the agents in the market. The second model has the advantage to be much simpler to estimate. Additionally, asymmetric, leverage effects, and fat tails should also be taken into account. Martens et al. (2009) specified a flexible unrestricted high-order AR model. They also considered leverage effects, days of the week effects and macroeconomics announcement. Differently, Corsi et al. (2008) presented a HAR model and they introduced two important extensions specifying a GARCH component modeling the volatility of volatility and assuming non Gaussian errors. Their results suggested an improvement in the accuracy in the point forecasting and a better density forecast.

In this work, we model and forecast volatility through the *Realized Range Volatility*. Our main objective is to study the prediction performance of the range as a proxy of the volatility. An accurate forecast of financial variability should have important implication in asset and derivative pricing, asset allocation, and risk management. Moreover, we try to fill a gap in the literature comparing the performance of the realized range with the more common realized volatility. In the first part of this paper we construct and analyze the realized range series, correct it from the microstructure noise following Martens and van Dijk (2007). In the second part, we implement time series techniques to model and capture the stylized facts within the volatility equation to gain in forecasting accuracy. In details, we consider an HAR model, we introduce leverage effects with respect to the return and the volatility, and a GARCH a GJR-GARCH specification for the volatility of volatility. Furthermore, in order to capture the statistical feature of the residuals of our model, we also consider a Normal Inverse Gaussian (NIG) distribution. The remainder of this paper is structured as follows. In section 2, we present the data and the correction procedure. In section 3, we present the model and we discuss the results for the estimation and forecast in section 4. Finally, section 5 presents the results and futures steps.

2 Data and correction procedure

Under the assumption that there are no market frictions and there is continuous trading, the *RRV* is five time more efficient than *RV*. In the reality, there are evidences against these assumptions and realized estimator became inconsistent and unbiased. Hence, a corrected version for the *RRV* should restore the efficiency of this estimator over the *RV*. In this paper, we follow Martens and van Dijk (2007) that proposed a correction based on scaling the range with the daily range. Basically, the scaling bias correction is not difficult to implement it does not require the availability of tick by tick data. The idea of Martens and van Dijk (2007) is based on the fact that the *daily range* is almost not contaminated by market frictions. The simulation results of Martens and van Dijk (2007) confirm the theory that the range is more efficient than the RV and in the presence of market frictions the scaling correction removes the bias and restores the efficiency of the Realized Range estimator over the Realized Volatility. The *RRV* is defined as

$$RRV_t^{\Delta} = \frac{1}{\lambda^2} \sum_{i=1}^{n} (ln\ p_{t,\Delta}^{hg} - ln\ p_{t,\Delta}^{lo})^2$$

where $p_{t,\Delta}^{hg}$ and $p_{t,\Delta}^{lo}$ are the high and low prices for day t, λ is a scaling factor and Δ indicates the sampling frequency. Therefore, the *scaled RRV* is defined as:

$$RRV_{scaled,t}^{\Delta} = \left(\frac{\sum_{l=1}^{q} RRV_{t-l}}{\sum_{l=1}^{q} RRV_{t-l}^{\Delta}}\right) RRV_t^{\Delta}$$

where q is the number of previous trading days used to compute the scaling factor. If the trading intensity and the spread do not change, q must be set as large as possible. However, in the reality only recent history should be taking into consideration.

Our database consists in more than seven year of 1 minute high, low, open and close prices for 16 stocks quoted on the NYSE. Because of space limitation, we concentrate on the analysis and present the detailed results for Procter & Gamble Company. However, similar conclusions emerge from the other series. The original sample covers the period from January 2, 2003 to March 30, 2010, from 09:30 trough 16:00 and a total of 1887 trading days. We constructed the series for the range for the one, five, thirty minutes and daily sampling frequency. We correct them on one, two and three previous months (i.e. 22, 44 or 66 days). The results of the corrections show that, after scaling, the volatility stabilized across the different sampling frequencies and scaled factors. Finally, we choose to sample every five minutes and to correct with the 66 previous days, the same election of the authors. A statistical analysis of the return and volatility series confirms the presence of the stylized facts vastly documented in the literature. The distribution of the returns exhibit excess of kurtosis while the square returns presents a slow decay in the autocorrelation function. The long-memory pattern in the hyperbolic and slowly decay of the ACF is much more pronounced for the *RRV* series.

3 Models for the observed volatility sequences

Different models have been presented to capture the stylized facts that financial series exhibit. Based on the statistical features briefly mentioned before, we consider the HAR model of Corsi (2009) to capture the long-memory pattern. We account for asymmetric effects with respect to the volatility and the returns. Moreover, following Corsi et al. (2008) we also include a GARCH specification to account for heteroskedasticity in observed volatility sequences and a standardized Normal Inverse

Gaussian (NIG) distribution to deal with the observed skewness of the residuals. Finally, to account for asymmetric effects in the variance equation or Volatility of the Volatility we consider a GJR specification.

We thus estimate the following model:

$$
\begin{gathered}
h_t = \alpha + \delta_s I_s(h_{t-1})h_{t-1} + \beta_d h_{t-1} + \beta_w h_{(t-1:t-5)} + \beta_m h_{(t-1:t-22)} + \\
+\gamma_R R_{t-1} + \gamma_{IR} I(R_{t-1})R_{t-1} + \sqrt{\sigma_t}\varepsilon_t \\
\sigma_t = \omega + \beta_1 \sigma_{t-1} + \alpha_1 u_{t-1}^2 + \phi_1 u_{t-1}^2 I(u_{t-1}) \\
\varepsilon_t | \Omega_{t-1} \sim d(0,1)
\end{gathered}
$$

where h_t is the $log\ RRV_{scaled,t}$, $h_{(t-1:t-j)}$ is the HAR component defined as

$$
h_{(t-1:t-j)} = \frac{1}{j}\sum_{k=1}^{j} h_{t-k}
$$

with $j = 5$ and 22 in order to capture the weekly and monthly component. $I_s(h_{t-1})$ is an indicator for $RRV_{scaled,t-1}$ bigger than the mean over $s = 5,10,22,44$ and 66 previous days and the unconditional mean (um) up to $t-1$.These variables capture the asymmetric effects with respect to the volatility. $R = ln(p_t^{cl}/p_{t-1}^{cl})$ is the return, with p^{cl} the closure price for the day t and $I(R_{t-1})$ is an indicator for negative returns in $t-1$, that captures the asymmetric effects with regard to the lagged return. $u_t = \sqrt{\sigma_t}\varepsilon_t$ is the error term. The full specification for σ_t is a GJR-GARCH to account for the asymmetric effect in the volatility of the volatility, where $I(u_{t-1})$ is an indicator for $u_{t-1} < 0$. Finally, we have 21 model specifications for the mean equation, three variance equations and two distributions for the residuals. In total, 126 models are considered. The estimation and forecast analysis is carried out for different horizons.

4 Estimation and forecast results

Firstly, we estimate the model for the entire sample from January 2003 to March 2010. The aim is to assess the impact and significance of our different variables in our models. Secondly, we compute one-day-ahead out-of-sample rolling forecast from January 3, 2006 to March 30, 2010 for a total of 1067 periods. We have estimated the models until December 30, 2005 and then, we re-estimate each model at each recursion. To evaluate the performance, we compute the Root mean square error (*RMSE*) and the Mean absolute error (*MAE*). We compare the different performances of the models with the Diebold Mariano Test based on the *RMSE*, on the *MAE* and on the *Qlike*, that is a *robust* loss function introduced by Patton (2008). Besides, we consider the Model Confidence Set approach of Hansen et al. (2010)

Table 1 Estimation results for the 2003-2010

	Normal dist.						NIG dist.					
	Constant var.			GARCH			Constant var.			GJR		
	II	VII	XIV	II	VII	XIV	II	VII	XIV	II	VII	XIV
α	-0.281 ***	-0.492 ***	-0.465 ***	-0.343 ***	-0.490 ***	-0.478 ***	-0.299 ***	-0.501 ***	-0.430 ***	-0.264 ***	-0.447 ***	-0.390 ***
	(0.070)	(0.091)	(0.079)	(0.087)	(0.106)	(0.090)	(0.069)	(0.091)	(0.075)	(0.076)	(0.096)	(0.078)
β_d	0.363 ***	0.329 ***	0.281 ***	0.350 ***	0.327 ***	0.274 ***	0.342 ***	0.313 ***	0.284 ***	0.342 ***	0.318 ***	0.291 ***
	(0.024)	(0.026)	(0.031)	(0.031)	(0.033)	(0.041)	(0.023)	(0.026)	(0.030)	(0.028)	(0.031)	(0.039)
β_w	0.460 ***	0.469 ***	0.522 ***	0.476 ***	0.482 ***	0.538 ***	0.437 ***	0.443 ***	0.483 ***	0.459 ***	0.464 ***	0.498 ***
	(0.042)	(0.042)	(0.049)	(0.048)	(0.048)	(0.058)	(0.036)	(0.036)	(0.043)	(0.042)	(0.041)	(0.052)
β_m	0.119 ***	0.109 ***	0.114 ***	0.104 ***	0.096 **	0.101 ***	0.159 ***	0.151 ***	0.154 ***	0.144 ***	0.134 ***	0.139 ***
	(0.035)	(0.035)	(0.035)	(0.038)	(0.038)	(0.037)	(0.030)	(0.031)	(0.030)	(0.034)	(0.033)	(0.033)
δ_{full}	-	0.003	-	-	0.003	-	-	0.000	-	-	0.000	-
		(0.005)			(0.005)			(0.005)			(0.005)	
δ_5	-	-	-0.011 *	-	-	-0.010 *	-	-	-0.007	-	-	-0.006
			(0.006)			(0.006)			(0.005)			(0.005)
γ_{RT}	-	1.630	-	-	1.291	-	-	2.441 *	-	-	2.045	-
		(1.670)			(1.827)			(1.485)			(1.608)	
γ_{IRT}	-	-11.861 ***	-9.598 ***	-	-9.872 ***	-8.073 ***	-	-12.122 ***	-8.532 ***	-	-10.762 ***	-7.738 ***
		(2.472)	(1.141)		(2.792)	(1.365)		(2.449)	(1.277)		(2.597)	(1.319)
ω	0.182 ***	0.177 ***	0.177 ***	0.009 ***	0.010 ***	0.010 ***	0.179 ***	0.174 ***	0.174 ***	0.011 ***	0.014 ***	0.012 ***
	(0.004)	(0.004)	(0.004)	(0.002)	(0.003)	(0.003)	(0.008)	(0.008)	(0.008)	(0.004)	(0.005)	(0.005)
β_1	-	-	-	0.901 ***	0.897 ***	0.901 ***	-	-	-	0.893 ***	0.873 ***	0.887 ***
				(0.018)	(0.021)	(0.020)				(0.031)	(0.039)	(0.035)
α_1	-	-	-	0.047 ***	0.046 ***	0.044 ***	-	-	-	0.065 ***	0.071 ***	0.066 ***
				(0.008)	(0.009)	(0.008)				(0.018)	(0.021)	(0.019)
ϕ_1	-	-	-	-	-	-	-	-	-	-0.040 *	-0.050 *	-0.045 *
										(0.023)	(0.026)	(0.024)
α_{NIG}	-	-	-	-	-	-	1.470 ***	1.440 ***	1.449 ***	1.662 ***	1.613 ***	1.620 ***
							(0.167)	(0.160)	(0.162)	(0.198)	(0.187)	(0.188)
β_{NIG}	-	-	-	-	-	-	0.379 ***	0.329 ***	0.329 ***	0.483 ***	0.426 ***	0.425 ***
							(0.106)	(0.099)	(0.099)	(0.125)	(0.116)	(0.116)
LLF	-982.5	-960.6	-959.8	-950.0	-933.8	-933.0	-907.0	-888.8	-889.2	-881.6	-866.3	-866.6
AIC	1975.1	1937.3	1933.5	1914.0	1887.7	1883.9	1827.9	1797.5	1796.5	1783.1	1758.6	1757.3
BIC	2002.4	1980.9	1971.8	1952.2	1942.3	1933.0	1866.2	1852.1	1845.6	1837.7	1829.6	1822.8
Lj_{30}	0.328	0.467	0.399	0.518	0.634	0.583	0.198	0.362	0.320	0.312	0.430	0.405
Lj_{40}	0.577	0.752	0.704	0.743	0.864	0.836	0.423	0.664	0.628	0.585	0.735	0.715
JB t	0.001	0.001	0.001	0.001	0.001	0.001	-	-	-	-	-	-
KS t	0.000	0.000	0.000	0.000	0.000	0.000	-	-	-	-	-	-
LL t	0.001	0.001	0.001	0.001	0.001	0.001	-	-	-	-	-	-

Note: Estimation results for the whole sample from January 2003 to May 2010. In this short version, we only present some of the results. LLF is the Log-likelihood function, *AIC* is the Akaike Information Criteria and *BIC* is the Bayesian information criterion . Standard errors in bracket. LJ_{30} and LJ_{40} are the Ljung Box test for 30 and 40 lags. $JB-t$ is the Jarque-Bera test for Normality, $KS-t$ is the Kolmogorov-Smirnov and $LL-t$ is the Lilliefors test. ”*”, ”**” and ”***” indicates significance at the 10%, 5% and 1%.

based on the same three loss function[2]. Table 1 presents the result for the 2003-2010 estimation, whereas table 2 and 3 present the forecast performance evaluation.

Estimation results for the full sample period (2003-2010) suggest that HAR components are significant for the three variance specifications and the two different distributions. The asymmetric effects with respect to the return and the volatility improve the goodness of fit of the model. The first one is highly significant and it increases the volatility after a negative return. On the contrary, when considering the full specification in the mean equation, the asymmetric effects with respect to the volatility, in the different horizons, are not significant. The asymmetric ef-

[2] In this version, we only present the results based on the *Qlike* loss function. Similar results are obtained with the other two loss functions.

Table 2 Out-of-sample forecast evaluation Diebold Mariano test based on the Qlike

	Full sample															
Model	I NI Co	II NI Co	VII NI Co	XIV NI Co	I NI Gj	II NI Gj	VII NI Gj	XIV NI Gj	I NO Co	II NO Co	VII NO Co	XIV NO Co	I NO Ga	II NO Ga	VII NO Ga	XIV NO Ga
I - NI - Co	-															
II - NI - Co	2.98*	-														
VII - NI - Co	2.69*	1.65	-													
XIV - NI - Co	2.75*	1.76	-0.11	-												
I - NI - Gj	2.43*	-3.09*	-2.70*	-2.77*	-											
II - NI - Gj	2.97*	1.10	-1.63	-1.75	3.08*	-										
VII - NI - Gj	2.68*	1.68	1.67	0.85	2.69*	1.67	-									
XIV - NI - Gj	2.74*	1.78	0.55	1.44	2.77*	1.78	-0.45	-								
I - NO - Co	2.11*	-3.16*	-2.72*	-2.78*	0.98	-3.14*	-2.70*	-2.77*	-							
II - NO - Co	2.86*	1.47	-1.38	-1.42	2.95*	1.32	-1.45	-1.48	3.02*	-						
VII - NO - Co	2.67*	1.59	0.10	0.12	2.69*	1.57	-0.33	-0.11	2.73*	1.51	-					
XIV - NO - Co	2.66*	1.65	0.91	0.81	2.67*	1.64	0.24	0.50	2.68*	1.49	0.54	-				
I - NO - Ga	-1.86	-3.02*	-2.74*	-2.81*	-2.55*	-3.01*	-2.74*	-2.81*	-2.19*	-2.90*	-2.72*	-2.72*	-			
II - NO - Ga	2.97*	0.57	-1.66	-1.78	3.08*	-0.65	-1.70	-1.81	3.13*	-1.35	-1.59	-1.67	3.01*	-		
VII - NO - Ga	2.71*	1.70	0.06	0.12	2.73*	1.69	-0.95	-0.37	2.76*	1.51	-0.10	-1.00	2.76*	1.72	-	
XIV - NO - Ga	2.71*	1.67	-0.08	0.04	2.73*	1.66	-0.96	-1.48	2.73*	1.34	-0.11	-0.95	2.78*	1.69	-0.10	-
	Crisis															
Model	I NI Co	II NI Co	VII NI Co	XIV NI Co	I NI Gj	II NI Gj	VII NI Gj	XIV NI Gj	I NO Co	II NO Co	VII NO Co	XIV NO Co	I NO Ga	II NO Ga	VII NO Ga	XIV NO Ga
I - NI - Co	-															
II - NI - Co	3.68*	-														
VII - NI - Co	3.32*	2.06*	-													
XIV - NI - Co	3.37*	2.03*	-0.80	-												
I - NI - Gj	3.20*	-3.67*	-3.24*	-3.28*	-											
II - NI - Gj	3.66*	0.73	-2.09*	-2.06*	3.65*	-										
VII - NI - Gj	3.30*	2.05*	1.54	1.31	3.22*	2.08*	-									
XIV - NI - Gj	3.36*	2.02*	-0.37	1.06	3.27*	2.05*	-1.17	-								
I - NO - Co	2.66*	-3.69*	-3.21*	-3.22*	1.22	-3.66*	-3.18*	-3.21*	-							
II - NO - Co	3.40*	1.19	-1.68	-1.50	3.36*	1.10	-1.74	-1.54	3.43*	-						
VII - NO - Co	3.19*	1.69	-0.10	0.13	3.13*	1.69	-0.37	0.01	3.15*	1.71	-					
XIV - NO - Co	3.19*	1.83	0.23	0.58	3.10*	1.84	-0.26	0.41	3.07*	1.67	0.34	-				
I - NO - Ga	-2.77*	-3.84*	-3.48*	-3.54*	-3.57*	-3.83*	-3.46*	-3.53*	-2.91*	-3.55*	-3.33*	-3.35*	-			
II - NO - Ga	3.65*	0.10	-2.15*	-2.13*	3.64*	-1.12	-2.15*	-2.12*	3.64*	-1.17	-1.74	-1.89	3.82*	-		
VII - NO - Ga	3.30*	1.96*	-0.45	0.16	3.24*	2.00*	-1.19	-0.12	3.22*	1.73	-0.09	-0.74	3.46*	2.06*	-	
XIV - NO - Ga	3.31*	1.89	-0.96	-0.31	3.20*	1.92	-1.66	-1.60	3.14*	1.40	-0.18	-0.74	3.48*	1.99*	-0.29	-

Note: Forecast performance for the full out-of-sample period (1067 observation) and the financial crisis period (200 observations). Model I is a $AR(1)$ specification, II is an $AR(1)+HAR$, VII is an $AR(1)+HAR+I_{um}(h_{t-1})h_{t-1}+R_{t-1}+I(R_{t-1})R_{t-1}$, $VIII$ is an $AR(1)+HAR+R_{t-1}+I(R_{t-1})R_{t-1}$, IX is an $AR(1)+HAR+I(R_{t-1})R_{t-1}$ and XIV is an $AR(1)+HAR+I_5(h_{t-1})h_{t-1}+I(R_{t-1})R_{t-1}$. NI indicates Normal Inverse Gaussian distribution, NO is Normal distribution and Co is a constant variance specification, Ga is a $GARCH$ and Gj is a GJR variance specification. The *Diebold Mariano* is a test for equal predictive accuracy between two models based on the Qlike loss function. Under Ho, both models have the same performance. T-statistic in the table. "*" rejects Ho at the 5%. Positive T-statistic favors the column model.

fect on the previous five days is marginally significant for some models. The sign and significance of the coefficients in the mean equation remain stable for the different specifications in the variance equation. The inclusion of the GARCH and GJR specifications improve the fitting of the models. The models that best fit the series are the ones that include the HAR and leverage effects, with GARCH and GJR variances. Diagnostic tests for the residuals present different results. Only for the models that include the HAR components we cannot reject the null hypothesis of no serial correlation in the residual, implying a good performance. Normality Tests for the residuals are rejected for all the models, which is an argument to in-

Table 3 Out-of-sample forecast evaluation Model Confidence set based on the Qlike

	Full Sample				Crisis			
Model	MAE	RMSE	$Qlike_R$	$Qlike_{SQ}$	MAE	RMSE	$Qlike_R$	$Qlike_{SQ}$
I - NI - Co	0.370	0.253	0.35	0.16	0.499	0.480	0.19	0.09
II - NI - Co	0.325	0.198	0.48	0.32	0.341	0.267	0.43	0.29
VII - NI - Co	0.322	0.192	0.48	0.70	0.328	0.236	0.67	0.66
VIII - NI - Co	0.322	0.191	0.63	0.82	0.327	0.235	0.67	0.79
IX - NI - Co	0.321	0.191	0.48	0.70	0.329	0.237	0.43	0.45
XIV - NI - Co	0.321	0.191	0.62	0.71	0.328	0.237	0.67	0.65
I - NI - Gj	0.363	0.241	0.30	0.11	0.461	0.421	0.19	0.13
II - NI - Gj	0.325	0.197	0.48	0.46	0.341	0.266	0.43	0.22
VII - NI - Gj	0.321	0.191	0.72	0.92	0.326	0.232	0.98	0.99
VIII - NI - Gj	0.322	0.190	0.98	0.99	0.326	0.232	1.00	1.00
IX - NI - Gj	0.321	0.190	0.63	0.82	0.328	0.234	0.67	0.57
XIV - NI - Gj	0.321	0.190	0.72	0.92	0.327	0.234	0.67	0.66
I - NO - Co	0.362	0.240	0.28	0.10	0.458	0.414	0.16	0.07
II - NO - Co	0.323	0.194	0.48	0.58	0.332	0.252	0.43	0.35
VII - NO - Co	0.320	0.190	0.72	0.84	0.321	0.228	0.67	0.68
VIII - NO - Co	0.320	0.189	1.00	1.00	0.322	0.228	0.98	0.99
IX - NO - Co	0.320	0.189	0.97	0.99	0.323	0.229	0.67	0.71
XIV - NO - Co	0.321	0.190	0.98	0.99	0.326	0.232	0.94	0.93
I - NO - Ga	0.373	0.257	0.35	0.13	0.518	0.503	0.13	0.05
II - NO - Ga	0.325	0.197	0.48	0.38	0.344	0.267	0.43	0.19
VII - NO - Ga	0.322	0.191	0.62	0.71	0.327	0.234	0.67	0.57
VIII - NO - Ga	0.322	0.191	0.62	0.71	0.328	0.235	0.43	0.45
IX - NO - Ga	0.321	0.190	0.62	0.71	0.329	0.237	0.43	0.41
XIV -NO - Ga	0.322	0.191	0.62	0.71	0.332	0.239	0.43	0.49

Note: Forecast performance for the full out-of-sample period (1067 observation) and the financial crisis period (200 observations). Model I is a $AR(1)$ specification, II is an $AR(1)+HAR$, VII is an $AR(1)+HAR+I_{um}(h_{t-1})h_{t-1}+R_{t-1}+I(R_{t-1})R_{t-1}$, $VIII$ is an $AR(1)+HAR+R_{t-1}+I(R_{t-1})R_{t-1}$, IX is an $AR(1)+HAR+I(R_{t-1})R_{t-1}$ and XIV is an $AR(1)+HAR+I_5(h_{t-1})h_{t-1}+I(R_{t-1})R_{t-1}$. NI indicates Normal Inverse Gaussian distribution, NO is Normal distribution and Co is a constant variance specification, Ga is a $GARCH$ and Gj is a GJR variance specification. MAE is the Mean Absolute Error. $RMSE$ is the Root Mean Square Error. The $Model\ Confidence\ Set$ is a procedure to determine the "best" models from a collection of models. It recursively eliminates the models that worst perform. Based on the Qlike loss function. $Qlike_R$ and $Qlike_{SQ}$ are the p-value for the $range$ and the $semi-qadratic$ deviation method.

troduce a non Gaussian distribution. As we said, the estimated parameters of the mean equation for the models with NIG distribution are similar to the models with Normal distribution. However, the introduction of this flexible distribution results in an improvement of the fitness of the models compared to the Gaussian distribution. The estimated parameters of the NIG distribution (α_{NIG} and β_{NIG}) capture the right skewness and excess of kurtosis displayed in the residuals.

We have analyzed the results for the out-of-sample forecast in two different periods. In particular, we study the accuracy of the our models for the full sample (1067 observations) and during the financial crisis, from September 15, 2008 to July 30, 2009 (200 observation).

For the full sample forecast, the model that performs better, based on the MAE and RMSE, is the autoregressive with HAR components, lagged and asymmetry over the return, with constant variance and Normal distribution. Other models that include asymmetric effects with respect to the volatility over the five previous days and the unconditional mean perform similarly. Models with different specifications for the variance and distribution for the innovation perform as the models with constant variance. Although, GARCH and GJR improve the goodness of fitness in the estimation, they do not have impact in the forecast. The Diebold-Mariano tests suggest that models with symmetric effects with respect to the volatility and the returns perform as HAR models. For the full sample the introduction of the HAR components seem to be the most important variable. Statistically, there is no difference between the performance of models with alternative variables, variance specifications or distributions. This result is confirmed by the Model confidence set, an approach to recursively eliminate the models that worst perform. In particular, only the AR(1) models (with different variance specification and distribution) are excluded for the set of best models.

During the financial crisis, the model that perform better is the autoregressive with HAR component with lagged return and asymmetric effect over the returns and the unconditional mean volatility. The results of the Diebold Mariano Test, based on the *Qlike* loss function, display some evidence in favor of models with asymmetric effects with respect to the volatility and the returns. However, the results of the Model confidence set approach are similar to the ones of the full sample. The set of best models include the HAR component of Corsi (2009) with different distribution and variance specifications.

5 Conclusions and future steps

In this paper, we have modeled and forecasted price variation through the *Realized Range Volatility* introduced by Martens and van Dijk (2007) and Christensen and Podolskij (2007). We have estimated the series for different sampling frequencies and corrected them with the scaling procedure of Martens and van Dijk (2007). After the corrections, the volatility stabilizes across different sampling frequencies and scaling factors which suggest that the bias caused by the microstructure friction was removed, restoring the efficiency of the estimator. We have considered a model which approximates long memory, has asymmetric effects with respect to the return and the volatility in the mean equation, and includes GARCH and GJR-GARCH specifications for the variance equation (which models the volatility of the volatility). A non Gaussian distribution was also considered for the innovations.

The results suggest that the HAR model with the asymmetric effect with respect to the volatility and returns is the one that better fit the data. The analysis of the forecast performances of the different models provides similar results for the two considered periods, the full sample and the financial crisis. The introduction of asymmetric effects improves the point forecasting performance. However, following the different evaluation approaches adopted, there is no evidence to state that these models perform statistically better than the simple HAR. As we expected, models with GARCH and GJR-GARCH specifications and different distributions for the innovations do not lead to more accurate point forecasts than models with constant variance.

In our opinion, the HAR components are able to capture most of the variability during the out-of-sample prevision. Then, in order to improve this performance, the introduction of financial and macroeconomics variables should be considered.

Other future steps are the possible correction for jumps in the volatility series and an economic analysis of the performances of the models forecast.

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