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OLD TECHNOLOGY UPGRADES,  
INNOVATION, AND COMPETITION IN  
VERTICALLY DIFFERENTIATED MARKETS

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# Old Technology Upgrades, Innovation, and Competition in Vertically Differentiated Markets\*

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## Abstract

We study how the migration from an old to a new technology is affected by the access price to the old technology. We show that both the incumbent and the regulator are willing to set a very high access price to accelerate consumers' migration to the new technology. When the quality of the old technology is exogenous and the entrant dominates investment in the new technology, the old technology is completely switched off in equilibrium, whereas the old technology persists when the incumbent dominates investment. When the incumbent can decide on an endogenous upgrade of the old technology, the migration to the new technology is slowed down, and the entrant might be foreclosed.

*Keywords:* Access; Investment; Vertical differentiation; Multi-product firms.

*JEL Codes:* L1, L51, L96.

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# 1 Introduction

In network industries, infrastructure investments are necessary to maintain or improve the quality of service provided to consumers. Such investments can involve either upgrades of the old generation network or the deployment of new infrastructures, i.e., next generation networks. For example, in the telecommunications industry, operators have started to invest in high-speed fibre next generation access networks, to replace the old legacy copper networks. At the same time, some historical telecom operators are planning to introduce a new technology, called “vectoring,” which allows to upgrade copper networks to provide higher speeds for Internet access.<sup>1</sup>

As network infrastructures are expected to be a strong contributor to economic activity and growth,<sup>2</sup> a fast transition from old network technologies to new ones is a key challenge for policy makers. For example, the European Commission has set up a “Digital Agenda 2020” with very ambitious objectives for the migration of consumers from standard broadband (based on the old generation copper network) to very-high speed broadband (based on next generation fibre networks).<sup>3</sup> A relevant and important question is then which type of regulatory intervention could accelerate the transition.

The migration from an old to a new network technology can indeed be a slow process, due in particular to the large investments necessary to deploy next generation networks, and to the competition on the consumers’ side between the two technologies, when they coexist in a transitory phase. Access regulations, which oblige the owner of the legacy network to provide access to competitors at a given price, can affect both investment incentives and the competition between the old and the new technologies, and hence, shape the transition process. In this paper, we study how the terms of access to the old generation network affect the competition between the old and the new generation networks, and hence, the migration from the old to the new technology.

Aside from access obligations, we also consider two other forms of regulatory intervention: (i) committing to switch off the old generation network after the new network has been deployed, and (ii) allowing or forbidding an upgrade of the old generation network. Switching off the old network forces consumer migration to the new network, and hence, reinforces firms’ incentives to invest in

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<sup>1</sup>Fibre networks provide a higher speed than the standard DSL broadband technology. Vectoring is an engineering technique that enables traditional copper lines to achieve speeds that are close to the theoretical limits, and therefore, also close to the speed of first-generation fibre networks. For vectoring to be effective, however, all copper lines in the relevant area have to be under the control of a single provider, which implies that vectoring is not compatible with sub-loop unbundling, a wholesale service mandated by most (17) European National Regulatory Agencies.

<sup>2</sup>See Czernich et al. (2011) for empirical evidence that the diffusion of broadband has a positive impact on growth.

<sup>3</sup>The European Commission’s objective is that half of European households subscribe to a broadband offer above 100 Mbps by 2020.

next generation networks. In Australia, for example, a public company (NBN Co) is rolling out a national fibre infrastructure and has started a countdown for the switch-off of the old copper network. The European Commission also sees a switch-off of the copper network as a means of providing proper investment incentives to operators.<sup>4</sup> An upgrade of the old generation network has a priori an ambiguous effect on investment incentives. It could either slow down the transition because of the tougher competition between the old and the new technologies after the upgrade, or alternatively, spur investment because operators could be willing to escape the competition from the old technology. Therefore, it is not surprising that in most European countries regulators are still wondering whether they should authorize the deployment of the vectoring technology.<sup>5</sup>

We then investigate the following questions. Should the regulator commit to switch off the old generation network once the next generation networks are in place? Is a formal switch-off necessary to trigger the transition to the new technology? Should the regulator allow the owner of the legacy network to upgrade it?

In the stream of literature which studies the interplay between regulation and investment in network industries (see Cambini and Jiang (2009) for a recent survey), the new technology always replaces the old technology. Some papers analyze investment from an incumbent firm only, which can upgrade its old network (e.g., see Foros (2004), Kotakorpi (2006), etc.). Others focus on entrants' incentives to bypass the old network by investing in a new alternative infrastructure (e.g., see Bourreau and Doğan (2005 and 2006), Avenali et al. (2010), Klumpp and Su (2010), etc.). Finally, some authors analyze investment races where firms compete to deploy new infrastructures, which completely replace the old one (e.g., Gans (2001), Hori and Mizuno (2006), Vareda and Hoernig (2010), etc.). In all these studies, consumers cannot choose between the old to the new technology, and therefore, the migration issue is absent.

We depart from this standard set-up by building a framework where two firms, an incumbent which owns the legacy network, and an entrant that leases access to the old generation network (OGN), can offer multiple differentiated services based on the old and the new technology. The

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<sup>4</sup>For example, Neelie Kroes, the Vice-President of the European Commission declared that “the gradual switch-off of copper could reduce the cost to such a degree that new fibre investments break even in under 10 years. And thus align the interests of investors and long-term financing providers.” (See: Investing in digital networks: a bridge to Europe's future, 3 October 2011, [http://europa.eu/rapid/press-release\\_SPEECH-11-623\\_en.htm](http://europa.eu/rapid/press-release_SPEECH-11-623_en.htm)). However, to date, in Europe, there is no example of copper switch-off. One notable exception is a public-private-partnership project in Italy, where a local authority is planning to invest in a next generation fibre network and to buy the incumbent's local copper network in the relevant area in order to switch it off. This project is still pending a decision of the European Commission with regard to its compatibility with EU state-aid rules.

<sup>5</sup>The only two European regulators that have withdrawn the sub-loop unbundling obligation to allow the deployment of vectoring are those of Belgium and Ireland, but other NRAs are also considering the possibility of at least partially removing this obligation.

migration from the old technology to the new one is endogenous to consumers' decisions. It depends on firms' pricing decisions, their initial investment decisions, and the upgrade of the OGN.

We start by analyzing a baseline model, where the OGN cannot be upgraded. The incumbent and the entrant initially decide on the quality of their next generation networks (NGNs). Once investments have taken place, the access charge to the OGN is set either by the incumbent or the regulator. Finally, firms compete with vertically-differentiated multiple products.

In this setting, there are two equilibria: a leapfrogging equilibrium, where the entrant is leader in NGN investments, and a persistence-of-leadership equilibrium, where the incumbent is the leader. In the persistence-of-leadership equilibrium, the migration to the new technology is complete: whether or not access to the OGN is regulated, in equilibrium the two firms offer only NGN services, with the entrant providing the highest-quality NGN. The OGN is switched off, without any formal regulatory intervention. By contrast, in the leapfrogging equilibrium, migration is only partial. The incumbent offers both OGN and NGN services, while the entrant offers only NGN services. However, this market outcome is inefficient: welfare would increase if the migration to the NGNs were complete. Therefore, a legal and formal switch-off of the OGN, once NGNs have been deployed, would be welfare-enhancing.

When the incumbent can upgrade the quality of the OGN at the same time as firms decide on their NGN investments, two additional equilibria emerge: a leapfrogging equilibrium with upgrade, and a persistence-of-leadership equilibrium with upgrade. In the leapfrogging equilibrium with upgrade, the incumbent renounces to invest in the NGN, and upgrades its OGN, whereas the entrant offers only NGN services. If the quality achievable with the upgrade is not too high, welfare is lower than under the leapfrogging equilibrium without upgrade. In the persistence-of-leadership equilibrium with upgrade, the incumbent offers both upgraded OGN services and NGN services, whereas the entrant offers only NGN services of low quality. However, welfare is lower than under the persistence-of-leadership equilibrium without upgrade. Therefore, forbidding the incumbent to upgrade its OGN would be welfare-enhancing.

Very few papers analyze the migration from an old to a new technology in network industries. De Bjil and Peitz (2009) study the transition from the old telephony technology (PSTN) to the new one (VoIP). In their setting, the incumbent can offer both technologies, while the entrant offers only the new one. Besides, they model market segmentation in a different fashion than we do, using a two-dimensional horizontal differentiation demand model. Brito, Pereira and Vareda (2012) and Bourreau, Cambini and Doğan (2012) explore the migration from the OGN to the NGNs

in the broadband market. Brito et al. (2012) study a setting where an incumbent can invest in an NGN and give voluntarily access to it, while access to the OGN is regulated. They then analyze the entrant's technology choice. Bourreau et al. (2012) study the impact of access to the old generation network on an incumbent and an entrant's investment incentives. However, in both papers, though the two technologies can coexist, when a firm deploys an NGN, it completely replaces its OGN. Therefore, these two papers ignore the migration issue at the retail level. In contrast with this earlier literature, we study the relation between access and investment in a setting where (i) both the incumbent and the entrant invest in NGNs, (ii) the NGN does not replace the OGN, (iii) the two firms can sell both OGN and NGN services (i.e., are multiproduct firms). We also consider different forms of regulatory intervention.

Finally, our paper is also related to the literature on multiproduct competition with vertical differentiation (see, for example, Gal-Or (1983), Champsaur and Rochet (1989), De Fraja (1996), Johnson and Myatt (2003 and 2006) and Chisholm and Norman (2012)). We study competition between multiproduct firms and vertical differentiation, when the two products correspond to two successive technological generations, and a regulator aims at influencing the migration from the old to the new technology.

The rest of the paper is organized as follows. In Section 2, we present the baseline model. We study two simple benchmark cases in Section 3, and then solve for the equilibrium in Section 4. We extend our setting to account for the possibility of an endogenous OGN upgrade in Section 5. Finally, we conclude.

## 2 The Model

**Firms.** There are two firms: an incumbent firm (I), which controls an old generation network (OGN),<sup>6</sup> and a rival firm (E). At the beginning of the game, the two firms are competing for OGN services, and firm E is leasing access at the per-unit price  $a$  to the incumbent's OGN. The access price is set either by the incumbent or a regulator. We denote by  $\delta_i > 0$  the quality of the OGN's service for firm  $i = I, E$ . Because the incumbent controls the OGN, we assume that it provides a higher quality of service on the OGN than the rival firm, that is,  $\delta_I > \delta_E$ .<sup>7</sup>

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<sup>6</sup>For example, in telecoms, the OGN is the copper infrastructure that is necessary to provide broadband DSL services.

<sup>7</sup>The quality levels for the OGN can be viewed as a legacy from a market equilibrium where the NGN was absent. If only the OGN were present, the two firms would choose to differentiate their qualities as much as possible. Given that the incumbent controls the OGN, we can assume that it was the first mover for OGN services, and chose to produce the high quality. We also solved our model when the entrant has OGN services of higher quality than the

Firm I and firm E may decide to invest in a next generation network (NGN). In order to build an NGN of quality  $\mu_i$ , firm  $i = I, E$  has to incur the quadratic investment cost  $c(\mu_i) = \mu_i^2/2$ . Note that in our setting the NGN does not replace the OGN, that is, the two technologies may coexist. We also rule out access to the NGN.

Finally, we consider that firms compete in prices and normalize the marginal costs to zero. We denote by  $d_i$  and  $m_i$  the price for the OGN's service and the NGN's service, respectively, for firm  $i = I, E$ . Besides, we denote by  $\pi_i$  firm  $i$ 's profit, gross of investment costs, and by  $\Pi_i$  its net profit, that is,  $\Pi_i = \pi_i - c(\mu_i)$ .

**Consumers.** A consumer is characterized by his type  $\theta$ , which represents his marginal willingness to pay for the quality of the service. Consumers' types are distributed on the interval  $[0, \bar{\theta}]$ , with density 1. To obtain analytical solutions, we set  $\bar{\theta} = 100$ . A consumer of type  $\theta$  buys at most one unit of service in order to maximize his net utility,  $U = \theta x - p$ , where  $x$  is the quality of the chosen service,  $p$  the price, and  $\{x, p\} \in \{\{\delta_I, d_I\}, \{\delta_E, d_E\}, \{\mu_I, m_I\}, \{\mu_E, m_E\}\}$ .

**Timing.** The timing of the game is as follows:

- Stage 1: the incumbent and rival firms choose their NGN's qualities  $\mu_I$  and  $\mu_E$ , respectively.
- Stage 2: the access charge to the OGN,  $a$ , is set either by the incumbent or by a welfare-maximizing regulator.
- Stage 3: the firms compete in prices with multiple products, and profits are realized.

Since our focus is how the access price to the OGN can affect the migration to NGN services, once NGN investments have been realized, rather than how access affects NGN investments, we assume that the regulator cannot commit to an access charge prior to the firms' investment decisions.<sup>8</sup>

Finally, we make the following assumption:

**A1.**  $\delta_i < 228.29$ , for  $i = I, E$ .

Assumption A1 ensures the existence of two equilibria, one where the incumbent is leader in NGN investments, and one in which the rival firm is the leader.<sup>9</sup>

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incumbent, that is,  $\delta_E > \delta_I$ . Our results were unchanged.

<sup>8</sup>This is in line with Foros (2004) and Kotakorpi (2006), for example. Brito et al. (2010) also consider the possibility that the regulator is unable to commit to an access price.

<sup>9</sup>If A1 does not hold, the equilibrium where the incumbent is leader in NGN investments may not exist.

### 3 Two Benchmarks

We start by considering two simple benchmark situations: (i) firm I and firm E compete in OGN services only, and (ii) firm I is a monopolist, which can provide both OGN and NGN services.

#### 3.1 Competition in OGN services

As a first benchmark, we consider a situation where the two firms do not invest in NGNs, which corresponds to the standard one-way access setting in the literature. Since we assume away investments, the game has only two stages: a first stage where the access charge is set either by the incumbent or the regulator, and a second stage where firms compete for OGN services.

At the last competition stage, firm  $i = I, E$  offers a quality  $\delta_i$  to consumers, at price  $d_i$ . The type of the marginal consumer who is indifferent between subscribing to firm I's service and firm E's service is given by  $\theta_L \delta_E - d_E = \theta_L \delta_I - d_I$ , that is,  $\theta_L = (d_I - d_E)/(\delta_I - \delta_E)$ . The marginal consumer who is indifferent between buying the lowest OGN quality  $\delta_E$  and not buying at all is characterized by  $\theta_S \delta_E - d_E = 0$ , and therefore,  $\theta_S = d_E/\delta_E$ . Firm I's and firm E's profit can then be written as

$$\pi_I = d_I (\bar{\theta} - \theta_L) + a (\theta_L - \theta_S) \quad \text{and} \quad \pi_E = (d_E - a) (\theta_L - \theta_S),$$

respectively. The two firms choose their prices to maximize their profits. The equilibrium prices are<sup>10</sup>

$$d_I^* = \frac{\delta_I [200 (\delta_I - \delta_E) + 3a]}{4\delta_I - \delta_E} \quad \text{and} \quad d_E^* = \frac{100\delta_E (\delta_I - \delta_E) + a(2\delta_I + \delta_E)}{4\delta_I - \delta_E}.$$

Both prices increase with the access charge, due to the entrant's higher perceived marginal cost, and to the strategic complementarity between the firms' prices. Replacing for the equilibrium prices, the entrant's demand is  $\theta_L - \theta_S = 2\delta_I (50\delta_E - a)/((4\delta_I - \delta_E)\delta_E)$ , and it is positive if and only if  $a \leq 50\delta_E$ .

We now turn to the first stage where the access charge is set. The effect of a higher access on the entrant's profit is given by

$$\frac{d\pi_E}{da} = \underbrace{\frac{\partial \pi_E}{\partial a}}_{(-)} + \underbrace{\frac{\partial \pi_E}{\partial d_I}}_{(+)} \underbrace{\frac{\partial d_I^*}{\partial a}}_{(+)}.$$

On the one hand, a higher access charge inflates the entrant's marginal cost, which hurts the

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<sup>10</sup>The second-order conditions are satisfied, as  $\delta_I > \delta_E$ .



entrant (direct effect). On the other, by strategic complementary, a higher access charge increases the incumbent's retail price, which increases the entrant's profit (indirect effect). Overall, we find that

$$\frac{d\pi_E}{da} = -\frac{8\delta_I(50\delta_E - a)(\delta_I - \delta_E)}{(4\delta_I - \delta_E)^2 \delta_E} < 0,$$

for  $a \leq 50\delta_E$ . The direct effect always dominates the indirect effect, which means that a higher access charge hurts the entrant.

Consider now that the access charge is set by the incumbent. As  $a \leq 50\delta_E$ , we have

$$\frac{d\pi_I}{da} = \frac{2\delta_I(50\delta_E - a)(8\delta_I + \delta_E)}{(4\delta_I - \delta_E)^2 \delta_E} \geq 0,$$

and therefore, since its profit increases with the access charge, the incumbent sets the access charge at a sufficiently high level so as to foreclose the entrant.

Finally, consider the case where the access charge is set by the regulator. The regulator sets the access charge to maximize the social welfare,  $W$ , which is defined as the sum of consumer surplus and industry profits. Consumer surplus is

$$CS = \int_{\theta_S}^{\theta_L} (x\delta_E - d_E)dx + \int_{\theta_L}^{\bar{\theta}} (x\delta_I - d_I)dx,$$

and we have  $W = CS + \pi_I + \pi_E$ . We find that

$$\frac{dW}{da} = -\frac{[400\delta_E(\delta_I - \delta_E) + (4\delta_I + 5\delta_E)a]\delta_I}{[4\delta_I - \delta_E]^2 \delta_E} < 0.$$

Therefore, welfare is maximized when the access charge is set at marginal cost (i.e., zero).

To sum up, without NGN investments, the access charge affects only the intensity of competition. Therefore, the incumbent would like to set a prohibitive access charge to foreclose its rival, whereas the regulator would like to set the access charge at marginal cost to reduce retail prices.

### 3.2 Monopoly with OGN and NGN services

As a second benchmark, we consider the case where firm I is a monopolist, which offers two different qualities: the OGN quality,  $\delta_I$ , and the NGN quality,  $\mu_I$ . The game has then only two stages: in a first stage, firm I decides on its NGN investment, and in a second stage, it sets the price for its low quality and high quality services.

At the second stage, firm I's profit, gross of investment cost, is

$$\pi_I(d_I, m_I) = \underbrace{d_I(\theta_L - \theta_S)}_{\text{OGN}} + \underbrace{m_I(\bar{\theta} - \theta_L)}_{\text{NGN}},$$

where  $\theta_S = d_I/\delta_I$  and  $\theta_L = (m_I - d_I)/(\mu_I - \delta_I)$ . Firm I chooses  $d_I$  and  $m_I$  to maximize its profit  $\pi_I$ . We find that  $d_I^* = 50\delta_I$  and  $m_I^* = 50\mu_I$ . Replacing for the equilibrium prices into the demands for the low quality and the high quality services, we find that  $\theta_L - \theta_S = 0$  and  $\bar{\theta} - \theta_L = 50$ . Hence, the incumbent monopolist sets prices so that only the high quality is active in equilibrium.

Moving backwards to the investment stage, the incumbent chooses an NGN quality  $\mu_I$  to maximize its profit,  $\Pi_I = 2500\mu_I - (\mu_I)^2/2$ . The optimal NGN investment is  $\mu_I^* = 2500$ .

To sum up, if the incumbent does not face competition, it invests in an NGN infrastructure, and shuts down its OGN. The result that the monopolist may find it optimal to provide only the high quality is well-known in the literature on vertical differentiation with multiple products firms.<sup>11</sup> Note however that it is not a general result; it is true in our setting, but it might not be true in other settings.

## 4 The Equilibrium

In this section, we solve for the equilibrium of the game, when the incumbent and the rival firm invest in NGN infrastructures, and can offer both OGN and NGN services in the retail market.

As we will detail below, under our assumptions, there are two market equilibria, which correspond to the two following configurations:<sup>12</sup>

- $\delta_E < \delta_I < \mu_I < \mu_E$ : firm E is the high quality NGN provider (“leapfrogging” case);
- $\delta_E < \delta_I < \mu_E < \mu_I$ : firm I is the high quality NGN (“persistence of leadership” case).

We start by considering that firm E is the high quality provider (“leapfrogging”), and then consider the other case (“persistence of leadership”).

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<sup>11</sup>For example, see Acharyya (1998) and Bhargava and Choudary (2008).

<sup>12</sup>At the end of Section 4, we will discuss/show that [other equilibria?]

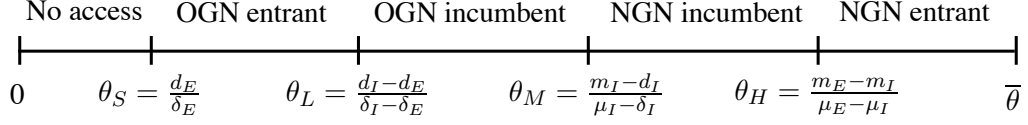


Figure 1: market segmentation

## 4.1 “Leapfrogging”: Firm E is the high quality NGN provider

### 4.1.1 Competition stage

At the competition stage (stage 3), four different network qualities are available to consumers, with the following ranking:  $\delta_E < \delta_I < \mu_I < \mu_E$ .

Since consumers have a unit demand, each consumer chooses at most one quality. The marginal consumer of type  $\theta_S$  who is indifferent between buying the lowest (OGN) quality  $\delta_E$  and not buying at all is characterized by  $\theta_S \delta_E - d_E = 0$ , and therefore,  $\theta_S = d_E / \delta_E$ . The consumer of type  $\theta_L$  who is indifferent between buying firm E’s OGN service and firm I’s OGN service is given by  $\theta_L \delta_E - d_E = \theta_L \delta_I - d_I$ , hence,  $\theta_L = (d_I - d_E) / (\delta_I - \delta_E)$ . Similarly, we can find the consumer of type  $\theta_M$  who is indifferent between firm I’s OGN service and firm I’s NGN service and the consumer of type  $\theta_H$  who is indifferent between firm I’s and firm E’s NGN services. Formally, we have  $\theta_M = (m_I - d_I) / (\mu_I - \delta_I)$  and  $\theta_H = (m_E - m_I) / (\mu_E - \mu_I)$ .

The demands for firm E’s and firm I’s OGN services are then

$$D_E^O = \theta_L - \theta_S = \frac{d_I - d_E}{\delta_I - \delta_E} - \frac{d_E}{\delta_E} \quad \text{and} \quad D_I^O = \theta_M - \theta_L = \frac{m_I - d_I}{\mu_I - \delta_I} - \frac{d_I - d_E}{\delta_I - \delta_E},$$

respectively, while the demands for the NGN services of firm E and firm I are

$$D_E^N = \bar{\theta} - \theta_H = \bar{\theta} - \frac{m_E - m_I}{\mu_E - \mu_I} \quad \text{and} \quad D_I^N = \theta_H - \theta_M = \frac{m_E - m_I}{\mu_E - \mu_I} - \frac{m_I - d_I}{\mu_I - \delta_I},$$

respectively. Figure 1 shows the different market segments that emerge. Note that a firm’s demand for a given service decreases with the price it sets for this service, but increases with the other prices.

The firms’ profits are as follows:

$$\Pi_I = (\theta_H - \theta_M)m_I + (\theta_M - \theta_L)d_I + a(\theta_L - \theta_S) - c(\mu_I),$$

and

$$\Pi_E = (\bar{\theta} - \theta_H)m_E + (\theta_L - \theta_S)(d_E - a) - c(\mu_E).$$

The two firms set their prices,  $\{d_E, m_E\}$  and  $\{d_I, m_I\}$ , to maximize their profits. Solving for the system of first order conditions,<sup>13</sup> we obtain the equilibrium prices.<sup>14</sup>

The following general result gives the effect of the OGN access charge on prices, and holds for all the market configurations that we consider in this paper.

**Lemma 1.** *The equilibrium OGN and NGN prices are increasing with the OGN access charge.*

*Proof.* See Appendix A1. □

A higher OGN access charge inflates the entrant's perceived marginal cost for the OGN service, and hence, its OGN price. When the incumbent has a service that competes directly with the entrant's OGN service, a higher access charge also inflates firm I's price for this service—this is because the incumbent has an opportunity cost from a price decrease, that is, foregone wholesale revenues. Finally, since OGN and NGN prices are strategic complements, an initial lift-up of the entrant's OGN price (and possibly of the incumbent direct rival's service price) percolates to all OGN and NGN prices.

This result shows that a low OGN access price leads to low NGN retail prices, which reduces NGN investment incentives.

We now replace for the equilibrium prices into the demand functions, and obtain the demand for OGN services for the entrant and the incumbent, respectively, at the equilibrium of the subgame:

$$D_E^O = \frac{2\delta_I [100\delta_E(\mu_E - \mu_I) - (4\mu_E - \mu_I)a]}{\eta\delta_E} \quad \text{and} \quad D_I^O = \frac{100\delta_E(\mu_E - \mu_I) - (4\mu_E - \mu_I)a}{\eta},$$

where  $\eta = 4\mu_E(\delta_I - \delta_E) + 4\delta_I(\mu_E - \mu_I) + 8\delta_I(\mu_E - \delta_E) + \delta_E(\mu_I - \delta_I) > 0$ , as  $\delta_E < \delta_I < \mu_I < \mu_E$ . Similarly, the demand for NGN services for the incumbent and the entrant are

$$D_I^N = \frac{100\mu_E(\delta_I - \delta_E) + 300\delta_I(\mu_E - \delta_E) + 3a\delta_I}{\eta},$$

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<sup>13</sup>The second order derivatives for the incumbent are  $\partial^2 \pi_I / \partial d_I^2 = -2/(\mu_I - \delta_I) - 2/(\delta_I - \delta_E) < 0$  and  $\partial^2 \pi_I / \partial m_I^2 = -2/(\mu_E - \mu_I) - 2/(\mu_I - \delta_I) < 0$ , and the determinant of the Hessian matrix is  $4(\mu_E - \delta_E)/((\mu_E - \mu_I)(\delta_I - \delta_E)(\mu_I - \delta_I)) > 0$ . Similarly, the second order derivatives for the entrant are  $\partial^2 \pi_E / \partial d_E^2 = -2/(\delta_I - \delta_E) - 2/\delta_E < 0$  and  $\partial^2 \pi_E / \partial m_E^2 = -2/(\mu_E - \mu_I) < 0$ , and the determinant of the Hessian matrix is  $4\delta_I/(\delta_E(\delta_I - \delta_E)(\mu_I - \mu_E)) > 0$ . Therefore, the solutions to the system of first order conditions correspond to a maximum.

<sup>14</sup>We omit the equilibrium prices to simplify the exposition.

and

$$D_E^N = \frac{2[100\mu_E(\delta_I - \delta_E) + 300\delta_I(\mu_E - \delta_E) + 3a\delta_I]}{\eta},$$

respectively. The firms' demands for OGN services decrease with the access charge  $a$ , and there is a threshold value for the access charge,  $\bar{a}_L = 100(\mu_E - \mu_I)\delta_E/(4\mu_E - \mu_I)$ , above which there is no demand for OGN services. Whereas, the demands for NGN services increase with the access charge.

#### 4.1.2 Access pricing stage

Moving backwards, the access charge is set either by the incumbent or by the regulator. Let us start with the former case, where the incumbent sets the access charge to maximize its profit.

**Profit-maximizing access charge.** At the equilibrium of the competition stage, the incumbent's profit is  $\Pi_I = \Pi_I(a, d_I^*, d_E^*, m_I^*, m_E^*)$ . Using the envelope theorem, the effect of a higher access charge on firm I's profit is then given by

$$\frac{d\Pi_I}{da} = \underbrace{\frac{\partial\Pi_I}{\partial a}}_{(+)} + \underbrace{\frac{\partial\Pi_I}{\partial m_E}}_{(+)} \underbrace{\frac{\partial m_E^*}{\partial a}}_{(+)} + \underbrace{\frac{\partial\Pi_I}{\partial d_E}}_{(+)} \underbrace{\frac{\partial d_E^*}{\partial a}}_{(+)}. \quad (1)$$

The first term in (1) represents the direct effect of a higher access charge on profit. As  $\partial\Pi_I/\partial a = D_E^O \geq 0$ , this direct effect is always positive; taking prices as given, a higher access charge means higher wholesale profits for the incumbent.

The second and third terms represent two indirect effects, which go through firm E's prices for the NGN and OGN services, respectively. From Lemma 1, we know that  $\partial m_E^*/\partial a > 0$  and that  $\partial d_E^*/\partial a > 0$ , that is, a higher access charge increases the rival firm's prices. Besides, we have

$$\left. \frac{\partial\Pi_I}{\partial m_E} \right|_{(M^*, D^*)} = d_I^* \frac{\partial D_I^O}{\partial m_E} + m_I^* \frac{\partial D_I^N}{\partial m_E} + a \frac{\partial D_E^O}{\partial m_E},$$

where  $M^* = (m_I^*, m_E^*)$  and  $D^* = (d_I^*, d_E^*)$ . As a given service's demand is increasing in the rival services' prices, we have  $\partial\Pi_I/\partial m_E|_{(M^*, D^*)} > 0$ . This proves that the first indirect effect is always positive; a higher access charge inflates the entrant's NGN price, which leads both to higher (OGN and NGN) retail and (OGN) wholesale sales for the incumbent.

Finally, we have

$$\frac{\partial \Pi_I}{\partial d_E} \Big|_{(M^*, D^*)} = \underbrace{d_I^* \frac{\partial D_I^O}{\partial d_E}}_{(+)} + \underbrace{m_I^* \frac{\partial D_I^N}{\partial d_E}}_{(+)} + \underbrace{a \frac{\partial D_E^O}{\partial d_E}}_{(-)}. \quad (2)$$

A higher OGN retail price from the rival firm has conflicting effects on the incumbent's profit. On the one hand, it benefits the incumbent's retail operations—this corresponds to the first and second terms in equation (2). On the other, it reduces the rival firm's demand for the OGN service, and therefore, it decreases the incumbent's wholesale revenues. Replacing for equilibrium prices into (2), we find that

$$\frac{\partial \Pi_I}{\partial d_E} \Big|_{(M^*, D^*)} = \frac{4\delta_I [100\delta_E (\mu_E - \mu_I) - a (4\mu_E - \mu_I)]}{\eta\delta_E} \geq 0,$$

which proves that the second indirect effect is always positive. Hence, we can state the following Lemma.

**Lemma 2.** *Assume that  $\delta_E < \delta_I < \mu_I < \mu_E$  (“leapfrogging”). Then, firm I sets an access charge which leads to a complete “switch-off” of the OGN.*

*Proof.* From the analysis above, we have  $d\Pi_I/da > 0$  for all  $a < \bar{a}_L$ . Therefore, the incumbent sets  $a \geq \bar{a}_L$ , and forecloses the entrant's OGN operations. At the same time, since  $a \geq \bar{a}_L$ , the demand for the incumbent's OGN services is equal to zero. Therefore, everything is as if the incumbent “switched off” the OGN.  $\square$

Compared to the benchmark where the incumbent and the entrant compete in OGN services only, the incumbent has an additional incentive to foreclose the entrant's OGN operations. When the access charge is low, the entrant offers OGN services at a low price. Due to the strategic complementarity between OGN services and NGN services, this drives down the equilibrium prices for NGN services. Therefore, the incumbent has an incentive to foreclose the entrant's OGN service to soften competition in NGN services. This is why the entrant also benefits from a high access charge to the OGN, since its NGN service remains active.

This effect is reminiscent of the retail-level business migration effect in Bourreau et al. (2012), except that in Bourreau et al., firms I and E each offer a single service, whereas in the present setting, each firm offers both OGN and NGN services. Besides, in Bourreau et al., the access charge is set ex ante, and it affects NGN investments. Whereas, here, the access charge is set ex post, after investments have taken place; it does not affect investments, but rather the migration

to the NGN at the retail level.

Finally, at the level of access charge that forecloses the entrant's OGN service, the demand for the incumbent's own OGN service is also equal to zero. Therefore, in the equilibrium of the access subgame, the OGN is no longer active.

**Welfare-maximizing access charge.** Let us now consider the case where the access charge is set by a welfare-maximizing regulator. We denote by  $CS_i^O$  (resp.,  $CS_i^N$ ) the surplus of the consumers who consume firm  $i$ 's OGN (resp., NGN) service.<sup>15</sup> Total welfare is defined as the sum of consumers' surplus and industry profits, that is,  $W = CS_E^O + CS_I^O + CS_E^N + CS_I^N + \Pi_I + \Pi_E$ . We have the following result.

**Lemma 3.** *Assume that  $\delta_E < \delta_I < \mu_I < \mu_E$  ("leapfrogging"). Then, the regulator sets an access charge which leads to a complete "switch-off" of the OGN.*

*Proof.* See Appendix A3. □

In contrast to the benchmark without NGN investments, the regulator sets a very high access charge to accelerate the migration from the old to the new technology. To understand the positive effect of a higher access charge on welfare, we discuss below the effect on consumers and on the rival firm.

As retail prices increase with the access charge (from Lemma 1), consumers are always worse off with a higher access charge. The effect of a higher access charge on the entrant's profit is given by

$$\frac{d\Pi_E}{da} = \underbrace{\frac{\partial\Pi_E}{\partial a}}_{(-)} + \underbrace{\frac{\partial\Pi_E}{\partial d_I}}_{(+)} \underbrace{\frac{\partial d_I^*}{\partial a}}_{(+)} + \underbrace{\frac{\partial\Pi_E}{\partial m_I}}_{(+)} \underbrace{\frac{\partial m_I^*}{\partial a}}_{(+)}. \quad (3)$$

The first term in (3) represents the direct effect of a higher access charge on the entrant's profit. We have  $\partial\Pi_E/\partial a = -D_E^O$ , and therefore it is always negative; taking prices as given, a higher access charge inflates the entrant's perceived marginal cost, and hence, lowers its profit. The second and third terms represent two indirect effects which operate via firm I's prices. From Lemma 1, we have  $\partial d_I^*/\partial a > 0$  and  $\partial m_I^*/\partial a > 0$ . Besides, we have

$$\left. \frac{\partial\Pi_E}{\partial d_I} \right|_{(M^*, D^*)} = (p_E^* - a) \frac{\partial D_E^O}{\partial d_I} + m_E^* \frac{\partial D_E^N}{\partial d_I} \geq 0,$$

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<sup>15</sup>See Appendix A2 for the expressions of consumer surplus.

and

$$\left. \frac{\partial \Pi_E}{\partial m_I} \right|_{(M^*, D^*)} = (p_E^* - a) \frac{\partial D_E^O}{\partial m_I} + m_E^* \frac{\partial D_E^N}{\partial m_I} \geq 0.$$

Therefore, the two indirect effects are always positive.<sup>16</sup> In other words, a higher access charge increases the entrant's profit, due to its softening effect on the incumbent's retail prices.

Since we have a negative direct effect and two positive indirect effects, the overall effect of an increase in the access charge on firm E's profit is a priori ambiguous. In our setting, however, we find that we always have<sup>17</sup>

$$\frac{d\Pi_E}{da} > 0.$$

In words, the entrant always benefits from a higher OGN access charge. This result contrasts with our benchmark where the two firms competed in OGN services only, and where the entrant was hurt by a higher access charge. By contrast, when firms are competing with multiple products, a high access charge on the OGN is not necessarily detrimental to the entrant, as it also offers NGN services.

To sum up, an increase in the access price has two opposite effects on social welfare. On the one hand, higher retail prices lead to a lower consumer surplus. On the other, both firms benefit from an increase in the access charge, as it softens competition for NGN services. We find that, overall, the positive effect of a larger access charge on producers' surplus dominates the negative effect it has on consumers' surplus. This result explains why the regulator finds it optimal to set a large access price to induce firms to switch off their OGN operations (Lemma 3).

Note that while the two firms benefit if they both abandon the old technology (since it reduces cannibalization within their product lines), each firm has no unilateral incentive to shut down its OGN service (otherwise it would set an infinite retail price for it, which would be tantamount to closing the service). The OGN switch-off can only be achieved via some form of coordination. Our results show that access requirements to the OGN can be such a coordination instrument. Since the incumbent controls the access terms, which affect the entrant's OGN demand and its own OGN demand, it can shut down all OGN services, to the benefit of both firms.

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<sup>16</sup>The indirect effect with respect to price  $d_I$  (resp.,  $m_I$ ) is strictly positive if firm I's OGN (resp., NGN) service is in direct competition with at least one of firm E's service. Otherwise, it is equal to zero. Since at least one of firm I's services competes directly with firm E's services, at least one of the two indirect effects is strictly positive.

<sup>17</sup>See Appendix A4.



### 4.1.3 Investment stage

We now determine the firms' investment decisions. Assume that  $\delta_E < \delta_I < \mu_I < \mu_E$  holds in equilibrium. Lemmas 2 and 3 show that in both the regulated and unregulated cases, the access charge is set at  $a \geq \bar{a}_L$ , which implies that firms I and E compete for NGN services only. We determine the equilibrium NGN investments in this configuration, and show that it is an equilibrium of the game. The following result summarizes our analysis.

**Proposition 1.** *There is a “leapfrogging” equilibrium, such that  $\delta_E < \delta_I < \mu_I^* < \mu_E^*$ . In both the regulated and unregulated cases, the firms compete in NGN services only, and the OGN is switched off.*

*Proof.* See Appendix A5. □

When the entrant is the leader in NGN investments, the migration from the OGN to the NGN takes place completely, that is, there is full migration. The OGN is replaced by two competing NGN infrastructures, and switched off. The OGN switch-off is realized through a very high access price, set by either the incumbent or the regulator, which allows the two competing firms to coordinate on shutting down their OGN operations. Therefore, we can state the following corollary.

**Corollary 1.** *In the “leapfrogging” equilibrium, there is no need to implement a formal switch-off of the OGN.*

In this equilibrium, the unregulated outcome coincides with the social optimum. Therefore, the regulator could decide to unregulate access to the OGN after NGN investments have taken place.

Finally, note that an exogenous upgrade of the incumbent's OGN has no effect on NGN investments, since the equilibrium does not depend on the incumbent's OGN quality  $\delta_I$ . Therefore, whether or not the regulator should allow an exogenous upgrade of the OGN is irrelevant.

## 4.2 “Persistence of leadership”: Firm I is the high quality provider

### 4.2.1 Competition stage

When the incumbent is leader in NGN investments, four different network qualities are available to consumers, with the following ranking:  $\delta_E < \delta_I < \mu_E < \mu_I$ . Following the same procedure as in Section 4.1, the demands for the OGN services of firm E and I can be written as

$$D_E^O = \theta_L - \theta_S = \frac{d_I - d_E}{\delta_I - \delta_E} - \frac{d_E}{\delta_E} \quad \text{and} \quad D_I^O = \theta_M - \theta_L = \frac{m_E - d_I}{\mu_E - \delta_I} - \frac{d_I - d_E}{\delta_I - \delta_E},$$

respectively, while the demands for the NGN services of firm I and firm E are

$$D_I^N = \bar{\theta} - \theta_H = \bar{\theta} - \frac{m_I - m_E}{\mu_I - \mu_E} \quad \text{and} \quad D_E^N = \theta_H - \theta_M = \frac{m_I - m_E}{\mu_I - \mu_E} - \frac{m_E - d_I}{\mu_E - \delta_I},$$

respectively. Firm I's and firm E's profits are as follows:

$$\Pi_I = m_I D_I^N + d_I D_I^O + a D_E^O - c(\mu_I) \quad \text{and} \quad \Pi_E = m_E D_E^N + (d_E - a) D_E^O - c(\mu_E).$$

The two firms set their prices to maximize their profits. Solving for the system of first order conditions, we obtain the equilibrium prices.<sup>18</sup> Replacing for the equilibrium prices into the demand functions, we find that the demand for the entrant's OGN service is decreasing in the access charge<sup>19</sup> and equals zero if and only if  $a \geq \bar{a}_P$ , where<sup>20</sup>

$$\bar{a}_P = \frac{50\delta_E [\delta_I(\mu_I - \mu_E) + \mu_E(\mu_E - \delta_I)]}{\mu_I[\delta_I - 4\mu_E] + \mu_E[\mu_E + 2\delta_I]}.$$

The demand for the incumbent's OGN service at  $a = \bar{a}_P$  is equal to

$$D_E^O|_{a=\bar{a}_P} = \frac{50\mu_E(\mu_I - \mu_E)}{4\mu_I\mu_E - (\mu_E)^2 - \mu_I\delta_I - 2\mu_E\delta_I},$$

which is strictly positive as  $\mu_I > \mu_E > \delta_I > \delta_E$ . Therefore, in contrast to the “leapfrogging” case, when there is “persistence of leadership” and the incumbent forecloses the entrant's OGN operations, it does not shut down its own OGN operations. The intuition is that the incumbent's high quality NGN service is not in direct competition with OGN services, and hence, the cannibalization of its NGN sales by its own OGN service is less severe than in the “leapfrogging” case.

#### 4.2.2 Access pricing

We now move backwards and determine the optimal access charge, which is set by either the incumbent or the regulator.

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<sup>18</sup>We omit the equilibrium prices to simplify the exposition. The second order derivatives for the incumbent are  $\partial^2 \Pi_I / \partial d_I^2 = -2/(\mu_E - \delta_I) - 2/(\delta_I - \delta_E) < 0$ ,  $\partial^2 \Pi_I / \partial m_I^2 = -2/(\mu_I - \mu_E) < 0$ , and the determinant of the Hessian matrix is  $4(\mu_E - \delta_E)/((\mu_I - \mu_E)(\mu_E - \delta_I)(\delta_I - \delta_E)) > 0$ . The second order derivatives for the entrant are  $\partial^2 \Pi_E / \partial d_E^2 = -2/(\delta_I - \delta_E) - 2/\delta_E < 0$ ,  $\partial^2 \Pi_E / \partial m_E^2 = -2/(\mu_I - \mu_E) - 2/(\mu_E - \delta_I) < 0$ , and the determinant of the Hessian matrix is  $4(\mu_I - \delta_I)\delta_I/((\mu_E - \delta_I)(\mu_I - \mu_E)(\delta_I - \delta_E)\delta_E) > 0$ . Therefore, the solutions to the first order conditions correspond to a maximum.

<sup>19</sup>See Appendix A6.

<sup>20</sup>There is no clear-cut ordering between  $\bar{a}_L$  and  $\bar{a}_P$ .

**Profit-maximizing access charge.** Using the envelope theorem, the effect of a higher access charge on firm I's profit can be written as

$$\frac{d\Pi_I}{da} = \underbrace{\frac{\partial\Pi_I}{\partial a}}_{(+)} + \underbrace{\frac{\partial\Pi_I}{\partial d_E}}_{(+)} \underbrace{\frac{\partial d_E^*}{\partial a}}_{(+)} + \underbrace{\frac{\partial\Pi_I}{\partial m_E}}_{(+)} \underbrace{\frac{\partial m_E^*}{\partial a}}_{(+)}. \quad (4)$$

The first term in (4) represents the direct effect of a higher access charge on profit, which is positive, as  $\partial\Pi_I/\partial a = D_E^O \geq 0$ . The second and third terms are the two indirect effects, which go through firm E's prices for OGN and NGN services, and are both positive. Indeed, from Lemma 1, we have  $\partial d_E^*/\partial a > 0$  and  $\partial m_E^*/\partial a > 0$ . Besides, we find that

$$\frac{\partial\Pi_I}{\partial d_E} = \frac{d_I^* - (\delta_I/\delta_E) a}{\delta_I - \delta_E}.$$

Since  $d_I^* - (\delta_I/\delta_E) \bar{a}_P = 0$ , we have  $\partial\Pi_I/\partial d_E > 0$  for all  $a < \bar{a}_P$ . Finally, we find that

$$\frac{\partial\Pi_I}{\partial m_E} = \frac{m_I^*}{\mu_I - \mu_E} + \frac{d_I^*}{\mu_E - \delta_I} > 0.$$

Therefore, we can state the following result.

**Lemma 4.** *Assume that  $\delta_E < \delta_I < \mu_E < \mu_I$  ("persistence of leadership"). Then, firm I sets a sufficiently high access charge so as to foreclose the entrant's OGN service. The incumbent's OGN service remains active in equilibrium.*

*Proof.* From the analysis above, we have  $d\Pi_I/da > 0$  for all  $a < \bar{a}_P$ . Therefore, the incumbent sets  $a \geq \bar{a}_P$ , and forecloses the entrant's OGN operations. However, at  $a \geq \bar{a}_P$ , the demand for the incumbent's OGN services is strictly positive.  $\square$

Because the competition from firm E's OGN services drives down the profitability of firm I's operations, firm I has an incentive to foreclose firm E's OGN service. However, as we have already stated, firm I's OGN service is not in direct competition with its own NGN service. Therefore, firm I has no incentive to shut it down.

**Welfare-maximizing access charge.** We now consider the case where the access charge is set by a welfare-maximizing regulator. We have the following result.

**Lemma 5.** *Assume that  $\delta_E < \delta_I < \mu_E < \mu_I$  (“persistence of leadership”). Then, the regulator sets the access charge so as to foreclose the entrant’s OGN service. The incumbent’s OGN service remains active in equilibrium.*

*Proof.* See Appendix A7. □

We know that the incumbent always benefits from a higher access charge and that, since the access charge inflates retail prices (from Lemma 1), consumers are always worse-off with a higher access charge. The effect of a higher access charge on the entrant’s profit is given by

$$\frac{d\Pi_E}{da} = \underbrace{\frac{\partial\Pi_E}{\partial a}}_{(-)} + \underbrace{\frac{\partial\Pi_E}{\partial d_I}}_{(+)} \underbrace{\frac{\partial d_I^*}{\partial a}}_{(+)} + \underbrace{\frac{\partial\Pi_E}{\partial m_I}}_{(+)} \underbrace{\frac{\partial m_I^*}{\partial a}}_{(+)}. \quad (5)$$

The first term in (5) represents the direct effect of a higher access charge on the entrant’s profit, which is negative. The second and third terms represent the two indirect effects, which go through firm I’s prices. There are both positive, as from Lemma 1,  $\partial d_I^*/\partial a > 0$  and  $\partial m_I^*/\partial a > 0$ , and

$$\frac{\partial\Pi_E}{\partial d_I} = \frac{m_E^*}{\mu_E - \delta_I} + \frac{d_E^* - a}{\delta_I - \delta_E} > 0 \quad \text{and} \quad \frac{\partial\Pi_E}{\partial m_I} = \frac{m_E^*}{\mu_I - \mu_E} > 0.$$

Though the effect of the access charge on the entrant’s profit is a priori ambiguous, as in the “leapfrogging” case, we find that the entrant benefits from a higher OGN access charge, that is,  $d\Pi_E/da > 0$ .<sup>21</sup>

All in all, the positive effect of the access charge on industry profits dominates the negative effect on consumer surplus, and therefore, the regulator chooses to set a very high access price to accelerate migration from the old to the new technology.

### 4.2.3 Investment stage

We now determine the firms’ investment decisions at the first stage of the game. Assume that  $\delta_E < \delta_I < \mu_E < \mu_I$  holds in equilibrium. From Lemmas 4 and 5, we know that in both the regulated and unregulated cases, the access charge is set at  $a \geq \bar{a}_P$ . Therefore, when firms decide on their NGN investments, they anticipate competition between a multiproduct incumbent (with OGN and NGN services) and a monoprodukt entrant (with NGN services only). We determine the

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<sup>21</sup>See Appendix A8.

equilibrium NGN investments in this configuration, and show that it is indeed an equilibrium of the game. The following result summarizes the analysis.

**Proposition 2.** *There is a “persistence of leadership” equilibrium, such that  $\delta_E < \delta_I < \mu_E^* < \mu_I^*$ . In both the regulated and unregulated cases, firm I offers OGN services of quality  $\delta_I$  and NGN services of quality  $\mu_I^*(\delta_I)$ , whereas firm E offers only NGN services of quality  $\mu_E^*(\delta_I)$ . We have  $\mu_I^*(\delta_I)$  decreasing with  $\delta_I$ , and  $\mu_E^*(\delta_I)$  increasing with  $\delta_I$ . The OGN is not switched off.*

*Proof.* See Appendix A9. □

When the incumbent is the leader in NGN investments, the migration from the OGN to the NGN is only partial; some consumers from the incumbent still use OGN services. However, Figures 4 and 5 show that the incumbent’s profit and total welfare decrease with  $\delta_I$ . Therefore, it would be both profit-enhancing for the incumbent and welfare-enhancing to shut down firm I’s OGN. This can be done if, ex-ante, the incumbent or the regulator can commit to switch off the OGN after NGN investments have taken place.<sup>22</sup> We can therefore state the following Corollary.

**Corollary 2.** *When there is “persistence of leadership”, a formal switch-off of the OGN is socially desirable.*

In contrast to the “leapfrogging” case, when there is “persistence of leadership” in equilibrium, an exogenous upgrade of the OGN affects the equilibrium outcome, in an ambiguous way. On the one hand, it decreases the incumbent’s NGN investment, but on the other it increases the entrant’s investment (see Figures 2 and 3). However, as an OGN upgrade hurts total welfare (see Figure 5), when it expects “persistence of leadership”, the regulator should not allow an exogenous upgrade of the OGN. We can then state the following result.

**Corollary 3.** *In the “persistence of leadership” equilibrium, an exogenous upgrade of the incumbent’s OGN decreases social welfare.*

Finally, in Propositions 1 and 2 we have shown that the game has two “leapfrogging” and “persistence of leadership” equilibria. In Appendix A10 we show that there are no other equilibria to this game.

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<sup>22</sup>This can also be done if firm I is allowed to exit the OGN market after NGN investments have taken place.

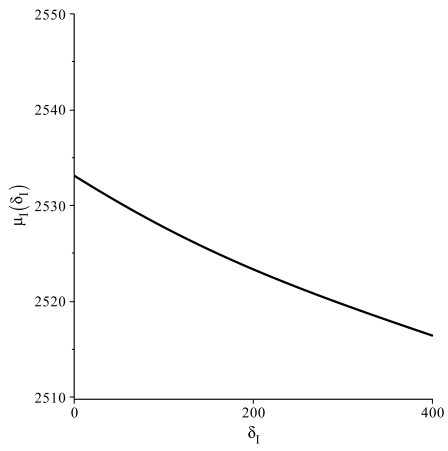


Figure 2: Incumbent's investment

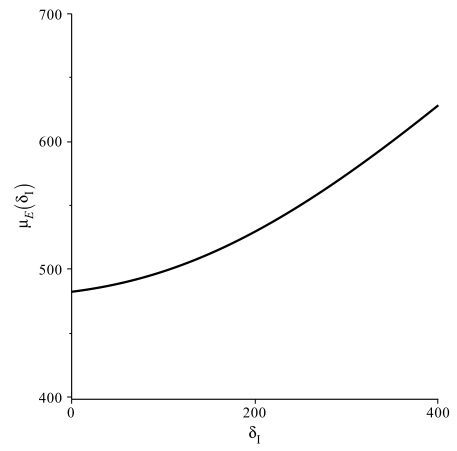


Figure 3: Entrant's investment

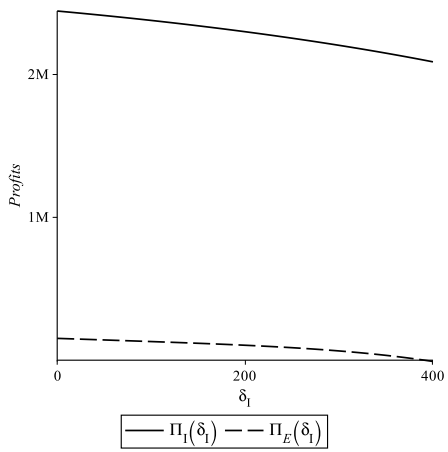


Figure 4: Incumbent's and entrant's profit

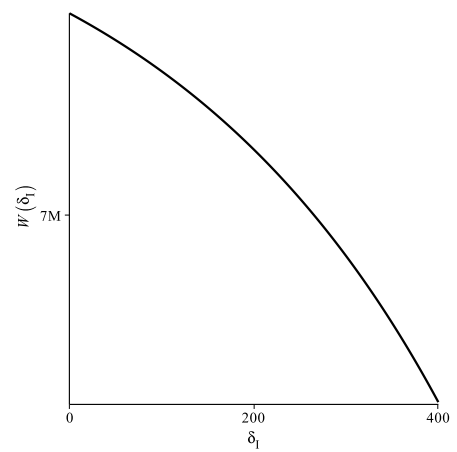


Figure 5: Total welfare

## 5 The Impact of an Old Technology Upgrade

In our baseline model, we considered that the quality of the OGN service was fixed, and therefore, the only investment decision for the incumbent was related to the NGN. If the OGN may be improved, the incumbent will face a trade-off between improving the old technology and developing the new one. For example, in the telecommunications industry, incumbent firms can invest in the so-called “vectoring” technology to upgrade the quality of the broadband services that use the old generation copper network.<sup>23</sup> Therefore, in a given area, an incumbent has to decide whether to upgrade its OGN with the vectoring technology and/or to roll out an NGN (fibre) infrastructure. In this section, we extend our baseline model to allow the incumbent to upgrade the quality of its OGN service, and investigate how this possibility of upgrade affects the firms’ incentives to invest in NGNs.

We assume that in the same period as the firms decide on their NGN investments (i.e., stage 1), firm I can also increase the quality of its OGN service, up to  $\tilde{\delta}_I \in [\delta_I, \bar{\delta}]$ . The upper bound  $\bar{\delta} > \delta_I$  is the maximum quality achievable via the upgrade. We assume that the incumbent incurs no cost for this upgrade. We assume furthermore that the incumbent’s quality upgrade does not spill over to the entrant’s OGN service, and that the incumbent does not provide access to its upgraded OGN, but only to the standard OGN. Therefore, when the incumbent upgrades its old generation network,  $\delta_E$  remains unchanged.

We assume that Assumption A1 still holds, and introduce the following additional assumptions:

**A2.**  $\bar{\delta} \in [284.8, 1250]$ .

**A3.**  $\delta_I > 92.35$ .

Assumptions A1, A2 and A3 ensure that with an endogenous OGN upgrade, there is both a “leapfrogging” equilibrium and a “persistence of leadership” equilibrium. Assumption A2 also implies that there is no equilibrium where the quality of the upgraded OGN is higher than the quality achievable via the best NGN technology.<sup>24</sup>

Note that with our assumptions, the quality achievable through the OGN’s upgrade can be either lower or larger than the quality of the NGN, that is, we can have either  $\tilde{\delta}_I > \mu_I$  or the reverse. For example, in telecoms, upgraded copper broadband lines are capable of achieving speeds that are

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<sup>23</sup>The vectoring technology allows operators to provide Internet access services with a higher speed than traditional broadband DSL services.

<sup>24</sup>We exclude this possibility because we are interested in situations where the new technology provides an improvement over the old technology, even if the latter is upgraded.

either lower or higher than those available on a low-quality NGN, depending on the length of the line.

Under our assumptions and according to the firms' NGN investments and the incumbent's OGN quality upgrade, at the competition stage, the market can be in one of the six following potential configurations:<sup>25</sup>

$$\begin{aligned}
(\text{S1}): \delta_E < \tilde{\delta}_I < \mu_I < \mu_E, & \quad (\text{L2}): \delta_E < \mu_E < \tilde{\delta}_I < \mu_I, \\
(\text{S2}): \delta_E < \tilde{\delta}_I < \mu_E < \mu_I, & \quad (\text{L3}): \delta_E < \mu_E < \mu_I < \tilde{\delta}_I, \\
(\text{L1}): \delta_E < \mu_I < \tilde{\delta}_I < \mu_E, & \quad (\text{L4}): \delta_E < \mu_I < \mu_E < \tilde{\delta}_I.
\end{aligned}$$

Cases (S1) and (S2) correspond to situations where the OGN is not upgraded in equilibrium (i.e.,  $\tilde{\delta}_I = \delta_I$ ), or the incumbent chooses a very small quality improvement for its OGN service (i.e.,  $\tilde{\delta}_I < \min\{\mu_I, \mu_E\}$ ). These two cases are equivalent to the “leapfrogging” and “persistence of leadership” cases, respectively, that we studied in Section 4. In Appendix B0 we show that the equilibria found in these two configurations persist also when firm I is allowed to upgrade the OGN when  $\bar{\delta}$ , the maximum quality achievable with the OGN, is not too large.

Cases (L1) to (L4) correspond to situations where the quality of the upgraded OGN service is higher than the quality of at least some of the NGN services. However, as we show in Appendix B1, (L3) and (L4) cannot be equilibrium outcomes. In both cases, firm I sets  $\tilde{\delta}_I = \bar{\delta}$ : under Assumption A2, firm E then profitably deviates by setting an NGN quality larger than  $\tilde{\delta}_I^*$ .

Therefore, in what follows, we focus on cases (L1) and (L2). Note that (L1) corresponds to the “leapfrogging” case, and (L2) to the “persistence of leadership” case, but with a large OGN upgrade.

## 5.1 Leapfrogging with OGN upgrade

In this configuration, two OGN qualities and two NGN qualities are available to consumers, with  $\delta_E < \mu_I < \tilde{\delta}_I < \mu_E$ . We proceed as in Section 4 to determine the marginal consumers between the different available qualities, which yields the firms' demands

$$D_E^O = \frac{m_I - d_E}{\mu_I - \delta_E} - \frac{d_E}{\delta_E}, \quad D_I^N = \frac{d_I - m_I}{\tilde{\delta}_I - \mu_I} - \frac{m_I - d_E}{\mu_I - \delta_E}, \quad D_I^O = \frac{m_E - d_I}{\mu_E - \tilde{\delta}_I} - \frac{d_I - m_I}{\tilde{\delta}_I - \mu_I}, \quad D_E^N = \bar{\theta} - \frac{m_E - d_I}{\mu_E - \tilde{\delta}_I}.$$

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<sup>25</sup>For the same reasons as in Section 4, there is no equilibrium where  $\mu_i < \delta_E$ . See Appendix A10.



### 5.1.1 Competition stage

The firms' profits, gross of investment costs, are

$$\pi_I = d_I D_I^O + m_I D_I^N + a D_E^O \quad \text{and} \quad \pi_E = (d_E - a) D_E^O + m_E D_E^N.$$

Firms I and E choose their prices to maximize their profits. Solving for the system of four first order conditions,<sup>26</sup> we find the equilibrium prices. Replacing for the equilibrium prices into the demands, we find:

$$D_E^O = \frac{2\mu_I [100\delta_E(\mu_E - \tilde{\delta}_I) - (4\mu_E - \tilde{\delta}_I)a]}{\kappa\delta_E}, \quad D_I^O = \frac{100\mu_E(\mu_I - \delta_E) + 300\mu_I(\mu_E - \delta_E) + 3a\mu_I}{\kappa},$$

$$D_E^N = \frac{2[100\mu_E(\mu_I - \delta_E) + 300\mu_I(\mu_E - \delta_E) + 3a\mu_I]}{\kappa}, \quad D_I^N = \frac{100\delta_E(\mu_E - \tilde{\delta}_I) - (4\mu_E - \tilde{\delta}_I)a}{\kappa},$$

where  $\kappa = 16\mu_I\mu_E - 4\mu_I\tilde{\delta}_I - 8\mu_I\delta_E - 4\mu_E\delta_E + \delta_E(\tilde{\delta}_I - \mu_I) > 0$ , as  $\delta_E < \mu_I < \tilde{\delta}_I < \mu_E$ .

The entrant's demand for OGN services and the incumbent's demand for NGN services decrease with the access charge  $a$ , and there is a threshold value for the access charge,  $\bar{a}'_L = 100(\mu_E - \tilde{\delta}_I)\delta_E/(4\mu_E - \tilde{\delta}_I)$ , above which there is no demand for the two low quality services. Whereas, the demands for the entrant's NGN service and the incumbent's upgraded OGN service increase with the access charge.

### 5.1.2 Access pricing

Moving backwards, the access charge is set either by the incumbent or by the regulator.

**Profit-maximizing access charge.** We begin by studying the case where the access charge is set by the incumbent. With a similar analysis than in Section 4, we obtain the following result.

**Lemma 6.** *Assume that  $\delta_E < \mu_I < \tilde{\delta}_I < \mu_E$  ("leapfrogging with upgrade"). Then, firm I sets the access charge so as to foreclose the entrant's OGN service, which also shuts down its own NGN operations.*

*Proof.* See Appendix B2. □

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<sup>26</sup>The second order derivatives for the incumbent are  $\partial^2\pi_I/\partial d_I^2 = -2/(\mu_E - \delta_I) - 2/(\delta_I - \mu_I) < 0$  and  $\partial^2\pi_I/\partial m_I^2 = -2/(\delta_I - \mu_I) - 2/(\mu_I - \delta_E) < 0$ , and the determinant of the Hessian matrix is  $4(\mu_E - \delta_E)/((\mu_I - \delta_E)(\delta_I - \mu_I)(\mu_E - \delta_I)) > 0$ . Similarly, the second order derivatives for the entrant are  $\partial^2\pi_E/\partial d_E^2 = -2/(\mu_I - \delta_E) - 2/\delta_E < 0$  and  $\partial^2\pi_E/\partial m_E^2 = -2/(\mu_E - \delta_I) < 0$ , and the determinant of the Hessian matrix is  $4\mu_I/(\delta_E(\mu_I - \delta_E)(\mu_E - \delta_I)) > 0$ . Therefore, the solutions to the first order conditions correspond to a maximum.

This result resembles Lemma 2 in the “leapfrogging” case without upgrade, except that firm I here shuts down its NGN operations rather than its OGN operations. Firm I has an incentive to shut down its low quality service (i.e., the NGN service), as it is in direct competition with its high quality service (i.e., the upgraded OGN service). Besides, firm I has an incentive to foreclose the entrant’s low-quality OGN service to soften price competition for high-quality services.

**Welfare-maximizing access charge.** We now consider the case where the access charge is set by the regulator.<sup>27</sup> We have the following result.

**Lemma 7.** *Assume that  $\delta_E < \mu_I < \tilde{\delta}_I < \mu_E$  (“leapfrogging with upgrade”). Then, the regulator sets the access charge so as to foreclose the entrant’s OGN service and the incumbent’s NGN service.*

*Proof.* See Appendix B4. □

Like the incumbent, the regulator is willing to have only the two highest qualities active in equilibrium. Therefore, it sets an access charge which leads to a zero demand for the two low quality services. The effect of a higher access charge on the entrant’s profit is given by

$$\frac{d\Pi_E}{da} = \underbrace{\frac{\partial\Pi_E}{\partial a}}_{(-)} + \underbrace{\frac{\partial\Pi_E}{\partial d_I}}_{(+)} \underbrace{\frac{\partial d_I^*}{\partial a}}_{(+)} + \underbrace{\frac{\partial\Pi_E}{\partial m_I}}_{(+)} \underbrace{\frac{\partial m_I^*}{\partial a}}_{(+)}. \quad (6)$$

The first term in (6) represents the negative direct effect of a higher access charge on the entrant’s profit. The second and third terms are the two indirect effects, which are positive, as we have  $\partial d_I^*/\partial a > 0$  and  $\partial m_I^*/\partial a > 0$  from Lemma 1, and  $\partial\Pi_E/\partial d_I = m_E^*/(\mu_E - \tilde{\delta}_I) > 0$  and  $\partial\Pi_E/\partial m_I = (d_E^* - a)/(\mu_I - \delta_E) > 0$ .

Though the sign of (6) is a priori ambiguous, we find that, overall,  $d\Pi_E/da > 0$ .<sup>28</sup> In words, the entrant benefits from a higher OGN access charge.

### 5.1.3 NGN investments and OGN upgrade

Finally, we solve for the investment decisions at stage 1 of the game. Assume that  $\delta_E < \mu_I < \tilde{\delta}_I < \mu_E$  holds in equilibrium. The firms anticipate that once investments have been realized, in the continuation game, there will be no demand for firm E’s OGN service and firm I’s NGN service. Therefore, at stage 1, everything is as if firm I and firm E were deciding on  $\tilde{\delta}_I$  and  $\mu_E$ , respectively.

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<sup>27</sup>See Appendix B3 for the expressions of consumer surplus and welfare in the leapfrogging and persistence of leadership cases with upgrade.

<sup>28</sup>See Appendix B5.

We determine the optimal investments in this case, and check that this candidate equilibrium is indeed a Nash equilibrium of the game. The following result summarizes the analysis.

**Proposition 3.** *There is a “leapfrogging with upgrade” equilibrium, such that  $\delta_E < \mu_I^* < \tilde{\delta}_I^* < \mu_E^*$ . In equilibrium, firm I upgrades its OGN at the maximum achievable quality ( $\tilde{\delta}_I^* = \bar{\delta}$ ), but does not invest in an NGN ( $\mu_I^* = 0$ ), while firm E invests in an NGN of quality  $\mu_E^*(\bar{\delta})$ . We have  $\tilde{\delta}_I^*$  and  $\mu_E^*(\bar{\delta})$  increasing with  $\bar{\delta}$ .*

*Proof.* See Appendix B6. □

In the “leapfrogging with upgrade” equilibrium, the old and new technologies coexist. Therefore, in contrast to the leapfrogging case without upgrade, the migration from the old to the new technology is only partial. This is because the incumbent find it less expensive to upgrade its OGN than to invest in an NGN.

An interesting question is how welfare compares in the “leapfrogging” and “leapfrogging with upgrade” equilibria. That is, provided that the entrant is the leader in NGN investments, should the regulator allow or forbid OGN upgrade? We have the following result.

**Proposition 4.** *If  $\bar{\delta} < 413.25$ , welfare is higher under the “leapfrogging” equilibrium than under the “leapfrogging with upgrade” equilibrium. Otherwise, the reverse is true.*

*Proof.* See Appendix B8. □

This result suggests that if the quality upgrade is limited, the regulator should forbid an OGN upgrade, because it would result in a partial migration to the NGN and lower quality levels. Note that this is true even though the upgrade is costless in our setting. Otherwise, if the maximum quality achievable under the OGN quality is sufficiently high, the regulator should allow the upgrade.

## 5.2 Persistence of leadership with OGN upgrade

In this configuration, two OGN qualities and two NGN qualities are available to consumers, with  $\delta_E < \mu_E < \tilde{\delta}_I < \mu_I$ . As in the previous case, we determine the marginal consumers between the different options, and find the firms’ demands,

$$D_E^O = \frac{m_E - d_E}{\mu_E - \delta_E} - \frac{d_E}{\delta_E}, D_E^N = \frac{d_I - m_E}{\tilde{\delta}_I - \mu_E} - \frac{m_E - d_E}{\mu_E - \delta_E}, D_I^O = \frac{m_I - d_I}{\mu_I - \tilde{\delta}_I} - \frac{d_I - m_E}{\tilde{\delta}_I - \mu_E}, D_I^N = \bar{\theta} - \frac{m_I - d_I}{\mu_I - \tilde{\delta}_I}.$$

### 5.2.1 Competition stage

The firms' profits are

$$\pi_I = d_I D_I^O + m_I D_I^N + a D_E^O \quad \text{and} \quad \pi_E = (d_E - a) D_E^O + m_E D_E^N.$$

Solving for the four first order conditions,<sup>29</sup> we find the demands in the price equilibrium. The entrant's demand for OGN services in this price equilibrium, is

$$D_E^O = \frac{-a\mu_E}{2\delta_E(\mu_E - \delta_E)}.$$

As  $D_E^O \leq 0$  for any  $a \geq 0$ , there is no interior equilibrium to the competition stage subgame where the entrant's demand for OGN services is strictly positive. This is because, when the entrant prices its OGN service, it seeks to minimize the cannibalization with its own NGN service. Since the entrant's OGN service cannot be active in equilibrium, we compute the equilibrium when the entrant offers only an NGN service, while the incumbent offers both OGN and NGN services. In equilibrium, the demands for the firms' services are then:

$$D_E^O = 0, \quad D_I^N = 50, \quad D_I^O = \frac{50\mu_E}{4\tilde{\delta}_I - \mu_E}, \quad \text{and} \quad D_E^N = \frac{100\delta_I}{4\tilde{\delta}_I - \mu_E}.$$

Note that demands are independent of the access charge.

### 5.2.2 Access pricing

Since equilibrium prices and demands at the competition stage do not depend on the access charge, the choice of the access price is irrelevant.

### 5.2.3 NGN investments and OGN upgrade

Assume that  $\delta_E < \mu_E < \tilde{\delta}_I < \mu_I$  holds in equilibrium. At stage 1, firm I and firm E decide on  $\tilde{\delta}_I$ ,  $\mu_I$  and  $\mu_E$ , respectively, and firms anticipate that once investments have been realized, in the continuation game there will be no demand for firm E's OGN services. We determine the firms' optimal investment in this case, and check that the outcome corresponds to an equilibrium. We can then state the following result.

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<sup>29</sup>As in the previous cases, we have verified that the Hessian matrix is negative definite, which ensures that the system of first order conditions identifies a maximum.

**Proposition 5.** *There is a “persistence of leadership with upgrade” equilibrium, such that  $\delta_E < \mu_E^* < \tilde{\delta}_I^* < \mu_I^*$ . Firm I upgrades its OGN at the maximum achievable quality ( $\tilde{\delta}_I^* = \bar{\delta}$ ), and invests in an NGN of quality  $\mu_I^* = 2500$ . Firm E sells only NGN services of quality  $\mu_E^*(\bar{\delta})$ .*

*Proof.* See Appendix B7. □

This result shows that, when it is leader in NGN investments, and can upgrade its OGN, the incumbent fully upgrades the OGN (i.e., to the maximum achievable quality), and also invests in an NGN of very high quality.<sup>30</sup> The entrant invests in an NGN, but becomes the provider with the lowest quality of service.

This is in contrast with the “persistence of leadership” equilibrium without upgrade, where the entrant was offering the second highest quality. Therefore, the possibility of upgrade allows the incumbent to restrict its competitor to the lowest market segment.

A relevant question is then whether the regulator should forbid the OGN upgrade. To see that, we compare the welfare under the “persistence of leadership” equilibrium and the “persistence of leadership with upgrade” equilibrium. We have the following result.

**Proposition 6.** *The social welfare is always lower under the “leapfrogging with upgrade” equilibrium than under the “leapfrogging” equilibrium.*

*Proof.* See Appendix B8. □

This result suggests that an OGN upgrade leads to an inefficient market outcome. Forbidding the incumbent to upgrade its OGN would therefore be welfare-enhancing.

**Risk of foreclosure.** The fact that the entrant is relegated to the lowest segment of the market, with only one active variety, suggests that there might be a risk of foreclosure. Assume that there is a fixed cost for the entrant of building an NGN of strictly positive quality (e.g., due to duct trenching).<sup>31</sup> Instead of upgrading its OGN to the maximum quality, the incumbent could rather set its OGN quality  $\tilde{\delta}_I$  at a sufficiently low level, close to the entrant’s NGN quality  $\mu_E$ , to deter the entrant from investing in an NGN: due to the intense competition from the incumbent’s OGN, the entrant might not generate enough profits to cover its fixed cost.

In such a case, the entrant would then decide not to invest in an NGN, and to offer only OGN services. However, if the entrant offers only OGN services, the incumbent has an incentive to set

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<sup>30</sup>The incumbent’s NGN investment level corresponds actually to the monopoly level

<sup>31</sup>The same argument would hold if we assumed a minimal quality for the NGN.

the access charge to the OGN at a prohibitive level to foreclose completely its competitor—as in our benchmark of Section 2.

To sum up, if the incumbent is allowed to upgrade its OGN infrastructure and is the leader in NGN investments, a risk of foreclosure might emerge, which calls for a regulation of the access to the OGN. Note that this is in contrast with the other cases, where the incumbent always sets the OGN access charge at the socially optimal level.

## 6 Conclusion

In this paper, we analyze the migration from an old to a new technology, when an incumbent and an entrant can use both technologies, and the entrant leases access to the incumbent’s old generation network. The two firms have to decide, first, on their investment in a new generation network, and then on the prices they charge consumers for using the old and the new technologies. Due to a lack of commitment, the access charge to the old network is set by the regulator or the incumbent after investments have taken place.

When the quality of the old generation network is given and exogenous, we show that two equilibria emerge: a persistence-of-leadership equilibrium, where the incumbent remains the leader with the new technology, and a leapfrogging equilibrium, where the entrant becomes the new leader. In the leapfrogging equilibrium, the migration to the new technology is complete, and the old network is switched off, via a very high access price set either by the incumbent or the regulator. By contrast, in the persistence-of-leadership equilibrium, the old network remains active, and we show that it is an inefficient outcome. A formal intervention of the regulator to switch off the old network after investments would be desirable. In this equilibrium, an exogenous upgrade of the old generation network is also welfare-decreasing.

We then extend our setting to allow the incumbent to upgrade the old network, as the same time as the two firms invest in the new networks. Allowing for upgrade yields two additional potential equilibria: a persistence-of-leadership equilibrium with upgrade, and a leapfrogging equilibrium with upgrade.

In the leapfrogging equilibrium with upgrade, the incumbent renounces to invest in a next generation network, and rather upgrades the old network. If the quality achievable with the upgrade is not too high, welfare is lower than in the leapfrogging equilibrium without upgrade. In such a case, forbidding the incumbent to upgrade its network would be welfare-enhancing. In the persistence-

of-leadership equilibrium with upgrade, the incumbent upgrades its old network to the maximum achievable quality, and also invests in an new network of very high quality. We show that this market outcome leads to a lower welfare than the persistence-of-leadership equilibrium without upgrade. We also argue that because the entrant is relegated to the lowest market segment, a risk of foreclosure might arise. This suggests that the regulator should not allow the OGN upgrade.

From a policy perspective, our results suggest that the access price to the old network might not be enough to achieve efficient outcomes, for the migration from an old technology to a new one. Additional instruments, such a legal switch-off of the old network or forbidding upgrades to the old network, might be necessary for the regulator to orient the market towards efficient outcomes.

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## Appendix

### Appendix A1: Proof of Lemma 1

*Proof.* We start by showing that the access charge has a positive direct effect on the entrant’s OGN price. The entrant’s OGN service always offers the lowest quality,  $\delta_E$ . Let us denote by  $\sigma$  (resp.,  $s$ ) the quality (resp., price) of the closest rival product, where  $\sigma > \delta_E$ . The entrant’s best-response for the OGN service,  $d_E^{BR}$ , then satisfies the FOC

$$\frac{\partial \pi_E}{\partial d_E} = 0 = \frac{s - d_E^{BR}}{\sigma - \delta_E} - \frac{d_E^{BR}}{\delta_E} - \frac{\sigma (d_E^{BR} - a)}{(\sigma - \delta_E) \delta_E}. \quad (7)$$

From the implicit function theorem, since the SOC holds, we have  $d_E^{BR} = d_E^{BR}(a, s, \sigma, \delta_E)$ , and the sign of  $\partial d_E^{BR} / \partial a$  is the same as the sign of the derivative of (7) with respect to  $a$ , that is,  $s / ((s - \delta_E) \delta_E)$ , which is strictly positive. Therefore,  $\partial d_E^{BR} / \partial a > 0$ .

Besides, the access price can have a direct effect on the price of one of the incumbent’s services, because it enters the incumbent’s profits via the wholesale revenues  $a(\theta_L - \theta_S)$ . However, it is the case only if the incumbent commercializes the service which is contiguous to firm E’s OGN service. Let us consider that it is the case, that is, firm I offers the second lowest quality, which we denote by  $\sigma_I$  for a price of  $s_I$ . The FOC of profit maximization with respect to  $s_I$  can be written as

$$\frac{\partial \pi_I}{\partial s_I} = \frac{a}{\sigma_I - \delta_E} + [\dots] = 0, \quad (8)$$

where the term into brackets is independent of  $a$ . Let us define by  $s_I^{BR}$  the solution of the FOC. From the implicit function theorem, since the SOC holds, the sign of  $\partial s_I^{BR} / \partial a$  is the same as the sign of the derivative of (8) with respect to  $a$ , that is,  $1 / (\sigma_I - \delta_E)$ , which is strictly positive. Therefore,  $\partial s_I^{BR} / \partial a > 0$ .

We now prove that the prices for the network services are strategic complements. From (7), using the implicit function theorem, we have  $\partial d_E^{BR} / \partial s > 0$ . For the other prices, we begin by considering firm I’s services. Firm I’s profit can be written as

$$\pi_I = d_I D_I^O(d_I, \vec{r}_I^O) + a D_E^O(d_E, \vec{r}_E^O) + m_I D_I^N(m_I, \vec{r}_I^N),$$

where  $\vec{r}_i^\tau$  represents the vector of prices for the services which compete with firm  $i$ 's technology  $\tau = O, N$ . We have

$$\frac{\partial \pi_I}{\partial d_I} = D_I^O(d_I, \vec{r}_I^O) + d_I \frac{\partial D_I^O}{\partial d_I} + a \frac{\partial D_E^O}{\partial d_I} + m_I \frac{\partial D_I^N}{\partial d_I}.$$

From the implicit function theorem, the effect of a given price  $p$  on  $d_I^{BR}$  is given by the sign of  $\partial^2 \pi_I / \partial d_I \partial p$ . Since demands are linear,  $\partial D_i^\tau / \partial p_j^{\tau'}$  is a constant, where  $i, j = I, E$ ,  $\tau, \tau' = O, N$ , and  $p_j^{\tau'}$  designates the price of firm  $j$ 's technology  $\tau' = O, N$ . Therefore, we have

$$\frac{\partial^2 \pi_I}{\partial d_I \partial p} = \frac{\partial D_I^O}{\partial p}.$$

Besides, remark that  $\partial D_i^\tau / \partial p_j^{\tau'} > 0$  if  $\tau \neq \tau'$ . That is, the demand for a firm's service increases in the prices of the rival services (controlled by the same firm or the rival firm). It follows that  $\partial^2 \pi_I / \partial d_I \partial p > 0$ , which implies that  $d_I$  and  $p$  are strategic complements. Using the same methodology, we prove that all prices are strategic complements.

To sum up, when the access charge  $a$  increases, this pushes up  $d_E^{BR}$  and possibly  $d_I^{BR}$ , due to the direct effects. Then, since at least one price is directly increasing, due to the strategic complementarities, all prices increase.  $\square$

## Appendix A2: Consumer surplus and total welfare

Total welfare is defined as the sum of consumer surplus and firms' profits, that is,  $W = CS_E^O + CS_I^O + CS_E^N + CS_I^N + \Pi_I + \Pi_E$ , where  $CS_i^\tau$  denotes the surplus of the consumers of firm  $i$ 's service that relies on technology  $\tau = O, N$ . Below, we provide the expressions for consumer surplus in the two possible cases.

### “Leapfrogging” case:

$$CS_E^O = \int_{\theta_S}^{\theta_L} (x\delta_E - d_E)dx = \frac{(d_E\delta_I - d_I\delta_E)^2}{2\delta_E(\delta_I - \delta_E)^2},$$

$$CS_I^O = \int_{\theta_L}^{\theta_M} (x\delta_I - d_I)dx = \frac{\delta_I}{2} \left( \frac{(d_I - m_I)^2}{(\delta_I - \mu_I)^2} - \frac{(d_I - d_E)^2}{(\delta_I - \delta_E)^2} \right) - \left( \frac{d_I - m_I}{\delta_I - \mu_I} - \frac{d_I - d_E}{\delta_I - \delta_E} \right) d_I,$$

$$CS_I^N = \int_{\theta_M}^{\theta_H} (x\mu_I - m_I)dx = \frac{\mu_I}{2} \left( \frac{(m_I - m_E)^2}{(\mu_I - \mu_E)^2} - \frac{(d_I - m_I)^2}{(\delta_I - \mu_I)^2} \right) - \left( \frac{m_I - m_E}{\mu_I - \mu_E} - \frac{d_I - m_I}{\delta_I - \mu_I} \right) m_I,$$

$$CS_E^N = \int_{\theta_H}^{\bar{\theta}} (x\mu_E - m_E)dx = \frac{\mu_E}{2} \left( \bar{\theta}^2 - \frac{(m_I - m_E)^2}{(\mu_I - \mu_E)^2} \right) - \left( \bar{\theta} - \frac{m_I - m_E}{\mu_I - \mu_E} \right) m_E.$$

”Persistence of leadership” case:

$$CS_E^O = \int_{\theta_S}^{\theta_L} (x\delta_E - d_E)dx = \frac{(d_E\delta_I - \delta_E d_I)^2}{2\delta_E(\delta_I - \delta_E)^2},$$

$$CS_I^O = \int_{\theta_L}^{\theta_M} (x\delta_I - d_I)dx = \frac{\delta_I}{2} \left( \frac{(d_I - m_E)^2}{(\delta_I - \mu_E)^2} - \frac{(d_I - d_E)^2}{(\delta_I - \delta_E)^2} \right) - \left( \frac{d_I - m_E}{\delta_I - \mu_E} - \frac{d_I - d_E}{\delta_I - \delta_E} \right) d_I,$$

$$CS_E^N = \int_{\theta_M}^{\theta_H} (x\mu_E - m_E)dx = \frac{\mu_E}{2} \left( \frac{(m_E - m_I)^2}{(\mu_E - \mu_I)^2} - \frac{(d_I - m_E)^2}{(\delta_I - \mu_E)^2} \right) - \left( \frac{m_E - m_I}{\mu_E - \mu_I} - \frac{d_I - m_E}{\delta_I - \mu_E} \right) m_E,$$

$$CS_I^N = \int_{\theta_H}^{\bar{\theta}} (x\mu_I - m_I)dx = \frac{\mu_I}{2} \left( \bar{\theta}^2 - \frac{(m_E - m_I)^2}{(\mu_E - \mu_I)^2} \right) - \left( \bar{\theta} - \frac{m_E - m_I}{\mu_E - \mu_I} \right) m_I.$$

### Appendix A3: Proof of Lemma 3

*Proof.* Replacing for the equilibrium prices into the welfare function, we derive welfare as a function of the access charge,  $a$ . This function  $W(a)$  is a second degree polynomial in  $a$ , which is therefore either strictly concave or strictly convex.<sup>32</sup> To begin with, assume that  $W(a)$  is strictly concave. To prove that the regulator sets an access charge above  $\bar{a}_L$ , it is sufficient to check that the slope of the welfare function is strictly positive at  $a = \bar{a}_L$ . We have

$$\left. \frac{dW(a)}{da} \right|_{a=\bar{a}_L} = \frac{600\mu_I(\mu_E - \mu_I)\delta_I}{(4\mu_E - \mu_I)\eta} > 0,$$

as  $\delta_I > \delta_E$  and  $\mu_E > \mu_I$ . Now, assume that  $W(a)$  is strictly convex. We always have  $dW/da|_{a=0} > 0$ . Indeed,

$$\left. \frac{dW(a)}{da} \right|_{a=0} = \frac{100(\mu_I - \mu_E)[-11\mu_I(\delta_I - \delta_E) - 9\delta_I\mu_I - 16\delta_I\mu_E - 20\delta_E\mu_E + 45\delta_E\delta_I]}{\eta^2},$$

which is always positive for  $\mu_E > \mu_I > \delta_I > \delta_E$ . Therefore,  $W(a)$  is strictly increasing over  $[0, \bar{a}_L]$ , which proves that the regulator sets  $a \geq \bar{a}_L$ . Therefore, in equilibrium, the regulator “switches off” the OGN.  $\square$

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<sup>32</sup>Due to its algebraic complexity, we omit  $W(a)$  here.

## Appendix A4: $d\Pi_E/da > 0$ (“leapfrogging” case)

A sufficient condition for  $d\Pi_E/da > 0$  to hold is

$$\frac{\partial \Pi_E}{\partial a} + \frac{\partial \Pi_E}{\partial m_I} \frac{\partial m_I}{\partial a} > 0.$$

Since the LHS of this inequality is linear in  $a$ , in order for the inequality to be satisfied, it suffices that the expression on the LHS has a positive slope and a positive intercept. We find that the intercept is equal to  $(200(\mu_E - \mu_I)\delta_I(24\delta_I(\mu_E - \delta_E) + 3\delta_I(\mu_I - \delta_I) + \mu_I(\delta_I - \delta_E) + 8\mu_E(\delta_I - \delta_E)))/\eta^2$ , while the slope is  $(2\delta_I(16\delta_I(\mu_E^2 - \delta_E\mu_I) + 7\delta_E\mu_I(\mu_E - \delta_I) + 4\delta_I\mu_I(\mu_I - \delta_E) + 32\delta_I\mu_E(\mu_E - \mu_I) + 16\mu_E^2(\delta_I - \delta_E) + \mu_I\delta_E(\mu_E - \mu_I)))/(\delta_E\eta^2)$ . Both expressions are positive for  $\delta_E < \delta_I < \mu_I < \mu_E$ .

## Appendix A5: Proof of Proposition 1.

*Proof.* Firm I’s and firm E’s profits are

$$\Pi_I = (\theta_H - \theta_L)m_I - \frac{\mu_I^2}{2}, \quad \text{and} \quad \Pi_E = (100 - \theta_H)m_E - \frac{\mu_E^2}{2},$$

where  $\theta_H = (m_E - m_I)/(\mu_E - \mu_I)$ , and  $\theta_L = m_I/\mu_I$ . Replacing for the equilibrium prices, the firms’ profits can be written as

$$\Pi_I(\mu_I, \mu_E) = 10000 \frac{(\mu_E - \mu_I)\mu_I\mu_E}{(4\mu_E - \mu_I)^2} - \frac{\mu_I^2}{2}, \quad \text{and} \quad \Pi_E(\mu_I, \mu_E) = 40000 \frac{(\mu_E - \mu_I)\mu_E^2}{(4\mu_E - \mu_I)^2} - \frac{\mu_E^2}{2}.$$

Firms choose their quality levels to maximize their profits. Solving for the system of first order conditions, we find that  $\mu_I^* = 482.3$  and  $\mu_E^* = 2533.1$ .<sup>33</sup> The firms’ equilibrium profits are  $\Pi_I^* = 152741.2$  and  $\Pi_E^* = 2443858.8$ .

In order to prove that this is a Nash equilibrium, we need to verify that both firms do not have incentive to unilaterally deviate from  $(\mu_I^*, \mu_E^*)$ . The entrant does not have incentive to deviate; the entrant, in fact, has only a possible deviation, that is, to choose  $\mu_E < \mu_E^*$ , and it is simple to check that it is not profitable to become the low quality operator. Consider now the incumbent. Given  $\mu_E^*$  and also given the quality of its OGN, here exogenously given, the incumbent might deviate by setting  $\mu_I > \mu_E^*$ . In this case, the ranking of the offered services would be  $\delta_I < \mu_E^* < \mu_I$ . This corresponds to the “persistence of leadership” scenario analyzed in 4.2. From Lemma 4, we know

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<sup>33</sup>The second order conditions are  $\partial^2 \Pi_I / \partial \mu_I^2 = -2/(\mu_E - \mu_I) - 2/\mu_I < 0$  and  $\partial^2 \Pi_E / \partial \mu_E^2 = -2/(\mu_E - \mu_I) < 0$ , which clearly identifies a maximum.

that the high quality incumbent does not switch off the OGN at the retail level. Given  $\delta_I$  and  $\mu_E$ , firm I's profits from deviation are:

$$\frac{A\mu_I^4 + B\mu_I^3 + C\mu_I^2 - D\mu_I + E}{2(2\delta_I\mu_E - 4\mu_I\mu_E + \mu_E^2 + \delta_I\mu_I)^2},$$

where  $A = (\delta_I - 4\mu_E)^2$ ,  $B = 2((2\mu_E - 2500)\delta_I + (10000 + \mu_E)\mu_E)(\delta_I - 4\mu_E)$ ,  $C = \mu_E((\mu_E - 5000)4\delta_I^2 + (75000\delta_I + \mu_E^2)\mu_E + (\delta_I + 20000)4\mu_E^2)$ ,  $D = 5000\delta_I\mu_E^2(22\mu_E + 5\delta_I)$  and  $E = 5000\delta_I\mu_E^3(10\delta_I - \mu_E)$ . It is possible to check that for  $\mu_E = \mu_E^*$ , these profits are always negative. Therefore, the incumbent does not have incentive to unilaterally deviate from  $(\mu_I^*, \mu_E^*)$ .  $\square$

### Appendix A6: $D_E^O$ is decreasing with $a$ (“persistence of leadership”)

We find that

$$\frac{\partial D_E^O}{\partial a} = \frac{2(2\delta_I\mu_E + \delta_I\mu_I - 4\mu_I\mu_E + \mu_E^2)\delta_I}{H(\delta_E)\delta_E},$$

where  $H(\delta_E) = (2\delta_I\mu_E - 8\delta_I\mu_I + 9\delta_I^2 + \mu_E^2 - 4\mu_I\mu_E)\delta_E - 8\delta_I^2\mu_E - 4\delta_I\mu_E^2 + 16\delta_I\mu_I\mu_E - 4\delta_I^2\mu_E$ . The numerator of  $\partial D_E^O/\partial a$  is negative, as  $\delta_E < \delta_I < \mu_E < \mu_I$ . The sign of the denominator is given by the sign of  $H(\delta_E)$ , which is linear in  $\delta_E$ . The slope of  $H(\delta_E)$  is equal to  $2\delta_I\mu_E - 8\delta_I\mu_I + 9\delta_I^2 + \mu_E^2 - 4\mu_E\mu_I$ , which is negative (using that  $\delta_E < \delta_I < \mu_E < \mu_I$ ). Furthermore,  $H(\delta_E = \delta_I) = 3\delta_I(\delta_I - \mu_E)(3\delta_I - 4\mu_I + \mu_E) > 0$  for  $\mu_I > \mu_E > \delta_I$ . This proves that  $H(\delta_E)$  is positive and that  $D_E^O$  decreases with  $a$ .

### Appendix A7: Proof of Lemma 5

*Proof.* Replacing for the equilibrium prices into the welfare function, we derive welfare as a function of the access charge,  $a$ . This function  $W(a)$  is a second degree polynomial in  $a$ , which is therefore either strictly concave or strictly convex.<sup>34</sup> To begin with, assume that  $W(a)$  is strictly concave. To prove that the regulator sets an access charge above  $\bar{a}_P$ , it is sufficient to check that the slope of the welfare function is strictly positive at  $a = \bar{a}_P$ . We have

$$\left. \frac{dW(a)}{da} \right|_{a=\bar{a}_P} = \frac{150\delta_I(\mu_I - \mu_E)(\delta_I - \mu_E)(\mu_I\delta_I - 6\delta_I\mu_E + \mu_E^2 + 4\mu_I\mu_E)}{G(\delta_E)(\mu_I\delta_I + \mu_E^2 + 2\delta_I\mu_E - 4\mu_I\mu_E)},$$

where  $G(\delta_E) = (\mu_E^2 + 9\delta_I^2 - 8\mu_I\delta_I + 2\delta_I\mu_E - 4\mu_I\mu_E)\delta_E - 4\delta_I\mu_E^2 - 4\mu_E\delta_I^2 - 8\delta_I^2\mu_E + 16\mu_E\delta_I\mu_E$ .

As  $\delta_E < \delta_I < \mu_E < \mu_I$ , the numerator of  $dW(a)/da|_{a=\bar{a}_P}$  is negative. The denominator is

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<sup>34</sup>Due to its algebraic complexity, we omit  $W(a)$ .

negative if  $G(\delta_E) > 0$ .  $G(\delta_E)$  is linear and decreasing in  $\delta_E$ , since  $G'(\delta_E) = \mu_E + 9\delta_I - 8\mu_I\delta_I + 2\delta_I\mu_E - 4\mu_E\mu_I > 0$ . On top of that, we have  $G(\delta_E = \delta_I) = \delta_I(\delta_I - \mu_E)(3\delta_I + \mu_E - 4\mu_I) > 0$ , which proves that  $dW(a)/da|_{a=\bar{a}_P} > 0$ .

Now, assume that  $W(a)$  is strictly convex. We always have  $dW/da|_{a=0} > 0$ . Indeed,

$$\left. \frac{dW(a)}{da} \right|_{a=0} = \frac{-400(\mu_I - \mu_E)(\delta_I - \mu_E)\delta_I J(\mu_I)}{(G(\delta_E))^2},$$

where  $J(\mu_I) = (8\delta_I\mu_E + \delta_I + \delta_E\mu_E - 10\delta_I\delta_E)\mu_I - \mu_E^2\delta_E - 10\delta_I^2\mu_E + 9\delta_I^2\delta_E + \delta_E\delta_I\mu_E + \delta_I\mu_E^2$ . The denominator of  $dW(a)/da|_{a=0}$  is strictly positive. As  $\mu_E > \mu_I > \delta_E > \delta_I$ , the numerator is positive if  $J(\mu_I) > 0$ . Since  $J'(\mu_I) = 8\delta_I\mu_E + \delta_I^2 + \delta_E\mu_E - 10\delta_I\delta_E > 0$ , and  $J(\mu_I = \mu_E) = 9\delta_I(\mu_I - \delta_I)(\mu_I - \delta_E) > 0$ , we have  $J(\mu_I) > 0$  for all  $\mu_I \geq \mu_E$ . This proves that  $dW(a)/da|_{a=0} > 0$ . Therefore, in equilibrium, the regulator switches off the OGN of the entrant firm.  $\square$

### Appendix A8: $d\Pi_E/da > 0$ (“persistence of leadership” case)

We find that  $\Pi_E(a)$  is a second degree polynomial in  $a$ , with

$$\frac{d^2\Pi_E(a)}{da^2} = \frac{K}{2\delta_E M^2},$$

where  $K = (-16\delta_I\mu_E^4 + (-64\delta_I^2 + 128\delta_I\mu_I)\mu_E^3 + (80\delta_I^2\mu_I - 256\delta_I\mu_I^2 + 80\delta_I^3)\mu_E^2 + (-64\delta_I^3\mu_I + 272\delta_I^2\mu_I^2 - 144\delta_I^4)\mu_E - 160\delta_I^3\mu_I^2 + 144\delta_I^4\mu_I)\delta_E + 16\delta_I^2\mu_E^4 + (64\delta_I^3 - 128\delta_I^2\mu_I)\mu_E^3 + (64\delta_I^4 - 224\delta_I^3\mu_I + 256\delta_I^2\mu_I^2)\mu_E^2 + (64\delta_I^4\mu_I - 128\delta_I^3\mu_I^2)\mu_E + 16\delta_I^4\mu_I^2$ , and  $M = -4\delta_E\mu_I\mu_E + 9\delta_E\delta_I^2 + \delta_E\mu_E^2 + 2\delta_E\delta_I\mu_E - 4\delta_I^2\mu_I - 8\delta_I^2\mu_E + 16\delta_I\mu_I\mu_E - 4\delta_I\mu_E^2 - 8\delta_E\delta_I\mu_I$ .

The denominator of  $d^2\Pi_E(a)/da^2$  is positive, hence the sign of the second order derivative is the same as the sign of  $K$ .  $K$  is a linear function in  $\delta_E$ , and it takes positive values at the extremes (namely, at  $\delta_E = 0$  and  $\delta_E = \delta_I$ ). Hence, we have  $K > 0$  for any admissible value of the quality parameters. This implies that  $\Pi_E(a)$  is a convex function. To prove that  $\Pi_E(a)$  increases with  $a$ , it is therefore enough to show that its derivative is positive at  $a = 0$ . We find that

$$\left. \frac{d\Pi_E(a)}{da} \right|_{a=0} = \frac{400(\mu_I - \mu_E)(\mu_E - \delta_I)\delta_I}{M^2} L(\delta_E),$$

where  $L(\delta_E) = (9\delta_I^2 - 10\delta_I\mu_I + \delta_I\mu_E + \mu_I\mu_E - \mu_E^2)\delta_E + \delta_I^2\mu_I - 10\delta_I^2\mu_E + 8\delta_I\mu_I\mu_E + \delta_I\mu_E^2$ .  $L(\delta_E)$  is linear in  $\delta_E$  and at the extremes it takes positive values. Hence,  $L(\delta_E)$  is positive, and  $d\Pi_E(a)/da|_{a=0} > 0$ . This proves that  $d\Pi_E/da > 0$  for all  $a$ .

## Appendix A9: Proof of Proposition 2

*Proof.* Firm I's and firm E's profits are

$$\Pi_I = (100 - \theta_H)m_I + (\theta_M - \theta_L)d_I - \frac{\mu_I^2}{2}, \quad \text{and} \quad \Pi_E = (\theta_H - \theta_M)m_E - \frac{\mu_E^2}{2},$$

where  $\theta_H = (m_I - m_E)/(\mu_I - \mu_E)$ ,  $\theta_M = (m_E - d_I)/(\mu_E - \delta_I)$ , and  $\theta_L = d_I/\delta_I$ . Replacing for the equilibrium prices, the firms' profits can be written as

$$\Pi_I(\mu_I, \mu_E) = 2500 \left( \frac{(\delta_I \mu_I - 4\mu_I \mu_E + 3\delta_I \mu_E)^2 (\mu_I - \mu_E)}{(2\delta_I \mu_E - 4\mu_I \mu_E + \mu_E^2 + \delta_I \mu_I)^2} - \frac{\delta_I (\delta_I - \mu_E) (\mu_I - \mu_E)^2 \mu_E}{(2\delta_I \mu_E - 4\mu_I \mu_E + \mu_E^2 + \delta_I \mu_I)^2} \right) - \frac{\mu_I^2}{2}, \quad (9)$$

and

$$\Pi_E(\mu_I, \mu_E) = 10000 \frac{\mu_E^2 (\delta_I - \mu_E) (\delta_I - \mu_I) (\mu_I - \mu_E)}{(2\delta_I \mu_E - 4\mu_I \mu_E + \mu_E^2 + \delta_I \mu_I)^2} - \frac{\mu_E^2}{2}.$$

Given  $\delta_I$ , firms choose their NGN investments to maximize their profits. The system of first order conditions cannot be solved analytically. Therefore, we revert to numerical simulations to obtain the optimal functions  $\mu_I^*(\delta_I)$  and  $\mu_E^*(\delta_I)$  shown in Figure 2. We use these equilibrium levels of investment in NGN quality to derive the equilibrium profits and the social welfare, for a given  $\delta_I$  (see Figure 3).

In order to prove that this is a Nash equilibrium, we need to verify that both firms do not have incentive to unilaterally deviate from  $(\mu_I^*(\delta_I), \mu_E^*(\delta_I))$ . The entrant may deviate by choosing  $\mu_E > \mu_I^*(\delta_I)$ , that is, by playing a “leapfrogging” game. In this case, the entrant's profit function, given  $\mu_I = \mu_I^*(\delta_I)$ , is

$$\frac{40000(\mu_E - \mu_I^*(\delta_I))\mu_E}{(4\mu_E - \mu_I^*(\delta_I))^2} - \frac{\mu_E^2}{2}.$$

It is possible to check that for any admissible value of  $\delta_I$  and  $\mu_E$ , this expression is always negative. Hence, the entrant does not have incentive to unilaterally deviate from the candidate equilibrium.

Let us now consider the incumbent. Its possible deviation is to offer an NGN of a quality lower than  $\mu_E^*(\delta_I)$ . The game would then become a “leapfrogging” game, where the incumbent switches off its OGN (see Lemma 2). Given that  $\mu_E = \mu_E^*(\delta_I)$ , the profit function of the incumbent in this case is

$$\frac{10000(\mu_E^*(\delta_I) - \mu_I)\mu_I\mu_E^*(\delta_I)}{(4\mu_E^*(\delta_I) - \mu_I)^2} - \frac{\mu_I^2}{2}.$$

The first order condition can be solved only through a numerical simulation. We obtain the

optimal deviation  $\mu_I^D(\delta_I)^{35}$  and the profits enjoyed by the incumbent from deviating. We then check that these profits are always lower than the profits at the candidate equilibrium  $\Pi_I^*(\delta_I)$ . We can therefore conclude that the incumbent does not have incentive to unilaterally deviate from  $(\mu_I^*(\delta_I), \mu_E^*(\delta_I))$ .  $\square$

### Appendix A10: Alternative equilibria.

We verify that there are no other equilibria than the ones highlighted in Propositions 1 and 2. We consider the following alternative market configurations: (1)  $\delta_E < \mu_I < \delta_I < \mu_E$ ; (2)  $\delta_E < \mu_E < \delta_I < \mu_I$ ; (3)  $\delta_E < \mu_E < \mu_I < \delta_I$ ; (4)  $\delta_E < \mu_I < \mu_E < \delta_I$ . Note that configuration (1) is identical to configuration (L1) in the second part of the paper, configuration (2) to (L2) and so on. The main difference is that here the quality of the incumbent's OGN is exogenous and does not exceed 228.29 (Assumption A1).

**Configuration (1):**  $\delta_E < \mu_I < \delta_I < \mu_E$ .

This scenario is similar to scenario (L1). From Subsection 5.1, we know that when  $\delta_E < \mu_I < \delta_I < \mu_E$ , at the equilibrium of the access price subgame, firm I does not offer NGN services, while firm E does not supply OGN services. Firm I and firm E's first-period profits are therefore identical to expressions (10) and (11) in Appendix B6, with  $\delta_I$  instead of  $\tilde{\delta}_I$ . The firms' investments at the candidate equilibrium,  $\mu^*(\delta_I)$  and  $\mu_I^*$ , respectively, are as shown in Figure 4, while firms' profits are as shown in Figure 5 (both these pictures must be considered for the relevant part, where  $\delta_I < 228.29$ ). We find that configuration (1) cannot be an equilibrium, as firm I finds it always optimal to deviate and to invest in an NGN of a quality larger than  $\delta_I$ .

**Configuration (2):**  $\delta_E < \mu_E < \delta_I < \mu_I$ .

This scenario is similar to scenario (L2). From Subsection 5.2, we know that when  $\delta_E < \mu_E < \delta_I < \mu_I$ , at the equilibrium of the access price subgame, firm E does not supply OGN services. Firm I's optimal OGN investment is then  $\mu_I^* = 2500$ . Figure 7 displays firms' investments at the candidate equilibrium (note that Assumption A1 still holds and the picture must be considered for the relevant values of  $\delta_I$ ). We find that configuration (2) cannot be an equilibrium: firm E finds it optimal to deviate and to invest in an NGN of a quality larger than  $\delta_I$ . By deviating, firm E prefers to play the game where  $\delta_I < \mu_E < 2500$  that is strategically identical to the one studied in Section 4.2. Firm E's profit in this game is similar to expression (10); a simple differentiation is enough to

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<sup>35</sup> Assumption A1 guarantees that the optimal deviation  $\mu_I^D(\delta_I)$  is lower than  $\delta_I$ .



prove that firm E obtains higher profits in case of deviation than at the candidate equilibrium.

**Configurations (3) and (4):**  $\delta_E < \mu_E < \mu_I < \delta_I$  and  $\delta_E < \mu_I < \mu_E < \delta_I$ .

These scenarios are similar to scenarios (L3) and (L4) in Section 5. Following similar arguments as above, we find that these two configurations cannot be Nash equilibria since, under Assumption A1, both firms may profitably deviate by investing in NGN services of a quality higher than  $\delta_I$ .

Finally, since under Assumption A1, investing in NGN services of a quality higher than  $\delta_I$  is always a profitable deviation, market configurations where  $\mu_i < \delta_E$  cannot be Nash equilibria.

### Appendix B0: Configurations (S1) and (S2) with endogenous upgrade.

Let us start with configuration (S1), characterized by the following ranking  $\delta_E < \tilde{\delta}_I < \mu_I < \mu_E$ . The candidate equilibrium is  $\mu_I^* = 482.3$  and  $\mu_E^* = 2533.1$ . In order to verify if this is a Nash equilibrium also when firm I is allowed to upgrade its OGN, we need to consider firms profits in case of unilateral deviations.

Let us start with firm I; given  $\mu_E^* = 2533.1$ , I may deviate as follows: *i*) by investing in a high quality NGN that is by playing a type (S2) game,  $\tilde{\delta}_I < \mu_E^* < \mu_I$ , *ii*) by upgrading the OGN, namely playing a type (L2) or (L3) game, that is  $\mu_E^* < \tilde{\delta}_I < \mu_I$  or  $\mu_E^* < \mu_I < \tilde{\delta}_I$  respectively, and *iii*) by playing a type (L1) game,  $\mu_I < \tilde{\delta}_I < \mu_E^*$ . In deviation *i*), we know that firm I does not sell OGN services; hence, the deviation collapses to  $\mu_I > \mu_E^*$ . In deviation *iii*) we know that firm I uses prices to shut down its NGN services (they are contiguous to OGN); hence this deviation collapses to  $\tilde{\delta}_I < \mu_E^*$ .

We find that deviation *i*) is never profitable due to the high investment cost, deviations *ii*) are impossible provided that  $\bar{\delta} < 1250$  while deviation *iii*) is profitable only for  $\bar{\delta} > 258.4$ , that is when firm I can sufficiently upgrade its OGN services.

Consider now firm E. Given  $\mu_I^* = 482.3$ , the entrant has only a possible deviation that is to produce  $\mu_E < \mu_I^*$ . This is obviously not profitable since it is never desirable to become low quality producer.

Let us now move to configuration (S2), with  $\delta_E < \tilde{\delta}_I < \mu_E < \mu_I$ . We know that in this market configuration, firm I invests in NGN and it does not switch off the OGN. Firm I first period profit function is similar to expression (9), with  $\tilde{\delta}_I$  instead of  $\delta_I$ . From the derivative of this profit function with respect to  $\tilde{\delta}_I$ , it is immediate to see that as  $\delta_E < \tilde{\delta}_I < \mu_E < \mu_I$ ,  $\partial \Pi_I(\tilde{\delta}_I, \mu_I, \mu_E) / \partial \tilde{\delta}_I < 0$ , hence firm I sets  $\tilde{\delta}_I = \delta_I$ , that is it does not upgrade its OGN.

Therefore the candidate equilibrium in the (S2) game with upgrade is exactly the same as without upgrade; hence, using Proposition 2, we know that firm I offers OGN services of quality  $\delta_I$  and NGN services of quality  $\mu_I^*(\delta_I)$ , whereas firm E offers only NGN services of quality  $\mu_E^*(\delta_I)$ , with  $\mu_E^*(\delta_I)$  and  $\mu_I^*(\delta_I)$  given in Figures 2 and 3.

We check if this is an equilibrium and let us start from firm I; its possible deviations are: *i*) to become the low quality provider, that is to offer very low quality NGN services by playing either game (S1), with  $\delta_I < \mu_I < \mu_E^*(\delta_I)$  or the game with configuration  $\mu_I < \delta_I < \mu_E^*(\delta_I)$ , *ii*) to upgrade the OGN by playing a type (L2) game, with  $\mu_I < \tilde{\delta}_I < \mu_E^*(\delta_I)$ , *iii*) to play a type (L4) game, with  $\mu_E^*(\delta_I) < \mu_I < \tilde{\delta}_I$  or, finally, *iv*) to play a type (L3) game by upgrading the OGN and by reducing the NGN investment at the same time,  $\mu_I < \mu_E^*(\delta_I) < \tilde{\delta}_I$ .

Deviations *i*) are not profitable; as always, it is not desirable to become the low quality provider. On the top of that, they are not admissible for  $\delta_I < 228.29$ . Consider now deviation *ii*). In this case, given  $\mu_E^*(\delta_I)$ , firm I upgrades its OGN to  $\tilde{\delta}_I \leq \bar{\delta}$ . Firm I first period profits in case of deviation are:

$$\Pi_I(\delta_I, \tilde{\delta}_I, \mu_I) = 2500 \frac{(4\mu_I \tilde{\delta}_I - 3\tilde{\delta}_I \mu_E^*(\delta_I) - \mu_I \mu_E^*(\delta_I))}{(4\tilde{\delta}_I - \mu_E^*(\delta_I))} + 10000 \frac{\mu_E^*(\delta_I)(\tilde{\delta}_I \mu_E^*(\delta_I)) \tilde{\delta}_I}{(4\tilde{\delta}_I - \mu_E^*(\delta_I))^2} - \frac{\mu_I^2}{2}$$

From the first order conditions, we find that firm I deviates by upgrading the OGN at the maximum level,  $\tilde{\delta}_I = \bar{\delta}$  and by investing  $\mu_I = 2500$ . Plugging these values into the above profit function we obtain the profits from deviation, as a function of  $\delta_I$  and  $\bar{\delta}$ , firm I's initial OGN quality and the OGN maximum quality respectively. We find that deviation *ii*) is profitable for  $\bar{\delta} > \bar{\delta}_L(\delta_I)$ , where  $\bar{\delta}_L(\delta_I)$  is the function that implicitly equates firm I profits at the candidate equilibrium and at deviation, formally  $\bar{\delta}_L(\delta_I) = \frac{0.058(\delta_I)^2 + 7.61\delta_I + 1264.21}{16.64 + 0.02\delta_I}$ .  $\bar{\delta}_L(\delta_I)$  is increasing in  $\delta_I$  and as  $\delta_I < 228.2$ , a sufficient condition for firm I to deviate is  $\bar{\delta} > 785.66$ .

Finally, it is possible to show that as  $\bar{\delta} \leq 1250$ , deviations *iii*) and *iv*) are never profitable.

## Appendix B1: Configurations (L3) and (L4) are not Nash equilibria.

The quality ranking in configuration (L3) is  $\delta_E < \mu_I < \mu_E < \tilde{\delta}_I$ . As in the other cases, we find that firm I (or the regulator) finds optimal to set the access charge such that firm I does not supply OGN services; hence, the market configuration collapses to  $\mu_I < \mu_E < \tilde{\delta}_I$ . Following the by now standard procedure, we can determine second period prices and then, back to the investment stage,

the first period profit functions:

$$\begin{aligned}\Pi_I(\tilde{\delta}_I, \mu_I, \mu_E) &= 2500 \frac{(\mu_I \tilde{\delta}_I - 4\tilde{\delta}_I \mu_E + 3\mu_I \mu_E)^2 (\tilde{\delta}_I - \mu_E)}{(2\mu_I \mu_E + \mu_I \tilde{\delta}_I - 4\tilde{\delta}_I \mu_E + \mu_E^2)^2} - 2500 \frac{\mu_I (\tilde{\delta}_I - \mu_E)^2 (\mu_I - \mu_E) \mu_E}{(2\mu_I \mu_E + \mu_I \tilde{\delta}_I - 4\tilde{\delta}_I \mu_E + \mu_E^2)^2} - \frac{\mu_I^2}{2} \\ \Pi_E(\tilde{\delta}_I, \mu_I, \mu_E) &= \frac{10000 \mu_E^2 (\mu_E - \mu_I) (\tilde{\delta}_I - \mu_I) (\tilde{\delta}_I - \mu_E)}{(2\mu_I \mu_E + \mu_I \tilde{\delta}_I - 4\tilde{\delta}_I \mu_E + \mu_E^2)^2} - \frac{\mu_E^2}{2}\end{aligned}$$

From firm I's first order condition, we find that  $\partial \Pi_I / \partial \mu_I < 0$  and  $\partial \Pi_I / \partial \tilde{\delta}_I > 0$ ; hence  $\mu_I^* = 0$  and  $\tilde{\delta}_I = \bar{\delta}$ . Plugging these values into firm E's first order condition, we find the optimal investment for firm E  $\mu_E^*(\bar{\delta})$ . Once the investment levels have been derived, we compute firms profits at the candidate equilibrium.

We find that this is not a Nash equilibrium. In fact, as  $\bar{\delta} = 1250$ , firm E finds optimal to deviate by increasing its NGN investment. More specifically, firm E optimally deviates by investing  $\mu_E > \bar{\delta}$ , that is by playing game (L1).

Let us now consider configuration (L4); in this case,  $\delta_E < \mu_E < \mu_I < \tilde{\delta}_I$ . As always, the model reduces to  $\mu_E < \mu_I < \tilde{\delta}_I$  as firm I (or the regulator) sets the access charge to shut down firm E's OGN operations. Solving the second period stage and then back to the investment stage, firms' profit functions are:

$$\begin{aligned}\Pi_I(\tilde{\delta}_I, \mu_I, \mu_E) &= 2500 \frac{(4\mu_I \tilde{\delta}_I - 3\mu_E \mu_I - \mu_E \tilde{\delta}_I)}{4\mu_I - \mu_E} + 10000 \frac{(\mu_I - \mu_E) \mu_E \mu_I}{(4\mu_I - \mu_E)^2} - \frac{\mu_I^2}{2} \\ \Pi_E(\mu_I, \mu_E) &= 10000 \frac{(\mu_I - \mu_E) \mu_E \mu_I}{(4\mu_I - \mu_E)^2} - \frac{\mu_E^2}{2}\end{aligned}$$

where  $\Pi_E$  does not depend on  $\tilde{\delta}_I$  since firm E's NGN services are not contiguous to firm I's upgraded OGN. Firm I's first order condition with respect to  $\tilde{\delta}_I$  is  $\partial \Pi_I / \partial \tilde{\delta}_I = 2500 > 0$ , hence  $\tilde{\delta}_I^* = \bar{\delta}$ . Solving the system of first order conditions, we find that firm I and firm E NGN investment levels are, respectively,  $\mu_I^* = 280.12$  and  $\mu_E = 136.4$ . Also in this case, we find that this is not a Nash equilibrium; as for scenario (L3), firm E finds it optimal to deviate by investing  $\mu_E > \tilde{\delta}_I^*$ .

## Appendix B2: Proof of Lemma 6

Using the envelope theorem, the effect of a higher access charge on firm I's profit is given by

$$\frac{d\Pi_I}{da} = \underbrace{\frac{\partial\Pi_I}{\partial a}}_{(+)} + \underbrace{\frac{\partial\Pi_I}{\partial d_E}}_{(+)} \underbrace{\frac{\partial d_E^*}{\partial a}}_{(+)} + \underbrace{\frac{\partial\Pi_I}{\partial m_E}}_{(+)} \underbrace{\frac{\partial m_E^*}{\partial a}}_{(+)}.$$

The first term represents the direct effect of a higher access charge on profit, which is positive. The second and third terms are the two indirect effects. From Lemma 1, we have  $\partial d_E^*/\partial a > 0$  and  $\partial m_E^*/\partial a > 0$ . Besides, we find that

$$\left. \frac{\partial\Pi_I}{\partial d_E} \right|_{(M^*, D^*)} = \frac{4\mu_I}{\delta_E} \frac{100\delta_E (\mu_E - \tilde{\delta}_I) - (4\mu_E - \tilde{\delta}_I)a}{\kappa}$$

is positive for  $a \leq \bar{a}'_L$ . Finally, we find that  $\partial\Pi_I/\partial m_E|_{(M^*, D^*)} = d_I^*/(\mu_E - \tilde{\delta}_I) > 0$ . Therefore, the two indirect effects are positive. It follows that  $d\Pi_I/da > 0$  for all  $a < \bar{a}'_L$ . Therefore, the incumbent sets  $a \geq \bar{a}'_L$ , and forecloses the entrant's OGN operations. Besides, for  $a \geq \bar{a}'_L$ , the demand for the incumbent's NGN service is equal to 0.

## Appendix B3: Consumer surplus and welfare with upgrade

“Leapfrogging with upgrade”:

$$CS_E^O = \int_{\theta_s}^{\theta_L} (x\delta_E - d_E)dx = \frac{(d_E\mu_I - \delta_E m_I)^2}{2\delta_E(\mu_I - \delta_E)^2},$$

$$CS_I^O = \int_{\theta_M}^{\theta_H} (x\tilde{\delta}_I - d_I)dx = \frac{\tilde{\delta}_I}{2} \left( \frac{(d_I - m_E)^2}{(\tilde{\delta}_I - \mu_E)^2} - \frac{(d_I - m_I)^2}{(\tilde{\delta}_I - \mu_I)^2} \right) - \left( \frac{d_I - m_E}{\tilde{\delta}_I - \mu_E} - \frac{d_I - m_I}{\tilde{\delta}_I - \mu_I} \right) d_I,$$

$$CS_E^N = \int_{\theta_H}^{\bar{\theta}} (x\mu_E - m_E)dx = \frac{\mu_E}{2} \left( \bar{\theta}^2 - \frac{(d_I - m_E)^2}{(\tilde{\delta}_I - \mu_E)^2} \right) - \left( \bar{\theta} - \frac{d_I - m_E}{\tilde{\delta}_I - \mu_E} \right) m_E,$$

$$CS_I^N = \int_{\theta_L}^{\theta_M} (x\mu_I - m_I)dx = \frac{\mu_I}{2} \left( \frac{(m_I - d_I)^2}{(\mu_I - \tilde{\delta}_I)^2} - \frac{(m_I - d_E)^2}{(\mu_I - \delta_E)^2} \right) - \left( \frac{d_I - m_I}{\tilde{\delta}_I - \mu_I} - \frac{d_E - m_I}{\delta_E - \mu_I} \right) m_I.$$

**“Persistence of leadership with upgrade”:**

$$CS_E^O = \int_{\theta_S}^{\theta_L} (x\delta_E - d_E)dx = \frac{(d_E\mu_E - \delta_E m_E)^2}{2\delta_E(\mu_E - \delta_E)^2},$$

$$CS_O^I = \int_{\theta_M}^{\theta_H} (x\tilde{\delta}_I - d_I)dx = \frac{\tilde{\delta}_I}{2} \left( \frac{(d_I - m_I)^2}{(\tilde{\delta}_I - \mu_I)^2} - \frac{(d_I - m_E)^2}{(\tilde{\delta}_I - \mu_E)^2} \right) - \left( \frac{d_I - m_I}{\tilde{\delta}_I - \mu_I} - \frac{d_I - m_E}{\tilde{\delta}_I - \mu_E} \right) d_I,$$

$$CS_I^N = \int_{\theta_H}^{\bar{\theta}} (x\mu_I - m_I)dx = \frac{\mu_I}{2} \left( \bar{\theta}^2 - \frac{(d_I - m_I)^2}{(\tilde{\delta}_I - \mu_I)^2} \right) - \left( \bar{\theta} - \frac{d_I - m_I}{\tilde{\delta}_I - \mu_I} \right) m_I,$$

$$CS_E^N = \int_{\theta_L}^{\theta_M} (x\mu_E - m_E)dx = \frac{\mu_E}{2} \left( \frac{(m_E - d_I)^2}{(\mu_E - \tilde{\delta}_I)^2} - \frac{(m_E - d_E)^2}{(\mu_E - \delta_E)^2} \right) - \left( \frac{d_I - m_E}{\tilde{\delta}_I - \mu_E} - \frac{d_E - m_E}{\delta_E - \mu_E} \right) m_E.$$

**Appendix B4: Proof of Lemma 7**

The proof is similar to the one for Lemma 3. Replacing for the equilibrium prices into the welfare function, we derive welfare as a function of the access charge. This function  $W(a)$  is a second degree polynomial in  $a$ , which is therefore either strictly concave or strictly convex.<sup>36</sup> To begin with, assume that  $W(a)$  is strictly concave. To prove that the regulator sets an access charge above  $\bar{a}'_L$ , it is sufficient to check that the slope of the welfare function is strictly positive at  $a = \bar{a}'_L$ . We have

$$\left. \frac{dW(a)}{da} \right|_{a=\bar{a}'_L} = \frac{600\mu_I (\tilde{\delta}_I - \mu_E) \tilde{\delta}_I}{(\tilde{\delta}_I - 4\mu_E) \kappa},$$

where  $\kappa = 16\mu_I\mu_E - 4\mu_I\tilde{\delta}_I - 8\mu_I\delta_E - 4\mu_E\delta_E + \delta_E(\tilde{\delta}_I - \mu_I)$ . As  $\delta_E < \mu_I < \tilde{\delta}_I < \mu_E$ , this expression is positive. Now, assume that  $W(a)$  is strictly convex. We always have  $dW/da|_{a=0} > 0$ . Indeed,

$$\left. \frac{dW(a)}{da} \right|_{a=0} = \frac{100 (\tilde{\delta}_I - \mu_E) \left[ -11\tilde{\delta}_I(\mu_I - \delta_E) - 9\mu_I\tilde{\delta}_I - 16\mu_I\mu_E - 20\delta_E\mu_E + 45\delta_E\mu_I \right]}{\kappa^2},$$

which is always positive as  $\delta_E < \mu_I < \tilde{\delta}_I < \mu_E$ . Therefore,  $W(a)$  is strictly increasing over  $[0, \bar{a}'_L]$ , which proves that the regulator sets  $a \geq \bar{a}'_L$ . Therefore, in equilibrium, the regulator “switches off” the OGN.

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<sup>36</sup>Due to its algebraic complexity, we omit  $W(a)$ .

### Appendix B5: $d\Pi_E/da > 0$ (“leapfrogging” case with upgrade)

We proceed exactly as in Appendix A4. A sufficient condition for  $d\Pi_E/da > 0$  to hold is

$$\frac{\partial \Pi_E}{\partial a} + \frac{\partial \Pi_E}{\partial \tilde{\delta}_I} \frac{\partial \tilde{\delta}_I}{\partial a} > 0.$$

The LHS of this inequality is linear in  $a$ ; in order for the inequality to be satisfied, it suffices that this function has a positive slope and a positive intercept. We find that the intercept is equal to  $\left(200(\mu_E - \tilde{\delta}_I)\mu_I(24\mu_I(\mu_E - \delta_E) + 3\mu_I(\tilde{\delta}_I - \mu_I) + \tilde{\delta}_I(\mu_I - \delta_E) + 8\mu_E(\mu_I - \delta_E))\right)/\kappa^2$ , while the slope is  $\left(2\mu_I(16\mu_I(\mu_E^2 - \delta_E\tilde{\delta}_I) + 7\delta_E\tilde{\delta}_I(\mu_E - \mu_I) + 4\mu_I\tilde{\delta}_I(\tilde{\delta}_I - \delta_E) + 32\mu_I\mu_E(\mu_E - \tilde{\delta}_I) + 16\mu_E^2(\mu_I - \delta_E) + \tilde{\delta}_I\delta_E(\mu_E - \tilde{\delta}_I))\right)/(\delta_E\kappa^2)$ . Both expressions are positive for  $\delta_E < \mu_I < \tilde{\delta}_I < \mu_E$ .

### Appendix B6: Proof of Proposition 3 (“leapfrogging” case with upgrade)

Assuming that  $\delta_E < \mu_I < \tilde{\delta}_I < \mu_E$  holds in equilibrium, at the access pricing stage, the access charge is set at such a level that only the incumbent’s upgraded OGN and the entrant’s NGN are active. Replacing for the equilibrium prices into the profit functions, firm I’s and firm E’s first-stage profits can be written as

$$\Pi_I(\tilde{\delta}_I, \mu_I, \mu_E) = 10000 \frac{(\mu_E - \tilde{\delta}_I) \tilde{\delta}_I \mu_E}{(4\mu_E - \tilde{\delta}_I)^2} - \frac{\mu_I^2}{2}, \quad (10)$$

$$\Pi_E(\tilde{\delta}_I, \mu_I, \mu_E) = 40000 \frac{(\mu_E - \tilde{\delta}_I) \mu_E^2}{(4\mu_E - \tilde{\delta}_I)^2} - \frac{\mu_E^2}{2}. \quad (11)$$

A visual inspection of  $\Pi_I(\tilde{\delta}_I, \mu_I, \mu_E)$  reveals that firm I does not invest in an NGN, that is,  $\mu_I^* = 0$ . Therefore, everything is as if firm I decided on  $\tilde{\delta}_I$  only and firm E on  $\mu_E$  only. Note that the incumbent is constrained by the maximum level of quality for the OGN,  $\bar{\delta}$ . The unconstrained solutions of the system of the first-order conditions  $\partial \Pi_I / \partial \tilde{\delta}_I = 0$  and  $\partial \Pi_E / \partial \mu_E = 0$  are  $\tilde{\delta}_I = 5000/3$  and  $\mu_E = 8750/3$ . As  $\bar{\delta} < 5000/3$  from Assumption A2, in equilibrium the constraint  $\tilde{\delta}_I \leq \bar{\delta}$  is binding, and therefore,  $\tilde{\delta}_I^* = \bar{\delta}$ . Firm E’s optimal NGN investment can then be derived from the

first order condition, evaluated at  $\tilde{\delta}_I^* = \bar{\delta}$ ,<sup>37</sup>

$$\left. \frac{\partial \Pi_E}{\partial \mu_E} \right|_{\tilde{\delta}_I = \bar{\delta}} = \frac{40000\mu_E^2}{(4\mu_E - \bar{\delta})^2} - \frac{80000(\bar{\delta} - \mu_E)\mu_E}{(4\mu_E - \bar{\delta})^2} - \frac{320000(\bar{\delta} - \mu_E)\mu_E^2}{(4\mu_E - \bar{\delta})^3} - \mu_E = 0.$$

Solving this expression for  $\mu_E$ , we find that

$$\mu_E^*(\bar{\delta}) = \frac{H}{6} - \frac{1250}{H} \left( \bar{\delta} - \frac{10000}{3} \right) + \frac{2500}{3} + \frac{\bar{\delta}}{4},$$

where

$$H = 5 \sqrt[3]{3^4 10 \bar{\delta}^2 - 300^2 5 \bar{\delta} + 10^9 + 30 \bar{\delta} \sqrt{3(5^2 10^6 23 - 10^5 26 \bar{\delta} + 243 \bar{\delta}^2)}}.$$

Firms' investments in the "leapfrogging with upgrade" case are shown in Figure 4. Note that in order to differentiate its services from those offered by the incumbent, firm E invests in a very high quality NGN,  $\mu_E^* > 2533$ .

We now have to check that this configuration corresponds to a Nash equilibrium, namely that both the incumbent and the entrant do not have incentive to deviate unilaterally from  $(\mu_I^*, \tilde{\delta}_I^*, \mu_E^*)$ . Consider the entrant first. Firm E's possible deviation is to invest in an NGN of quality lower than  $\tilde{\delta}_I^*$  and eventually to offer also OGN services. In line with our previous findings, we find that independently of the access charge, firm E is not willing to offer OGN services, since they would cannibalize its NGN services. Therefore, firm E's optimal deviation is to offer NGN services only of a quality lower than  $\tilde{\delta}_I^*$ . We find that firm E's first period profit function from this deviation is:

$$\Pi_E^d(\mu_E) = 10000 \frac{\mu_E \tilde{\delta}_I^* (\tilde{\delta}_I^* - \mu_E)}{(4\tilde{\delta}_I^* - \mu_E)^2} - \frac{(\mu_E)^2}{2}.$$

This is a concave function. Solving for the first order condition we find firm E's optimal NGN investment in case of deviation, and hence, its profits. Obviously, the profits that firm E obtains at the candidate equilibrium and those in case of deviation both depend on  $\tilde{\delta}_I^* = \bar{\delta}$ . In Figure 6, we plot the two profit functions; it is immediate to see that for  $\bar{\delta} < 1250$ , firm E does not find it optimal to deviate.

Now, consider firm I's deviations. Given  $\mu_E^*$ , firm I has two alternatives: either to invest in an NGN of a quality higher than  $\mu_E^*$  or to invest in an NGN of a quality lower than  $\mu_E^*$ . In the former

<sup>37</sup>The second order condition is  $\partial^2 \Pi_E / \partial (\mu_E)^2 |_{\tilde{\delta}_I = \bar{\delta}} = -80000\bar{\delta}(5\mu_E + \bar{\delta}) / (4\mu_E - \bar{\delta})^4 - 1 < 0$ , which clearly identifies a maximum.

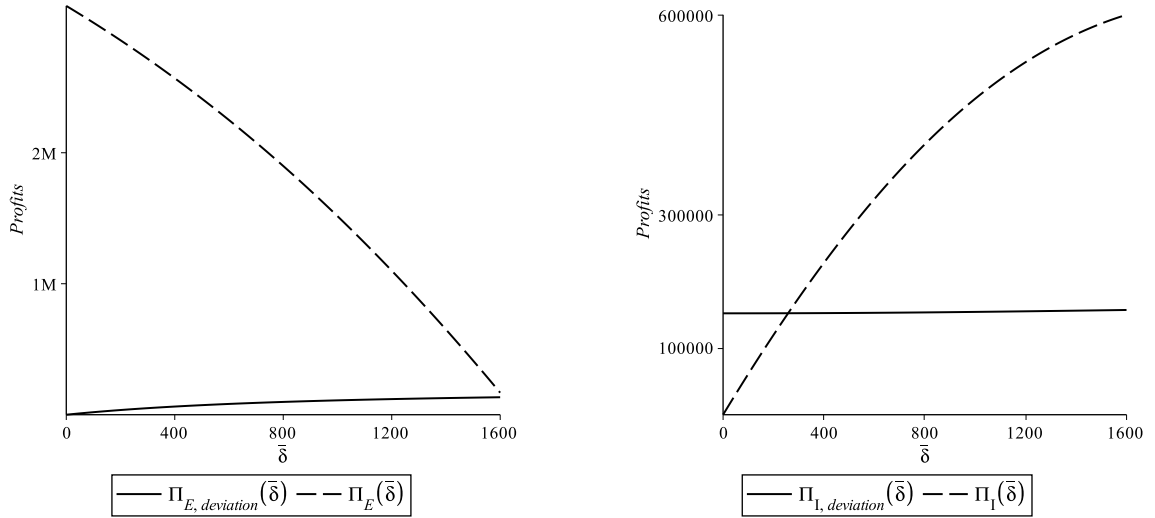


Figure 6: E deviation in the leapfrogging with Figure 7: I deviation in the leapfrogging with upgrade case

case, the only possible service quality ranking is  $\tilde{\delta}_I < \mu_E^* < \mu_I$ ;<sup>38</sup> we already know that in this case firm I does not switch off its OGN (Lemma 4) and that firm I's profit function is the same as in expression (9). We find that, due to the high costs of NGN investments, the amount of profits firm I obtains from deviating are always negative. In case of a deviation characterized by an NGN service of quality lower than  $\mu_E^*$ , the only possible ranking is  $\tilde{\delta}_I^* < \mu_I < \mu_E^*$ . From our previous analysis, we know that in this case firm I would like to switch off its OGN, hence the best deviation is to offer only NGN services of a lower quality than  $\mu_E^*$ . Firm I's first period profit function is then

$$\Pi_I^d(\mu_I) = 10000 \frac{\mu_I \mu_E^* (\mu_E^* - \mu_I)}{(4\mu_E^* - \mu_I)^2} - \frac{(\mu_I)^2}{2},$$

This function is concave in  $\mu_I$ . Solving for the first order condition, we obtain firm I's optimal NGN investment in case of deviation and the associated profits. As before, the profits at the candidate equilibrium and those in case of deviation both depend on  $\tilde{\delta}_I^* = \bar{\delta}$ . In Figure 7, we plot the two profit functions; it is immediate to see that for  $\bar{\delta}$  sufficiently large, firm I gets lower profits from deviation. Formally, we find that for  $\bar{\delta} > 257.98$  firm I does not deviate. This completes the proof.

<sup>38</sup>Note that due to Assumption A2, upgraded OGN services cannot be of a quality higher than  $\mu_E^*$ .



## Appendix B7: Proof of Proposition 5 (“persistence of leadership” case with upgrade)

Assume that  $\delta_E \leq \mu_E \leq \tilde{\delta}_I \leq \mu_I$  holds in equilibrium. Under this assumption, at the investment stage, firm I sets the quality for its OGN and NGN services, while firm E sets the quality of its NGN service. The three first-order conditions of profit maximization are<sup>39</sup>

$$\frac{\partial \Pi_I}{\partial \mu_I} = 2500 - \mu_I,$$

$$\frac{\partial \Pi_I}{\partial \tilde{\delta}_I} = \frac{2500(\mu_E)^2 (20\tilde{\delta}_I + \mu_E)}{(4\tilde{\delta}_I - \mu_E)^3},$$

and

$$\frac{\partial \Pi_E}{\partial \mu_E} = \frac{(\mu_E)^2(6\tilde{\delta}_I - \mu_E)^2 - 4(\tilde{\delta}_I)^2(17500 - 3\mu_E + 16\tilde{\delta}_I)\mu_E + 40000(\tilde{\delta}_I)^3}{(4\tilde{\delta}_I - \mu_E)^3}.$$

Note that  $\partial \Pi_I / \partial \tilde{\delta}_I > 0$  as  $\delta_E \leq \mu_E \leq \tilde{\delta}_I \leq \mu_I$ ; hence  $\tilde{\delta}_I^* = \bar{\delta}$ . From  $\partial \Pi_I / \partial \mu_I = 0$  it follows immediately that  $\mu_I^* = 2500$ . Finally, we solve numerically the first order condition  $\partial \Pi_E / \partial \mu_E = 0$  and we obtain the solution  $\mu_E^*(\bar{\delta})$ . Figure 7 represents the investment levels that solve the system of first order conditions as a function of  $\bar{\delta}$ .

In order to check if this is a Nash equilibrium, we need to determine firms' profits with unilateral deviations. Consider firm E first; the entrant has two possible deviations: *i*) to invest  $\mu_E \in (\bar{\delta}, 2500)$  or *ii*) to invest  $\mu_E > 2500$ . Deviation *ii*) is never profitable due to the large cost of the NGN investment that yields to negative profits. With deviation *i*) firm E plays a game similar to game (S2) (persistence of leadership with no upgrade); from Appendix A9 we take the first period profit function for the entrant. Plugging in this function firm I's investments  $\mu_I^* = 2500$  and  $\tilde{\delta}_I^* = \bar{\delta}$  we obtain firm E profit function from deviation. Maximizing this function we derive the optimal profits E can obtain by deviating; it is possible to show that for sufficiently large levels of  $\bar{\delta}$ , E does not find optimal to deviate; formally, E does not find optimal to deviate for  $\bar{\delta} > 284.8$ .

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<sup>39</sup>The second order derivatives for the incumbent are  $\partial^2 \Pi_I / \partial \tilde{\delta}_I^2 = -80000(\mu_E)^2(5\tilde{\delta}_I + \mu_E) / (4\tilde{\delta}_I - \mu_E)^4 < 0$  and  $\partial^2 \Pi_I / \partial \mu_I^2 = -1 < 0$ , and the determinant of the Hessian matrix is  $80000(\mu_E)^2(5\tilde{\delta}_I + \mu_E) / (4\tilde{\delta}_I - \mu_E)^4 > 0$ . The second order condition for the entrant is  $\partial^2 \Pi_E / \partial \mu_E^2 = -\left(20000(\tilde{\delta}_I)^2(8\tilde{\delta}_I + 7\mu_E) + 256(\tilde{\delta}_I)^3(\tilde{\delta}_I - \mu_E) + 16\tilde{\delta}_I(\mu_E)^2(6\tilde{\delta}_I - \mu_E) + (\mu_E)^4\right) / (4\tilde{\delta}_I - \mu_E)^4 < 0$ . Therefore, the first order conditions correspond to a maximum.

Let us now consider firm I. Firm I first period profit function is:

$$\Pi_I^*(\mu_I, \mu_E, \tilde{\delta}_I) = \frac{2500(\mu_I(4\tilde{\delta}_I - \mu_E) - 3\tilde{\delta}_I\mu_E)}{4\tilde{\delta}_I - \mu_E} + \frac{10000(\tilde{\delta}_I - \mu_E)\mu_E\tilde{\delta}_I}{(4\tilde{\delta}_I - \mu_E)^2} - \frac{(\mu_I)^2}{2}. \quad (12)$$

The candidate equilibrium is  $\mu_E^* = \mu_E^*(\bar{\delta})$ ,  $\tilde{\delta}_I^* = \bar{\delta}$  and  $\mu_I^* = 2500$ ; replacing these investments levels in expression (12) we obtain firm I's level of profits at the candidate equilibrium.

Firm I's deviation is to supply OGN services of quality lower than  $\mu_E^*(\bar{\delta})$  and, at the same time, to select a different level of NGN investment. Therefore, as for firm E, deviation implies to play a type (S2) game with  $\tilde{\delta}_I < \mu_E^* < \mu_I$ ,<sup>40</sup> firm I's first period profit function in case of deviation is given in expression (9); from our previous analysis, we know that these profits decrease with the quality of firm I's OGN services;<sup>41</sup> hence, the optimal deviation is not to upgrade the OGN at all and to set  $\tilde{\delta}_I^* = \delta_I$ . Using this fact and plugging E's NGN investments at the candidate equilibrium,  $\mu_E^*(\bar{\delta})$ , in expression (9) and solving for the optimal  $\mu_I$  in case of deviation, it is possible to show that a sufficient condition for firm I not to have incentive to deviate is  $\delta_I > 92.35$ .

## Appendix B8: Welfare comparisons

**Proof of Proposition 4** This result can be easily proved graphically. In Figure 8 we plot the welfare levels with and without OGN upgrade as a function of the quality of the upgraded OGN  $\bar{\delta}$ ; without upgrade, the OGN is switched off and the equilibrium is independent on its quality while with OGN upgrade, welfare increases with  $\bar{\delta}$ . From the plot, the Proposition follows.

**Proof of Proposition 5** In Figure 9 we plot the welfare levels with and without OGN upgrade as a function of the quality of the upgraded OGN,  $\bar{\delta}$ . Without upgrade, firm I provides both OGN and NGN services and their quality depends on  $\delta_I$ ; therefore, the welfare also depends on  $\delta_I$ : in Figure 9, we plot the welfare levels at the two extremes vales,  $\delta_I = 92.35$  and  $\delta_I = 228.29$ ; when  $\delta_I \in (92.35, 228.29)$  the welfare takes intermediate values. With OGN upgrade, welfare increases with  $\bar{\delta}$ . From the plot, it is immediate to see that allowing for upgrade never increases welfare.

<sup>40</sup>We do not consider deviation  $\tilde{\delta}_I < \mu_I < \mu_E^*$ : this would imply that firm I becomes the low quality provider and it cannot be profitable.

<sup>41</sup>Formally, the derivative of expression (9):

$$\frac{\partial \Pi_I}{\partial \delta_I} = \frac{2500\mu_E^2(\mu_I - \mu_E)^2(\delta_I\mu_I + 20\mu_I\mu_E - 22\delta_I\mu_E + \mu_E^2)}{(-4\mu_I\mu_E + 2\delta_I\mu_E + \mu_E^2 + \delta_I\mu_I)^3},$$

is negative as  $\delta_I < \mu_E < \mu_I$ .

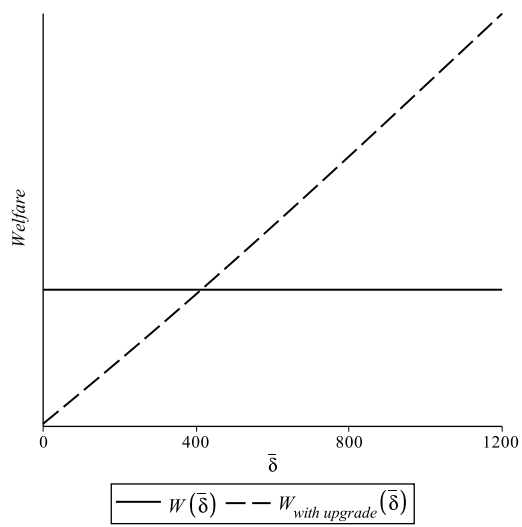


Figure 8: Welfare with leap frogging

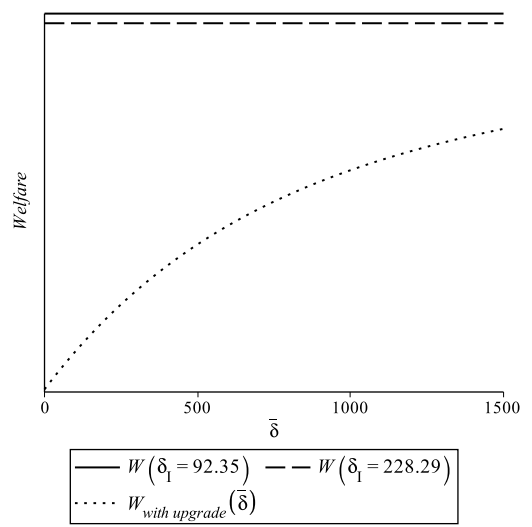


Figure 9: Welfare with persistence of leadership