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CRIME, IMMIGRATION AND THE LABOR MARKET:
A GENERAL EQUILIBRIUM MODEL

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Crime, Immigration and the Labor Market: A General Equilibrium Model

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Abstract

Does immigration cause crime? To answer this question, we build a two-country general equilibrium model with search costs in which the migration (in/out-)flows, the crime rates and the equilibrium wages in the two countries are determined by the interaction between the labor market, the crime market and the decision to migrate. The main result of our model is that, in equilibrium, the relationship between immigration and crime depends on the conditions of both the labor and crime markets of the two countries. In particular, when the tightness of the labor market is sufficiently elastic relative to that of the crime market, immigration causes a reduction in the domestic crime rate of the host country. An implication of this result is that migration flows from countries with strong work rigidities to societies characterized by more elastic labor markets are mutually benefic in terms of reducing the corresponding crime rates.

Keywords: Crime Rate, Labor Market, Immigration.

JEL classification: J61, J64, K42.

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1 Introduction. Immigration and Crime: A Controversial Relationship

*'Do immigrants make us safer?'*¹ Among the "hot" issues faced by policymakers in industrialized countries, the relationship between immigration and crime is one of the most controversial. Natives in host countries generally perceive immigration as a source of criminality. By analyzing data from the National Identity Survey during the period 1995-2003, Bianchi et al. (2008) report that the majority of the population in OECD countries is worried that immigrants increase crime, with the proportion of respondents in line with this view ranging from a low of 40% in the United Kingdom to a high of 80% in Norway (see also Martinez and Lee, 2000; Bauer et al. 2001). Despite public opinion, the nature of the relationship between immigration and crime is still an open question for social scientists.

The recent empirical literature is not conclusive. While some studies find a positive correlation between immigration and the crime rate of the host country, other empirical investigations report opposite conclusions. In addition, very few (if any) theoretical contributions give any convincing explanation for this puzzling evidence. Existing models either focus on the relationship between (un)employment and crime or analyze how natives' decision to migrate abroad depends on the economic conditions of the domestic labor market. In light of traditional theories of rational choice (Becker, 1968; Sah, 1991), agents decide to commit crime when the expected benefits from engaging in criminal activities overcome the associated expected costs. Similarly, agents migrate to foreign countries when the expected net benefits from moving abroad are higher than the expected earnings from remaining in the home country and participating in the domestic labor market. Interestingly, there are no theoretical contributions that build up a unified framework analyzing the simultaneous interplay between immigration, the (domestic and foreign) labor market and the (domestic and foreign) crime market. Introducing both migration and crime as available economic alternatives to detrimental labor conditions in the home country has two main advantages. First, it offers a richer and more realistic setting to account for migration flows. Indeed, in addition to better job opportunities, the decision of rational agents to move abroad can also be stimulated by the profitability of the crime market of the host country. Second, in this general setting the relationship between

¹New York Times Magazine, December 3rd, 2006.

immigration and crime ultimately depends on the structural characteristics of the labor market of the host country. Consider two countries, A and B , that are identical in all respects except for the elasticity of the tightness of the domestic labor market such that the probability of a job seeker being hired is high in country A and low in country B . Suppose that both countries register migration inflows, meaning that there is somebody abroad who finds it more profitable to move to country A (B) than remain in her own country. Due to the different level of rigidity in the labor market, it is reasonable to expect that the probability of an immigrant engaging in criminal activities is higher in country B than in country A . In a nutshell, countries with an elastic labor market are more likely to exhibit a negative relationship between immigration and crime than countries with strong work rigidities.

Several empirical observations are in line with the previous intuition (see, e.g., Engelhardt, 2010). Figure 1 plots the growth rates of the number of crimes per inhabitant (CPG) and the growth rates of the foreign-born population per inhabitant (FBG) of 19 European countries that registered positive migration inflows during the period 2005-2007².

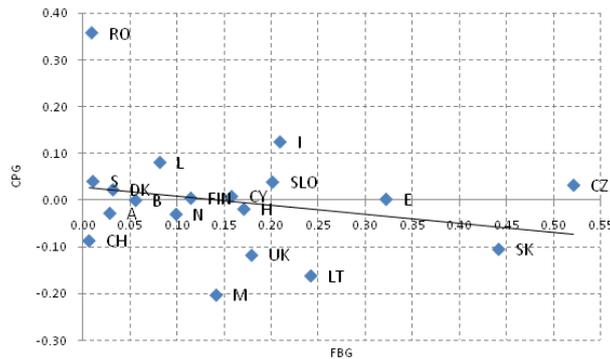


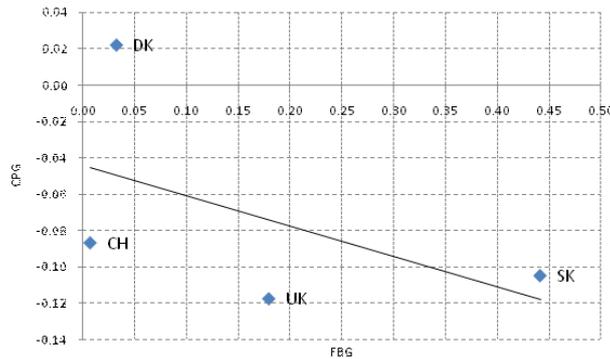
Figure 1. Immigration and crime in European countries.

While in 9 countries (Malta, Lithuania, the United Kingdom, the Slovak Republic, Switzerland, Norway, Austria, Hungary and Belgium) immigration is associated with a negative growth rate of crime per inhabitant, in the other 10 countries (Spain, Finland, Cyprus, Denmark, the Czech Republic, Slovenia, Sweden, Luxembourg, Italy and Romania) the sign of the relationship is reversed.³ The previous figure does not account for the possible effects that differences in the structure of the labor market across countries might exert on the immigration-crime

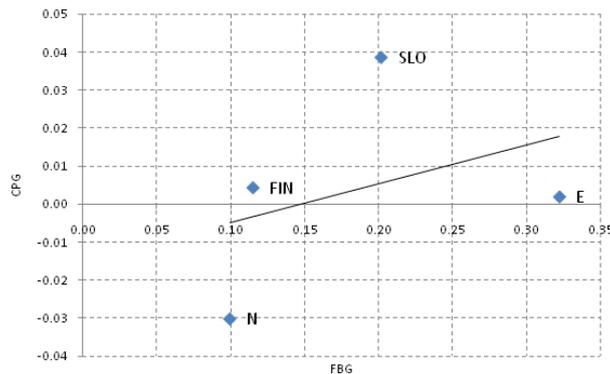
²Data drawn from the Eurostat website.

³Overall countries, there is a negative correlation between the growth rates of crime per inhabitant and immigration per inhabitant (Pearson's correlation coefficient = -0.23).

relationship. To account for these differences across countries, we employ the 2005 *Index of Freedom in the Labor Market*⁴ (henceforth, IFLM) as a proxy of the elasticity of the tightness. Based on the IFLM index, Figure 2 shows the relationship between immigration and crime of the four European countries with the highest IFLM (Figure 2) and the four European countries with the lowest IFLM.



(a) Highest IFLM



(b) Lowest IFLM

Figure 2. FBG and CPG in European countries with the highest (a) and the lowest (b) IFLM.

As seen in the previous figure, the relationship between immigration and crime seems to be negative for countries with the highest IFLM⁵ and positive for countries with the lowest

⁴The IFLM is built by the Heritage Foundation. It is built upon six quantitative factors of the labor market that are equally weighted: (a) Ratio of minimum wage to the average value added per worker; (b) Hindrance to hiring additional workers; (c) Rigidity of hours; (d) Difficulty of firing redundant employees; (e) Legally mandated notice period; (f) Mandatory severance pay. For further references on methodological issues, <http://www.heritage.org/index/>

⁵With the only exception of Denmark, the four host countries with the highest IFLM exhibit a negative growth rate of crime per inhabitant. Moreover, the higher the growth rate of immigrants per inhabitant, the lower the growth rate of crime per inhabitant (Pearson's correlation coefficient = -0.52).

IFLM⁶. As far as we know, no other study has modeled this aspect, and our main aim is to fill this gap in the literature.

In this paper, we present a two-country general equilibrium model with search costs in which the migration (in/out)flows, the crime rates and the equilibrium wages are simultaneously determined by the interaction between immigration, crime and the labor markets of the two countries. In each country, the labor market is characterized by the presence of search costs for both workers and firms. As in the standard matching theory, these costs lead to frictional unemployment and a non (perfectly) competitive wage that is the result of a Nash bargaining process between firms and job-seekers. The crime markets of the two countries are modeled as monopolistic competition in which criminals are price-makers. Criminal activities impose victimization costs on residents that are assumed to increase in the domestic crime rate. The free entry condition in the crime market implies that a marginal agent will be indifferent between committing a crime and participating in the labor market if and only if the expected benefits of a job-seeker are equal to earnings from criminal activities. After having analyzed the interaction between the crime market and the labor market in the autarkic case, we enrich the model by allowing the agents to migrate to the other country. Again, rational agents will migrate if and only if the expected gains from moving abroad are higher than the expected benefits from remaining in the home country.

We begin by stating the conditions for the existence and stability of a (general) international equilibrium. Then, we study how the relationship between immigration and crime in a country depends on the characteristics of the domestic labor market. Our main results can be summarized as follows. First, criminal activities are endemic to economic systems, meaning that there are no equilibria in which one country registers a null crime rate. Second, there exists a negative relationship between the domestic crime rate and the tightness of the national labor market such that a reduction in the domestic crime rate will lead to higher employment opportunities for residents. At the same time, by reducing the unemployment duration, an increase in the tightness of the domestic labor market implies a higher probability of finding a job. Third, within the country, the relationship between immigration and crime depends on the elasticity of both the labor and crime markets. In particular, when the tightness of the

⁶With the only exception of Norway, the four host countries with the lowest IFLM exhibit a positive growth rate of crime per inhabitant. Moreover, the higher the growth rate of immigrants per inhabitant, the higher the growth rate of crime per inhabitant (Pearson's correlation coefficient = 0.32).

labor market is sufficiently elastic relative to that of the crime market, immigration causes a reduction in the domestic crime rate. The intuition behind this result proceeds as follows. Consider a country that in equilibrium registers migration inflows. *Ceteris paribus*, by increasing the size of the population, immigration causes a reduction in the domestic crime rate of the host country. This effect modifies the equilibrium conditions of both the labor and crime markets. In the former, given the reduction in the victimization costs, firms offer higher wages and create more vacancies while job-seekers demand lower wages. If the tightness of the labor market is sufficiently elastic, the Nash bargaining process leads to an equilibrium characterized by a higher number of vacancies relative to the number of job-seekers than what was observed in the initial equilibrium. Thus, the expected benefits from participating in the labor market as job-seekers increases. In the crime market, the reduction in the crime rate weakens competition among criminals. Depending on the elasticity of the demand of crime, the expected benefits from entering the crime market increases. If the labor market over-reacts with respect to the crime market, then the economy reaches a new equilibrium in which immigration is associated with a lower domestic crime rate in the host country.

The rest of the paper is organized as follows. In the next section, we discuss the novelty of our contribution by relating it to the existing empirical and theoretical literature. In Section 3, we state the assumptions and solve the model in autarchy, namely, assuming the existence of a single, closed economy. In Section 4, we extend our analysis to the two-country context, and we derive the conditions for open economy equilibria. In Section 5, we sketch some extensions of the original model by relaxing some assumptions. Finally, in Section 6, we conclude and discuss the policy implications of our findings.

2 Literature Review

Although several empirical papers analyze the relationship between immigration and crime, the empirical evidence is ambiguous, and results seem to depend on the institutional and political portraits of countries as well as the interval of time that is considered. In some cases, immigrants' inflows are found to be positively correlated with the domestic crime rate. For instance, by using data from the 1960–2000 US censuses, Borjas et al. (2006) find that a 10% increase in the labor supply due to immigration in a particular skill group reduced the

black wage of that group by 2.5%, lowered the employment rate by 5.9%, and increased the incarceration rate by 1.3%. Similarly, by focusing on the Spanish situation in the period 1999–2006, Alonso et al. (2008) find that, although both immigrants and natives contributed to the increase in the crime rate, the immigrants' contribution is relatively higher. As opposed to these findings, several studies report opposite conclusions. Bianchi et al. (2008) analyze the relationship between immigration and crime across Italian provinces during the period 1990–2003. By using instrumental variables based on immigration to other European countries to identify the causal effect that exogenous changes in the composition of immigrant population in Italy, the authors find that the overall effect of immigration on crime is not significantly different from zero. Apart from the previous study and a few other exceptions, most of the papers that criticize the existence of a positive relationship between immigration and crime focus on data from the US. For instance, there is a wide consensus among economists and sociologists on the so-called Latino Paradox, namely, the fact that, despite Hispanic immigrants' socioeconomic situation, they are less inclined to commit crime than white Americans. By analyzing data on violent acts committed by young whites, blacks and Hispanics from 180 neighborhoods in Chicago between 1995 and 2003, Sampson (2008) reports a significantly lower rate of violence among Hispanic immigrants compared to blacks and whites, with this effect being mainly driven by the low attitude of first-generation immigrants (those born outside the United States) to engage in criminal activities. Extending the analysis to a wider range of criminal activities and given the puzzling evidence that immigrants are relatively less prone to commit crime with respect to natives, Butcher and Piehl (2007) estimate a model to disentangle three possible causes: deportation, immigrant self-selection or deterrence. By using data from the 5% Public Use Microsamples of the US Census in 1980, 1990 and 2000, the authors find that immigrants appear to be self-selected to have low criminal propensities, and this has increased over time. Reid et al. (2005) combine 2000 US Census data and 2000 Uniform Crime Report data to explore how the foreign-born population influences criminal offending across a sample of metropolitan areas. After controlling for a host of demographic and economic characteristics, the authors find that immigration does not increase crime rates, and some aspects of immigration lessen crime in metropolitan areas. Finally, evidence also shows that the relationship between immigration and crime is strongly volatile over time. By analyzing US data, Moehling and Piehl (2007) find that while immigrants were slightly more inclined than

natives to commit crime at the beginning of the 20th century, by 1930 the trend had reversed such that immigrants were less likely than natives to be committed to prisons at all ages 20 and older. The relative decline of criminal activities of foreign-born subjects was driven by a sharp increase in the commitment rates of the native born as opposed to the stable commitment rate of immigrants.

To explain this contradictory evidence, we develop a model of search in the labor market. Seminal contributions to this approach go back to Diamond (1981, 1982a, b), Mortensen (1982a, b), and Pissarides (1984a, b).⁷ We propose a general equilibrium model in which the sign of the relationship between immigration and crime depends on the tightness of the domestic labor market, namely, on the probability of an immigrant finding a job in the host country. As far as we know, no theoretical papers have explored this relationship.

Although not dealing with the relationship between immigration and crime, Ortega (2000) is probably the paper most related to ours. The author presents a two-country labor matching model, with no crime, in which domestic firms offer job-vacancies to residents, taking into account the average search costs of the population, while job-seekers look for a position either in their own country or, by bearing mobility costs, abroad. In each country, the equilibrium wage is the outcome of a Nash bargaining between firms and job-seekers based on a constant returns to scale matching function. Finally, countries can differ in their structural characteristics. Two main results are derived. First, the model generally admits multiple equilibria: a no-migration equilibrium, where job-seekers look for a position in their country exclusively; a full-migration equilibrium where all the natives in the country with worse structural conditions migrate and look for a job abroad; and an intermediate-migration equilibrium, where only a fraction of the natives in the country with worse structural conditions migrate. Second, the equilibria are Pareto-ranked along with the level of migration, such that the full-migration and the no-migration equilibria are the Pareto-superior and Pareto-inferior outcomes, respectively. Our model differs from Ortega's (2000) in several respects. First, while in his model countries differ from each other in the probability faced by workers of losing their job, we consider cross-country differences in the expected costs of being victims of crime. In particular, while in Ortega (2000) firms observe whether a worker is immigrant or native and pay different wages accordingly, we assume firms in one country are more vulnerable to crime (suffer larger victimization costs) than

⁷Excellent surveys of the literature until the 80s and the 90s are provided by Mortensen (1986) and Mortensen and Pissarides (1999), respectively.

firms in the other country. Second, in our model, firms do not observe origins of job-seekers and pay the same wage to all workers. Third, unlike Ortega (2000), we study the interplay between immigration and crime.

Leaving aside agents' decision to migrate, Burdett et al. (2003) build up a search model in a closed economy to analyze the interaction between crime, inequality and unemployment. Each firm posts a (fixed) wage and hires all the job-seekers who are willing to work at that wage. Crime is introduced as an opportunity to steal resources from someone else. The probability of an agent engaging in criminal activities differs according to her labor status and depends on the wage opportunity she encounters. With a given probability, criminals are caught and sent to jail. Finally, everyone can also fall victim to crime, and the probability of victimization depends on the likelihood of an agent engaging in criminal activities. Given this setting, the authors show that introducing crime as an alternative opportunity implies both wage dispersion and multiplicity of equilibria in terms of the crime rate and the unemployment rate. In a subsequent paper (Burdett et al. 2004), the authors extend their framework to incorporate on-the-job search. Our setting differs from these contributions in several respects. First, in our model, wages are determined through a bargaining mechanism between firms and workers (Pissarides, 2000) such that, in equilibrium, the wages reflect bargaining power and costs borne by both parts. Second, Burdett et al. (2003, 2004) introduce crime as an activity agents can commit at any time and state (employed or unemployed). Unlike those authors, we model crime as an occupational choice. An agent can be either employed, unemployed or criminal. Finally, differently from these papers, we deal with an open economy with migration across countries.

Engelhardt et al. (2008) build up a model that differs from that by Burdett et al. (2003) in the assumptions about the labor market. As in Pissarides (2000), the authors explicitly model a bilateral bargaining between workers and employers to determine the terms of the employment contract. Moreover, they endogenize the job-finding rate by assuming free entry for firms. Thus, in the Engelhardt et al. model, a worker's decision to commit a crime depends on both her bargaining strength and the chance of an unemployed worker finding a job. After having studied the conditions of the existence and uniqueness of an equilibrium, the authors show that agents' propensity towards crime is ranked according to their labor force status, with unemployed workers being the most likely to engage in criminal activities. Given this setting, they analyze the effects of labor and crime policies on the crime rate. In particular, while

labor policies (such as unemployment insurance, small wage subsidies, hiring subsidies) reduce the crime rate to the cost of altering the labor market conditions, crime policies significantly affect the crime rate, implying only negligible effects on the labor market. Although based on the same wage determination process, our model extends the analysis to a more general open economy framework with migration (in/out-)flows. As a consequence, policy interventions that positively affect the elasticity of the tightness of the labor market turn out to be the most influential instrument for reducing the crime rate of host countries.

A final remarkable difference between our model and the existing literature mentioned above concerns the assumptions used to model the crime market. Indeed, while in other studies the structure of the crime market is exogenously imposed and both the subjective probability of committing a crime as well as the expected profits from criminal activities are fixed by assumption, in our contribution the crime market is modeled as a monopolistic competition, and the corresponding equilibrium conditions are clearly derived.

3 The One-Country Model

Country A is a closed economy with population, P_A , that is made up of a continuum of agents and is fixed over time. Agents live forever and can be either employed (L_A), unemployed (U_A) or criminals (N_A). It follows that $P_A = L_A + U_A + N_A$. At any instant of time, unemployed agents decide whether to participate in the labor market as job-seekers or enter the crime market by engaging in criminal activities.

3.1 The Labor Market

The labor market of country A is characterized by the presence of search frictions. This means that, due to some source of imperfect information in the labor market, the matching process between vacancies and job-seekers is costly in terms of both time and economic resources. Given these costs, the interaction between firms and job-seekers generates an equilibrium level of frictional unemployment. In particular, suppose that the following expression describes the matching function in the labor market:

$$M_A = M(U_A, V_A), \quad \frac{\partial M_A}{\partial U_A}, \frac{\partial M_A}{\partial V_A} > 0, \quad (1)$$

where V_A is the number of vacancies in country A . Following the standard literature, we assume that the matching function is homogenous of degree 1. Therefore, we will have

$$m_A \equiv \frac{M_A}{V_A} = q(\phi_A), \quad (2)$$

where $\phi_A \equiv \frac{V_A}{U_A}$ measures the tightness of the labor market. Since $M_A \leq V_A$ and $M_A \leq U_A$, $q(\phi_A)$ represents the probability for a vacancy to be covered, and it is decreasing in ϕ_A . Therefore, the corresponding instantaneous probability of covering a vacancy is $q(\phi_A)dt$. Assuming a Poisson distribution, the average arrival time of a match for a vacancy is $\int_0^{\infty} e^{-q(\phi_A)t} dt = \frac{1}{q(\phi_A)}$.

Similarly, the probability of finding a job is $\phi_A q(\phi_A)$, with an instantaneous probability of $\phi_A q(\phi_A)dt$. This means that the average time for a worker to find a job is $\frac{1}{\phi_A q(\phi_A)}$. As usual, the probability of finding a job is increasing in ϕ_A . Therefore, by considering the constraint on the population size, $L_A = P_A - U_A - N_A$, we can write the level of frictional unemployment (the ratio between unemployed inhabitants and the size of the population) as a function of the equilibrium crime rate (the ratio between criminals and the size of the population):

$$u_A(n_A) = \frac{\delta_A(1 - n_A)}{\delta_A + \phi_A q(\phi_A)}, \quad (3)$$

where $\delta_A > 0$ is the instantaneous probability of an employed worker losing her job and n_A is the crime rate.

Let us consider the problem faced by a generic value-maximizer firm in country A when entering the search process. Let $J_{A,0}$ and $J_{A,1}$ be the value of an uncovered and covered vacancy in country A , respectively. The two no arbitrage conditions for hiring and losing a job-seeker faced by the firm are

$$\begin{cases} r_A J_{A,0} = q(\phi_A)(J_{A,1} - J_{A,0}) - \Omega_A(n_A) \\ r_A J_{A,1} = \Lambda_A - w_A - \delta_A(J_{A,1} - J_{A,0}) - k(n_A), \end{cases} \quad (4)$$

where r_A is the interest rate, Λ_A is the marginal productivity of labor assumed to be constant, $\delta_A(J_{A,1} - J_{A,0})$ is the turnover cost in terms of the firm's value, while $\Omega_A(n_A) = c_A + k(n_A)$ represents the total cost borne by firms at each moment. The total cost includes the cost of searching for a new employee in country A , $c_A > 0$, and the expected victimization cost of crime, $k(n_A)$, which is strictly increasing in the crime rate, n_A , and assumes value 0 when

$n_A = 0$. Thus, when $n_A = 0$, our setting collapses into a traditional search model. Finally, to avoid trivial results, we assume $\Lambda_A > c_A$.

Given the free entry condition in the market, $J_{A,0}$ must be null. Therefore, system (4) implies that the expected (total) cost of hiring an employee must be equal to the present value of firm's net income: $\frac{\Omega_A(n_A)}{q(\phi_A)} = \frac{\Lambda_A - w_A - k(n_A)}{r_A + \delta_A}$. From this equality, we obtain the (so-called) job-creation (JC) curve, that is, the relationship between the tightness of the labor market and the wages offered by the firms:

$$w_A^d = \Lambda_A - \frac{(r_A + \delta_A)\Omega_A(n_A)}{q(\phi_A)}. \quad (5)$$

Moving to the labor force, let $W_{0,A}$ and $W_{1,A}$ be the current values of being unemployed and employed in country A , respectively. Thus, similarly to system (4), we can write two no arbitrage conditions for unemployed inhabitants. In particular, the first imposes that the current value of being job-seeker is equal to the expected value of finding a job. Similarly, the second condition imposes that the current value of being employed is equal to the expected value of losing the job and moving back to the status of job-seeker:

$$\begin{cases} r_A W_{0,A} = \phi_A q(\phi_A)(W_{1,A} - W_{0,A}) - z_A - k(n_A) \\ r_A W_{1,A} = w_A - \delta_A(W_{1,A} - W_{0,A}) - k(n_A), \end{cases} \quad (6)$$

where z_A is the search cost faced by an unemployed inhabitant and $\phi_A q(\phi_A)(W_{1,A} - W_{0,A})$ and $\delta_A(W_{1,A} - W_{0,A})$ are the expected gains of passing from unemployed to employed and from employed to unemployed in country A , respectively. For simplicity, we assume henceforth that $z_A = 0$. Moreover, as shown by previous equations, we assume that inhabitants and firms incur the same victimization costs.

Given the presence of search costs, the equilibrium expression of the wage in the labor market is the result of a negotiation process between firms and job-seekers. In particular, by assuming a Nash bargaining process (NBP), we have that

$$w_A = \arg \max (W_{1,A} - W_{0,A})^\gamma (J_{A,1} - J_{A,0})^{1-\gamma}, \quad \gamma \in (0, 1),$$

where γ measures the relative bargaining power of workers. Therefore, the total surplus $H_A = J_{A,1} - J_{A,0} + W_{1,A} - W_{0,A}$ is allocated between job-seekers and firms as follows:

$W_{1,A} - W_{0,A} = \gamma H_A$. By solving the maximization problem and considering systems (4) and (6) together with the fact that $J_{A,0} = 0$, we get the current value of being a job-seeker:

$$x_A(n_A) \equiv r_A W_{0,A} = \frac{\gamma}{1-\gamma} \Omega_A(n_A) \phi_A - k(n_A). \quad (7)$$

The value of being job-seekers is increasing in the tightness and the workers' bargaining power. From Equation (7) and the result of the maximization problem, we obtain the labor supply curve in terms of ϕ_A :

$$w_A^s = \gamma \Lambda_A + \gamma \Omega_A(n_A) \phi_A - (1-\gamma)k(n_A). \quad (8)$$

By equalizing Equation (8) to (5), we obtain the expression of ϕ_A as a function of the other parameters of the model. Formally,

$$\gamma \Lambda_A + \gamma \Omega_A(n_A) \phi_A - (1-\gamma)k(n_A) = \Lambda_A - \frac{(r_A + \delta_A) \Omega_A(n_A)}{q(\phi_A)}. \quad (9)$$

Equation (9) implicitly defines $\phi_A(n_A)$ as a function of n_A . Therefore, the corresponding wage is given by

$$w_A(n_A) = \gamma \Lambda_A + \gamma \Omega_A(n_A) \phi_A(n_A) - (1-\gamma)k(n_A). \quad (10)$$

The first result we derive concerns the relationship between the tightness of the labor market, $\phi_A(n_A)$, and the crime rate, n_A .

Lemma 1. *There exists a strictly negative relationship between $\phi_A(n_A)$ and n_A , $\frac{d\phi_A(n_A)}{dn_A} < 0$.*

Proofs are left to the Appendix. The previous lemma has an obvious implication. Suppose the crime rate increases. Then, for any level of the tightness, the corresponding increase in the victimization costs induces job-seekers to demand higher wages and firms to offer lower wages. Thus, the ratio between vacancies and unemployed inhabitants decreases. The previous result combined with Equation (3) imply that the effects on the unemployment rate of an increase in n_A are ambiguous. On the one hand, it directly reduces the unemployment rate. On the other hand, by indirectly reducing the probability of finding a job, it stimulates the unemployment rate.

3.2 The Crime Market

We model the crime market as a monopolistic competition. Criminals are price-makers and face a demand of crime (drugs, stolen goods, prostitution, etc.). We assume the existence of a single criminal good, Q_A . Let $Q(P_A)$ and $\bar{p}_{-j,A}$ be the total demand of crime and the average price applied by all the criminals but j , respectively. We assume the demand of crime will increase in the size of the population. As in the standard models of monopolistic competition, the demand of crime faced by criminal j in country A is $q_{j,A} = \frac{Q(P_A)}{n_A P_A} + b(\bar{p}_{-j,A} - p_{j,A})Q(P_A)$, where $b > 0$ is a parameter capturing the degree of competition in the crime market and $N_A = n_A P_A$ represents the number of criminals in the market. For simplicity, we assume that criminals incur the same costs of committing crimes. In particular, the cost function of criminal activities is $C(q_{j,A}) = F_A + a_A q_{j,A}$, where F_A represents the fixed component of the costs of crime and a_A is the marginal cost. Consistently with the existing literature, we can interpret the cost function of crime in expected terms such that it also incorporates the risk of being captured and arrested by police. In this case, we assume that, ceteris paribus, the risk of criminal j of being arrested increases in j 's share of the demand of crime. In other words, the risk is a part of the marginal cost.

The equilibrium price applied by criminal j will be $p_{j,A} = \frac{1}{bn_A P_A} + a_A$, meaning that, at time t , for each criminal, the profit from crime net of the victimization costs is

$$\Pi(n_A, P_A) = \frac{Q(P_A)}{b(n_A P_A)^2} - F_A - k(n_A). \quad (11)$$

The marginal agent will be indifferent to enter the crime market or not when the expected profit from committing crime is equal to the expected revenue from participating in the labor market as a job-seeker: $\Pi(n_A, P_A) = x_A(n_A)$. By combining Equations (11) and (7), we obtain the following expression:

$$\frac{Q(P_A)}{b(n_A P_A)^2} - F_A = \frac{\gamma}{1 - \gamma} \Omega_A(n_A) \phi_A^*(n_A). \quad (12)$$

Given our theoretical framework, the next section provides an equilibrium analysis of the one-country model.

3.3 The Autarkic Equilibrium

Starting from the assumptions of the model, we now state the definition of an interior equilibrium in autarky.

Definition 1. *Given the size of the population, P_A , an autarkic equilibrium is a list $\{n_A^*, \phi(n_A^*), w(n_A^*), u_A(n_A^*)\}$ such that $\phi(n_A^*)$ satisfies Equation (9), $w(n_A^*)$ satisfies Equation (10), $u_A(n_A^*)$ satisfies Equation (3) and n_A^* satisfies $\Pi(n_A^*, P_A) \geq x_A(n_A^*)$.*

In other words, the economy is in equilibrium when no agent has an incentive to move from the labor market to the crime market and vice versa.

Notice that the case in which $\Pi(n_A^*, P_A) > x_A(n_A^*)$ defines a corner solution in autarky. In this case, it is always profitable for an agent to engage in criminal activities. Formally, $n_A^* = 1$, $\phi(n_A^*) = \phi(1)$, $w(n_A^*) = w(1)$, $u_A(n_A^*) = 0$.

The model can be solved recursively. Once the equilibrium crime rate, n_A^* , is determined, Equations (9), (10) and (3) are used to derive $\phi(n_A^*)$, $w_A(n_A^*)$ and $u_A(n_A^*)$, respectively. The main result on existence is presented as follows.

Proposition 1. *An autarkic equilibrium always exists.*

Proposition 1 states that the one-country model always admits an autarkic equilibrium. Given the hypotheses on the structure of the crime market, when the proportion of criminals is small enough, criminal activities are more profitable than productive activities, and agents have an incentive to move from the labor market to the crime market. By Definition 1, this process may even drive the system to converge to an equilibrium associated with $n_A^* = 1$. The following corollary states the conditions for this corner solution to be the unique equilibrium of the model.

Corollary 1. *The corner solution, $n_A^* = 1$, is the unique autarkic equilibrium if and only if $\Pi(P_A, n_A) > x(n_A)$, $\forall n_A \in (0, 1]$.*

The structure of the labor market plays a crucial role in determining the number of autarkic equilibria. Let $\eta_{\phi(n_A), k(n_A)} = \frac{d\phi(n_A)}{dn_A} \frac{1}{\phi(n_A)} \Omega_A(n_A)$ represent the elasticity of the tightness of the labor market with respect to the victimization costs of crime. There is a direct linkage between $\eta_{\phi(n_A), k(n_A)}$ and the elasticity of the labor market. A decrease in the crime rate reduces the search costs of firms. The higher the reaction of the tightness of the labor market to the change in the search costs, the higher the capacity of the system to create vacant positions for job-seekers. In general, the model might exhibit multiple equilibria. However, as shown by the

next proposition, we can state a sufficient condition for the value of $\eta_{\phi(n_A),k(n_A)}$ such that the autarkic equilibrium is unique.

Proposition 2. *If $\eta_{\phi(n_A),k(n_A)}$ is always larger than a certain critical value $\tilde{\eta}_{\phi(n_A),k(n_A)}$, then $\frac{dx_A(n_A)}{dn_A} > \frac{\partial \Pi(P_A, n_A)}{\partial n_A}, \forall n_A \in (0, 1]$, and the autarkic equilibrium is unique.*

When the condition for the value of $\eta_{\phi(n_A),k(n_A)}$ stated in Proposition 2 does not hold, uniqueness cannot be assured. Interestingly, as stated by the following corollary, crime is endemic in the one-country model, and there is no equilibrium with a null crime rate.⁸

Corollary 2. *There is no autarkic equilibrium with $n_A^* = 0$.*

Our next step is to study the (local) stability of the autarkic equilibria. We start by stating the conditions for an interior equilibrium to be stable, and then we relate these conditions with the uniqueness property.

Proposition 3. *An interior equilibrium is stable if and only if $\frac{dx_A(n_A^*)}{dn_A} > \frac{\partial \Pi(P_A, n_A^*)}{\partial n_A}$.*

The intuition behind the previous proposition is straightforward. An interior equilibrium is locally stable if a (sufficiently) small increase in n_A^* makes unemployment more valuable than crime. If not, a higher crime rate will induce more agents to enter the crime market, making the economy diverge from the initial equilibrium. The next result is intuitive since it states that when uniqueness holds, the equilibrium is also stable, that is, the final state of the economy is not affected by temporary perturbations. In this case, the equilibrium is not only locally stable, but it becomes globally stable.

Corollary 3. *If the autarkic equilibrium is unique, then the equilibrium is stable.*

Finally, we show that whenever multiplicity of equilibria occurs, there always exists at least one interior equilibrium that is stable. In particular, the equilibrium associated with the lowest crime rate is always stable.

Corollary 4. *Let the equilibria of the model be ordered according to the corresponding crime rates in $[\underline{n}_A^*, \overline{n}_A^*]$, with $\underline{n}_A^* > 0$ and $1 \geq \overline{n}_A^* > \underline{n}_A^*$, representing the highest and the lowest equilibrium crime rates, respectively. The equilibrium associated with \underline{n}_A^* is stable.*

According to Corollary 4, adequate policy interventions are potentially able to perturb the system towards the equilibrium with the lowest crime rate. As discussed below, some policies are more effective than others in terms of crime reduction.

⁸Indeed, "all societies have crime and deviance - and - [...] crime may be a necessary price to pay for a certain social freedom" (Macdonis and Plummer, 2008, pp. 543).

4 Open Economy: The Two-Country Model

We now extend our analysis to an open economy. Suppose there are two countries, A and B , with initial population sizes P_A and P_B , respectively. We assume the world population, P , is fixed. Thus, the size of the population in country B can be expressed as the difference between the world population and the size of population in country A , $P_B = P - P_A$. As before, populations in the two countries are composed of a continuum of agents. Unless explicitly mentioned, countries are identical in all other respects, and all the assumptions above continue holding in this context. The key difference with respect to the closed economy case is that in the open economy inhabitants of country A can move to country B and vice versa. Migration has two main implications. First, differently from the autarkic case, the size of the country population is not fixed: it increases if the country registers migration inflows and decreases when natives migrate abroad. Second, in an open economy, agents face a higher number of economic activities they can engage in. Indeed, in addition to participating in both the labor and crime markets in their own country, agents can also decide to work or commit a crime in the host country.

Assuming that agents do not bear any mobility cost from migration,⁹ in an interior international equilibrium the following no arbitrage conditions must be satisfied:

$$\Pi(P_A, n_A) = x_A(n_A), \quad (13)$$

$$\Pi(P_B, n_B) = x_B(n_B), \quad (14)$$

$$x_A(n_A) = x_B(n_B). \quad (15)$$

The previous conditions imply the following no arbitrage condition between the crime markets of the two countries:

$$\Pi(P_A, n_A) = \Pi(P_B, n_B). \quad (16)$$

Expressions (13) and (14) are no arbitrage conditions stating that, within each country,

⁹In the extensions, we will discuss how introducing positive mobility costs affects the equilibrium analysis.

entering in the crime market must be as profitable as being job-seekers in the labor market. Expressions (15) and (16) describe no arbitrage conditions between countries and characterize the international equilibrium: when the value of being job-seekers and the profits from crime are the same in the two countries, agents are indifferent between migrating and remaining in their own country. When these two conditions are satisfied, countries do not register any migration flow. In the following equilibrium analysis, we assume that conditions (13) and (14) always hold such that we restrict our attention to situations in which either Equation (16) or (15) is not satisfied and agents have an incentive to migrate. That is, if (say) $\Pi_A(P_A, n_A) > \Pi_B(P_B, n_B)$, we will observe agents moving from country B to country A , because they will be attracted by higher profits in the crime market and, given the first two no arbitrage conditions, by the higher value of being job-seekers. As long as migration flows occur, the size of the population in the two countries as well as the corresponding equilibrium conditions in the labor and crime markets change.

We study an equilibrium in an open economy. In particular, the definition of autarkic equilibria can be generalized as follows:

Definition 2. *An equilibrium in an open economy is a list $\{P_i^*, n_i^*, \phi(n_i^*), w(n_i^*), u_i(n_i^*)\}$, with $i = A, B$, such that $\phi(n_i^*)$ satisfies Equation (9), $w(n_i^*)$ satisfies Equation (10), $u_i(n_i^*)$ satisfies Equation (3), $\{P_i^*, n_i^*\}$ represents a domestic equilibrium (as defined in Definition 1) and one of the following conditions holds: (i) $\Pi(P_A^*, n_A^*) = \Pi(P_B^*, n_B^*)$; (ii) $\Pi(P_A^*, n_A^*) > \Pi(P_B^*, n_B^*)$ and $P_A^* = P$; (iii) $\Pi(P_A^*, n_A^*) < \Pi(P_B^*, n_B^*)$ and $P_B^* = P$.*

As stated by the previous definition, an international equilibrium is a situation in which there is no incentive to migrate abroad and, within countries, no agents have an incentive to move from the labor market to the crime market and vice versa.

Notice that the cases in which $\Pi(P_A^*, n_A^*) \geq \Pi(P_B^*, n_B^*)$ define two (symmetric) corner solutions in the open economy. When $\Pi(P_A^*, n_A^*) > \Pi(P_B^*, n_B^*)$, $P_A^* = P$ and $P_B^* = 0$ where the pair $\{P, n_A^*\}$ satisfies Definition 1. The same analysis holds for the case in which $\Pi(P_A^*, n_A^*) < \Pi(P_B^*, n_B^*)$.

To conduct our analysis, we introduce two fundamental relationships: the domestic locus and the international locus. The domestic locus of country i describes the combinations $\{P_i, n_i^D\}$ that imply equilibrium in the domestic (labor and crime) markets (conditions (13) and (14)). The international locus is given by the combinations $\{P_i, n_i^I\}$, such that the no

arbitrage condition (15) is satisfied. Hereafter, indices D and I denote the values assumed by the variables when they are on the domestic and international loci, respectively.

Since the world population is fixed, we can focus on the domestic and international loci of one country only. With no loss of generality, we will refer to country A . An international equilibrium is described by a combination $\{P_A^*, n_A^*\}$ that simultaneously belongs to the domestic and international loci. By symmetry, we show that any equilibrium pair $\{P_A^*, n_A^*\}$ is associated with a unique combination $\{P_B^*, n_B^*\}$ in country B . Given these preliminary considerations, we analyze the existence and properties of an international equilibrium. We proceed in two steps. First, we derive the properties of the domestic and international loci of country A . Second, we study the existence, stability and (non-)uniqueness of an international equilibrium as a function of the slopes of the domestic and international loci.

4.1 The Domestic Locus

The domestic locus describes the effects of a change in the size of the population of a country on the corresponding equilibrium crime rate in autarky. As seen before, for a given population size, there can be multiple equilibria. The existence of multiple equilibria can explain why countries with similar characteristics, in terms of labor market institutions and population size, exhibit different crime rates. In a closed economy, the stability of an equilibrium (see Proposition 3) represents a valid property for selecting those equilibria that are likely to be observed. Moreover, from Corollaries 3 and 4 we know that, for any population size, there always exists at least one stable equilibrium in autarky. Below, we show that the slope of the domestic locus can be used to discriminate between stable and unstable autarkic equilibria.

Given Equation (9), the effects of a variation of n_A on $x_A(n_A)$ are ambiguous (see the proof of Proposition 2): a higher crime rate reduces the tightness of the labor market but increases the total costs of firms. Using Equation (11), we obtain the following derivatives:

$$\left\{ \begin{array}{l} \frac{\partial \Pi(P_A, n_A)}{\partial n_A} = -\frac{2Q(P_A)}{bP_A^2 n_A^3} - \frac{dk(n_A)}{dn_A} < 0, \\ \frac{\partial \Pi(P_A, n_A)}{\partial P_A} = \frac{dQ(P_A)}{dP_A} \frac{1}{(n_A P_A)^2} - \frac{2Q(P_A)}{n_A^2 P_A^3} \geq 0 \iff \eta_{Q(P_A), P_A} \geq 2. \end{array} \right. \quad (17)$$

Where $\eta_{Q(P_A), P_A} \equiv \frac{dQ(P_A)}{dP_A} \frac{1}{Q(P_A)} P_A$ is the elasticity of the demand of crime with respect to the population size. Since we have assumed $\frac{dQ(P_A)}{dP_A} > 0$, then $\eta_{Q(P_A), P_A}$ is always positive.

To study the relationship between P_A and n_A , we totally differentiate Equation (13):

$$\frac{dn_A^D(P_A)}{dP_A} = \frac{\frac{\partial \Pi(P_A, n_A)}{\partial P_A}}{\frac{dx(n_A)}{dn_A} - \frac{\partial \Pi(P_A, n_A)}{\partial n_A}}. \quad (18)$$

The derivative in (18) is well-defined when $\frac{dx(n_A)}{dn_A} \neq \frac{\partial \Pi(P_A, n_A)}{\partial n_A}$, implying that the locus is continuous.¹⁰ By Proposition 3, if the domestic equilibrium is stable, it follows that

$$\begin{cases} \frac{dn_A^D(P_A)}{dP_A} \geq 0 & \iff \eta_{Q(P_A), P_A} \geq 2 \\ \frac{dn_A^D(P_A)}{dP_A} < 0 & \iff \eta_{Q(P_A), P_A} < 2. \end{cases} \quad (19)$$

In particular, since for any population size the elasticity of the demand for crime with respect to the size of the population can assume only one value, stable equilibria belong to the decreasing (increasing) part of the domestic locus when the elasticity $\eta_{Q(P_i), P_i}$ is lower (higher) than 2, with this critical value being implied by the functional form used to describe the demand of crime faced by a criminal, $q_{j,A}$.

4.2 The International Locus

The international locus is defined over the space of combinations (P_A, n_A) that satisfy the no migration condition (15), with $P_A \in (0, P)$ and $n_A \in (0, 1]$. Rewriting (15) using (7) and (9), we obtain

$$\frac{\Omega(k_A(n_A))}{q(\phi(k_A(n_A)))} = \frac{\Omega(k_B(n_B))}{q(\phi(k_B(n_B)))}. \quad (20)$$

By Lemma 1, there is always a negative relationship between n_i and $\phi(k_i(n_i))$. Since $k(n_i)$ monotonically increases in n_i , it follows that there is a negative relationship between $k_i(n_i)$ and $\phi(k_i(n_i))$. Given the previous considerations, $\Omega(k_i(n_i))$ monotonically increase in n_i . By assuming the same search technology for both countries, we can write (20) as $\psi(k_A(n_A)) = \psi(k_B(n_B))$, where $\psi(\cdot)$ is a function describing the search technology. The assumption of homogeneous search technology implies that countries have the same functional form for the matching function, $q(\phi(k_i(n_i)))$, and the same search cost, $c_A = c_B$. Therefore, to have an international equilibrium, the victimization costs of the two countries must coincide: $k_A(n_A) = k_B(n_B)$. This equality leads to the expression of the international locus:

¹⁰Notice that in case of $\frac{dx(n_A)}{dn_A} > \frac{\partial \Pi(P_A, n_A)}{\partial n_A}$, $\forall n_A \in (0, 1]$, Proposition 2 implies the existence of only one equilibrium level of n_A for any given level of P_A and consequently the domestic locus will be a function $n_A^D(P_A)$ defined in the space (P_A, n_A) .

$$n_A^I(P_A) = k_A^{-1}(k_B(n_B^D(P_B))). \quad (21)$$

$n_A^I(P_A)$ indicates the value of n_A that satisfies the no migration condition expressed by Equation (15) while $n_B^D(P_B)$ is the equilibrium crime rate of country B satisfying Equation (14), with $P_B = P - P_A$. In other words, this locus gives the value that the crime rate in A should assume in order to guarantee the absence of migration flows from one country to the other. Since in both countries the victimization costs increase in the crime rate, we have that $\frac{dn_A^I(P_A)}{dn_B^D(P_B)} > 0$.

When countries are identical in all respects, the international locus of country A corresponds to the domestic locus of country B . In this case, we will have $n_A = n_B$, and the crime rate in the two countries is determined by the domestic loci at $P_A = P_B = \frac{P}{2}$.

To avoid trivial results, we assume that countries differ in victimization costs. In particular, we assume that $k_A(n) < k_B(n)$, $\forall n \in (0, 1)$ and $k_A(0) = k_B(0)$ and $k_A(1) = k_B(1)$. These assumptions imply that, for a given value of the crime rate, the expected loss from crime is smaller in country A than in country B . For instance, insurance services as well as health aids may be more effective in country A than in country B . Nonetheless, when all agents are criminals, no insurance service or health institutions can exist, implying that the cost is the same in both countries.

Since the domestic locus of B takes values $n_B \in (0, 1]$, for $P_B \in (0, P]$, given our assumptions $k_A(0) = k_B(0)$ and $k_A(1) = k_B(1)$ and the continuity of $k_A^{-1}(k_B(n_B^D(P - P_A)))$, due to the fact that $k_A(\cdot)$ is monotonically increasing, we can conclude that the international locus always exists for $(P_A, n_A) \in [0, P) \times (0, 1]$.

Taking into account the population constraint, the slope of the international locus is

$$\frac{dn_A^I(P_A)}{dP_A} = - \frac{dn_A^I(P_A)}{dn_B^D(P_B)} \frac{dn_B^D(P_B)}{dP_B}. \quad (22)$$

Therefore, the international locus presents a positive (negative) slope when the slope of the domestic locus of country B is negative (positive). Notice that the continuity of the domestic locus of country B in the interval $P_A \in [0, P)$ implies continuity of the international locus of country A in the same interval. Moreover, by the assumption on the victimization costs in the two countries, for any $P_A \in (0, P)$, we have that $n_A^I(P_A) > n_B^D(P_B)$, with $P_B = P - P_A$.

4.3 International Equilibrium

By definition, an international equilibrium is a combination $\{P_A^*, n_A^*\}$ that simultaneously belongs to the domestic and international loci. Therefore, given our loci, we turn our attention to the equilibrium analysis. As before, we start with the existence of an international equilibrium, and then we focus on the properties of (non-)uniqueness and stability.

Proposition 4. *An international equilibrium exists.*

The model might exhibit multiple equilibria. In this respect, several results concerning the multiplicity of equilibria stated above can be extended to the open economy model. For instance, in any interior equilibrium, $P_A^* > 0$, $P_B^* = P - P_A^* > 0$, and $n_A^* > 0$ implies $n_B^* = k_B^{-1}(k_A(n_A^*)) > 0$. In the trivial case of an equilibrium characterized by full migration from country B to country A , $P_A^* = P$ and $n_A^* > 0$ while $P_B^* = 0$. Similarly, when profits from crime are (always) higher than the value of being job-seekers in both countries, an equilibrium with full crime emerges such that $n_A^* = n_B^* = 1$ while Equations (11) and (16) imply that P_A^* and P_B^* are determined according to the following condition:

$$\frac{P_A^*}{P - P_A^*} = \sqrt{\frac{Q(P_A^*)}{Q(P - P_A^*)}}. \quad (23)$$

As in autarky, in general the model admits multiple equilibria. As follows, we provide conditions to have uniqueness of the international equilibrium. First, Proposition 5 relates the uniqueness of the international equilibrium to the elasticity of the demand of crime in the two countries.

Proposition 5. *If $\eta_{Q(P_A), P_A}, \eta_{Q(P_B), P_B} > 2$ or $\eta_{Q(P_A), P_A}, \eta_{Q(P_B), P_B} < 2$, $\forall P_A, P_B \in [0, P]$ with $P_B = P - P_A$ and $\frac{dx_A(n_A)}{dn_A} > \frac{\partial \Pi(P_A, n_A)}{\partial n_A}, \forall n_A \in (0, 1], \frac{dx_B(n_B)}{dn_B} > \frac{\partial \Pi(P_B, n_B)}{\partial n_B}, \forall n_B \in (0, 1]$, there is a unique international equilibrium.*

According to Proposition 5, if the two countries present persistent and similar characteristics of the labor and crime markets, then there exists a unique international equilibrium. As shown by the next proposition, when the international equilibrium is unique, the domestic crime rates in both countries are unequivocally determined.

Proposition 6. *If the international equilibrium is unique, then it is associated with a unique pair (n_A^*, n_B^*) .*

We now turn to the stability properties of international equilibria. First notice that, by

Corollary 3, given P_A^* , the uniqueness of the pair (n_A^*, n_B^*) also implies stability of the domestic equilibria. Second, the next proposition states necessary and sufficient conditions for an international equilibrium to be stable.

Proposition 7. *Let (P_A^*, n_A^*) be an interior international equilibrium. This equilibrium is locally stable if and only if $\frac{dn_A^D(P_A^*)}{dP_A} > \frac{dn_A^I(P_A^*)}{dP_A}$.*

As in the one-country model, the uniqueness of the international equilibrium implies its stability. This is formally stated in the following corollary.

Corollary 5. *If the international equilibrium is unique, then it is stable.*

As shown by Equations (21) and (22), victimization costs play a crucial role in determining the stability properties of an international equilibrium. Indeed, by affecting the magnitude of $\frac{dn_A^I(P_A)}{dn_B^D(P_B)}$, victimization costs define the reaction of $n_A^I(P_A^*)$ to migration for any given slope of the domestic locus of country B . Figure 3 offers a graphic intuition of the results in Propositions 4 and 7.

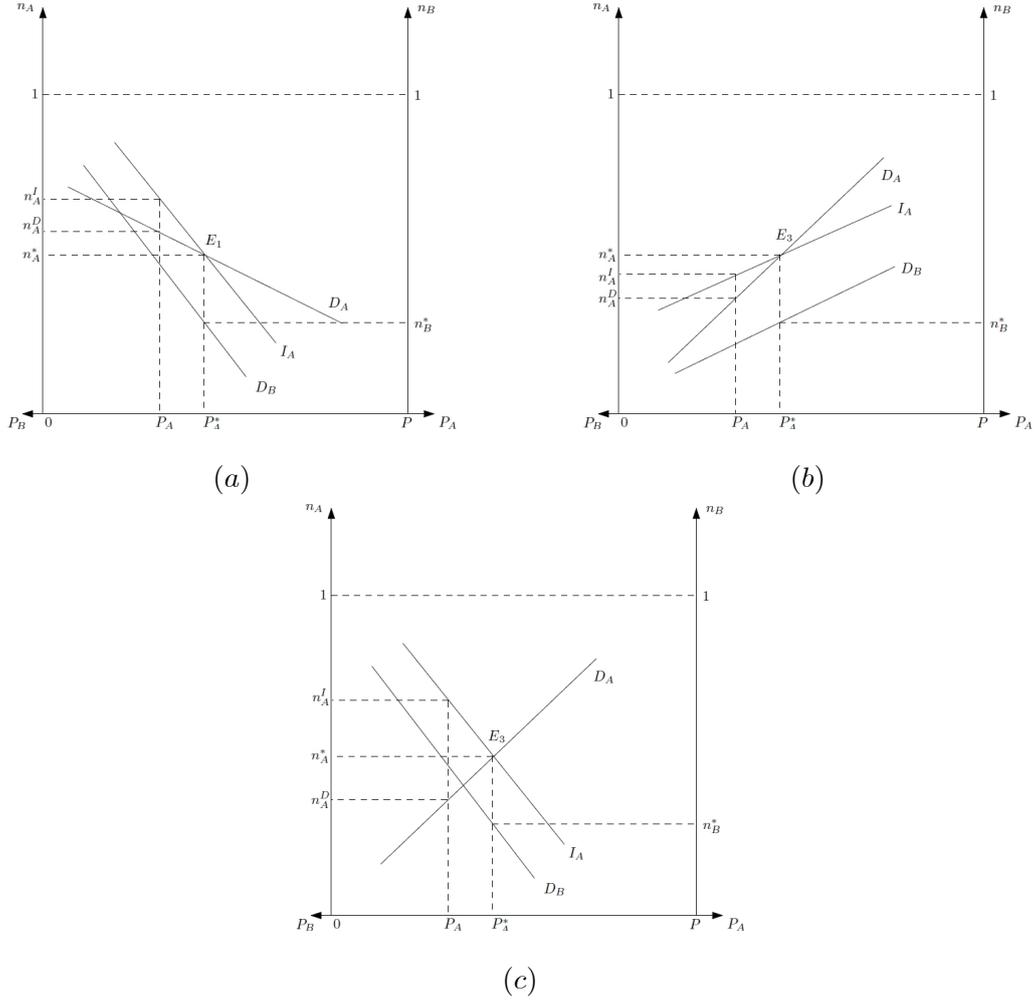


Figure 3. International equilibria and slopes of the (domestic and international) loci.

Let D_i and I_i represent the domestic and international loci of country $i = A, B$. Given the constraint on the population, the intersection of these two curves represents the international equilibrium in the space (P_i, n_i) . As shown in Proposition 7, the stability of this equilibrium depends on the slopes of the domestic and international loci.

Figure 3.a shows the case in which A presents a negative relationship between immigration and crime and B presents a positive relationship between immigration and crime. Let us focus on the stability of the international equilibrium E_1 . Starting from E_1 , suppose the population of country A decreases below the equilibrium level. By assumption, the domestic markets adjust instantaneously. Thus, the crime rate in country A jumps to the level implied by the domestic locus, $n_A^D(P_A)$, which is lower than that associated with the international locus, $n_A^I(P_A)$. In

this situation, profits from crime in country A are higher than those in country B , which, on the other hand, implies that the expected revenue from participating in the labor market as a job-seeker is higher in country A than in country B . Thus, inhabitants of country B will find it convenient to migrate to country A . The economy moves along the domestic locus until it moves back to the international equilibrium, E_1 . During this adjustment process, as shown in Proposition 3, the crime rate of country A decreases. The intuition behind this result can be explained as follows. Migration flows from country B to country A have two effects. First, as P_A increases, the demand of crime increases. Second, given the initial number of criminals, the increase in P_A reduces the crime rate in country A . If the labor market is flexible enough, the increase in the value of being a job-seeker will overcome the variation in the profits from crime, causing a reduction in the number of criminals and a consequent further reduction in the crime rate, n_A .

Figure 3.b shows the opposite case of Figure 3.a. That is, this represents the situation in which B is characterized by a negative relationship between immigration and crime, whereas A is characterized by a positive relationship between immigration and crime. Finally, Figure 3.c describes the case in which both countries have a positive relationship between immigration and crime. There, migration causes an increase in the crime rate of the host country and a decrease in the crime rate of the other.¹¹

By taking advantage of Figure 3, the next corollary states the conditions such that migration flows are beneficial for both countries.

Corollary 6. *Suppose that in a stable international equilibrium $\frac{dn_A^D(P_A^*)}{dP_A} < 0$. Then $\frac{dn_B^D(P_B^*)}{dP_B} > 0$.*

Suppose that country A and country B exhibit a positive and negative relationship between the size of the population and the domestic crime rates, respectively (see Figure 3.a). The previous corollary states that migration flows from country A to country B are mutually beneficial whereby they decrease the crime rates in both countries. Vice versa, in the case of migration flows from country B to country A , the crime rates of both countries increase.

The last result concerns the effects of the institutional characteristics of a country on the relationship between immigration and the domestic crime rate. As shown in Figure 2, European host countries characterized by highly elastic labor markets exhibit a negative relationship

¹¹The equilibrium in which both countries are characterized by a negative relationship between immigration and crime is unstable, therefore we have omitted the graphical representation.

between migration inflows and the domestic crime rate. Our analysis offers an economic-based explanation for this empirical observation. Suppose country A registers immigration. Migration inflows generate an increase in the size of the population, P_A , and, given the initial number of criminals, a reduction in the crime rate, n_A . Depending on the elasticity of the demand of crime, $\eta_{Q(P_A),P_A}$, the former effect makes participation in the crime market more profitable.¹² Similarly, depending on the elasticity of the tightness of the labor market, $\eta_{\phi(n_A),k(n_A)}$, the second effect increases the value of participating in the labor market as job-seekers. Since agents' economic decisions are based on the no arbitrage condition between committing a crime and looking for a (legal) job, the net effect of immigration on the domestic crime rate is likely to depend on the elasticities of both the crime and labor markets of country A . This is formally stated in the next proposition.

Proposition 8. *If the absolute value of $\eta_{\phi(n_A),k(n_A)}$ is sufficiently high, then there is a negative relationship between n_A and P_A .*

The threshold value of $\eta_{\phi(n_A),k(n_A)}$ such that n_A is negatively related to P_A depends on the variation of the demand for crime with respect to the population level. Thus, we might expect such a value to depend on country-specific institutional characteristics. Nevertheless, in Figure 4, we implement the smoothing techniques suggested by Bowman and Azzalini (2003)¹³ on data referring to the 19 European countries mentioned in the introduction to show that the relationship between immigration and crime depends on the characteristics of the labor market of the host country. In line with our theoretical results, the sign of the relationship

¹²In his seminal work, Becker (1968) uses the elasticity of crime with respect to the expected punishment as a measure of the individual propensity to commit a crime. In our model, the "propensity" to engage in criminal activities is related to the elasticity of the demand of crime. Indeed, the elasticity captures two effects. First, there is a positive relationship between the elasticity of the demand of crime and agents' propensity to commit crimes. Second, when the expected punishment increases, the average costs beared by criminals as well as the equilibrium price of crime increase, implying a reduction of the number of criminals.

¹³The analysis uses a kernel smoothing method in order to calculate a nonparametric estimate of the following regression curve:

$$CPG = f(FPG, IFLM) + \varepsilon,$$

where CPG is the Crime Per-capita Growth rate, FPG is the Foreign Population Growth rate and IFLM is the Index of Freedom in the Labor Market, $f(.,.)$ is an unknown (a priori) function and ε is an error term. The aim of the nonparametric regression is to provide an estimate of the smooth function $f(.,.)$.

The local mean estimator is a simple kernel estimator of $f(.,.)$, where the kernel function, $w(x_i - \bar{x}, h)$, is a smooth positive function, which gives weights that decrease monotonically as the difference $x_i - \bar{x}$ increases in size. In particular, two univariate normal kernels, one for each explanatory variable (x_i), are used and h (the smoothing parameter) is the relative standard deviation. When h goes to zero, then the estimates simply interpolates the points; when h goes to infinity, the estimate is a constant function that assigns the sample mean of CPG to each covariate.

between CPG and FBG changes according to the $IFLM$ value.¹⁴ In particular, as revealed by the graph, the sign of the relationship appears to be positive for low $IFLM$ values while it becomes negative in correspondence to high $IFLM$ values.

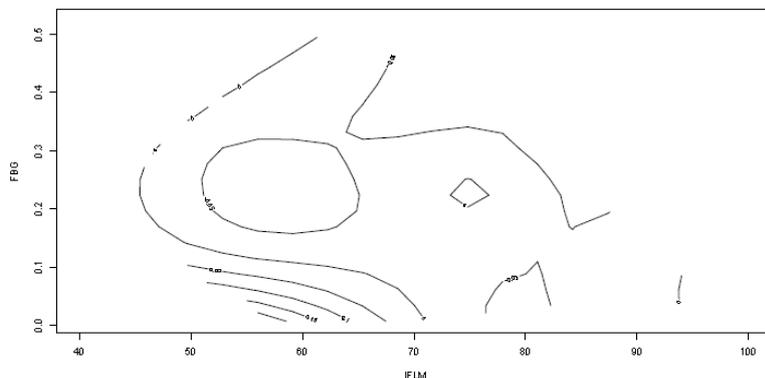


Figure 4. Smoothing techniques and the relationship between FBG , $IFLM$ and the CPG .

To better detect the relationship between FBG , $IFLM$ and the CPG , Figure 5 provides a linear representation of the regression.

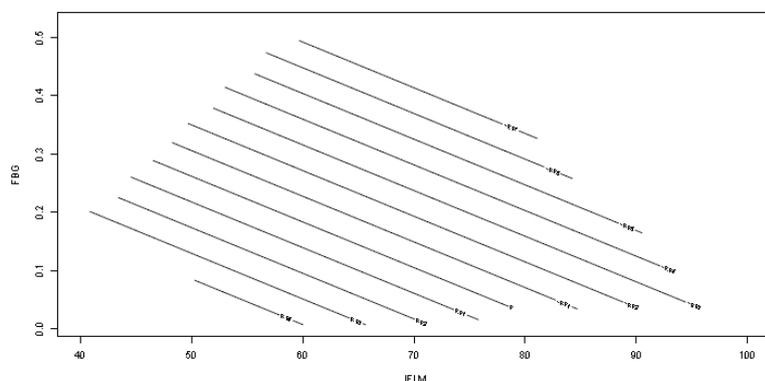


Figure 5. Linear estimate between FBG , $IFLM$ and the CPG .

The level curve associated with $CPG = 0$ includes the pairs $(FBG, IFLM)$ such that the growth rates of the number of crimes per inhabitant is null. Let us denote these combinations with $(FBG, IFLM)_{CPG=0}$. The level curves positioned on the right of $(FBG, IFLM)_{CPG=0}$ are associated with $CPG < 0$ such that, in line with the results reported in Proposition 8, the

¹⁴By conducting a p-value test for linearity (as null hypothesis), we find that the p-value is 0.094, then the linear hypothesis can be rejected at 10% of confidence.

relationship between immigration and crime is negative. Moreover, our model predicts that when immigration increases, the *IFLM* threshold level decreases, meaning that immigrants reduce the minimum level of flexibility in the labor market necessary to have a positive effect of immigration on crime reduction. This result supports the so-called Latino Paradox (Sampson, 2008), whereby immigration negatively affects the crime rate by reducing the average demand for criminal activities.

5 Extensions

In this section, we discuss possible extensions of our theoretical framework. In particular, by starting from an international equilibrium that satisfies Proposition 7, we show the effects on the equilibrium crime rates and the migration flows of relaxing specific assumptions of the model. We study the effects of the following variations in country A : (i) a change in the (relative) bargaining power of job-seekers and firms; (ii) changes in the structure of the crime market such as an increase in the degree of competitiveness of the crime market and an increase in the fixed costs of committing criminal activities; (iii) an increase in the labor productivity; and (iv) a change in the structure of mobility costs.

5.1 The Role of γ_A

Using Proposition 7, we show that an increase in the bargaining power of the job-seekers in country A , γ_A , reduces the probability of observing a negative relationship between n_A and P_A . Namely, countries strongly unionized are more likely to have a positive relationship between the crime rate and the size of the population. By Equation (7), to have $\frac{dx(n_A)}{dn_A} > 0$, it must be that

$$\phi(n_A)[1 + \eta_{\phi(n_A)k(n_A)}] > \frac{1 - \gamma_A}{\gamma_A}. \quad (24)$$

If γ_A increases, then the *RHS* of the inequality (24) decreases, going from infinity to zero. *Ceteris paribus*, the set of values of $\eta_{\phi(n_A)k(n_A)}$ and $\phi(n_A)$ that satisfy the previous inequality gets larger. In other words, countries with unionized labor markets are more likely to exhibit a positive relationship between immigration and the crime rate.

Using Proposition 7, we analyze the effects of a (marginal) increase in the bargaining power

of workers in country A , γ_A , on the initial international equilibrium. We refer to the following lemma.

Lemma 2. *If $\gamma_A \in (\underline{s}(n_A), \bar{s}(n_A))$, with $\underline{s}(n_A) = \frac{\Lambda_A + k_A(n_A)}{\Lambda_A + k_A(n_A) + \Omega_A(n_A)\phi_A(n_A) - \frac{\Omega_A(n_A)(r_A + \delta_A)}{\phi_A(n_A) \frac{dq(\phi_A(n_A))}{d\phi_A(n_A)}}}$ and $\bar{s}(n_A) = \frac{\Lambda_A + k_A(n_A)}{\Lambda_A + k_A(n_A) + \Omega_A(n_A)\phi_A(n_A)}$, then $x_A(n_A)$ strictly decreases in γ_A .*

Notice that when $\phi_A(n_A)$ goes to zero, $\bar{s}(n_A)$ goes to one, while the limit of $\underline{s}(n_A)$ depends on the term $\phi_A(n_A) \frac{dq(\phi_A(n_A))}{d\phi_A(n_A)}$. If this term is increasing in $\phi_A(n_A)$ (as in the case of a Cobb-Douglas function with decreasing returns to scale, the specification most frequently used for $q(\phi_A(n_A))$; see Stevens (2007) for a detailed discussion on matching functions), the term converges to 0, that is, an economy with a low number of vacancies with respect to the unemployment rate is more likely to exhibit a negative relationship between γ_A and $x_A(n_A)$. On the contrary, countries with a high number of vacancies with respect to the unemployment rate are more likely to exhibit a positive relationship between γ_A and $x_A(n_A)$. In this case, an increase in the workers' bargaining power leads to an increase in the expected wage rate that overcomes the effect on $x_A(n_A)$ caused by a reduction in the probability of finding a job.

Given the previous considerations, Lemma 2 shows that an increase in γ_A may lead to a decrease or an increase in the value of being job-seekers according to the characteristics of the labor market. If the tightness of the labor market is small enough and the usual assumptions on the shape of the matching function hold, for a wide range of $\gamma_A \in [0, 1]$, we have that $\frac{dx_A(n_A)}{d\gamma_A} < 0$. In this case, when γ_A increases, for any level of n_A , we will observe a reduction in $x_A(n_A)$. For the domestic market, this implies the following inequality $x_A(n_A) < \Pi(P_A, n_A)$ and, by the assumption that the domestic markets adjust instantaneously, n_A increases and $\Pi(P_A, n_A)$ decreases. In terms of no migration conditions, the previous considerations imply that $x_A(n_A) = \Pi(P_A, n_A) < \Pi(P_B, n_B) = x_B(n_B)$. Thus, inhabitants in country A have an incentive to migrate to country B . The adjustment process triggered by the migration flows drives the system to converge to an international equilibrium that is associated with higher crime rates in both countries. In terms of Figure 3, the D_A curve shifts up. At the end of the process, we can have higher or lower values of the crime rate n_A^* . For instance, in the case of Figure 3.b, an upward shift in curve D_A will cause a decrease in n_A^* , while in the other two cases we will observe a higher level of the equilibrium crime rate n_A^* . Vice versa, for countries in which $\frac{dx_A(n_A)}{d\gamma_A} > 0$, an increase in the bargaining power of workers, through a downward

shift in curve D_A , will cause the opposite effects on the equilibrium crime rate n_A^* .

5.2 The Form of the Crime Market

We now turn to the crime market, and we consider two variations. First, we analyze the effects of an increase in the degree of competitiveness of the crime market of country A . As we have already said, parameter b measures the degree of competitiveness of the crime market, that is, the parameter measures the power of price-makers in the crime market. Given $p_{j,A} = \frac{1}{bn_A P_A} + a_A$, it is possible to build the following measure of concentration:

$$\frac{p_{j,A} - a_A}{p_{j,A}} = \frac{1}{1 + bn_A P_A a_A}. \quad (25)$$

As shown by Equation (11), when b increases, *ceteris paribus*, the profits from criminal activities decrease. For a given crime rate, this makes $x_A(n_A) > \Pi(P_A, n_A)$. Given the assumption of instantaneous adjustment of the domestic markets, both the number of criminals and the crime rate decrease. As a consequence, if $\frac{dx_A(n_A)}{dn_A} < 0$, then the previous considerations imply an increase in both $x_A(n_A)$ and $\Pi(P_A, n_A)$. Thus, $x_A(n_A) = \Pi(P_A, n_A) > \Pi(P_B, n_B) = x_B(n_B)$, and, by the no migration conditions, inhabitants of country B have an incentive to migrate to country A . If country A is characterized by high job creation rates (i.e., high values of $\phi(n_A)$ and $\eta_{\phi(n_A)k(n_A)}$), migration inflows will lead to a further decrease in the crime rate in country A . Similarly, country B registers migration outflows and, by (22) and Proposition 7, a reduction in the crime rate. The opposite is true when country A is characterized by low job creation rates (i.e., low values of $\phi(n_A)$ and $\eta_{\phi(n_A)k(n_A)}$). By the same reasoning, we obtain opposite results when $\frac{dx_A(n_A)}{dn_A} > 0$. In terms of Figure 3, the D_A curve shifts down. At the end of the process, we can have higher or lower values for the crime rate n_A^* . For instance, in the case of Figure 3.b, a downward shift in D_A will cause an increase in n_A^* , while in the other two cases we will observe a lower level of the equilibrium crime rate n_A^* .

Second, we show how results change when the fixed costs of committing crimes in country A increase. For any pair (P_A, n_A) , higher fixed costs of crime imply a lower value of $\Pi(P_A, n_A)$ and then a lower crime rate in country A due to an increase in the number of job-seekers. Given P_A , in the new domestic equilibrium, country A registers higher (lower) values of $x_A(n_A)$ and $\Pi(P_A, n_A)$ when $\frac{dx_A(n_A)}{dn_A} < 0$ ($\frac{dx_A(n_A)}{dn_A} > 0$). The conclusions (as well as the graphic

representation) coincide with those associated with an increase in b .

5.3 The Role of Labor Productivity

Now, we consider the effects of an increase in the labor productivity of country A . The analysis is based on the following lemma.

Lemma 3. $\phi_A(n_A)$ is an increasing function of Λ_A .

Lemma 3 shows that $\frac{d\phi_A^*(n_A)}{d\Lambda_A} \geq 0$. Thus, the higher the productivity of country A , the higher the tightness of the domestic labor market. *Ceteris paribus*, this implies an increase in the value of being job-seekers such that $x_A(n_A) > \Pi(P_A, n_A)$. As a consequence, n_A decreases. The reduction in the crime rate in country A increases both $x_A(n_A)$ and $\Pi(P_A, n_A)$. By the no migration conditions, agents in country B move to country A . If country A is characterized by high values of $\phi(n_A)$ and $\eta_{\phi(n_A)k(n_A)}$ (i.e., high rates of job creation), migration inflows imply a further decrease of n_A . Moreover, by Equation (22) and Proposition 7, the previous considerations also imply a reduction in the crime rate in country B . Again, as in the other extensions, an increase in the labor productivity of country A implies a downward shift in the domestic locus of country A . In other words, public policies as well as technological innovations that increase labor productivity reduce crime.

5.4 The Role of Mobility Costs

Suppose that migration is associated with some mobility costs such that agents living in country B who move to country A bear costs mc_B while agents migrating from country A to country B incur costs mc_A . Let us assume that $mc_B > mc_A$. Then $m(c_B - c_A) \equiv m\Delta c > 0$. The no mobility conditions (16) and (15) become:

$$x(n_A) = x(n_B) + m\Delta c, \quad (26)$$

$$\Pi(P_A, n_A) = \Pi(P_B, n_B) + m\Delta c. \quad (27)$$

Let $(\widehat{P}_A, \widehat{n}_A, \widehat{P}_B, \widehat{n}_B)$ be the population sizes and the crime rates of the two countries in the international equilibrium when $m\Delta c > 0$. It follows that $\Pi(\widehat{P}_A, \widehat{n}_A) = x(\widehat{n}_A) > x(\widehat{n}_B) = \Pi(\widehat{P}_B, \widehat{n}_B)$, implying $\widehat{P}_A < P_A^*$, where P_A^* still indicates the equilibrium size of the population

when there are no mobility costs. Now, since $\hat{P}_A < P_A^*$, when the relationship between P_A and n_A is negative (see Figure 3.a), we will have $\hat{n}_A > n_A^*$; moreover, Corollary 6 implies that $\hat{n}_B > n_B^*$. That is, when the domestic locus of one country presents a negative slope, then the mobility costs increase the equilibrium crime rates of both countries. Similarly, when the relationship between P_A and n_A is positive (Figures 3.b and 3.c.) we will have $\hat{n}_A < n_A^*$. That is, when the mobility costs to migrate from B to A are relatively higher than the mobility costs to migrate from A to B , country A will experience a lower crime rate, while we will have $\hat{n}_B < n_B^*$ when $\frac{dn_B^D(P_B)}{dP_B} < 0$ (Figure 3.b) and $\hat{n}_B > n_B^*$ when $\frac{dn_B^D(P_B)}{dP_B} > 0$ (Figure 3.c).

6 Conclusion

Does immigration cause crime? The empirical evidence is puzzling. We highlight the role of the structure of labor and crime markets in defining the nature of this relationship. To analyze the interplay between immigration, unemployment and crime, we have developed a two-country, general equilibrium model in which agents can choose between looking for legal jobs and committing crime either in their country or abroad. Our main result draws attention to the role of the elasticity of the tightness of the domestic labor market with respect to the victimization cost on defining how immigration and crime relate. If this elasticity is sufficiently high relative to that of the demand of crime, an increase in the population size due to migration inflows is associated with a decrease in the crime rate of the host country. The opposite holds when the demand of crime is more elastic than the tightness of the labor market. As far as we know, this is the first contribution to underline the interplay between the elasticity of the labor market of the host country and the sign of the relationship between immigration and crime.¹⁵

Our model offers several additional insights to better understand the relationship between immigration and crime. First, crime is endemic to any economic system such that there are no equilibria in which the crime rate of a country is null. Second, migration flows from countries with strong work rigidities to societies characterized by more elastic labor markets are mutually benefic in terms of reducing the corresponding crime rates. Finally, although highly stylized, our results contribute to the debate on the efficacy of restrictive immigration policies. The

¹⁵Engelhardt (2010) studies the effects of rigidities of the labor market on the incarceration rate. He finds that unemployed are incarcerated two times faster than low wage workers and four times faster than high wage workers.

controversial Bossi-Fini law¹⁶ aimed at reforming the Italian immigration system is a valid example of such institutional interventions. According to the law, only those immigrants who prove they have a regular and permanent job in Italy are entitled to apply for a visa. Our model questions the efficacy of this legislative intervention by sharing the idea that *‘to crack down on crime, closing the nation’s doors is not the answer.’*¹⁷ Indeed, in the most optimistic scenario, the inflows of *regular* foreign workers induced by the law would exert pressure on both the labor and crime markets of the host country. In the former, the lower number of available positions would reduce the expected profits of native job-seekers. In the latter, the increase in the size of the population would stimulate the demand of crime and therefore the expected profits of criminal activities. For the marginal native agent, committing a crime would become more profitable than looking for a job. As a result, rather than producing significant effects on the size of the crime rate, the law would only modify the composition of the criminal population, with an increase in the share of natives compared to foreigners. On the contrary, policies aimed at improving the flexibility of the labor market and/or the productivity of workers are more effective in terms of crime dissuasion.

Several aspects of our model are worthy of further research. For instance, it might be interesting to relax the assumption of homogeneous labor. Introducing heterogeneity in labor skills will allow us to study the differences in the propensity of both migrating and committing crime across workers’ skills. Second, it might also be interesting to study the effects of introducing differences in the unemployment duration of foreigners and natives on migration flows and crime rates. Last (but not least), we propose a number of theoretical insights that can be tested on empirical grounds.

¹⁶ July 30th, 2002, n. 189.

¹⁷ R. Sampson, New York Times, March 11th, 2006.

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Appendix

A Proofs

Proof of Lemma 1. By Equation (9), let $G(\phi_A(n_A), n_A)$ be given by

$$G(\phi_A(n_A), n_A) = (1 - \gamma)\Lambda_A q(\phi_A(n_A)) - (r_A + \delta_A)\Omega_A(n_A) + q(\phi_A(n_A))(1 - \gamma)k(n_A) - q(\phi_A(n_A))\gamma\Omega_A(n_A)\phi_A(n_A). \quad (\text{A0})$$

By the implicit function theorem, $\frac{d\phi_A(n_A)}{dn_A} = -\frac{\partial G(\phi_A(n_A), n_A)/\partial n_A}{\partial G(\phi_A(n_A), n_A)/\partial \phi_A(n_A)}$, with $\frac{\partial G(\phi_A(n_A), n_A)}{\partial \phi_A(n_A)} \neq$

0. It follows,

$$\frac{d\phi_A(n_A)}{dn_A} = \frac{[\gamma\phi_A(n_A)q(\phi_A(n_A)) + (r_A + \delta_A) - (1 - \gamma)q(\phi_A(n_A))]\frac{dk(n_A)}{dn_A}}{[(1 - \gamma)(\Lambda_A + k(n_A)) - \gamma\Omega_A(n_A)\phi_A(n_A)]\frac{dq(\phi_A(n_A))}{d\phi_A(n_A)} - \gamma\Omega_A(n_A)q(\phi_A(n_A))}. \quad (\text{A1})$$

Since a sensible wage bargaining requires $W_{1,A} > W_{0,A}$, then $\Lambda_A - w_A > 0$. By Equation (10), the term in squared brackets at the denominator of Equation (A1) is positive. Since $\frac{dq(\phi_A(n_A))}{d\phi_A(n_A)} < 0$, the denominator is negative. The sign of the numerator of (A1) is positive when

$$\gamma\phi_A(n_A)q(\phi_A(n_A))\frac{dk(n_A)}{dn_A} + (r_A + \delta_A)\frac{dk(n_A)}{dn_A} > (1 - \gamma)q(\phi_A(n_A))\frac{dk(n_A)}{dn_A}.$$

The previous inequality can be rewritten as

$$\gamma\phi_A(n_A)q(\phi_A(n_A)) + (r_A + \delta_A) > (1 - \gamma)q(\phi_A(n_A)). \quad (\text{A2})$$

By Equation (9), we have

$$\gamma\phi_A(n_A)q(\phi_A(n_A)) = \frac{(1 - \gamma)q(\phi_A(n_A))}{\Omega_A(n_A)}(\Lambda_A + k(n_A)) - (r_A + \delta_A). \quad (\text{A3})$$

By replacing Equation (A3) in (A2), it follows that inequality in (A2) holds when $\Lambda_A > c_A$. Therefore $\frac{d\phi(n_A)}{dn_A} < 0$. ■

Proof of Proposition 1. The equilibrium crime rate, n_A^* , is determined by functions $\Pi(P_A, n_A)$ and $x(n_A)$. As n_A goes to zero, $\lim_{n_A \rightarrow 0^+} \Pi(P_A, n_A) = \infty$ and $\lim_{n_A \rightarrow 0} x_A(n_A) = \frac{\gamma}{1-\gamma}c_A\phi_0 <$

∞ , where $\phi_0 < \infty$ denotes the value of the market tightness when $n_A = 0$. From Equation (11), $\Pi(P_A, n_A)$ is decreasing in n_A , with $\Pi(P_A, 1) = \frac{Q(P_A)}{bP_A^2} - F_A - k(1)$; moreover, both functions $\Pi(P_A, n_A)$ and $x_A(n_A)$ are continuous on the interval $n_A \in (0, 1]$. Therefore, since $\Pi(P_A, 0) > \frac{\gamma}{1-\gamma}c_A\phi_0, \forall P_A \in (0, P]$, two cases are possible:

- a) $\exists n_A^* \in (0, 1]$ such that $\Pi(P_A, n_A^*) = x(n_A^*)$
- b) $\Pi(P_A, n_A) > x(n_A), \forall n_A \in (0, 1]$

In the first case, an interior equilibrium, n_A^* , exists. The second case implies the existence of a corner solution in which the expected profit from crime is higher than the value of being job-seekers for any admissible and strictly positive crime rate. Thus, it is profitable for all agents to engage in criminal activities implying $n_A^* = 1$. ■

Proof of Corollary 1. The proof of the necessary condition proceeds by contradiction. Suppose the corner solution is the unique autarkic equilibrium but $\exists \bar{n}_A \in (0, 1]$ such that $\Pi(\bar{n}_A, P_A) \leq x(\bar{n}_A)$. If $\Pi(\bar{n}_A, P_A) = x(\bar{n}_A)$, \bar{n}_A is an equilibrium contradicting the initial assumption. If $\Pi(\bar{n}_A, P_A) < x(\bar{n}_A)$, since both functions are continuous and $\Pi(n_A, P_A)$ monotonically decreases in n_A , $\Pi(0, P_A) > \frac{\gamma}{1-\gamma}c_A\phi_0, \forall P_A \in (0, P]$ implies that there exists another equilibrium $n_A^* \in (0, \bar{n}_A)$ such that $\Pi(P_A, n_A^*) = x(n_A^*)$. The proof of the sufficient condition is trivial. If $\Pi(P_A, n_A) > x(n_A), \forall n_A \in (0, 1]$, then $\nexists n_A^* \in (0, 1]$ such that $\Pi(P_A, n_A^*) = x(n_A^*)$. ■

Proof of Proposition 2. First, we prove that if $\frac{dx_A(n_A)}{dn_A} > \frac{\partial \Pi(P_A, n_A)}{\partial n_A}, \forall n_A \in (0, 1]$, the autarkic equilibrium is unique. Let n_A^* be the equilibrium crime rate in country A when population is P_A such that $x(n_A^*) = \Pi(P_A, n_A^*)$. Since $\Pi(0, P_A) > \frac{\gamma}{1-\gamma}c_A\phi_0, \forall P_A \in (0, P]$, if $\frac{dx_A(n_A)}{dn_A} > \frac{\partial \Pi(P_A, n_A)}{\partial n_A}$, we have $x(n_A) > \Pi(P_A, n_A) \forall n_A \in (n_A^*, 1]$ and $x(n_A) < \Pi(P_A, n_A) \forall n_A \in (0, n_A^*)$. Then, by continuity of $x(n_A)$ and $\Pi(P_A, n_A)$ in $n_A \in (0, 1]$, it follows that the equilibrium is unique.

Now, we prove that $\frac{dx_A(n_A)}{dn_A} > \frac{\partial \Pi(P_A, n_A)}{\partial n_A}, \forall n_A \in (0, 1]$, implies the existence of a threshold value $\tilde{\eta}_{\phi(n_A), k(n_A)}$ such that for $\eta_{\phi(n_A), k(n_A)} > \tilde{\eta}_{\phi(n_A), k(n_A)}, \forall n_A \in (0, 1]$, the autarkic equilibrium is unique. By differentiating (7) with respect to n_A and imposing $\frac{dx_A(n_A)}{dn_A} > \frac{\partial \Pi(P_A, n_A)}{\partial n_A}, \forall n_A \in (0, 1]$, it follows that

$$\frac{dk(n_A)}{dn_A} \left\{ \frac{\gamma}{1-\gamma} \left[\phi(n_A) + \Omega_A(n_A) \frac{d\phi(n_A)}{dn_A} \right] - 1 \right\} > -\frac{2Q(P_A)}{bP_A^2 n_A^3} - \frac{dk(n_A)}{dn_A}. \quad (\text{A4})$$

That is,

$$\eta_{\phi(n_A),k(n_A)} > \tilde{\eta}_{\phi(n_A),k(n_A)} \quad (\text{A5})$$

where $\tilde{\eta}_{\phi(n_A),k(n_A)} \equiv \frac{1-\gamma}{\gamma} \frac{1}{\phi(n_A)} \left(-\frac{2Q(P_A)}{bP_A^2 n_A^3} \frac{1}{\frac{dk(n_A)}{dn_A}} - 1 \right) - 1$. Therefore, if $\eta_{\phi(n_A),k(n_A)} > \tilde{\eta}_{\phi(n_A),k(n_A)}$, $\forall n_A \in (0, 1]$, the following inequality holds: $\frac{dx_A(n_A)}{dn_A} > \frac{\partial \Pi(P_A, n_A)}{\partial n_A}$, $\forall n_A \in (0, 1]$. ■

Proof of Corollary 2. By contradiction, suppose that there exists an equilibrium in which $n_A^* = 0$. By Equation (11), $\lim_{n_A \rightarrow 0^+} \Pi(P_A, n_A) = \infty$. Moreover, by Equation (9), $\lim_{n_A \rightarrow 0} x_A(n_A) = \frac{\gamma}{1-\gamma} c_A \phi_0$. This implies that, as n_A goes to zero, $\Pi_A(P_A, n_A) > x_A(n_A)$, $\forall P_A$. Therefore, for some agents it is profitable to enter the crime market. ■

Proof of Proposition 3. First, we focus on the sufficient condition. Let $n_A^* \in (0, 1)$ be the equilibrium crime rate. Consider an increase from n_A^* to $n_A^* + \varepsilon$, with $\varepsilon > 0$ small enough. If $x_A(n_A^* + \varepsilon) > \Pi(n_A^* + \varepsilon, P_A)$, at $n_A^* + \varepsilon$, unemployment is more profitable than crime. Thus, both the number and the proportion of criminals decrease and the economy moves back to the initial equilibrium. Now, consider a reduction of the crime rate from n_A^* to $n_A^* - \varepsilon$. It is easy to check that n_A^* is stable if $x_A(n_A^* - \varepsilon) < \Pi(n_A^* - \varepsilon, P_A)$. Since functions $x_A(n_A^*)$ and $\Pi(P_A, n_A^*)$ are differentiable, we can consider the limit as ε goes to zero. The two conditions collapse into the following expression:

$$\frac{dx_A(n_A^*)}{dn_A} > \frac{\partial \Pi(P_A, n_A^*)}{\partial n_A} \quad (\text{A6})$$

Moving to the necessary condition, by contradiction, suppose that the domestic equilibrium is stable and that $\frac{dx_A(n_A^*)}{dn_A} < \frac{\partial \Pi(P_A, n_A^*)}{\partial n_A}$. Since the equilibrium is (locally) stable, after any small perturbation, the economy must go back to the initial equilibrium. Consider a negative perturbation that makes the economy to pass from n_A^* to $n_A^* - \varepsilon$. Since the equilibrium is stable the proportion of criminals must increase from $n_A^* - \varepsilon$ to n_A^* . But since we have assumed that $\frac{dx_A(n_A^*)}{dn_A} < \frac{\partial \Pi(P_A, n_A^*)}{\partial n_A}$, the reduction in the value of unemployment, $x_A(n_A^*)$, is smaller than the reduction in the value of crime, $\Pi(P_A, n_A^*)$, i.e. $x_A(n_A^* - \varepsilon) > \Pi(n_A^* - \varepsilon, P_A)$. This contradicts the hypothesis of stability. ■

Proof of Corollary 3. Recall that $\lim_{n_A \rightarrow 0^*} \Pi(P_A, n_A) = \infty$ and $\lim_{n_A \rightarrow 0} x_A(n_A) = \frac{\gamma}{1-\gamma} c_A \phi_0$ (see Equations (11) and (9), respectively). This implies that, as n_A goes to zero, $\Pi_A(P_A, n_A) > x_A(n_A)$, $\forall P_A$. When an interior equilibrium is unique, $\exists n_A^* \in (0, 1) : \Pi(P_A, n_A^*) = x(n_A^*)$ and, by continuity of $\Pi(P_A, n_A)$ and $x_A(n_A)$, $\forall \varepsilon \in (0, n_A^*)$, $\Pi(n_A^* - \varepsilon, P_A) > x(n_A^* - \varepsilon)$.

That is,

$$\frac{x(n_A^*) - x(n_A^* - \varepsilon)}{\varepsilon} > \frac{\Pi(P_A, n_A^*) - \Pi(n_A^* - \varepsilon, P_A)}{\varepsilon} \quad (\text{A7})$$

as ε goes to zero, the previous expression collapses into

$$\frac{dx_A(n_A^*)}{dn_A} > \frac{\partial \Pi(P_A, n_A^*)}{\partial n_A} \quad (\text{A8})$$

which is the condition to have stability of an interior equilibrium.

Assume $\Pi(P_A, n_A) > x(n_A)$, $\forall n_A \in (0, 1]$, then the unique equilibrium is the corner solution, $n_A^* = 1$ (see Proposition 3.3). Thus, $\forall \varepsilon \in (0, n_A^*)$, $\Pi(n_A^* - \varepsilon, P_A) > x(n_A^* - \varepsilon)$ and some agents find profitable to enter the crime market implying that the corner solution is stable. ■

Proof of Corollary 4. For a given size of the population of country A , P_A , we have multiple equilibria if and only if there are at least two crime rates, $\underline{n}_A^*, \bar{n}_A^* \in (0, 1]$ that satisfy Equation (12). Without loss of generality, suppose \underline{n}_A^* is the lowest equilibrium level of the crime rate: $\underline{n}_A^* < \bar{n}_A^*$. As n_A goes to zero, $\Pi_A(P_A, n_A) > x_A(n_A)$. Thus, by the continuity of $\Pi(P_A, n_A)$ and $x_A(n_A)$ and the fact that $x(\underline{n}_A^*) = \Pi(P_A, \underline{n}_A^*)$, it follows that $\exists \varepsilon > 0$: $\Pi(\underline{n}_A^* - \varepsilon, P_A) > x(\underline{n}_A^* - \varepsilon)$. That is,

$$x(\underline{n}_A^*) - x(\underline{n}_A^* - \varepsilon) > \Pi(P_A, \underline{n}_A^*) - \Pi(\underline{n}_A^* - \varepsilon, P_A). \quad (\text{A9})$$

By taking the limit of this inequality as ε goes to zero,

$$\frac{dx_A(\underline{n}_A^*)}{dn_A} > \frac{\partial \Pi(P_A, \underline{n}_A^*)}{\partial n_A}, \quad (\text{A10})$$

which is the condition to have stability of an interior equilibrium. ■

Proof of Proposition 4. By Definition 2, an equilibrium is associated to both a population size, P_A^* , and a crime rate, n_A^* . Since the domestic locus of country A is continuous on $(0, P] \times (0, 1]$ and the international locus is defined on $[0, P] \times (0, 1]$, three cases are possible:

1) $\exists P_A^* \in (0, P) : n_A^* = n_A^I(P_A^*) = n_A^D(P_A^*)$, thus (P_A^*, n_A^*) will be an interior international equilibrium.

2) $n_A^I(P_A) < n_A^D(P_A)$, $\forall P_A \in (0, P)$. Since $n_A^I(P_A)$ represents the crime rate of country A that satisfies the no migration condition (15) for given population $P_B = P - P_A$ and crime rate $n_B^D(P - P_A)$ in country B , then it follows that $\Pi(P_A, n_A^D(P_A)) < \Pi(P_A, n_A^I(P_A)) =$

$\Pi(P_A, n_B^D(P - P_A)), \forall P_A \in (0, P)$. Therefore, through the migration flows from country A to country B , the international equilibrium collapses into a situation in which the crime rate of country B is determined by the domestic locus of country B at $P_B = P$.

3) $n_A^I(P_A) > n_A^D(P_A), \forall P_A \in (0, P)$. Since $n_A^I(P_A)$ represents the crime rate of country A that satisfies the no migration condition (15) for given population $P_B = P - P_A$ and crime rate $n_B^D(P - P_A)$ in country B , then it follows that $\Pi(P_A, n_A^D(P_A)) > \Pi(P_A, n_A^I(P_A)) = \Pi(P_A, n_B^D(P - P_A)), \forall P_A \in (0, P)$. Therefore, through the migration flows from country B to country A , the international equilibrium collapses into a situation in which the crime rate of country A is determined by the domestic locus of country A at $P_A = P$. ■

Proof of Proposition 5. When $\frac{dx_A(n_A)}{dn_A} > \frac{\partial \Pi(P_A, n_A)}{\partial n_A}, \forall n_A \in (0, 1]$, by Proposition 2, the equilibrium crime rate is unique for any size of the population. At the same time, if $\eta_{Q(P_A), P_A} > 2$ (or < 2) $\forall P_A \in (0, P]$, the profit function is always increasing (or decreasing) in P_A implying the monotonicity of the domestic locus for country A . If $\eta_{Q(P_B), P_B} > 2$ (or < 2) $\forall P_A \in (0, P]$ also the domestic locus of B is monotonically increasing (or decreasing) in P_B , thus from Equation (22) we can conclude that the international locus is always decreasing (or increasing) in P_A . Thus, the international equilibrium is also unique. ■

Proof of Proposition 6. Let (P_A^*, n_A^*) be the size of the population and the crime rate of country A associated with the unique international equilibrium. In country B there is a unique equilibrium population size, $P_B^* = P - P_A^*$, and a unique equilibrium crime rate, $n_B^* = k_B^{-1}(k_A(n_A^*))$. By contradiction, suppose there is another equilibrium crime rate n_B^{**} that is compatible with P_B^* :

$$x(n_A^*) = x(n_B^*) = \Pi(P_B^*, n_B^*) = x(n_B^{**}) = \Pi(P_B^*, n_B^{**}) \quad (\text{A11})$$

This equality violates the monotonicity of $\Pi(P_B^*, n_B)$ with respect to n_B . ■

Proof of Proposition 7. Suppose $\frac{dn_A^D(P_A^*)}{dP_A} > 0$ and $\frac{dn_A^D(P_A^*)}{dP_A} > \frac{dn_A^I(P_A^*)}{dP_A}$. For $\varepsilon > 0$ small enough, it follows that $n_A^D(P_A^* + \varepsilon) > n_A^I(P_A^* + \varepsilon)$. Since the profits from crime monotonically decrease in the crime rate, it follows that $\Pi(P_A^* + \varepsilon, n_A^D(P_A^* + \varepsilon)) < \Pi(P_A^* + \varepsilon, n_A^I(P_A^* + \varepsilon)) = \Pi(P - P_A^* - \varepsilon, n_B^D(P - P_A^* - \varepsilon))$, with $n_A^I(P_A^* + \varepsilon) = k_A^{-1}(k_B(n_B^D(P - P_A^* - \varepsilon)))$. Therefore, migrations from country A to country B occur until $P_A = P_A^*$ and $n_A^* = n_A^D(P_A^*) = n_A^I(P_A^*)$. The proof of the proposition when $\frac{dn_A^D(P_A^*)}{dP_A} < 0$ proceeds in an analogous way. We prove the

necessary condition by contradiction. Suppose that the domestic equilibrium is stable and $\frac{dn_A^D(P_A^*)}{dP_A} < \frac{dn_A^I(P_A^*)}{dP_A}$. Consider a negative perturbation of P_A^* to $P_A^* - \varepsilon$. Since the equilibrium is stable, the size of the population must converge back to P_A^* . However, since $\frac{dn_A^D(P_A^*)}{dP_A} < \frac{dn_A^I(P_A^*)}{dP_A}$, then $n_A^D(P_A^* - \varepsilon) > n_A^I(P_A^* - \varepsilon)$. Given that profits from crime are monotonically decreasing in the crime rate, there will be migration flows from country A to country B , contradicting the hypothesis of stability. ■

Proof of Corollary 5. By contradiction, suppose that the unique international equilibrium is unstable. If this equilibrium is an interior solution, we have

$$\frac{dn_A^D(P_A^*)}{dP_A} < \frac{dn_A^I(P_A^*)}{dP_A} \quad (\text{A12})$$

Then, $\forall \varepsilon \in (0, P - P_A^*)$, we get $n_A^D(P_A^* + \varepsilon) < n_A^I(P_A^* + \varepsilon)$. This implies $\Pi(P_A^* + \varepsilon, n_A^D(P_A^* + \varepsilon)) > \Pi(P_A^* + \varepsilon, n_A^I(P_A^* + \varepsilon)) = \Pi(P - P_A^* - \varepsilon, n_B^D(P - P_A^* - \varepsilon))$ such that full migration from country B to country A occurs. Thus, the model admits another equilibrium in which $P_A^* = P$ and $n_A^* = n_A^D(P)$. If the unique equilibrium is characterized by full migration, say $P_A^* = P$ and $n_A^* = n_A^D(P)$, and this equilibrium is unstable, it follows that $\Pi(P - \varepsilon, n_A^D(P - \varepsilon)) < \Pi(\varepsilon, n_B^D(\varepsilon))$, $\forall \varepsilon \in (0, P)$. Therefore, another full migration equilibrium in which $P_A^* = 0$ exists, contradicting uniqueness. The proof concerning the case in which the unique unstable equilibrium is $P_B^* = P$ and $n_B^* = n_B^D(P)$ proceeds in an analogous way. ■

Proof of Corollary 6. The stability condition requires $\frac{dn_A^D(P_A^*)}{dP_A} > \frac{dn_A^I(P_A^*)}{dP_A}$, that is $\frac{dn_A^I(P_A^*)}{dP_A} < 0$. Thus, given condition (22) we can conclude that $\frac{dn_B^D(P_B^*)}{dP_B} > 0$. ■

Proof of Proposition 8. By Equation (11), we have that

$$\frac{\partial \Pi(P_A, n_A)}{\partial P_A} = -\frac{2Q(P_A)}{bn_A^2 P_A^3} + P_A \frac{dQ(P_A)}{dP_A} \frac{1}{bn_A^2 P_A^3}, \quad (\text{A13})$$

and

$$\frac{\partial \Pi(P_A, n_A)}{\partial n_A} = -\frac{2Q(P_A)}{bn_A^3 P_A^2} - \frac{dk(n_A)}{dn_A} \quad (\text{A14})$$

Thus,

$$\frac{\partial \Pi(P_A, n_A)}{\partial P_A} = \frac{dQ(P_A)}{dP_A} \frac{1}{bn_A^2 P_A^2} + \left[\frac{\partial \Pi(P_A, n_A)}{\partial n_A} + \frac{dk(n_A)}{dn_A} \right] \frac{n_A}{P_A} \quad (\text{A15})$$

Then, the domestic locus has a negative slope if and only if

$$\frac{dQ(P_A)}{dP_A} \frac{1}{bn_A^2 P_A^2} + \left[\frac{\partial \Pi(P_A, n_A)}{\partial n_A} + \frac{dk(n_A)}{dn_A} \right] \frac{n_A}{P_A} < 0 \quad (\text{A16})$$

that is,

$$\frac{\partial \Pi(P_A, n_A)}{\partial n_A} < -\frac{dQ(P_A)}{dP_A} \frac{1}{bn_A^3 P_A} - \frac{dk(n_A)}{dn_A} \quad (\text{A17})$$

Since the stability condition requires $\frac{dx(n_A)}{dn_A} > \frac{\partial \Pi(P_A, n_A)}{\partial n_A}$, a sufficient condition for the previous inequality to hold is

$$\frac{dx(n_A)}{dn_A} < -\frac{dQ(P_A)}{dP_A} \frac{1}{bn_A^3 P_A} - \frac{dk(n_A)}{dn_A} \quad (\text{A18})$$

From the derivative of (7) w.r.t. n_A , given the values of γ_A and $\phi(n_A)$, the last inequality implies that the absolute value of $\eta_{\phi(n_A), k(n_A)}$ must be large enough. In this case, along the domestic locus, immigration reduces the crime rate. Moreover, the threshold value of both $\frac{dx(n_A)}{dn_A}$ and $\eta_{\phi(n_A), k(n_A)}$ depends on the variation of $Q(P_A)$ with respect to P_A . ■

Proof of Lemma 2. From Equation (9) we can write

$$q(\phi_A(n_A)) = \frac{\Omega_A(n_A)(r_A + \delta_A)}{(\Lambda_A + k_A(n_A))(1 - \gamma_A) + \gamma_A \Omega_A(n_A) \phi_A(n_A)}, \quad (\text{A19})$$

that is,

$$q(\phi_A(n_A)) = \Psi(\phi_A(n_A), \gamma_A). \quad (\text{A20})$$

By the implicit function theorem,

$$\frac{d\phi_A(n_A)}{d\gamma_A} = \frac{\frac{d\Psi(\phi_A(n_A), \gamma_A)}{d\gamma_A}}{\frac{dq(\phi_A(n_A))}{d\phi_A(n_A)} - \frac{d\Psi(\phi_A(n_A), \gamma_A)}{d\phi_A(n_A)}}. \quad (\text{A21})$$

Since $\frac{d\Psi(\phi_A(n_A), \gamma_A)}{d\gamma_A} \geq 0$, $\frac{d\Psi(\phi_A(n_A), \gamma_A)}{d\phi_A(n_A)} \geq 0$ and $\frac{dq(\phi_A(n_A))}{d\phi_A(n_A)} < 0$, we conclude that $\frac{d\phi_A(n_A)}{d\gamma_A} \leq 0$.

From (7), it follows that

$$\frac{dx_A(n_A)}{d\gamma_A} = \frac{1}{(1 - \gamma_A)} \Omega_A(n_A) \phi_A(n_A) \left[\frac{1}{(1 - \gamma_A)} + \frac{\gamma_A}{\phi_A(n_A)} \frac{d\phi_A(n_A)}{d\gamma_A} \right]. \quad (\text{A22})$$

From the last two equations, we have that $\frac{dx_A(n_A)}{d\gamma_A} = 0$ when $\gamma_A = \underline{s}(n_A)$ or $\gamma_A = \bar{s}(n_A)$, with $\underline{s}(n_A) \equiv \frac{\Lambda_A + k_A(n_A)}{\Lambda_A + k_A(n_A) + \Omega_A(n_A)\phi_A(n_A) - \frac{\Omega_A(n_A)(r_A + \delta_A)}{\phi_A(n_A) \frac{dq(\phi_A(n_A))}{d\phi_A(n_A)}}}$ and $\bar{s}(n_A) \equiv \frac{\Lambda_A + k_A(n_A)}{\Lambda_A + k_A(n_A) + \Omega_A(n_A)\phi_A(n_A)}$. Since $\frac{dq(\phi_A(n_A))}{d\phi_A(n_A)} < 0$, we have that $0 < \underline{s}(n_A) < \bar{s}(n_A) < 1$. Moreover we know that $\lim_{\gamma_A \rightarrow 0} \frac{dx_A(n_A)}{d\gamma_A} > 0$ and $\lim_{\gamma_A \rightarrow 1^-} \frac{dx_A(n_A)}{d\gamma_A} > 0$. Thus, when $\underline{s}(n_A) < \gamma_A < \bar{s}(n_A)$, then $\frac{dx_A(n_A)}{d\gamma_A} < 0$. ■

Proof of Lemma 3. As in Lemma 1, we have that $\frac{d\phi_A(n_A)}{d\Lambda_A} = -\frac{\partial G(\phi_A(n_A), n_A) / \partial \Lambda_A}{\partial G(\phi_A(n_A), n_A) / \partial \phi_A(n_A)}$.

Thus,

$$\frac{d\phi_A(n_A)}{d\Lambda_A} = -\frac{(1 - \gamma)q(\phi_A(n_A))}{(1 - \gamma)\Lambda_A \frac{dq(\phi_A(n_A))}{d\phi_A(n_A)} - \gamma(\Lambda_A + k(n_A))(q(\phi_A(n_A)) + \phi_A(n_A) \frac{dq(\phi_A(n_A))}{d\phi_A(n_A)})}. \quad (\text{A23})$$

From Lemma 1 we know that the denominator is negative, so it is easy to check that for a non trivial economy the expression is strictly positive. ■