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DYNAMIC PRINCIPAL COMPONENTS: A NEW CLASS OF MULTIVARIATE GARCH MODELS

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Dynamic Principal Components: a New Class of Multivariate GARCH Models

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Abstract

The OGARCH specification is the leading model for a class of multivariate GARCH (MGARCH) specifications that are based on linear combinations of univariate GARCH specifications. Most MGARCH models in this class adopt a spectral decomposition of the covariance matrix, allowing for heteroskedasticity on at least some of the principal components, while the loading matrix, which maps the conditional principal components to the asset returns, is constant over time. This paper extends the OGARCH model class to allow for time-varying loadings. Our approach closely parallels the DCC modelling approach, introduced as an extension of the CCC model, to allow for dynamic correlations. After introducing an auxiliary process that captures the relevant features of the unobservable loading dynamics, we compute the time-varying loading matrix from the auxiliary process, subject to the necessary orthonormality constraints. The resulting model (the Dynamic Principal Components, or DPC, model) preserves the OGARCH models ease of interpretation and feasibility. In particular, we show that the eigenvectors of the sample covariance matrix can consistently estimate the time-varying loadings intercept term. This property extends to the dynamic framework the well-known analogous property of the OGARCH model. Empirical examples demonstrate the benefits to the loading matrix of introducing time-varying properties.

Keywords: Spectral Decomposition, Principal Component Analysis, Orthogonal GARCH, Scalar BEKK, DCC, Multivariate GARCH, Two-step Estimation.

J.E.L. codes: C32, C58, C13, G10.

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1 Introduction

One strand of the financial econometrics literature has focused during the last decade on introducing new Multivariate Generalized Auto Regressive Conditional Heteroskedasticity (MGARCH) models and developing feasible estimation approaches for MGARCH specifications. The main purpose of the latter efforts has been to overcome the well-known *curse of dimensionality* that affects the MGARCH model class by maintaining the ease of estimation and the interpretability of the model outcomes.[1] In that framework, the use of linear transformations of univariate conditionally orthogonal GARCH processes has received increasing attention by researchers (e.g., Kariya 1988, Van der Weide 2002, Lanne and Saikkonen 2007, Fan et al. 2008, and Boswijk and Van der Weide 2011). The leading model in this class may be the Orthogonal GARCH (OGARCH) model, introduced by Alexander and Chibumba (1997) and Alexander (2001). In the OGARCH model the matrix that maps the orthogonal processes to the asset returns (the loading matrix) is orthonormal. Moreover, the orthogonal processes coincide with the principal components of the asset returns, the loading matrix coincides with the eigenvector matrix of the asset's unconditional covariance matrix, and the loading matrix can be consistently estimated with no real effort from the asset-sample covariance matrix (first estimation step). The principal components follow univariate GARCH models. Their parameters are estimated in a separate step by means of univariate quasi-maximum-likelihood (QML) methods, leading to the OGARCH estimator. The number of parameters estimated in the second step is $\mathcal{O}(N)$, where N is the number of assets.

As the principal components variances are recovered within a univariate framework, the second step is feasible irrespective of the number of assets. In addition, because of the orthogonality of the principal components, the second step is equivalent to the joint QML estimator of the component GARCH parameters, so the second step is QML-efficient, conditional on the first step. The OGARCH model is attractive because of the ease of estimation and the estimation outputs interpretability in terms of principal component analysis. However, as Ding and Engle (2001) pointed out, *"[...] it may not be reasonable to assume that the loading matrix is constant over time."* In this paper, we suggest an extension of the OGARCH model that allows for time-varying loadings. Our proposed extension has no effect on most of the desirable properties that characterize the OGARCH

[1]See Silvennoinen and Teräsvirta (2009) for a discussion of the optimal features of MGARCH models.

model.

The main challenge of building a model that allows for time-varying loadings lies in obtaining the two goals of ensuring the orthonormality of the time-varying loading matrix and getting a meaningful interpretation of the loading dynamics. We achieve these goals by means of a modelling approach that closely parallels the Dynamic Conditional Correlation (DCC) approach Engle (2002) adopted in extending Bollerslev's (1990) Constant Conditional Correlation (CCC) model. The DCC model introduces an auxiliary process, the *correlation driving process*, with the purpose of capturing the relevant features of the underlying correlation dynamics. Then the correlation-driving process is suitably rescaled to recover the proper conditional correlation matrices.

We introduce in the principal component framework an auxiliary process, the *loading-driving process*, to capture the relevant features of the underlying loading dynamics. Then we extract the time-varying loading matrix from the loading-driving process under the required orthonormality constraints. The simplest specification for the loading-driving process is an Exponentially Weighted Moving Average (EWMA) of the cross-products of the asset returns, which captures possible structural breaks in the OGARCH unconditional covariance matrix and provides a first answer to the Ding and Engle's (2001) doubts. In the EWMA case, the eigenvector matrix of the loading-driving process reflects the related structural breaks in the OGARCH loading matrix. A natural extension of the EWMA model is given by a loading-driving process structured as a BEKK recursion or filter[2] (e.g., Engle and Kroner 1995, Engle and Mezrich 1996, Ding and Engle 2001). Once dynamic loadings are available, they can be used to recover conditional principal components, which are later modelled as univariate GARCH processes. We call the resulting model Dynamic Principal Component (DPC), highlighting the term "dynamic "in the model name to refer to the time-varying nature of the linear mapping from the principal components to the asset returns. We stress that the components can follow any univariate GARCH specification, so the DPC model has a flexible modular structure that is similar to the modular structure of the DCC model.

We can estimate the DPC model parameters by a three-step estimator, which is similar to that used in the DCC model. In its simplest specification, which we call Scalar DPC, this estimator reduces to a set of $N+1$ $\mathcal{O}(1)$ estimations, thus overcoming the curse of dimensionality and main-

[2]BEKK stands for Baba, Engle, Kraft, and Kroner (Engle and Kroner 1995). We refer to this extension of the EWMA model as a recursion or filter, not as a BEKK model, for reasons explained later in the paper.

taining model feasibility irrespective of the number of variables. The first and second steps of the estimator consider fitting a (possibly misspecified) BEKK recursion, the loading-driving process, to a collection of asset returns under a *variance-targeting*-like constraint. The variance-targeting constraint fixes the BEKK intercept and takes into account a sample estimator. Despite being $\mathcal{O}(N^2)$, this first step is not computationally demanding.[3] Conditional on targeting, the BEKK estimation of the dynamic loading parameters might be made simple by imposing, for instance, a scalar parameterization. The BEKK recursion estimation, the second step of our approach, has a complexity of order $\mathcal{O}(1)$. The BEKK filter estimation is used just to recover the dynamic conditional loadings that are required to compute the conditional principal components. In the third step, the volatilities of the conditional principal components are fitted one at a time via univariate QML. Each univariate maximization is $\mathcal{O}(1)$, leading to the $\mathcal{O}(N)$ third step. As with the OGARCH estimator, the $\mathcal{O}(N)$ estimation of the component volatilities is feasible irrespective of the number of assets, and it is QML-efficient conditional on the estimation of the conditional loadings.

As a major theoretical result, we show that, under normal conditions the eigenvectors of the loading-driving process's intercept coincide with those of the asset unconditional covariance matrix. We call this property of the DPC model *loading targeting*. The name stems from the fact that the eigenvector matrix of the unconditional covariance matrix is the *unconditional* principal components' loading matrix. Loading targeting implies that the $\mathcal{O}(N^2)$ term of the DPC parameterization is consistently estimated by the sample covariance matrix's eigenvectors. This property extends the well-known analogous property of the OGARCH model to the dynamic framework. However, the first two steps of the DPC estimator might suffer inconsistency problems like those that affect Engle's (2002) DCC model (Aielli 2013.)[4] Nevertheless, as a second consequence of loading targeting, the inconsistency, if present, of the first two estimation steps would affect at most $\mathcal{O}(N)$ parameters: the eigenvalues of the intercept term and the loading dynamic parameters. Such an inconsistency problem can be at least partially mitigated by the flexibility of the $\mathcal{O}(N)$ third step of estimation. This property is an advantage of the DPC modelling approach with respect to the DCC modelling approach, where the inconsistency of the second step, which involves $\mathcal{O}(N^2)$ parameters (Aielli

[3] We assume that the sample size is sufficiently large to allow for an appropriate estimation of the unconditional covariance matrix.

[4] While the joint estimation of all model parameters will avoid inconsistency problems, it will expose the estimation to the curse of dimensionality.

2013), is generally followed by a less flexible $\mathcal{O}(1)$ third step of estimation. Allowing for feasible $\mathcal{O}(N)$ third steps in the DCC estimator is possible via, for example, composite likelihood (Engle et al. 2009) but at the cost of efficiency. In addition, we introduce a specification test to determine whether there is model misspecification in the loadings. The DPC properties imply that, under correct model specification and with consistent estimates, the conditional principal components are conditionally orthogonal. Therefore, accepting this null hypothesis implies both the appropriateness of the fitted model and the absence of inconsistency issues. A Monte Carlo experiment verifies the test's asymptotic properties. We also report results from a set of empirical applications that demonstrate the advantages of the DPC modelling and estimation approach. By using two datasets, each with its own cross-sectional dimension, we show that DPC specifications perform well in terms of model fit and provide easy-to-interpret model outcomes.

The remainder of the paper is organized as follows. The next section introduces the DPC modelling approach. Section 3 describes the DPC estimator and an easily implemented test of correctly specified loading dynamics that requires only the estimation of a restricted VAR. Section 4 reports a set of empirical applications. Finally, Section 5 concludes the paper. Figures, tables, some technical material, and the proofs of the propositions are reported in the appendices.

2 The DPC model

Let $y_t = [y_{1,t}, \dots, y_{N,t}]'$ denote the vector of the asset returns at time $t = 1, 2, \dots$. We assume that $E_{t-1}[y_t] = 0$, where $E_t[\,\cdot\,]$ is the expectation operator conditional on the information set at time t, denoted as $\mathcal{I}_t$. We set $H_t = [h_{i,j,t}] = E_{t-1}[y_t y_t']$, with H_t the conditional covariance matrix (CCM) of y_t. If H_t is finite, let

$$H_t = L_t D_t L_t' \tag{1}$$

denote a spectral decomposition (SD) of H_t. The diagonal elements of $D_t = \operatorname{diag}(d_{1,t}, d_{2,t}, \dots, d_{N,t})$ are the eigenvalues of H_t, and the columns of $L_t = [l_{i,j,t}]$ are the associated eigenvectors. By the properties of the SD of a positive semi-definite (PSD) matrix (see Gruber, 2013, among many others), L_t is orthonormal (i.e., $L_t L_t' = L_t' L_t = I_N$, where I_N denotes the N-dimensional identity matrix). The elements of the vector

$$u_t = L_t' y_t \tag{2}$$

are the conditional principal components (thereafter *components*) of y_t. The components are *conditional* since they are computed conditional on $\mathcal{I}_{t-1}$. The components are conditionally orthogonal with conditional covariance matrix given by D_t. From (2), the asset returns can be written as

$$y_t = L_t u_t. \tag{3}$$

We name the matrix L_t, which maps the conditional components into the asset returns, the *conditional loading* matrix. In correlated systems, a few components can explain most of the conditional volatility dynamics. We are interested in building a model for a SD of H_t, which is i) directly readable in terms of loading and component dynamics and ii) possibly endowed with desirable theoretical and empirical properties. Differently from the approach of Alexander (2001) and that of many other studies inspired by the OGARCH model, we aim to provide a specification for a SD of the conditional covariance matrix where both the eigenvalues (the diagonal elements of D_t) and the eigenvectors matrices (the L_t) are allowed to vary over time.

2.1 Conditional loadings modelling

In constructing the L_t model, we first introduce an auxiliary process, the *loading-driving process*, which can capture the relevant features of the underlying loading dynamics. Then we extract the time-varying loadings from the auxiliary process so the time-varying loading matrix has the required orthonormality property. We define the auxiliary process as a Scalar BEKK recursion based on the asset returns, or

$$Q_t = (1 - a - b)S + a y_{t-1} y_{t-1}' + b Q_{t-1}, \tag{4}$$

where $(y_0, Q_0) \in \mathcal{I}_0$, while the scalars a and b and the matrix S are parameters to be estimated. The matrix of the conditional loadings, L_t, is defined as the eigenvector matrix of Q_t, that is,

$$Q_t = L_t G_t L_t', \tag{5}$$

where $G_t = \text{diag}(g_{1,t}, \dots, g_{n,t})$ contains the eigenvalues of Q_t. We then make the following assumption:

Assumption 2.1.1. $0 \leqslant a$, $0 \leqslant b$, $a + b < 1$, *S and Q_0 are positive definite (PD).*

Under Assumption 2.1.1, the matrix Q_t is PD, so the SD of Q_t exists for any t. However, we require an identification condition to achieve the uniqueness of the SD of Q_t in (5). We start by imposing a uniqueness condition on the SD of S. Let

$$S = LDL' \tag{6}$$

denote the SD of S, where $L = [l_{i,j}]$ and $D = \text{diag}(d_1, d_2, \dots, d_N)$.

Assumption 2.1.2. *The eigenvalues of S are arranged in strictly decreasing order; the sign of the associated eigenvectors is such that the diagonal elements of L are positive.*

Assumption 2.1.2 requires that S has distinct eigenvalues and that the diagonal elements of L are different from zero. This is an inconsequential restriction, as the set of PD matrices ruled out by Assumption 2.1.2 has a null measure.

Assumption 2.1.3. *The eigenvalues of Q_t are arranged in strictly decreasing order; the sign of the associated eigenvectors is such that the diagonal elements of $L_t'L$ are positive.*

The uniqueness condition in Assumption 2.1.3 implicitly requires that Q_t has distinct eigenvalues and that the diagonal elements of $L_t'L$ are different from zero. Such a restriction is mild as, in the dynamic case, it holds almost surely (a.s.). In the constant case, which implies that $Q_t = S$, Assumption 2.1.3 reduces to Assumption 2.1.2. In Section 2.6 we show that the columns of L are the directions of $\mathcal{R}^N$, along which the *unconditional* principal components of y_t move. Therefore, the sign condition in Assumption 2.1.3 can be interpreted as minimizing the angle between the directions of $\mathcal{R}^N$, along which corresponding pairs of conditional and unconditional components move.

To provide a first motivation of the suggested model for L_t, consider a switching regime data-generating process (DGP) such that $H_t \in \{\bar{S}_i,\ i = 1, 2, \ldots, K\}$. In the i-th regime it holds that $H_t = \bar{S}_i = \bar{L}_i \bar{D}_i \bar{L}_i'$, where the right-hand side denotes a SD of $\bar{S}_i$. If $a + b = 1$ in (4), the loading-driving process approximates an EWMA of outer products of y_t. In that case, $Q_t \approx \bar{S}_i$, or $L_t \approx \bar{L}_i$, where $\bar{L}_i$ is the current loading matrix. Therefore, for $a + b = 1$, the matrix L_t can be seen as an approximation of the current regime of the conditional loading matrix. We can thus interpret model (4-5) as a natural extension of the EWMA model, including the EWMA model as a limiting case. Alternatively, we might consider model (4-5) as the true DGP of the loading process, with the aim of modelling smooth loading dynamics. The latter perspective is adopted in most of the present work.

The identification conditions in Assumption 2.1.3 do not ensure the existence of a unique loadings sequence for a given data set. We require an additional constraint on S. As an example, consider a value $c > 0$ be such that $ca + b < 1$. Multiplying both sides of (4) by c yields

$$Q_t^* = (1 - a^* - b)S^* + a^* y_{t-1} y_{t-1}' + b\, Q_{t-1}^*,$$

where $a^* = ca$, $S^* = Sc(1 - a - b)/(1 - ca - b)$, and $Q_t^* = cQ_t$, for $t = 0, 1, \ldots$. Given the observed series, the loading-driving processes Q_t and Q_t^* differ only by a scale factor, so they provide the same time-varying loadings. If the process is covariance-stationary,[5] a way to get identification is to restrict the magnitude of S by imposing that

$$\mathrm{trace}(S) = \mathrm{trace}(\bar{S}), \tag{7}$$

where

$$\bar{S} = E[y_t y_t']$$

($\bar{S}$ is the unconditional covariance matrix of y_t). The next section introduces a model for the components that, among other desirable properties, implies that the identification condition (7) holds.

[5] We discuss the structural properties of the model in Section 2.6.

2.2 Components conditional variance modelling

Following Alexander and Chibumba (1997) (see also Alexander, 2001), we propose that a possible model for the conditional variances of the components is Bollerslev's (1986) GARCH(1,1) model with variance targeting (Engle and Mezrich, 1996); that is:

$$E_{t-1}[u_{i,t}^2] = d_{i,t}, \quad d_{i,t} = (1-\alpha_i-\beta_i)\,\gamma_i + \alpha_i u_{i,t-1}^2 + \beta_i d_{i,t-1}, \quad i = 1,2,\ldots,N, \tag{8}$$

where $(u_{i,0}, d_{i,0}) \in \mathcal{I}_0$. If $a = b = 0$, by combining 8 with 4 ,we note that the eigenvalues of S are not identified. As a natural identification condition, we impose that $\gamma_i = d_i, i = 1,2,\ldots,N$, where d_i is the $i-$th largest eigenvalue of S. The restriction $a = b = 0$ in (4) leads to the OGARCH model. In this case, $Q_t = S$, which implies that $L_t = L$, or the loadings are constant. We then set the following assumption:

Assumption 2.2.1. $\gamma_i = d_i$, $\alpha_i \geqslant 0, \beta_i \geqslant 0$, $\alpha_i + \beta_i < 1$, *and* $d_{i,0} > 0$, *for* $i = 1,2,\ldots,N$.

Since $d_i > 0$, (from Assumption 2.2.1), it follows that $d_{i,t} > 0$, which ensures that $H_t = L_t D_t L_t'$ is PD. Moreover, since $\alpha_i + \beta_i < 1$, the components are covariance stationary with unconditional second moment equal to

$$E[u_{i,t}^2] = E[d_{i,t}] = d_i, \tag{9}$$

(Bollerslev 1986). Since the d_i's are arranged in strictly decreasing order (Assumption 2.1.2), it follows that

$$E[u_{1,t}^2] > E[u_{2,t}^2] > \cdots > E[u_{N,t}^2]. \tag{10}$$

Therefore, the components are arranged in decreasing order according to their *unconditional* variances.

Definition 2.2.1. *The model defined by equations (3), (4-5), and (8) and Assumptions 2.1.1-2.2.1 is called the Scalar DPC model.*

The term *scalar* in "Scalar DPC model"refers to the Scalar BEKK recursion[6], adopted for Q_t.

[6]The expression "Scalar BEKK recursion"for Q_t is used in place of "Scalar BEKK model"because Q_t is the CCM of y_t only if the loadings are constant and the components are conditionally homoskedastic.

The Scalar DPC model includes $\mathcal{O}(N^2)$ parameters in the loading-driving process (because of the presence of S in the intercept) and $\mathcal{O}(N)$ parameters in the conditional component volatility models. Section 2.6 introduces a more general specification of Q_t and discusses the curse of dimensionality issue. The following property holds for the Scalar DPC:

Proposition 2.2.1. *Under weak stationarity conditions, the loading process of the Scalar DPC model is identified.*

Proof. See Appendix D.

This paper focuses on the GARCH(1,1) specification for the components. However, the components could follow any univariate GARCH specification, possibly including exogenous regressors and/or leverage effects, (see, e.g., Bollerslev 2010 and Francq and Zakoïan 2010.) The resulting modularity of the Scalar DPC structure is a modelling advantage that is shared with the DCC model in Engle (2002). In the Engle model, the asset conditional variances, rather than the components, can follow any GARCH specification. Irrespective of the GARCH specification of the components, under stationarity (of the GARCH) both the ordering of the components according to equation (10) and Proposition 2.2.1 hold provided that the unconditional variances of the components are set equal to the eigenvalues of S.

2.3 An illustrative example

For $(a, b) = 0$ the Scalar DPC model yields the OGARCH model.[7]

With the OGARCH model we can write

$$\bar{S} = E[y_t y_t'] = E[L_t u_t u_t' L_t'] = LE[u_t u_t']L' = LDL' = S.$$

The constant loading matrix L, and the unconditional covariance matrix of the components D, are both provided by the SD of the second moment of the asset returns. Since Q_t can capture switching

[7] In its general definition the OGARCH model allows for reduced rank H_t. Since $d_{i,t} > 0$ for $i = 1, 2, \ldots, N$, the DPC model includes only the case of the full-rank OGARCH model. The defining equations of the DPC model can be modified easily to include the OGARCH model in its general specification. In order to simplify the exposition in this paper, we do not consider such an extension. In addition, the assumption of reduced rank H_t is difficult to motivate, and it typically results in poor empirical performances.

regimes in the second moment of y_t, we can interpret the DPC as a model that can capture switching regimes in the unconditional structure of the OGARCH model.

As an illustration, consider the impact of the Greek debt crisis that began at the end of 2009 on the Greek/German bond volatility/covolatility dynamics reported in Figure 1.[8] The sample statistics computed before and after the break date, which we fix at the end of 2009, show that the crisis resulted in a dramatic increase in the Greek bond index's volatility. (See Table 1.) The German bond index's volatility was unaffected by the crisis. The Greek debt crisis also led to Greek bond index dynamics' drifting away from the German bond index' dynamics, as shown by the sample unconditional cross-correlation; in fact, the correlation collapsed from 0.79 before the break date to -0.15 after the break date. The estimate of $l_{1,1}^2$ (*squared first loading* hereafter), which measures the contribution of the first conditional component volatility against the volatility of the Greek bond index, moved from 0.53 before the crisis to 0.99 during the crisis. Orthonormality implies that, during the crisis the loading matrix approaches the identity. As a consequence, during the crisis the bond series coincides with the components, which are conditionally uncorrelated, suggesting a kind of flight-to-safety effect for the German market.

The OGARCH estimation output computed from the whole sample period essentially ignores the break and is dramatically biased toward the crisis period. (The OGARCH squared first loading is close to 1.) On the other hand, the DPC estimation output correctly detects the impact of the crisis: the average DPC squared first loading before and after the break date is substantially equal to the corresponding sample statistics. Accordingly, the average DPC estimated cross-correlations drops to -0.14 during the crisis, as compared to the value of 0.85 observed before the crisis, and is similar to the corresponding sample statistics. The estimates of the loading dynamic parameters are equal to $(\hat{a}, \hat{b}) = (0.065, 0.935)$. The presence of a dramatic structural break in the loading process causes the estimated loading dynamic to be "integrated"$(\hat{a} + \hat{b} = 1)$.

With the OGARCH model, the component volatilities overlap before the crisis. The DPC model's correct detection of the break allows for a more realistic picture of the hierarchy of the component volatilities and simplifies their association with known/observed risk sources. In most applications the structural breaks in the OGARCH unconditional structure, if present, would be less

[8]The example uses changes in the five-year benchmark bond redemption yield for Germany and Greece. The time series were recovered from Thomson Datastream.

evident than they are in the Greek/German bond example. In addition, when structural breaks are absent, we would expect similar pictures from the DPC and the OGARCH estimation outputs. A second example supports this expectation. Similar to the previous case, we consider the UK/German bond index series before and during the Greek crisis. (See Figure 2.) The plots of the bond returns suggest the absence of structural breaks. The graphic comparison of the OGARCH and the DPC confirms that there is no apparent break in the evolution of squared conditional loadings or in the component volatilities. The OGARCH and DPC estimation outputs are similar apart from the presence of limited dynamic in the squared conditional loading.

2.4 Some remarks on the DGP of the Scalar DPC model

The DGP of the Scalar DPC model has the following structure:[9]

Definition 2.4.1. DGP of the Scalar DPC model. *Given the initial information,* $\mathcal{I}_0 = (Q_0, D_0, u_0)$, *set* $y_0 = L_0 u_0$, *where* L_0 *is the eigenvector matrix of* Q_0 *computed under the identification condition in Assumption 2.1.3. Then*

1. *for* $t = 1, 2, \ldots, T$, *set* $u_t = D_t^{1/2} z_t$, *where* $z_t | \mathcal{I}_{t-1} \sim N(0, I_N)$, *and the diagonal elements of* D_t *are defined as in (8);*
2. *for* $t = 1, 2, \ldots, T$, *set* $Q_t = (1 - a - b)S + a\, L_{t-1} u'_{t-1} u_{t-1} L'_{t-1} + b\, Q_{t-1}$, *where* L_{t-1} *is the eigenvector matrix of* Q_{t-1}, *computed under the identification condition in Assumption 2.1.3;*
3. *for* $t = 1, 2, \ldots, T$, *set* $y_t = L_t u_t$.

Remark 2.1. The DPC CCM is reconstructed as

$$H_t = L_t D_t L'_t. \tag{11}$$

There is a logical difference between (1) and (11). In (1), the pair (L_t, D_t) is defined as a function of H_t; to be precise, (L_t, D_t) is the pair of eigenvectors/eigenvalues of H_t. Based on the orthonormality of L_t and the diagonality of D_t, as ensured by the defining equations of the DPC model, $L_t D_t L'_t$ in (11) is a SD of H_t. Roughly speaking, we will never compute the SD of H_t with the DGP of the DPC model; on the contrary, we will always obtain H_t from its SD, as provided by

[9] We consider the Gaussian case for simplicity, but other distributions can be used.

the DPC recursions.

Remark 2.2. The conditional components are arranged in decreasing order according to their *unconditional* variances; see (10). This does not imply that the components are also arranged in decreasing order according to their *conditional* variances. The conditional variances of the components (i.e., the $d_{i,t}$) can fall in any order at each point in time. This property, which merely extends the analogous property of the OGARCH model to the dynamic framework, should not be interpreted as a drawback. In fact, in a conditional analysis, the interest points at the possibility that things are different conditionally than they are unconditionally. Consider, for example, a bivariate model in which the first component and the second component are given the interpretation of market factor and interest rate factor, respectively. Consistent with the factor model literature (e.g., Elton et al. 2009 and the references cited therein), the market factor will explain most of the unconditional system volatility. A conditional analysis should be able to reveal periods in which a crisis in the bond markets can temporarily become the main driver of the system's volatility dynamics. This behavior is accounted for by allowing the conditional variance of the second component to exceed the conditional variance of the first component temporarily.[10]

Remark 2.3. The i-th component comes from the eigenvector that is associated with the i-th largest eigenvalue of Q_t. Therefore, from a geometrical perspective, $u_{i,t}$ moves along the i-th longest axis of Q_t, which is time-varying. The fact that the conditional components move along time-varying axes does not prevent the interpretation of the components as economic, well-identified, underlying factors. Consider, for instance, a switching regime DGP of H_t. If one gives the first component the interpretation of market factor, the first eigenvector of H_t, which is used to compute the market factor, should change depending on the underlying regime. A natural extension of such a modelling perspective is to think of the eigenvectors of H_t (i.e., the definition of the components) as a smooth process. Most MGARCH models are implicitly characterized by smooth eigenvector and eigenvalue dynamics. The DGP of the Scalar DPC model is characterized by such a feature,

[10] A model in which the components are arranged in decreasing order according to their *conditional* variances can easily be implemented by modelling the increments between the component volatilities. For example, one can set $\kappa_{i,t} = d_{i,t} - d_{i+1,t}$, where $k_{i,t} = (1 - \alpha_i - \beta_i)\, p_i + \alpha_i u_{i,t-1}^2 + \beta_i k_{i,t-1}$, $p_i = d_i - d_{i+1}$, and $d_{N+1,t} = 0$. Since $p_i > 0$ (recall that the d_i's are arranged in decreasing order), the $k_{i,t}$s are positive, so the $d_{i,t}$'s are arranged in decreasing order.

with the additional property that the smooth eigenvector and eigenvalue dynamics are explicitly modelled.

2.5 Factor GARCH representation

In their Generalized Orthogonal Factor GARCH (GOF-GARCH) model, Lanne and Saikkonen (2007) suggested setting as conditionally homoskedastic a subset of factors in order to get a model that is closely related to a traditional factor representation with idiosyncratic and systematic errors (GOF-GARCH). Silvennoinen and Teräsvirta (2009) pointed out that the GOF-GARCH model can be seen as combining the advantages of both the factor models (which have a reduced number of heteroskedastic factors) and the orthogonal models (which have relative ease of estimation because of the orthogonality of factors). Similar advantages can be obtained by a factor representation of the DPC model. Let V_t and Z_t denote, respectively, the $N \times M$ and $N \times (N - M)$ matrices obtained by collecting the first M and last $N - M$ columns of L_t. M is the number of the unconditionally most volatile components that can explain a given (large) amount of unconditional volatility. Suppose that the last $N - M$ components are conditionally homoskedastic (i.e., $\alpha_i = \beta_i = 0$ for $i = M + 1, M + 2 \ldots, N$). The DPC model can be written as

$$y_t = V_t \xi_t + \epsilon_t, \tag{12}$$

where $\xi_t = [u_{1,t}, \ldots, u_{M,t}]'$, and $\epsilon_t = Z_t[u_{M+1,t}, \ldots, u_{N,t}]'$. The vector ξ_t can be seen as a vector of conditionally heteroskedastic factors. The vector ϵ_t, which is conditionally orthogonal to the factors, can be interpreted as a vector of idiosyncratic errors. If the loading process is constant (i.e., if $a = b = 0$), the model coincides with a static GARCH(1,1) factor model with a constant orthogonal weight matrix and a reduced number of idiosyncratic errors (Lanne and Saikkonen 2007). This model closely parallels the GOF-GARCH model. The basic difference is in the matrix's mapping of the factors to the asset returns: in the GOF-GARCH model this matrix comes from the polar, rather than spectral, decomposition of the second moment. Allowing for time-varying weights (i.e., for $a \geqslant 0$ and $b \geqslant 0$) is a natural extension of the constant weight model.

2.6 Extension and structural properties

This section demonstrates that, under usual conditions, the columns of L are the eigenvectors of the stationary second moment of y_t, or

$$\bar{S} = L\bar{D}L', \tag{13}$$

where $\bar{D}$ is diagonal. We refer to this property as the *loading targeting* of the DPC model. The loading targeting can be equivalently expressed by stating that the vector

$$\bar{u}_t = L'y_t, \tag{14}$$

is the vector of the *unconditional* principal components (unconditional components hereafter) of y_t. Taking advantage of the loading targeting, we can replace the $\mathcal{O}(N^2)$ term of the DPC parameterization in (4) and (6): those corresponding to the eigenvectors of S, by means of the eigenvectors of the asset's sample covariance matrix. The remaining model parameters, which we have to estimate - the eigenvalues of S, the loading dynamic parameters, and the component volatility parameters - are jointly $\mathcal{O}(N)$, thus avoiding the curse of dimensionality. Before proving the loading targeting, it is convenient to extend (4) in order to allow for more flexible loading dynamics. Let us set

$$Q_t = (S - \mathcal{A}S\mathcal{A}' - \mathcal{B}S\mathcal{B}') + \mathcal{A}y_{t-1}y_{t-1}'\mathcal{A}' + \mathcal{B}Q_{t-1}\mathcal{B}', \tag{15}$$

where

$$\mathcal{A} = LAL', \quad \mathcal{B} = LBL', \tag{16}$$

$$A = \text{diag}(\sqrt{a_1}, \sqrt{a_2}, \ldots, \sqrt{a_N}), \quad B = \text{diag}(\sqrt{b_1}, \sqrt{b_2}, \ldots, \sqrt{b_N}). \tag{17}$$

Equation (15) looks like a Full BEKK(1,1,1) recursion with restricted parameter matrices (Engle and Kroner 1995). The restrictions included in the constraints (16-17) act on the dynamic parameter matrices and give a peculiar expression of the intercept term.

Assumption 2.6.1. *In equation (15), Q_0 is PD, S is PD, $a_i \geqslant 0$, $b_i \geqslant 0$, and $a_i + b_i < 1$, for $i = 1, \ldots, N$.*

Clearly, the intercept of Q_t can be re-written as $L(D-ADA-BDB)L'$. Therefore, under Assumption

2.6.1, the intercept of Q_t is PD, ensuring that Q_t is PD (Engle and Kroner 1995) and that the SDs of Q_t exist. The dynamic parameter restrictions required by Assumption 2.6.1 are easy to impose. On the other hand, the PD constraints that would be required on a traditional Full BEKK equation with targeting would imply a non-linear (quadratic) function of the elements of A and B.[11] The following definition is based on the general specification just introduced:

Definition 2.6.1. *The model defined by equations (3), (5), (8), and (15-17) and Assumptions 2.1.2, 2.1.3, 2.2.1, and 2.6.1 is called the DPC model.*

By setting $a_i = a$ and $b_i = b$, for $i = 1, 2, \ldots, N$, we obtain the Scalar DPC model. We can then derive the structural properties of the DPC model.

Proposition 2.6.1. Stationarity. *In the DPC model (see def. 2.6.1), suppose that: i) the process y_t starts infinitely far in the past; ii) $u_{i,t} = d_{i,t}^{1/2} z_{i,t}$, where the processes $\{z_{i,t}\}_{t=-\infty,+\infty}$, for $i = 1, 2, \ldots, N$, are iid with zero mean and unit variance and mutually independent; and iii) $E[\log(\alpha_i z_{i,t}^2 + \beta_i)] < 0$, for $i = 1, 2, \ldots, N$. Then y_t is strictly and weakly stationary.*

Proof. See Appendix D.

Given the mutual independence of the components, the stationarity of the univariate component processes is the essential element for the stationarity of the vector process y_t. We focus on a simple GARCH(1,1) case, but the extension of the proof to other GARCH specifications for the components is straightforward, provided that the univariate independent processes, $(d_{i,t}, u_{i,t})$, for $i = 1, 2, \ldots, N$, are strictly and weakly stationary. (See the proof of the proposition.) Conditions for strict and weak stationarity of several univariate GARCH specifications adopted in practice can be found in Francq and Zakoïan (2010) and elsewhere. We also consider the introduction of targeting in the loading-driving process.

Proposition 2.6.2. Loading targeting. *In the DPC model (see def. 2.6.1), suppose that the assumptions of proposition 2.6.1 hold. In addition, suppose that the conditional distribution of $z_{i,t}$ is symmetric for $i = 1, 2, \ldots, N$. Then the columns of L are the eigenvectors of $\bar{S} = E\left[y_t y_t'\right]$.*

[11]A restriction strategy of the Full BEKK parameter matrices similar to (15-16) was first suggested by Noureldin et al. (2014).

Proof. See Appendix D.

The proof of the proposition uses symmetry arguments. The extension to the general case of GARCH specification of the components is straightforward provided that $d_{i,t}$ does not depend on the sign of $z_{i,t-m}$, $m = 1, 2, \ldots$. (See the proof of the proposition.) The latter condition rules out, for example, the possibility of asymmetry/leverage effects in the component variance equations. However, for the loading targeting to hold, the assumptions of the proposition are only sufficient, not also necessary. By modelling the loading-driving process as a restricted Full BEKK(P,Q,K) recursion, we obtain a general extension of the DPC model:

$$Q_t = C + \sum_{\mathrm{q}=1}^{\mathrm{Q}} \sum_{\mathrm{k}=1}^{\mathrm{K}} \mathcal{A}_{\mathrm{q,k}} \{y_{t-\mathrm{q}} y'_{t-\mathrm{q}}\} \mathcal{A}'_{\mathrm{q,k}} + \sum_{\mathrm{p}=1}^{\mathrm{P}} \sum_{\mathrm{k}=1}^{\mathrm{K}} \mathcal{B}_{\mathrm{p,k}} Q_{t-\mathrm{p}} \mathcal{B}'_{\mathrm{p,k}}, \tag{18}$$

$$C = S - \sum_{\mathrm{q}=1}^{\mathrm{Q}} \sum_{\mathrm{k}=1}^{\mathrm{K}} \mathcal{A}_{\mathrm{q,k}} S \mathcal{A}'_{\mathrm{q,k}} - \sum_{\mathrm{p}=1}^{\mathrm{P}} \sum_{\mathrm{k}=1}^{\mathrm{K}} \mathcal{B}_{\mathrm{p,k}} S \mathcal{B}'_{\mathrm{p,k}}. \tag{19}$$

$$\mathcal{A}_{\mathrm{q,k}} = L A_{\mathrm{q,k}} L', \quad \mathcal{B}_{\mathrm{p,k}} = L B_{\mathrm{p,k}} L', \tag{20}$$

where $A_{\mathrm{q,k}} = \mathrm{diag}(\sqrt{a_{\mathrm{q,k},1}}, \sqrt{a_{\mathrm{q,k},2}}, \ldots, \sqrt{a_{\mathrm{q,k},N}})$, $B_{\mathrm{p,k}} = \mathrm{diag}(\sqrt{b_{\mathrm{p,k},1}}, \sqrt{b_{\mathrm{p,k},2}}, \ldots, \sqrt{b_{\mathrm{p,k},N}})$, $0 \leqslant a_{\mathrm{q,k},i} < 1$, $0 \leqslant b_{\mathrm{p,k},i} < 1$, for $\mathrm{q} = 1, 2, \ldots, \mathrm{Q}$, $\mathrm{p} = 1, 2, \ldots, \mathrm{P}$, $\mathrm{k} = 1, 2, \ldots, \mathrm{K}$, $i = 1, 2, \ldots, N$. In this general model, we can easily verify that Q_t id PD by construction. Moreover, by $\mathrm{P} = \mathrm{Q} = \mathrm{K} = 1$, the model in (18)-(20) yields the DPC model.

2.7 Representation in terms of unconditional components

The DPC model has the following equivalent representation:

$$y_t = L\bar{u}_t, \quad \bar{u}_t = \bar{L}_t u_t, \tag{21}$$

where $\bar{L}_t$ is the eigenvector matrix of $\bar{Q}_t$,

$$\bar{Q}_t = (D - ADA - BDB) + A\bar{u}_{t-1}\bar{u}'_{t-1}A + B\bar{Q}_{t-1}B, \tag{22}$$

$\bar{Q}_0 = L'Q_0L$ is PD, and u_t is defined as in (8). In computing the SD of $\bar{Q}_t$, we must consider the following uniqueness conditions, which are coherent with those in Assumption 2.1.3:

Assumption 2.7.1. *The eigenvalues of $\bar{Q}_t$ are arranged in strictly decreasing order; the sign of the associated eigenvectors is such that the diagonal elements of $\bar{L}_t$ are positive.*

The loading-driving process and the conditional loadings are reconstructed as $Q_t = L\bar{Q}_t L'$ and $L_t = L\bar{L}_t$, respectively. The matrices Q_t and $\bar{Q}_t$ share the same eigenvalue matrix; see equation (5). In fact, the right-hand side of

$$\bar{Q}_t = L' Q_t L = L' L_t G_t L_t' L = \bar{L}_t G_t \bar{L}_t' \tag{23}$$

is the SD of $\bar{Q}_t$ computed under the identification conditions in Assumption 2.7.1. Noting that the CCM of $\bar{u}_t$ is $\bar{L}_t D_t \bar{L}_t'$, we have that the unconditional components follow a Diagonal DPC model with loading-driving process given by $\bar{Q}_t$ and a conditional loading matrix given by $\bar{L}_t$. The diagonality restriction imposes a diagonal form for the intercept and for the dynamic parameter matrices of $\bar{Q}_t$. Thus, the DPC model of y_t can be seen as a rotation of the Diagonal DPC model of the unconditional components of y_t, where the rotation matrix is L. The representation in (21)-(22) extends straightforwardly to the case of the general model in (18)-(20). A special case of the resulting specification is the Diagonal VECH form of $\bar{Q}_t$, which can be written in Hadamard form as

$$\bar{Q}_t = (\iota\iota' - \sum_{\mathrm{q}=1}^{\bar{\mathrm{Q}}} \bar{A}_{\mathrm{q}} - \sum_{\mathrm{p}=1}^{\bar{\mathrm{P}}} \bar{B}_{\mathrm{p}}) \odot D + \sum_{\mathrm{q}=1}^{\bar{\mathrm{Q}}} \bar{A}_{\mathrm{q}} \odot \{\bar{u}_{t-\mathrm{q}} \bar{u}_{t-\mathrm{q}}'\} + \sum_{\mathrm{p}=1}^{\bar{\mathrm{P}}} \bar{B}_{\mathrm{p}} \odot \bar{Q}_{t-\mathrm{p}},$$

where ι is the $N \times 1$ vector of unit elements, and $\odot$ denotes the element-wise matrix product. If the intercept and the parameter matrices are PD, $\bar{Q}_t$ is PD (Ding and Engle 2001).

3 Estimation and specification testing

3.1 Joint and two-step Quasi-Maximum-Likelihood

When discussing the estimation strategy, we focus on the DPC model of Definition 2.6.1. Let ψ denote the vector that stacks the diagonal elements of A and B. Recalling that $\mathcal{A} = LAL'$ and $\mathcal{B} = LBL'$, where L is the eigenvector matrix of S, we parameterize the model of the conditional loadings in terms of (S, ψ). The SD of the loading-driving process evaluated at (S, ψ) is denoted as

$$Q_t(S, \psi) = \{L_t(S, \psi)\} G_t(S, \psi) \{L_t(S, \psi)\}',$$

under the identification conditions 2.1.2 and 2.1.3. The CCM of the components evaluated at (S, ψ, ϕ) is denoted as $D_t(S, \psi, \phi)$, where $\phi = (\phi_1, \phi_2, \ldots, \phi_N)$, $\phi_i = (\alpha_i, \beta_i)$, $i = 1, \ldots, N$. The parameter (S, ψ) enters $D_t(S, \psi, \phi)$ through the eigenvalue matrix of S, or D, whose diagonal elements are the unconditional variances of the components, and $u_t = u_t(S, \psi) = \{L_t(S, \psi)\}' y_t$. See Assumption 2.2.1 and equations (3) and (8). Finally, the model of the CCM is written as

$$H_t(S, \psi, \phi) = \{L_t(S, \psi)\} D_t(S, \psi, \phi) \{L_t(S, \psi)\}'.$$

Given our semi-parametric specification, we suggest the use of a QML approach for the estimation of the DPC model parameters. Taking into account that $\{L_t(S, \psi)\}^{-1} = \{L_t(S, \psi)\}'$, and $|L_t(S, \psi)| = \pm 1$, the DPC quasi-log-likelihood (QLL) can be written as

$$\mathcal{L}_T(S, \psi, \phi) = \sum_{i=1}^{N} \left\{ \frac{1}{2} \sum_{t=1}^{T} N \log 2\pi + \log d_{i,t}(S, \psi, \phi) + u_{i,t}^2(S, \psi)/d_{i,t}(S, \psi, \phi) \right\}, \tag{24}$$

where the right-hand side is the sum of the components' (univariate) QLLs. The number of parameters entering the QLL is $N(N+1)/2 + 2N + 2N$, where $N(N+1)/2$ is the number of distinct elements of S, $2N$ is the number of diagonal elements in A and B, and $2N$ is the number of component volatility parameters. If the number of assets is small, the parameters can be estimated jointly via QML, and standard errors can be computed by means of the usual sandwich formula (e.g., Bollerslev and Wooldridge 1992).[12] For three reasons the QML estimation with more than a few assets becomes difficult. First, as is common in the MGARCH framework because of the presence of the $\mathcal{O}(N^2)$ intercept term, the number of model parameters increases rapidly with the number of assets. This problem can be alleviated, at the cost of efficiency, by replacing the eigenvectors of S with the eigenvectors of $\hat{S}$, the sample (*unconditional*) covariance matrix. Such a replacement is consistent with the loading targeting previously introduced. The remaining model parameters (i.e., the eigenvalues of S and (ψ, ϕ)) can be estimated in a second step via QML with a parameter number of order $\mathcal{O}(N)$. The standard errors for the second step, adjusted for the first step, can be computed following Newey and McFadden's (1994) and Pagan's (1986) method-of-moments approach. See Noureldin et al. (2014), among others, for empirical applications. Second, the esti-

[12]The bivariate estimation outputs in sections 2.3 were obtained using such an approach.

mated loading process is typically highly persistent, in which case S is poorly identified from the data. This is particularly evident in the case of the Scalar DPC model, where, for $a+b \approx 1$, the scale factor $(1-a-b)$, that multiplies S in the intercept term is close to zero. However, this identification problem is not peculiar to the DPC model; it also affects, for example, the joint estimation of the BEKK model's parameters in the variance-targeting re-parameterization when the volatility process is highly persistent. Similar problems usually affect all models that have a targeting-like intercept, including Engle's (2002) DCC model. Third, the QLLs of both the joint and the two-step QML estimators, though regular, are highly multimodal, which makes it difficult to maximize the QLL by means of standard gradient-based algorithms. In fact, with more than a few assets, the probability of nearly coincident eigenvalues of Q_t for some t increases rapidly, making the computation of Q_t's eigenvectors unstable. Even though maximization of the QLL can be addressed via gradient-free optimizers like the Nelder-Mead simplex method (Nelder and Mead 1965), computing standard errors remains difficult. Alternatively, the approach suggested by Paolella and Polak (2014) could be adopted, but computational time can easily explode.

3.2 Large-scale estimation: The DPC estimator

We suggest a three-step procedure as a solution to the *numerical* problems that arise when using (possibly two-step) QML estimators for the DPC model parameters with more than a few assets. We call this procedure the DPC estimator. The idea behind the DPC estimator stems from the empirical fit of Q_t, which we interpret to be a misspecified CCM of the asset returns. Then, the eigenvectors of $\hat{Q}_t$ are used as estimators of the conditional loadings. Therefore, the components are estimated as $\hat{u}_t = \hat{L}_t' y_t$, where $\hat{L}_t$ is the eigenvector matrix of $\hat{Q}_t$. Finally, the GARCH models of the components are fitted to the elements of $\hat{u}_t$ one at a time. Since Q_t is not usually the CCM of the asset returns, $\hat{L}_t$ can be inconsistent. However, if $\hat{L}_t$ is a good estimate of the loading process, the null hypothesis that the elements of $\hat{u}_t$ are conditionally orthogonal should not be rejected. The next section provides further insight on this aspect and introduces a test that serves as a tool for model specification. Let $\hat{S}$ denote the assets sample covariance matrix. Its spectral decomposition is

$$\hat{S} = \hat{L}\hat{D}\hat{L}',$$

where $\hat{D} = \mathrm{diag}(\hat{d}_1, \ldots, \hat{d}_N)$. The uniqueness is ensured by an identification condition similar to that in Assumption 2.1.2. The DPC estimator is then defined as follows.

Definition 3.2.1. DPC estimator

(1) Estimate S with $\hat{S}$;

(2) conditional on step 1, estimate (A, B), fitting a BEKK model by QML methods, and recover the Q_t sequence;

(3) conditional on steps 1-2, for $i = 1, \ldots, N$ estimate (α_i, β_i) via univariate QML.

Remark 3.1. Step (1) implies that the eigenvectors and the eigenvalues of S are replaced by the eigenvectors and the eigenvalues of $\hat{S}$. Since the eigenvectors of S are equal to the eigenvectors of $\bar{S}$ (loading targeting), the eigenvectors of $\hat{S}$ are consistent estimators of the eigenvectors of S. Since the eigenvalues of S are not usually equal to the eigenvalues of $\bar{S}$, step (1) may be inconsistent, although the inconsistency is, at most, $\mathcal{O}(N)$, (not $\mathcal{O}(N^2)$). The joint and two-step estimators in the previous section do not suffer from this inconsistency problem, as they do not replace the eigenvalues of S with the eigenvalues of $\hat{S}$; instead, they estimate the eigenvalues.

Remark 3.2. Conditional on step (1), step (2) may be inconsistent because, even for known (S, A, B), the matrix Q_t is not usually the CCM of y_t. However, since A and B are diagonal, the inconsistency is, at most, $\mathcal{O}(N)$, and in the Scalar DPC model, the inconsistency is, at most, $\mathcal{O}(1)$. Therefore, the appropriateness of steps (1) and (2) must be evaluated by means of the test introduced in the next section.

Remark 3.3. Because of the orthogonality of the components, conditional on step (2), step (3) coincides with the joint QML estimator of the components' GARCH(1,1) dynamic parameters, so it is QML-efficient. Conditional on step (2), the asymptotic properties of step (3) follow as a direct consequence of the univariate QML estimator of the dynamic parameters of the GARCH(1,1) model's asymptotic properties. (See Lee and Hansen 1994, Lumsdaine 1996, Berkes et al. 2003, Berkes and Horvàth 2003 and 2004, Francq and Zakoïan 2004, and Boussama et al. 2011).

Remark 3.4. The conditioning of step (2) on step (1) implies that $\hat{S}$ and $\hat{L}$, replace S and L in (15) and (16), respectively. The conditioning of step (3) on step (2) implies that $u_{i,t}$ and d_i are replaced by the corresponding estimated BEKK quantities. Therefore, if $\hat{L}_t$ denotes the eigenvector matrix of the estimated BEKK CCM, computed under the identification condition in Assumption 2.1.3, the components are replaced by the elements of $\hat{u}_t = \hat{L}_t' y_t$, and d_i is replaced by $\hat{d}_i$, for $i = 1, 2, \ldots, N$.

Remark 3.5. Steps (1) and (2) coincide with the variance-targeting QML estimator of the BEKK model $E_{t-1}[y_t y_t'] = Q_t$. The univariate estimations at step (3) are two-step QML GARCH(1,1) estimators of the dynamic conditional variance parameters, where the first step consists of replacing the GARCH(1,1) intercept term d_i with $\hat{d}_i$, the i-th eigenvalue of $\hat{S}$.

Remark 3.6. Since we replace S with $\hat{S}$, any problem associated with the identification of S when the loading process is near integrated is circumvented.

Remark 3.7. The QLLs that are maximized at steps (2) and (3) are smooth, so they can be efficiently maximized by standard gradient-based optimizers likes Newton-Raphson optimizers.

Remark 3.8. In the case of the Scalar DPC model, the DPC estimator reduces to $N + 1$ $\mathcal{O}(1)$ estimations, where only the estimation of the dynamic loading parameters requires matrix inversions.

Remark 3.9. If $A = B = 0$ (OGARCH model), step (2) is not required. In this case, the DPC estimator is similar to the two-step OGARCH estimator. The only difference is that the components' univariate GARCH(1,1) models are estimated via two-step variance-targeting QML, rather than via joint univariate QML.[13] The sample variances of the estimated components are given by the eigenvalues of $\hat{S}$. Since with the OGARCH model the sample variances of the estimated com-

[13] Francq et al. (2011) showed that, if the model is misspecified, the variance-targeting estimator of the GARCH(1,1) model can be superior to the QML estimator for long-term prediction or Value-at-Risk calculation.

ponents are given by the eigenvalues of $\hat{S}$, Step (1) is consistent because S coincides with $\bar{S}$.

The DPC estimator is attractive because of its intuitiveness and because the three steps coincide with estimators that are widely used in practice. Given any code for MGARCH estimations, a few minor changes will suffice to allow the DPC estimator to be computed. Alternatively, the DPC estimator can be computed from the representation in terms of unconditional components as follows. (See section 2.7.)

(a) estimate the unconditional components as $\tilde{u}_t = \hat{L}' y_t$;

(b) conditional on step (a), estimate (A, B) by fitting the BEKK model $E_{t-1}[\bar{u}_t \bar{u}_t'] = \bar{Q}_t$ via QML, subject to variance targeting;

(c) conditional on steps (a) and (b), estimate the component volatility parameters, (α_i, β_i), for $i = 1, \ldots, N$, via univariate QML.

The conditioning of step (b) on step (a) implies that $\bar{u}_t$ in (22) is replaced by $\tilde{u}_t$. The variance targeting at step (b) implies that D in (22) is replaced by $\hat{D}$. The conditioning of step (c) on steps (a) and (b) implies that u_t in (8) is replaced by $\hat{u}_t = \tilde{L}_t' \tilde{u}_t$, where $\tilde{L}_t$ is the eigenvector matrix of the estimated $\bar{Q}_t$ provided by step (b), and that d_i in (8) is replaced by $\hat{d}_i$. In computing the SD of the estimated $\bar{Q}_t$, identification conditions similar to that in Assumption 2.7.1 are applied. By loading targeting, step (a) is consistent. Step (b) is potentially inconsistent because i) D is not, in general, the second moment of $\bar{u}_t$, and ii) even for known (D, A, B), the matrix $\bar{Q}$ is not usually the CCM of $\bar{u}_t$. With the OGARCH model,step (b) coincides with the replacement of d_i by $\hat{d}_i$.

Following Engle (2009), we derive an approximate asymptotic distribution for the DPC estimator by focusing on the method-of-moments framework. (See Newey and McFadden 1994 and Pagan 1986). The moment conditions for the t-th observation are given by $m_t(\theta) = [m_{S,t}', m_{\psi,t}', m_{\phi,t}']'$, where $\theta = [\text{vech}(S)', \psi', \phi']'$, $m_{S,t} = \text{vech}(S - y_t y_t')$, $m_{\psi,t} = (\partial/\partial\psi)\mathcal{L}_t(S, \psi, \phi)$, and $m_{\phi,t} = (\partial/\partial\phi)\sum_{i=1}^N \mathcal{L}_{i,t}(S, \psi, \phi_i)$, where $\mathcal{L}_t(S, \psi, \phi)$ is the contribution to the joint DPC QLL by the t-th observation, and $\mathcal{L}_{i,t}(S, \psi, \phi_i)$ is the t-th contribution to the joint DPC QLL by the i-th component; see equation (24). Assuming that, at the true parameter values,

$$E_{t-1}[m_t(\theta)] = 0, \tag{25}$$

under standard regularity conditions, as $T \to \infty$ we have

$$\sqrt{T}(\hat{\theta} - \theta_0) \xrightarrow{d} N(0, \mathcal{I}^{-1}\mathcal{J}\mathcal{I}'^{-1}),$$

where

$$\mathcal{J} = VAR\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T} m_t(\theta_0)\right], \ \mathcal{I} = E\left[\frac{\partial}{\partial\theta} m_t(\theta_0)\right].$$

The estimation of $\mathcal{J}$ requires an HAC estimator. (See, e.g., Newey and West 1987). The upper left block of $\mathcal{I}$, corresponding to vech(S), is simply $-2I_{N(N+1)/2}$. In practice, using the OGARCH model does not require second step, as only the relevant moment conditions are considered in the construction of $m_t(\theta)$. Since the OGARCH model is a BEKK model, for the OGARCH model, Assumption (25) holds (Bollerslev and Wooldridge 1992). In the general case, Assumption (25) could be misspecified because of the first two blocks of the moment conditions. (See remarks (3.1 and 3.2)). The third block is correctly specified. (See remark 3.3.) However, since the third step estimator depends on the previous steps, the third step estimator could also be inconsistent. This problem does not affect the joint and two-step estimators. (See remark 3.1.)

To verify the small sample performances of the DPC estimator we conduct a Monte Carlo study. The data-generating process takes Gaussian innovations and simulates the Scalar DPC model for different cross-sectional dimensions. We set the latter to $N = 2, 3, 5, 10, 30$, while for the series length we consider $T = 500, 1000, 1500$. Moreover we define the loading dynamic parameters by combining $a = .005, .01, .02, .03, .05, .09$, with $b = 0, .90, .94, .95, .97, .98, .99$ and excluding the pairs (a, b), such that $a + b \geqslant 1$. The GARCH dynamic parameters have been extracted from a set of 30 pairs of α and β kept fix for all the cross-sectional sizes. The model unconditional covariance has been set equal to the unconditional covariance of the 30-asset empirical dataset described in Section (4.2). For each value of the set (N, T, a, b), we perform 500 replications to take a balance between computational time and the large number of parameter combinations. On each simulated series we estimate the Scalar DPC model. Tables with Biases and Mean Squared Errors are available upon request. They suggest a proper convergence to the true parameter values, and a presence of limited distortions on the parameter a that increase with a going to zero and $a + b$ approaching one. Therefore, the consistency problems we previously highlighted are present but have a reduced impact.

3.3 Testing for correctly specified conditional loadings

The main innovation of the DPC modeling framework is the introduction of dynamic loadings. However, despite the intuition in Ding and Engle (2001), there are no theoretical reasons to exclude the possibility of empirical data that is characterized by constant loadings. To allow one to distinguish between those two possibilities, we introduce a test designed to detect the null hypothesis of constant conditional loadings against an alternative hypothesis of dynamic conditional loadings. More generally, we test for possible misspecifications of the conditional loadings dynamic. Loading misspecifications might also arise as a product of the multi-stage estimation approach because of the potential inconsistency of the first two stages, and the test also sheds light on this issue. The testing approach we suggest mimics Engle and Sheppard's (2001) test for dynamic conditional correlations. We start by introducing a test that is robust to misspecifications in the components' conditional variances. Under the null hypothesis of correctly specified conditional loadings, the cross-product of the components has a conditionally zero mean at the true loading parameter values, that is, $E_{t-1}[u_{i,t}u_{j,t}] = 0$, for $i \neq j = 1, 2, \ldots, N$. Let U_t denote the $N(N-1)/2 \times 1$ vector, stacking the distinct cross-products of the components evaluated at the true parameter values. Moreover, the process U_t is orthogonal to any function of $\mathcal{I}_{t-1}$, given the construction of a loading dynamic that is conditional on the past. Therefore, we introduce a test by considering the following system of equations

$$U_t = \delta_0 + \delta_1 U_{t-1} + \cdots + \delta_K U_{t-K} + \xi_t, \tag{26}$$

where δ_0, δ_1, ..., δ_K are scalar parameters and ξ_t is a heteroskedastic innovation process.[14] Under the null hypothesis of a correctly specified loading process, the intercept and all the model parameters in the regression should be zero. In order to estimate the test statistic, for each element in U_t, $v_{i,t} = u_{j,t}u_{l,t}, \quad i = 1, 2, \ldots (N(N-1)/2, \quad j \neq l, and \quad j, l = 1, 2, \ldots N$, we need the $T \times 1$ vector, containing the cross-product time series, and the $T \times (K+1)$ matrix of explanatory variables (including the constant), containing lags of the cross-product. Then the parameters can be estimated by stacking the $N(N-1)/2$ univariate equations and performing a restricted, seemingly unrelated regression.[15] The test statistic, which is of a Wald-type, verifies the null hypothesis of

[14]Heteroskedasticity is a certainty, given that conditional components are heteroskedastic.

[15]The underlying SURE model is a restricted one since the parameters are common across equations.

zero coefficients in the auxiliary regression (26), where a heteroskedasticity consistent estimator (White 1980) is required for the covariance matrix of the estimated coefficients.[16] Under the null hypothesis, the test statistic asymptotically follows a chi-square distribution with $K+1$ degrees of freedom. However, the asymptotic distribution might deviate from a chi-square distribution since we replace the vector of conditional components, u_t, with the estimated conditional components, $\hat{u}_t = \hat{L}_t y_t$, where $\hat{L}_t$ is the estimated conditional loading matrix. We analyzed this possibility by means of Monte Carlo simulations. In order to verify the null hypothesis of constant conditional loadings, we set $\hat{L}_t = \hat{L}$, where $\hat{L}$ is the vector of unconditional loadings. Therefore, rejection of the null hypothesis will signal the presence of misspecification in the loadings, as the conditional components are not conditionally orthogonal, thus calling for the introduction of dynamism in the loadings. The rejection of the null hypothesis of correct model specification - the test focuses on estimated conditional components - can be due both to an incorrect model specification and to the presence of inconsistency in the first two stages of the DPC estimator. On the other hand, if the test does not reject the null hypothesis, then the model is correctly specified and there are no biases that are due to inconsistency. Under the null hypothesis, inconsistency problems arise only when the DPC estimator is used. However, those problems but are not present under the joint estimation of all model parameters. In addition, the test performances are not affected by inconsistency issues when one tests for constant conditional loadings, as the components are estimated from an OGARCH model. The test does not depend on the components' volatility, so it is robust to misspecifications of the components' conditional variances.[17] For this reason, we label *robust* the test in (26). Even though less efficient than other, more sophisticated procedures, our testing approach is simple to compute and is viable in most econometric software. Notably, if it is used in testing the null hypothesis that $a = b = 0$ in the DPC loading-driving process, our test circumvents the difficulties that arise when testing for parameters that are on the boundary and/or are not identified[18] (Andrews and Ploberger 1994). Following Engle and Sheppard (2001), we design an

[16]The White estimator is needed due to the potential heteroskedasticity of components.

[17]Heteroskedasticity is taken into account in the computation of the test statistic's parameters covariance matrix by means of White-type standard errors.

[18]If $a = 0$, the parameter b in Q_t is not identified.

alternative test by replacing U_t in the auxiliary regression (26) with the $N(N+1)/2$ vector

$$\tilde{U}_t = \text{vech}(z_t z_t' - I_N),$$

where $z_t = D_t^{-1/2} L_t' y_t$ is the vector of the variance standardized conditional components. Since z_t has, by construction, a conditionally zero mean with covariance matrix I_N, the vector $\tilde{U}_t$ is orthogonal to any function of $\mathcal{I}_{t-1}$. However, the resulting test depends on the models adopted for the conditional components variances, which might be misspecified. Even if the components conditional variances are correctly specified, the power of the test is likely to be affected by the need to replace D_t with an estimate. Given that the test is not robust to conditional component variance misspecification, we label it *non-robust*. We conduct a Monte Carlo study to assess the size and power of the robust and non-robust tests of constant conditional loadings proposed above. We set the data-generating process to a Scalar DPC model with Gaussian innovations, the same adopted to analyze the QML estimator small sample performances. We thus consider five cross-sectional dimensions, setting $N = 2, 3, 5, 10, 30$. For the loading dynamic parameters we consider the set of pairs obtained by combining $a = 0, .005, .01, .02, .03, .05, .09$, with $b = 0, .90, .94, .95, .97, .98, .99$ and excluding the pairs (a, b), such that $a = 0$ or $b = 0$ and $a + b \geqslant 1$. For each value of the triple (N, a, b), we generate 500 samples of length $T = 1000$. Then we estimate the OGARCH model on each simulation and test for dynamics in the loadings. For the robust test only we run a similar experiment with $T = 1500$ and $N = 2, 3, 5, 10, 30$, where 1500 is the length of the rolling window size of the large dataset used in the empirical analysis section. Table 2 reports the percentages of rejections for the null hypothesis of constant conditional loadings at the nominal 5% confidence level. The rejection frequency for the pair $(a, b) = (0, 0)$ gives the size of the test,[19] while all other frequencies evaluate the test's power. Given the size of the two tests, the non-robust test size worsens with increasing cross-sectional dimension. The size, which is close to the nominal level in the five-asset case, then increases, reaching an unacceptable 31.6% with thirty assets. For the robust test, the size is below the nominal level only for the $N = 2$ case, and it increases with the cross-sectional dimension ad for the non-robust test. However, the robust test performs much better than the non-robust test, with rejection frequencies always below 10%. For both tests the cross-

[19] In that case, the DGP collapses on the OGARCH model.

sectional dimension has a relevant impact on performances. In fact, results become acceptable for some parameter designs beginning when the simulations involve five assets. There is also a difference between the robust and non-robust tests, with the latter having generally better performances; the robust test is better only for small cross-sectional dimensions. However, the two tests have similar power when the ARCH coefficient is not too small; otherwise there is a clear preference for the non-robust test. As expected, the test power improves with the sample size. Finally, the deviations from the expected rejection rate might be due to the deviation of the test asymptotic distribution from the chi-square density. In summary, considering that ARCH coefficients below 0.03 are not common in the empirical analyses, we think that the robust test presents a good compromise in terms of size and power. Therefore, we consider only this test in the following sections' simulations and empirical examples. The previous simulations assessed the test performances by evaluating the size under an OGARCH data-generating process. However, we are also interested in the test outcomes when we estimate the correctly specified model using a DPC data-generating process. Table (3) reports the rejection frequencies of the test when the data-generating process is the Scalar DPC of the previous simulation. We set the sample size to $T = 1000$ and use only the robust test. The rejection frequencies worsen with increasing cross-sectional dimensions, in particular for values of the ARCH coefficient a above 0.03 and the GARCH coefficient above 0.98. Nevertheless, the test performances are acceptable for parameter combinations, suggesting a high level of persistence and a not-small impact of innovations on the loading dynamic. Therefore, we believe that the test provides accurate results in the empirical analyses. Moreover, as the test performances of Table (3) are overall acceptable, inconsistency issues, if present, have a limited impact on the model fit.

In order to shed further light on the model performances, we also fit a Full DPC on the simulated Scalar DPC series on cases $N = 2, 3, 5, 10$. (The case with $N = 30$ is excluded as too computationally demanding.[20]) Results are reported in Table (3). Test performances improve slightly, given the additional flexibility of the loading dynamic. We conclude the simulation experiments by assessing the tests' performances in detecting the model misspecification, so we conduct simulations under a Full DPC with cross-sectional dimensions $N = 2, 3, 5, 10, 30$ and sample size $T = 1000$. We

[20]The computational burden of this test comes from the parameter estimation of the loading dynamic. In fact, the loading-driving process corresponds to a diagonal BEKK with 60 free parameters. Although the single model estimation is feasible, the computational complexity increases if we consider the replication number (500) and the twenty-one specifications we consider for the loading parameters in the DGP

randomly generate the loading dynamic parameters and impose a number of persistence levels; that is, for a given data-generating process, we impose that $a_i + b_i = p$ for all $i = 1, 2, \ldots, N$, with a_i extracted from a uniform density $U(0.005, 0.2)$ (and $b_i = p - a_i$). We consider three persistence levels $p = 0.90, 0.95$, and 0.99. Table (4) reports the results that are associated with the estimation of both a misspecified Scalar DPC and the correctly specified model. As expected, for small cross-sectional dimensions, the rejection frequencies of the misspecified model and the correctly specified model are close. However, increasing the values of N improves the test's ability to detect model misspecification with a high level of persistence. Nevertheless, results still suggest a lack of power with large cross-sectional dimensions.

3.4 Explained-variance-based restrictions

In the DPC framework a natural restriction strategy consists of imposing common volatility dynamics on the less volatile components, which we call explained-variance-based, or EV-based, restrictions. This strategy is implemented by setting $(\alpha_i, \beta_i) = (\alpha_j, \beta_j)$ for $i, j > M$, where M can be either selected *a priori* or determined by means of a model-selection criterion (e.g., BIC or AIC). The next section includes among the possible specifications a DPC model with $M = 3$ in an example with thirty assets. With respect to the unrestricted model, reducing the number of dynamic components' conditional variance parameters from $2N$ to $2M + 2$ provides a computational advantage. By the orthogonality of the components and conditional on steps (1) and (2), the common volatility parameters can be efficiently estimated by maximizing the sum of restricted components' univariate QLLs. In addition, if the common dynamic parameters are set to zero, $\alpha_i = \beta_i = 0$ for $i, j > M$, we get a Factor DPC model with M factors. (See Section 2.5 for details on this specification.) In this case, step (3) reduces to running M univariate GARCH estimations. The EV-based restrictions can be combined with other restriction strategies. Following Hafner and Franses (2009), we can impose common smoothing by setting $\beta_i = \beta$ for all i. Noureldin et al. (2014) provide evidence that common persistence (CP) can improve on common smoothing. In the DPC model, the CP restriction is implemented by setting $\beta_i = \lambda - \alpha_i$ for $i = 1, 2, \ldots, N$, where $1 > \lambda \geqslant \max\{\alpha_i\}$. In that case, λ is the CP shared by the conditional volatilities of the components. If we impose CP, step (3) of the DPC estimator can no longer be split into N separate univariate maximizations. However, the maximization of the objective function can be significantly simplified by concentrating out the α_i

from the sum of the components' QLLs. The concentrated log-likelihood thus becomes a function of the CP parameter only. For the fixed CP parameter, concentrating out the α_is reduces the model complexity, leading to a collection of N scalar maximizations that are free from matrix inversions. Conditional on steps (1) and(2) of the DPC estimator, the estimated CP parameter and the final estimates of the α_is coincide with the component volatility parameters' joint QML estimates under CP.

4 Empirical comparisons with competing models

This section compares the empirical performances of the Scalar DPC model with that of the OG-ARCH, Scalar BEKK, CCC, and DCC models. It also considers the Corrected DCC (cDCC) model from Aielli (2013) and the related three-step estimation procedure.[21] We include the DCC model in the comparison because of its similarities with the DPC framework in terms of modelling strategy and with its three-step large-scale estimator (DCC estimator). In its scalar version the DCC is the natural competitor of the Scalar DPC. In the DCC model an auxiliary process (the correlation-driving process) is introduced to capture the relevant features of the underlying correlation dynamics. In its simplest specification, the Scalar DCC model (or DCC model), the correlation-driving process is defined as a Scalar BEKK recursion:

$$Q_t = (1 - a - b)S + a\,\varepsilon_{t-1}\varepsilon'_{t-1} + b\,Q_{t-1}, \tag{27}$$

where Q_0 is PD, S is PD with ones on the main diagonal and $a \geqslant 0$, $b \geqslant 0$, $a + b < 1$, and $\varepsilon_t = \Sigma_t^{-1/2} y_t$ is the vector of the univariate GARCH(1,1) standardized returns.[22] The conditional correlation matrix is computed as $R_t = Q_t^{*-1} Q_t Q_t^{*-1}$, where $Q_t^* = \mathrm{diag}(\sqrt{q_{1,1,t}}, \ldots, \sqrt{q_{N,N,t}})$. The CCM is reconstructed as $H_t = \Sigma_t^{1/2} R_t \Sigma_t^{1/2}$. Setting $a = b = 0$ yields Bollerslev's (1990) CCC model,

[21] Several MGARCH models that are related in various ways to the DPC model are not compared with the DPC model because of the feasibility and/or dis-homogeneity of their feasible estimators with respect to the DPC approach. Among others, we recall van der Weide's (2002) and Boswijk and van der Weide's (2011) GO-GARCH model, the Lanne and Saikkonen' (2007) GOF-GARCH model, the Fan et al.'s (2008) Conditional Uncorrelated Components (CUC) model, and Vrontos et al.'s (2003) Full Factor GARCH model. These models have a common feature in that they are all linear combinations of univariate orthogonal GARCH processes, where the linear map between the orthogonal processes and the asset returns can be partially recovered from unconditional information. See Bauwens et al. (2006) and Silvennoinen and Teräsvirta (2009) for details.

[22] For economy of notation, we use a unique symbol to denote quantities that play analogous roles in the DCC and DPC specifications.

where $R_t = S$ at each point in time. The first step of the DCC estimator is the estimation of the asset-conditional variances via univariate QML. To maintain a fair comparison with the DPC, we specify the conditional variances as GARCH(1,1). In the second step, the sample correlation matrix of the variances' standardized returns replaces the matrix S. Finally, in the third step, the dynamic correlation parameters are estimated via QML, conditional on the previous steps. The DCC and DPC estimators have a common motivation: simplifying (or making feasible) the estimation of large systems. Both estimators involve the estimation of $N(N+1)/2$ location parameters,[23] plus two dynamic parameters for the auxiliary process plus two dynamic parameters for each of the N univariate volatility processes. If we focus on the number of estimated parameters in each of the steps, we have a sequence of $\mathcal{O}(N^2)$ - $\mathcal{O}(1)$ - $\mathcal{O}(N)$ parameters with the DPC estimator and $\mathcal{O}(N)$ - $\mathcal{O}(N^2)$ - $\mathcal{O}(1)$ with the DCC estimator. As we noted in Section 3.2, the steps (1) and (2) of the DPC estimator might be affected by the inconsistency problems of $\mathcal{O}(N)$ parameters. This shortcoming could be compensated for in terms of empirical performances by the conditionally QML-efficient step (3), which has order $\mathcal{O}(N)$. On the other hand, with the DCC estimator, the inconsistency of step (2), which has order $\mathcal{O}(N^2)$ (Aielli 2013), is followed by a conditionally QML-efficient step (3), which has a typical size of $\mathcal{O}(1)$. With the DPC estimator there is more flexibility in the third step compared to the DCC estimator. This allows mitigating the inconsistency affecting the first two steps. If one allows for feasible $\mathcal{O}(N)$-order step (3) estimations in the DCC estimator via, for example, composite likelihood (Engle et al. 2009), step (3) is no longer QML-efficient conditional on steps (1) and (2). Aielli (2013) suggested replacing the DCC correlation-driving process with a proper Scalar BEKK model, defined as $E_{t-1}[\varepsilon_t^* \varepsilon_t^{*\prime}] = Q_t$, where

$$Q_t = (1 - a - b)S + a\,\varepsilon_{t-1}^* \varepsilon_{t-1}^{*\prime} + b\,Q_{t-1} \tag{28}$$

(Corrected DCC, or cDCC, model). By construction, it holds that $\varepsilon_t^* = Q_t^* \varepsilon_t$. The large-scale estimation of the cDCC model is carried out using a three-step profile estimation strategy that is computationally equivalent to the DCC estimator (cDCC estimator). Under a correctly specified model, the cDCC estimator solves the inconsistency problem of the DCC estimator's step (2).

[23]In the DPC model the location parameters are the $N(N+1)/2$ lower triangular elements of S; in the DCC model the location parameters are the $N(N-1)/2$ lower off-diagonal elements of S plus the N-asset GARCH intercepts. In the Scalar DCC model, the diagonal elements of S are 1.

4.1 A simulation study

We assess the performance of the DPC model under misspecification by running a bivariate simulation experiment similar to that proposed Engle (2002) for the DCC model. Figure 3 reports the conditional loadings' DGPs. Only $l_{1,1,t}$ is graphically represented because the remaining loadings are determined by orthonormality and identification conditions. We generate 250 bivariate series of length 2000 for each of the five loading paths. The component volatilities are randomly drawn from a set of paths, which, apart from a scale factor, is the same as that in Figure 3. The scale factor is such that the unconditional variances of the first and second components equal 7 and 3, respectively.[24] The simulated series are characterized by a variety of loading and component volatility dynamics that do not favor the DPC model, as would happen, for instance, if the components were generated as GARCH(1,1) processes. An example of simulated paths, together with the related DPC estimation output, is reported in Figure 4. On each simulated path, we estimate the OGARCH, Scalar BEKK (SBEKK), Scalar DPC (DPC), CCC, DCC, and cDCC models via QML, discarding the first 500 observations to eliminate the impact of the simulations' starting values. Given that the true values are known, the Mean Squared Error (MSE) is an optimal performance measure. The quantity is defined as

$$\text{MSE} = \frac{1}{1500}\sum_{t=1}^{1500} \text{vech}(\hat{H}_t - H_t)'\text{vech}(\hat{H}_t - H_t),$$

where $\hat{H}_t$ and H_t are the estimated and true CCMs, respectively. The average MSE is reported in Table 5 for the various models. Apart from the case of constant loadings, where all models except the Scalar BEKK perform similarly, the DPC model performs best.[25] The reduction in the MSE with respect to the second-best model is always around 20 percent. OGARCH performances are significantly worse than those of the DPC.

4.2 Data description and estimated specifications

In order to provide evidence that supports the introduction of the DPC model and its coherence with the optimal MGARCH properties, we provide two empirical examples with differing cross-sectional

[24]The scaling factors are purely arbitrary but do not play a role in the simulation experiments. Similar results can be obtained with other unconditional variance levels.

[25]As expected, in the case of constant loadings, the OGARCH is the best model.

dimensions. We consider two datasets of real data: one that has ten constituents of the DJIA index and is, thus, a small-scale example, and one that focuses on all thirty constituents of the DJIA index and is, thus, a medium-scale example. The ten-asset dataset is the same as that Noureldin et al. (2014) used. We recovered from Yahoo!Finance the adjusted close time series. The constituents of the DJIA index are those at the end of 2009. (See Appendix A.) The returns are computed as daily close-to-close log returns, and the sample covers the time period from 01/02/2001 to 02/01/2014. We compare the model performances using an out-of-sample framework and compute one-step-ahead CCM forecasts using a rolling estimation window of 1500 days. This approach leads to a total of 1750 one-step-ahead out-of-sample forecasts. The forecast period covers the whole time of the US financial crisis that exploded in the late 2008 after the Lehman default. We estimate the following specifications, where each line starts with the model acronym used in the results tables:

- OGARCH: full rank OGARCH model;
- SBEKK: Scalar BEKK model;
- DPC: Scalar DPC model (see def. 2.2.1);
- DPC_s: Scalar DPC model with components restricted so they have common volatility dynamics (i.e., $(\alpha_i, \beta_i) = (\alpha_j, \beta_j)$ for all i, j; see def. 2.2.1 and section 3.4);
- DPC_r: Scalar DPC model with the last $N-3$ components restricted so they have common volatility dynamics (i.e., $(\alpha_i, \beta_i) = (\alpha_j, \beta_j)$, for $i, j = 4, 5, \ldots, N$; see def. 2.2.1 and section 3.4);
- DPC_f: three-factor Scalar DPC model (i.e., $\alpha_i = \beta_i = 0$, for $i = 4, 5, \ldots, N$; see def. 2.2.1 and section 2.5);
- DCC: Scalar DCC model;
- cDCC: Scalar cDCC model;
- CCC: Scalar DCC (or Scalar cDCC) model, subject to $a = b = 0$.

For the Scalar DPC specifications, a superscript * denotes the presence of a common persistence (CP) restriction. This is implemented in the component volatility dynamics as $\beta_i = \lambda - \alpha_i$, for

$i = 1, 2, \ldots, M$. We set $M = 3$ in the factor specifications, and $M = N$ otherwise. We use the DPC estimator to recover the parameters for all of the DPC specifications, including the OGARCH model. The DPC estimator for the OGARCH model almost coincides with the two-step OGARCH estimator; the only difference is that the components fitted in step (2) are subject to a variance-targeting constraint. We estimated the SBEKK model via a two-step variance-targeting QML. We estimate the DCC and cDCC model parameters using the three-step estimation procedures discussed in Engle (2002) and Aielli (2013). In both cases, we use GARCH(1,1) models for the conditional variances, subject to variance targeting.[26] The CCC estimator coincides with the first two steps of the DCC (or, equivalently, the cDCC) estimator.

4.3 Model evaluation

We evaluate the model's performances based on one-step-ahead out-of-sample predictions. We use the predictive ability pairwise comparison tests (Diebold and Mariano 1995, Amisano and Giacomini 2007) based on several loss functions and the augmented Mincer-Zarnowitz (AMZ) regression-based tests (Mincer and Zarnowitz 1969; see also Patton and Sheppard 2009).

Let $\hat{H}_t^{(i)}$ and $\hat{H}_t^{(j)}$ denote the one-step-ahead out-of-sample estimate of H_t provided by models i and j, respectively, where $i \neq j$. The loss that is due to predicting H_t with $\hat{H}_t^{(i)}$ is denoted as $loss_t^{(i)}$, and is a function of both the true and the predicted covariances. The null hypothesis of equal predictive ability is defined as

$$\mathcal{H}_0 : E\left[loss_t^{(i)} - loss_t^{(j)}\right] = 0.$$

Under regularity conditions, the test statistics,

$$pcstat(i,j) = \frac{T^{-1}\sum_{t=1}^{T}(loss_t^{(i)} - loss_t^{(j)})}{\sqrt{\widehat{AVAR}[T^{-1}\sum_{t=1}^{T}(loss_t^{(i)} - loss_t^{(j)})]}}, \tag{29}$$

are asymptotically distributed as a standardized Normal (under $\mathcal{H}_0$). We use the Newey-West HAC estimator (Newey and West 1987) since the computation of $\widehat{AVAR}$ requires one. A positive (negative) significant value of $pcstat(i,j)$ will provide evidence in favor of model j (i). The empirical evaluations use the following loss functions:

[26]See footnote (13).

1. Predictive density (PDen) loss (Amisano and Giacomini 2007), defined as the negative of the QLL, or

$$loss_t^{(i)} = -\frac{1}{2}\left(N \log 2\pi + \log|\hat{H}_t^{(i)}| + y_t'\{\hat{H}_t^{(i)}\}^{-1} y_t\right). \tag{30}$$

2. Mean square error loss (MSE), defined as $loss_t = x_t' x_t$, where x_t is a vector of residuals specified either as

   i) $x_t = \text{vech}(y_t y_t' - \hat{H}_t)$ (generalized residuals, GR) or as

   ii) $x_t = \text{vech}(\hat{\varepsilon}_t \hat{\varepsilon}_t' - \hat{R}_t)$ (generalized standardized residuals, SR) or as

   iii) $x_t = \text{vech}(\hat{u}_t \hat{u}_t' - \hat{D}_t)$ (generalized component residuals, CR),

   where $\hat{\varepsilon}_t$, $\hat{R}_t$, $\hat{u}_t$, and $\hat{D}_t$ denote, respectively, the vector of univariate standardized residuals, the conditional correlation matrix, the vector of the components, and the CCM of the components, as implied by the one-step-ahead out-of-sample forecast of H_t.

3. Portfolio MSE loss (Diebold and Mariano 1995), defined as $loss_t = \{p_t^2 - E_{t-1}[p_t^2]\}^2$, where $p_t = \pi_t' y_t$, where π_t is a vector of portfolio weights. In that case, we consider the following alternative portfolio compositions:

   i) the equally weighted portfolio (EQW),

   ii) the minimum variance portfolio (MMV) with short selling, where $\pi_t = (H_t^{-1}\iota)/(\iota' H_t^{-1}\iota)$, and

   iii) a minimum-variance hedging portfolio (HDG) with weights $\pi_t = (H_t^{-1}\tau)/(\tau' H_t^{-1}\tau)$, where $\tau = [1, 0, \ldots, 0]'$.

   In the hedging portfolio the first asset is hedged against all other assets in the portfolio.[27] The EQW weights do not depend on H_t, so they are free of estimation errors. In computing the portfolio weights, which computation depends on H_t, we replace H_t with the estimated forecast.

4. Portfolio PDen loss, defined as the univariate version of (30) and computed from portfolio returns.

[27] The optimal weights are computed by setting the expected return vector such that the first entry is equal to one and all others are set to zero; see eq. (1-2) in Engle and Colacito (2006) or Harris and Nguyen (2013).

Under regularity conditions, if the forecast equals H_t, then the expected loss is minimum. In computing the MSE loss, the generalized residuals are appropriate for assessing the predictive performances in terms of CCM,[28] the generalized standardized residuals are suggested to assess the predictive performances in terms of the variance/correlation decomposition of H_t, and the generalized component residuals are appropriate for assessing the predictive performances in terms of the eigenvector/eigenvalue decomposition of H_t. The one-step-ahead returns that enter the loss functions are in deviations from the rolling mean, so the loss functions depend only on estimates extracted from the rolling window.

The AMZ regression for the i, j-th estimated conditional covariance is defined as

$$y_{i,t}y_{j,t} = c_0 + c_1\hat{h}_{i,j,t} + c_2 y_{i,t-1}y_{j,t-1} + \epsilon_{i,j,t}, \tag{31}$$

where $\hat{h}_{i,j,t}$ is the one-step-ahead out-of-sample estimated conditional covariance. If $\hat{h}_{i,j,t} = h_{i,j,t}$, it holds that $(c_0, c_1, c_2) = (0, 1, 0)$. The AMZ regression test is an F-test for the null hypothesis that $(c_0, c_1, c_2) = (0, 1, 0)$. Computing the test requires heteroskedasticity robust standard errors (White 1980). Including $y_{i,t-1}y_{j,t-1}$ among the regressors can improve the test's power, as discussed in Patton and Sheppard (2009). Therefore, we also consider AMZ regressions of the type

$$\hat{\varepsilon}_{i,t}\hat{\varepsilon}_{j,t} = c_0 + c_1\hat{\rho}_{i,j,t} + c_2\hat{\varepsilon}_{i,t-1}\hat{\varepsilon}_{j,t-1} + \bar{\epsilon}_{i,j,t}, \tag{32}$$

where $i \neq j$, and

$$\hat{u}_{i,t}^2 = c_0 + c_1\hat{d}_{i,t} + c_2\hat{u}_{i,t-1}^2 + \tilde{\epsilon}_{i,t}. \tag{33}$$

In these two equations, $\hat{\varepsilon}_{i,t}$, $\hat{\rho}_{i,j}$, $\hat{u}_{i,t}$, and $\hat{d}_{i,t}$ denote, respectively, the univariate standardized residuals, the conditional cross-correlations, the components, and the conditional variances of the components as derived from the one-step-ahead out-of-sample estimation output of a given model. The AMZ regression in (32) is appropriate for assessing the prediction performances of the variance/correlation decomposition of $\hat{H}_t$, while the regression in (33) is used to assess the prediction performances of the SD of $\hat{H}_t$. All of these comparisons are pairwise but we are comparing a large

[28] In that case, we are aware that returns' cross-products are a noisy proxy for the conditional covariances. However, the MSE loss is robust to noise in the proxy adopted for the true and unknown covariance. For additional details, see Patton and Sheppard (2009), Patton (2011), Laurent et al. (2013), and Caporin and McAleer (2014), among others.

set of models, so we use the Bonferroni correction when determining the rejections of the null hypothesis.[29]

4.4 Results

4.4.1 Testing for dynamic loadings

Before estimating the DPC specification, we address a fundamental question concerning whether the loadings are really dynamic. To this purpose, Figures (5) and (6) report the estimated parameters of the fitted Scalar DPC (as well as those of the corresponding DCC specifications) on the two datasets in a rolling analysis. The parameters suggest a high level of persistence and ARCH parameters around 0.02, indicating reasonable test properties. Figure (7) provides the rolling p-value of the test for constant loadings. The OGARCH fit on both datasets leads us to reject the null hypotheses on almost all of the sample, while the DPC leads us to accept the null hypothesis. The latter finding also suggests that inconsistency issues should have a limited impact but that a Scalar DPC specification might be sufficient to capture the loading dynamic.

4.4.2 Ten-asset dataset

Figure 8 reports the boxplots of the F-statistics computed from the AMZ regressions on the ten-asset dataset. The DPC estimators perform better overall than the dynamic correlation estimators, DCC and cDCC, do in predicting the variance/covariance processes. (See Figure 8 - Panel A.) As expected, DCC and cDCC provide better predictions of the conditional variances and conditional correlation processes than the DPC estimators do (Figure 8- Panels B and D). This result is not surprising, as the DCC and cDCC directly model the variances and correlations, while the variances and correlations are indirectly recovered in the DPC specifications we consider. Apart from the factor specifications, the DPC provide better predictions of the eigenvector/eigenvalue decomposition of the CCM than the alternative dynamic correlation models do. (See Figure 8 - Panel E.) Given this result, coupled with the result that DPC specifications focus on the eigenvectors and eigenvalues, it is not surprising that they beat models like DCC and cDCC, where those elements are indirectly recovered. However, if we compare OGARCH to DPC, recalling that DPC is the immediate extension

[29] We might also have used the Model Confidence Set from Hansen et al. (2003 and 2011), as in Caporin and McAleer (2014).

of OGARCH for dynamic loadings, DPC improves with respect to OGARCH when we focus on the AMZ results for the eigenvector/eigenvalue decomposition. Thus, we have first relevant results in support of the importance of having dynamic loadings. DPC_s performs better overall than SBEKK across all panels in Figure 8. With respect to SBEKK, the estimation of DPC_s requires only one additional $\mathcal{O}(1)$ estimation step that is free from matrix inversions so it is not computationally demanding. (See Section 3.) Notably, the SBEKK performs better overall than DCC or cDCC do, which is in line with the empirical evidence in Caporin and McAleer (2008 and 2014) on multivariate GARCH model ranking when the cross-sectional dimension is small and a noisy proxy of the true covariance matrix is used. Moving to the pairwise comparisons, there is a large collection of loss functions that would require a number of tables to report results and make a detailed evaluation and analyses of results difficult. Therefore, we decided to summarize the pairwise comparisons as follows.[30] For any loss function, $pcstat(i, j)$ (29) that suggests a rejection of the null hypothesis at the 5% confidence level provides evidence of differences in the predictive ability of models i and j. Then, if $pcstat(i, j)$ is positive (negative), the rejection is considered to favor model j (i) and is used to define an indicator variable. We then compute a score of the performances of j against i as the number of rejections in favor of j, less the number of rejections in favor of i, across all of the loss functions we consider. The resulting scores are reported in Table 6.[31] Based on this overall performance indicator, DPC_r is the best model/estimator (all scores are positive, with a total score of 22), followed by its CP-restriction specifications, DPC_r^*, and DPC_s, with scores of 19 and 17, respectively. The best correlation model is the cDCC, with a score of 15. There is no redundancy in the computation of the total scores because we are considering out-of-sample forecasts; therefore, depending on the estimation error, a largely parameterized model could perform worse than a parsimoniously parameterized model. The benchmark models, OGARCH, SBEKK, and CCC, perform worse than dynamic extensions when dynamic extensions are included in the set of fitted models. The score of DPC against OGARCH is 4, suggesting that allowing for dynamic loadings is somewhat relevant, given that DPC is the simplest extension of OGARCH for dynamic loadings. The factor specifications DPC_f and DPC_f^*, where some components are conditionally homoskedastic, have the smallest total scores among the DPC specifications. (See Section 2.5.) Therefore, we have a second

[30] The pairwise comparisons are based on rejections of null hypotheses with the Bonferroni correction.

[31] The web appendix includes the detailed tables for the various loss functions.

relevant result: in a small-scale empirical example, the introduction of dynamic loadings slightly improves the forecast models' performances compared to a case with constant loadings and provide better results than dynamic conditional correlation models do. If we focus on specific loss functions, some additional results emerge. For instance, the pairwise comparisons based on the GR MSE loss indicate that no DPC specifications lose against the dynamic correlation estimators and the sign of the test statistic is always negative, but none suggests a rejection of the null hypothesis. However, the DPC_s specification has only four dynamic parameters, as all components are restricted in order to share common dynamic parameters, whereas DCC and cDCC have twenty-two dynamic parameters. Therefore, a less parameterized model provides, in our example, forecasting performances that are equally as good as those of a much more parameterized specification. As for the pairwise comparisons based on the SR MSE loss, all comparisons between the DPC specifications and the correlation estimators are significant and favor the DPC. This is an unexpected result. In fact, the use of the SR MSE loss is expected to favor the estimators that are expressly designed to predict dynamic correlations, such as the DCC and cDCC estimators. However, the empirical evidence is opposite, and our proposal performs markedly better.

4.4.3 Thirty-asset dataset

The message from the thirty-asset dataset is in line with, but more explicit than, that from the ten-asset dataset. The DPC specifications perform better overall in terms of the F-statistics of the AMZ regressions (Figure 9), apart from the case of the asset variance processes, because the DCC and cDCC model estimate the conditional variances directly. Unlike the case with the ten-asset dataset, all DPC specifications outperform the correlation estimators in predicting the correlation processes, which is surprising, as DPC does not explicitly model correlations. We link this finding on the additional flexibility of DPC over that of DCC and cDCC to the construction of variance processes on the conditional components, rather than on the returns. The performances of SBEKK are no longer as good as they are with ten assets, confirming the results in Caporin and McAleer (2014), which suggest, in the presence of many assets, the adoption of more flexible parameterizations. The simplest DPC specification, the DPC_s, which requires only a $\mathcal{O}(1)$ step that is free from matrix inversions with respect to SBEKK, improves on the SBEKK performances. Therefore, even minor changes in the SBEKK model can improve the models' performances. Moving to the pairwise comparisons, the superiority of the DPC estimators is more marked with the thirty-asset dataset than it is with the ten-asset dataset, as shown in the summary results included in Table **??**. Results, particularly from the poor performances of the factor DPC specifications, DPC_f and DPC^*_f, confirm the importance of allowing for a fully conditionally heteroskedastic factor structure. Provided that all components are conditionally heteroskedastic, the better performances of DPC_r and DPC^*_r with respect to DPC and DPC* indicate that the adoption of EV-based DPC restrictions improves model performances compared to unrestricted DPC specifications. The cDCC correlation model is still the best of the correlation models, a result that is probably a by-product of the estimation approach that resolves the inconsistency issues of the DCC three-step estimator. With a pairwise score of 7, the simplest DPC specification, DPC_s, which has only two more dynamic parameters than SBEKK, largely improves on SBEKK, but it also performs better than the most parameterized models, DPC, DCC, and cDCC. Finally, when compared to those of the correlation models, the scores of the DPC specifications using the thirty-asset dataset are higher than they are in the ten-asset dataset. Overall, the evidence from the ten-asset example are confirmed when the cross-sectional dimension is enlarged, and the preference for DPC specifications is more marked. At the single loss-function level, the DPC specifications outperform the DCC and cDCC models also in the thirty-asset case when standardized residuals' MSE losses are considered.

4.4.4 Results summary

We have seen that the DPC specifications/estimators perform better overall than the other estimated models. Among the DPC specifications, the factor specifications DPC_f and DPC^*_f, perform worst, suggesting that allowing for a fully conditionally heteroskedastic factor structure is important. Our results also show that the imposition of EV-based restrictions can profitably reduce the estimation error without prejudice for estimation flexibility, provided that all components are allowed to be conditionally heteroskedastic. With the DCC/cDCC model, the imposition of similar restrictions on the volatilities of the asset returns would require an *ad hoc* estimation step. This result might follow, for instance, Aielli and Caporin's (2014) proposal. Caporin and McAleer (2014) show that the improved empirical performances of DCC/cDCC over that of SBEKK in the fitting of large systems are due primarily to the introduction of asset-specific conditional variance dynamics. The improvement of DPC's performance over those of SBEKK and DCC/cDCC suggest that allowing for specific components' variance dynamics is a better strategy than is allowing for specific assets' variance dynamics.

5 Conclusions

This paper introduces a new model class, the Dynamic Principal Component (DPC) Multivariate GARCH. Built upon a dynamic spectral decomposition of conditional covariance matrices, the model extends Alexander's (2001) OGARCH model and has some similarities with Engle's (2002) DCC modelling approach. Since the DPC specification improves either the model flexibility with respect to the data features, the ease of interpreting model outcomes, or the feasibility/scalability with large cross-sectional dimensions, we believe that the model has all the relevant features outlined by Silvennoinen and Teräsvirta (2009). Empirical analyses show the benefits of the DPC model compared to competing specifications. Future research should explore the possible advantages of the DPC model in empirical applications, taking into account, for instance, portfolio allocation strategies, risk management problems, or pricing issues.

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A Data description

The 10-dimensional dataset includes Alcoa (AA), American Express (AXP), Bank of America (BAC), Coca Cola (KO), Du Pont (DD), General Electric (GE), International Business Machines (IBM), JP Morgan (JPM), Microsoft (MSFT), and Exxon Mobil (XOM).
The 30-dimensional dataset includes the 10-dimensional dataset plus 3M (MMM), AT&T (T), Boeing (BA), Caterpillar (CAT), Chevron (CVX), Cisco (CSCO), Hewlett-Packard (HPQ), Home Depot (HD), Intel (INTC), Johnson & Johnson (JNJ), McDonald's (MCD), Merck & Co. (MRK), Pfizer (PFE), Procter & Gamble (PG), Travelers Companies (TRV), United Health Group (UNH), United Technologies (UTX), Verizon Communications (VZ), Wal-Mart Stores (WMT), and Walt Disney (DIS).

B Figures

Figure 1: An example on the comparison of DPC and OGARCH models: Greece versus Germany. Baseline data (upper panels) are the change in the 5 years benchmark bond redemption yields. The letter B in abscissa denotes the beginning of the Greek debt crisis; the letter J denotes the downgrade of the Greek bond to junk bond.

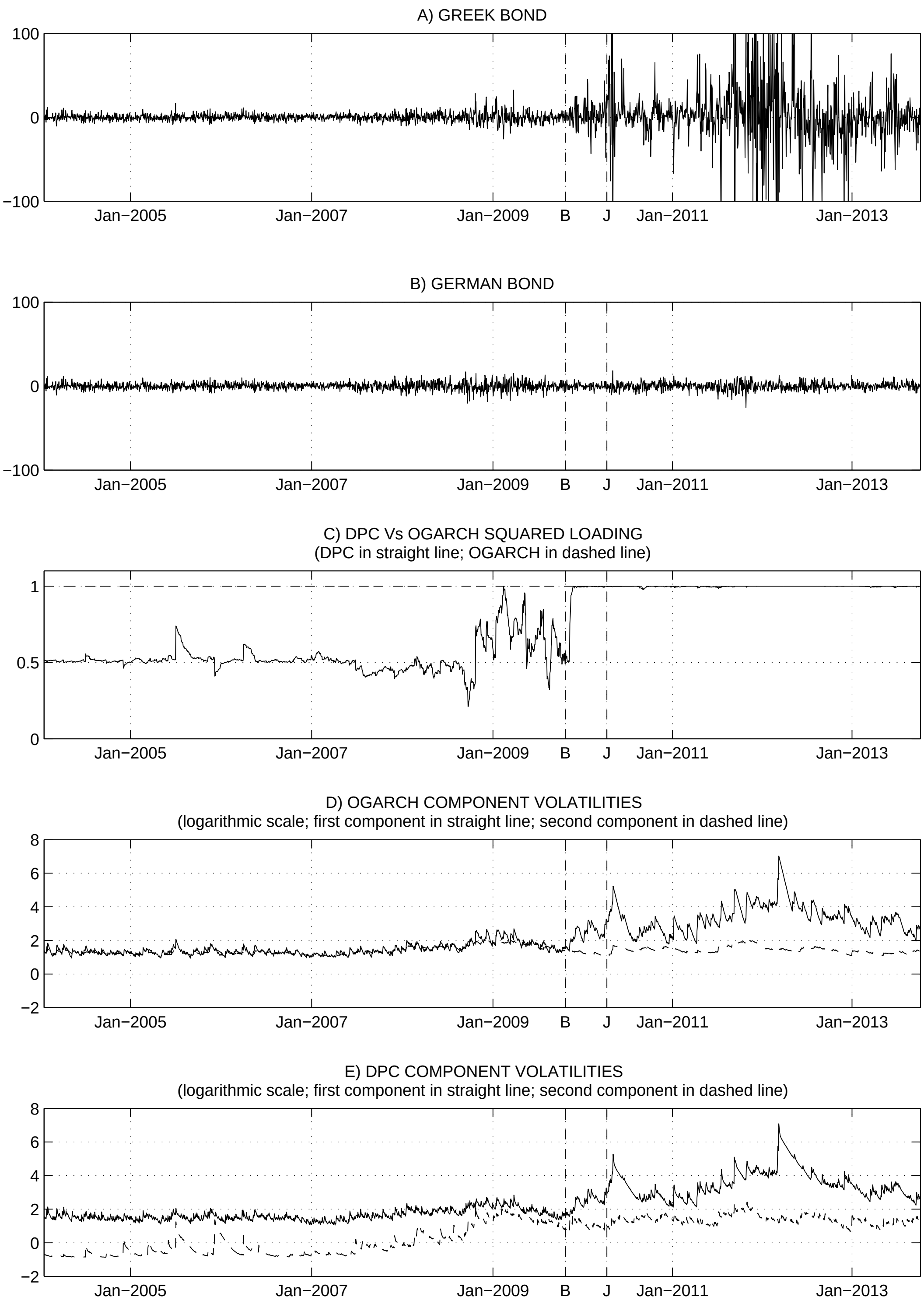


Figure 2: An example on the comparison of DPC and OGARCH models: UK versus Germany. Baseline data (upper panels) are the change in the 5 years benchmark bond redemption yields. The letter B in abscissa denotes the beginning of the Greek debt crisis; the letter J denotes the downgrade of the Greek bond to junk bond.

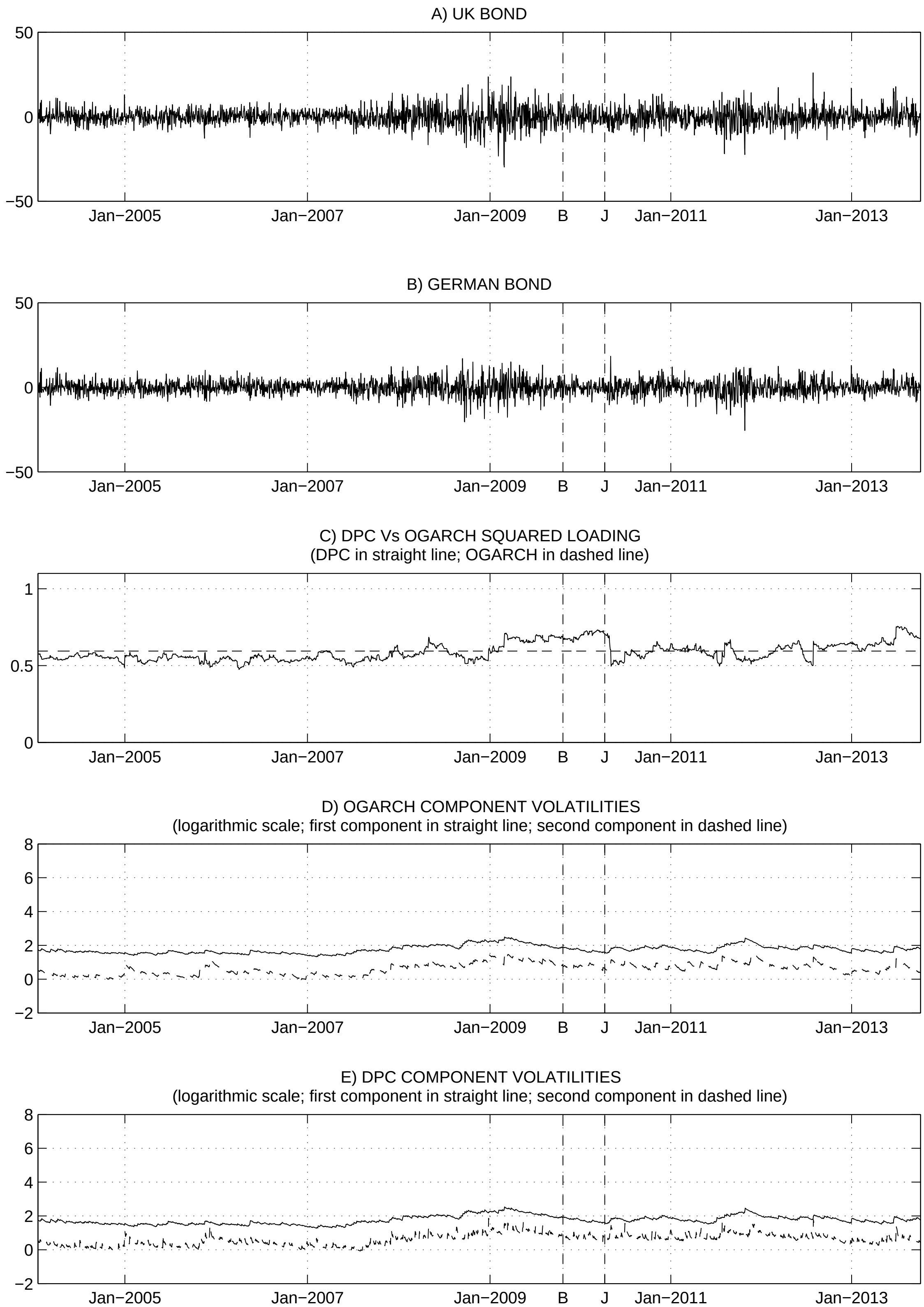


Figure 3: Conditional loadings paths used in simulation experiments.

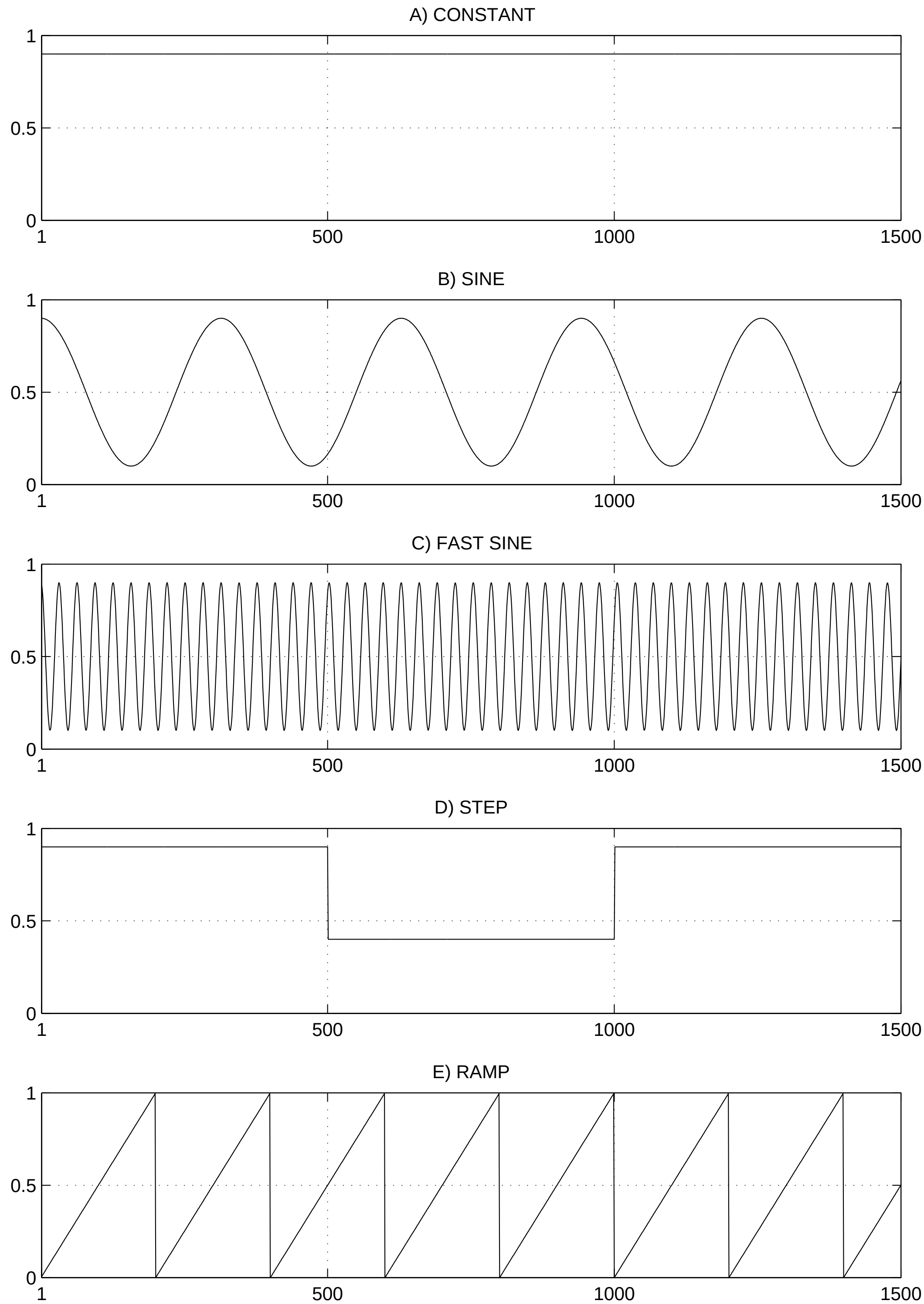


Figure 4: An example of a misspecified DPC estimation outputs from simulated data.

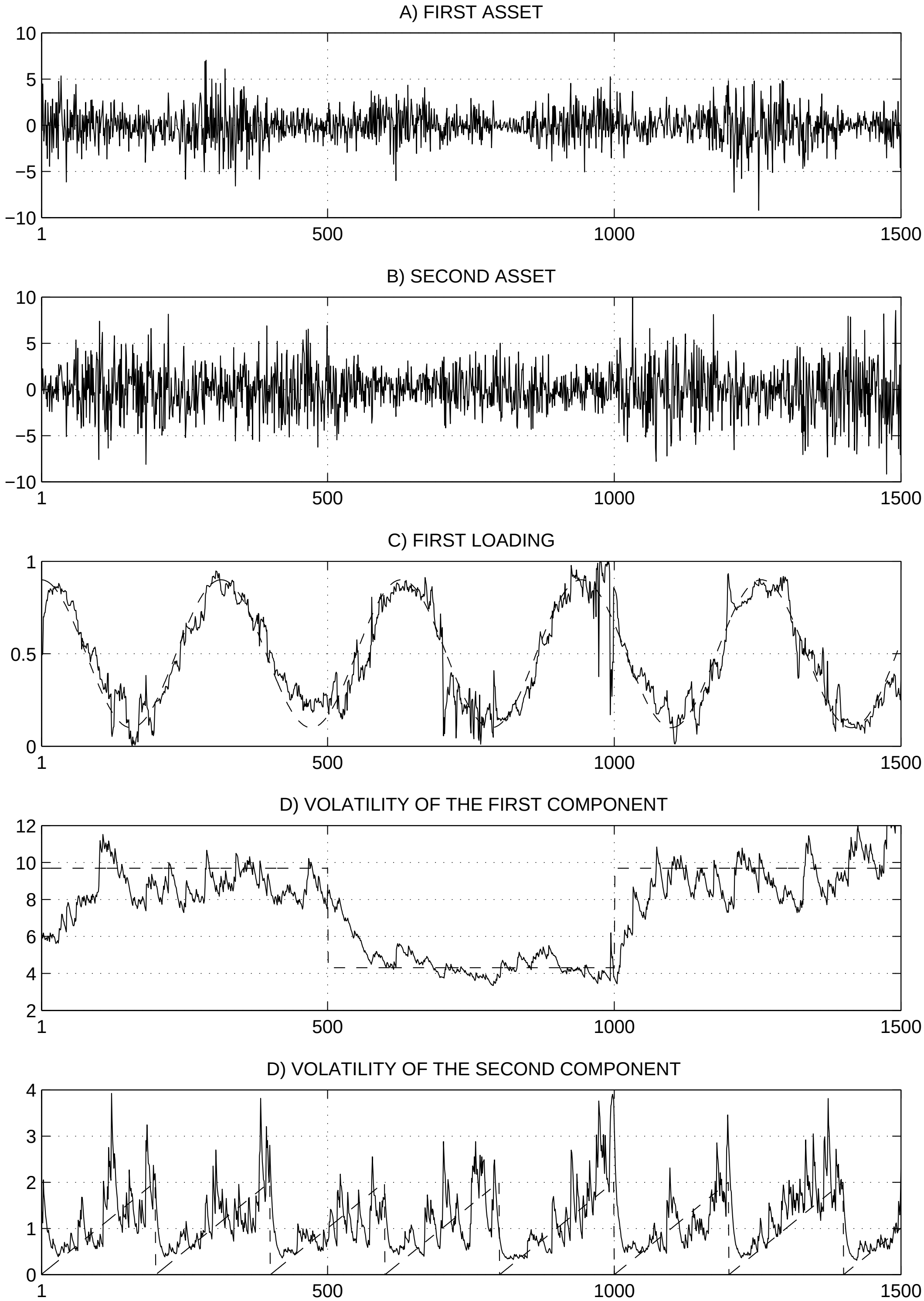


Figure 5: 10-assets dataset. Plots of the rolling window estimated ARCH parameter (a), GARCH parameter (b), and persistence parameter ($a + b$), of the DPC, DCC, and cDCC Q_t-processes.

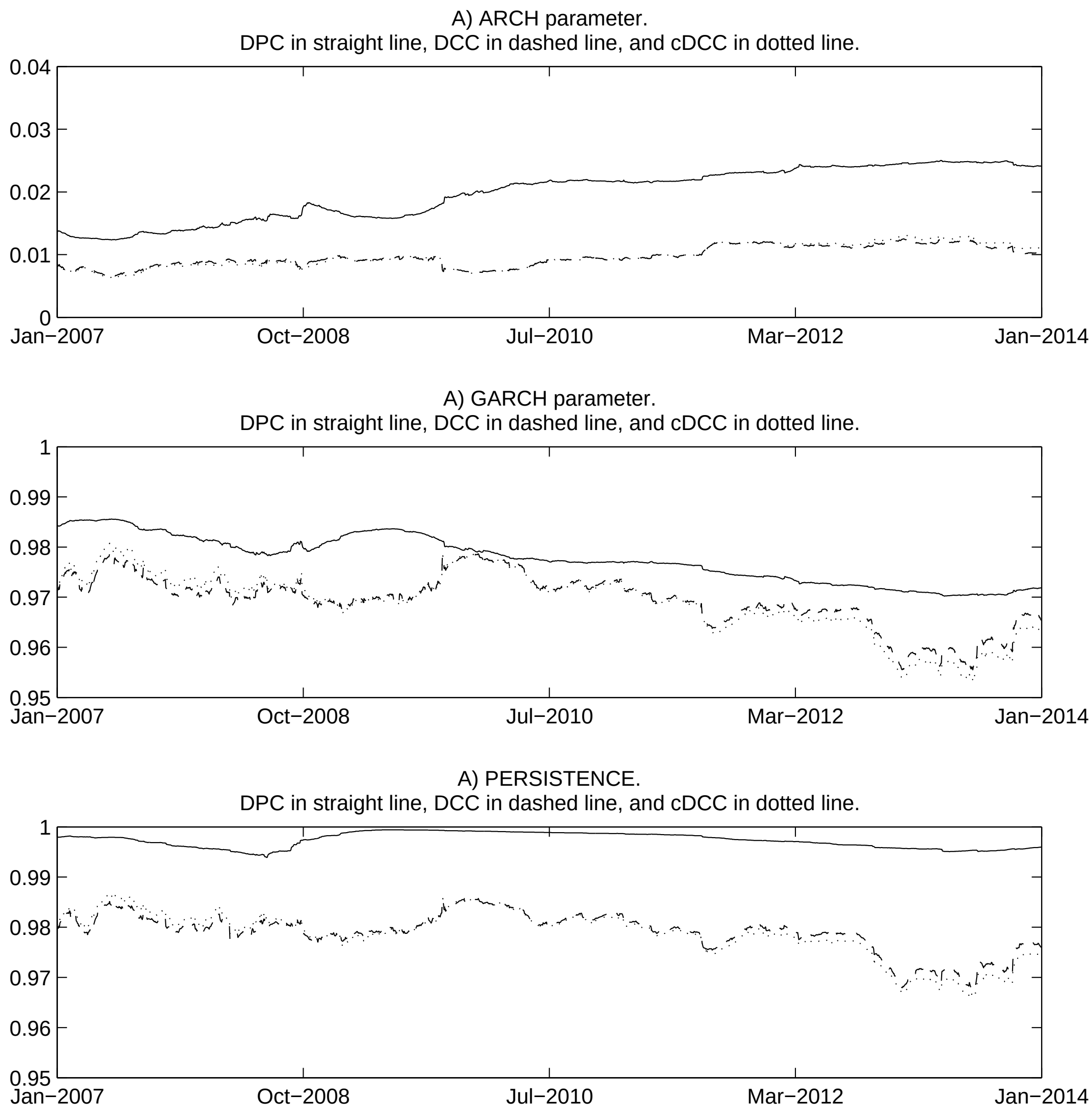


Figure 6: 30-assets dataset. Plots of the rolling window estimated ARCH parameter (a), GARCH parameter (b), and persistence parameter ($a+b$), of the DPC, DCC, and cDCC Q_t-processes.

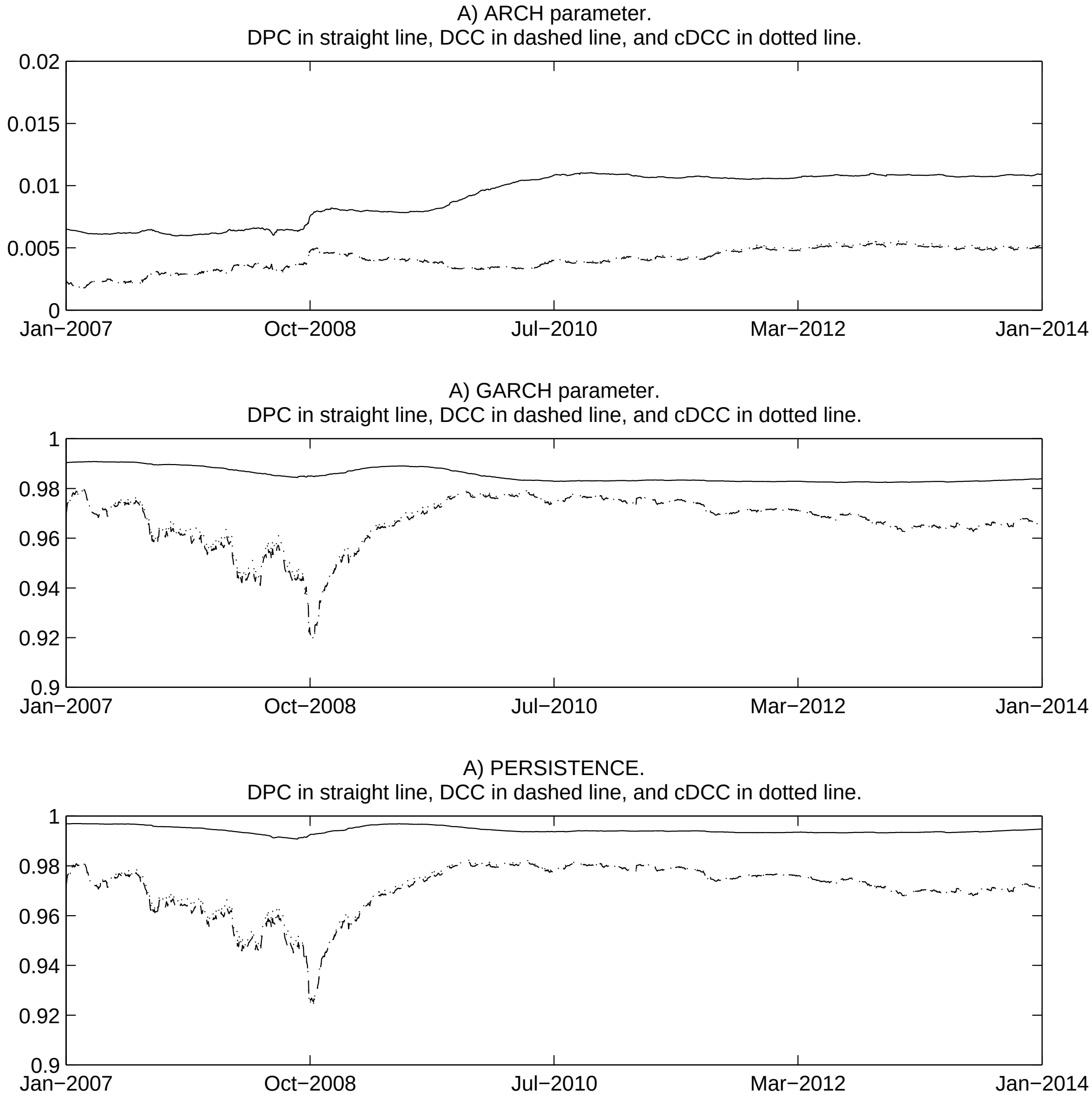


Figure 7: Robust Wald test statistic of correctly specified conditional loadings. OGARCH in dashed line, DPC in straight line, and 5% critical value in dotted line. Rolling window of 1500 observations shifted every 10 days, for a total of 175 tests over the whole sample period.

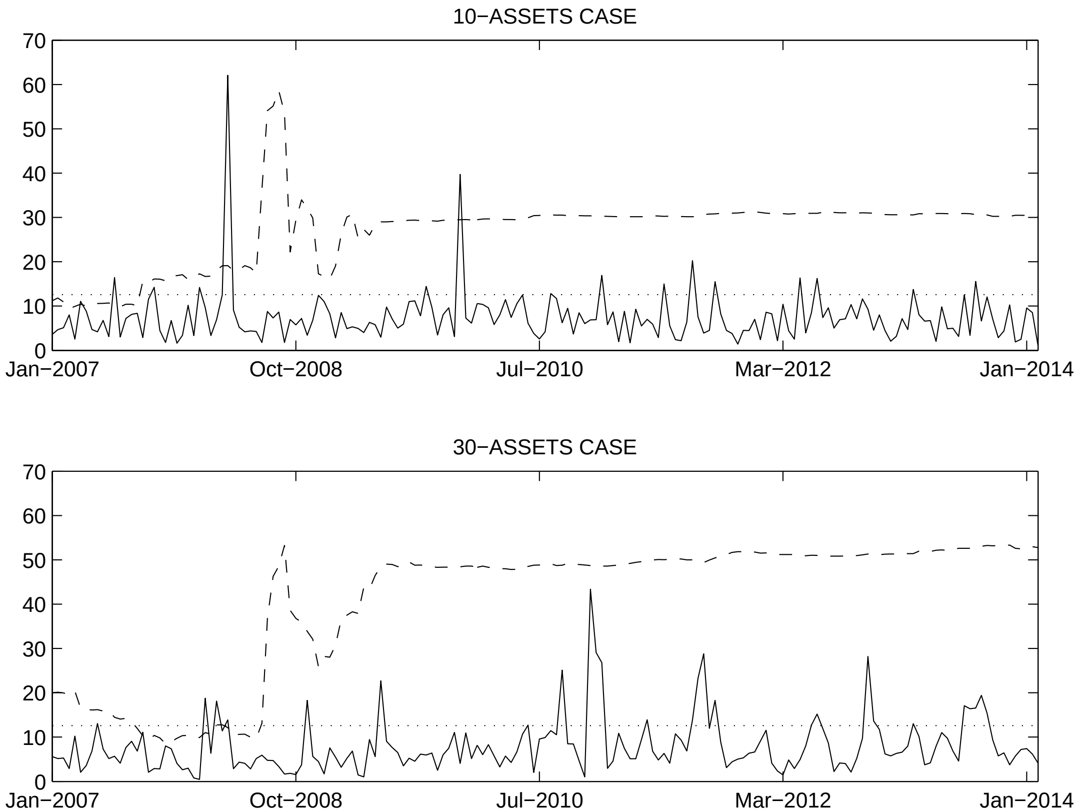


Figure 8: Augmented Mincer-Zarnowit regressions results for the 10-dimensional case. Boxplots of the F-statistics from the AMZ regressions in (31-33). Small F-statistics provide evidence in favour of the model specification and/or of the fitting performances. See the legend in Section 4.2 for the model/estimator acronyms.

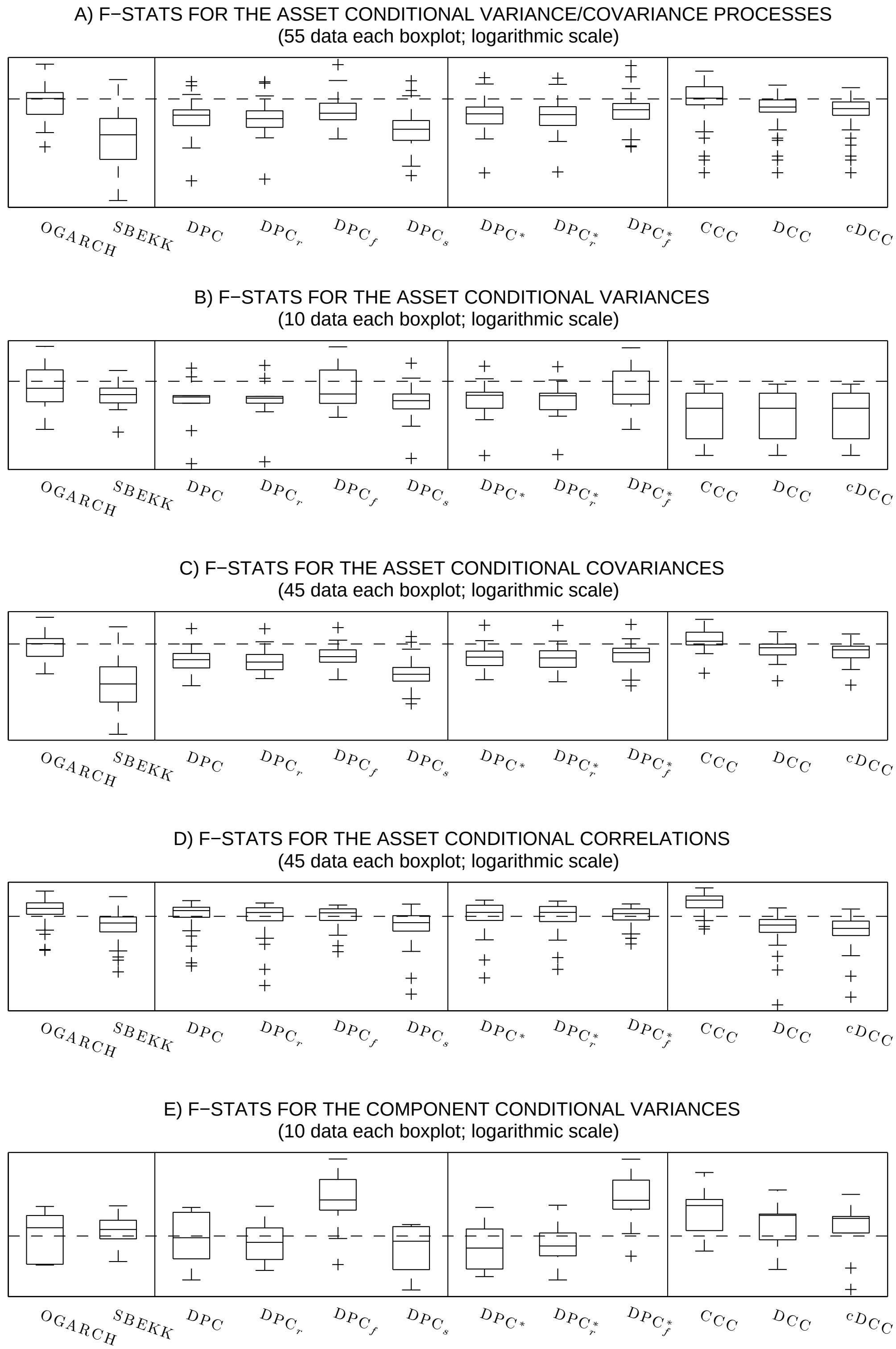
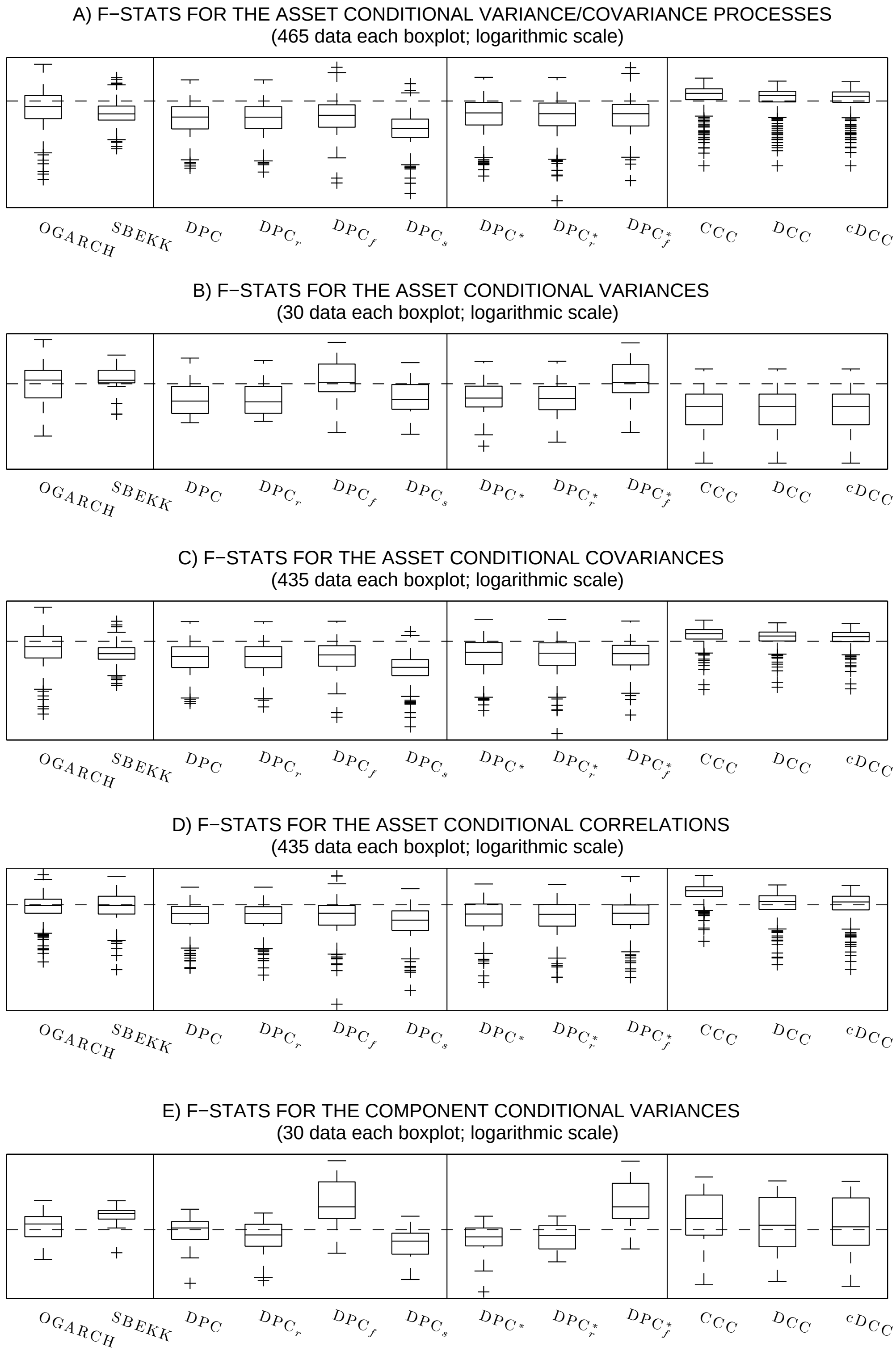


Figure 9: Augmented Mincer-Zarnowit regressions results for the 30-dimensional case. Boxplots of the F-statistics from the AMZ regressions in (31-33). Small F-statistics provide evidence in favour of the model specification and/or of the fitting performances. See the legend in Section 4.2 for the model/estimator acronyms.



C Tables

Table 1: Germany and Greece bond example

		Greece	Germany	Squared first loading	Cross-correlation
Before the break	Sample	4.50	4.29	0.53	0.79
	OGARCH	4.54	4.14	0.99	0.01
	DPC	4.42	4.09	0.53	0.85
After the break	Sample	38.50	4.33	0.99	-0.15
	OGARCH	31.96	4.31	0.99	0.07
	DPC	32.23	4.23	0.99	-0.14

The third and fourth columns report, for Greece and Germany, the standard deviations of the change in redemption yields for the 5 year benchmark bond indices (data source is Datastream). The fifth column provides the squared first loading and the sixth column includes the cross correlations. On the second and fifth rows the tables indicate the sample values. The third and sixth rows report the average values recovered from an OGARCH model, while the fourth and seventh rows provide averages coming from a DPC model. The sample data starts in January 2004 and ends in December 2013. The break date is placed at the end of 2009.

Table 2: Test of constant loadings: percentage of rejections at the 5% confidence level. The robust test is based on the artificial regression of U_t, and the non-robust test is based on the artificial regression of $\tilde{U}_t$ (see Section 3.3). The DGP is the Scalar DPC with the ARCH parameter of loading dynamic (a) reported in the rows and the GARCH parameter (b) in the columns. On the upper left corner ($a = b = 0$) the DGP collapses on the OGARCH model. In all cases we fit a OGARCH model on the simulated series.

		Non-robust test, T=1000							Robust test, T=1000							Robust test, T=1500						
N=2	$a\backslash b$	0	0.90	0.94	0.95	0.97	0.98	0.99	0	0.90	0.94	0.95	0.97	0.98	0.99	0	0.90	0.94	0.95	0.97	0.98	0.99
	0	2.4	–	–	–	–	–	–	4.0	–	–	–	–	–	–	4.2	–	–	–	–	–	–
	0.005	–	2.8	2.6	3.2	2.4	3.0	2.4	–	13.8	11.4	10.8	9.4	10.2	9.2	–	17.0	15.2	15.6	14.4	15.6	14.0
	0.01	–	2.4	3.2	2.8	2.8	1.8	–	–	24.4	27.0	22.6	18.6	17.2	–	–	34.6	35.2	29.6	27.8	33.2	–
	0.02	–	4.4	2.2	4.6	3.8	–	–	–	46.2	44.4	46.2	48.2	–	–	–	60.8	62.6	65.0	64.6	–	–
	0.03	–	4.6	4.4	3.2	–	–	–	–	63.2	66.2	63.2	–	–	–	–	75.4	79.0	79.6	–	–	–
	0.05	–	7.4	7.2	–	–	–	–	–	84.2	88.2	–	–	–	–	–	92.0	91.8	–	–	–	–
	0.09	–	17.4	–	–	–	–	–	–	94.4	–	–	–	–	–	–	97.6	–	–	–	–	–

		Non-robust test, T=1000							Robust test, T=1000							Robust test, T=1500						
N=3	$a\backslash b$	0	0.90	0.94	0.95	0.97	0.98	0.99	0	0.90	0.94	0.95	0.97	0.98	0.99	0	0.90	0.94	0.95	0.97	0.98	0.99
	0	2.8	–	–	–	–	–	–	6.2	–	–	–	–	–	–	4.2	–	–	–	–	–	–
	0.005	–	4.2	3.2	4.4	3.6	5.4	3.8	–	12.8	14.4	13.2	11.8	10.0	6.6	–	23.2	23.2	19.2	19.2	18.8	15.6
	0.01	–	4.6	5.2	3.2	4.8	3.6	–	–	29.4	31.6	23.6	28.2	21.9	–	–	43.6	41.4	40.6	40.4	37.4	–
	0.02	–	7.0	4.8	5.8	7.4	–	–	–	59.2	61.2	61.0	62.6	–	–	–	77.0	78.2	76.4	82.2	–	–
	0.03	–	9.0	13.8	10.2	–	–	–	–	77.2	82.8	78.8	–	–	–	–	91.2	90.0	82.2	–	–	–
	0.05	–	24.8	31.2	–	–	–	–	–	94.2	94.4	–	–	–	–	–	97.0	99.0	–	–	–	–
	0.09	–	54.6	–	–	–	–	–	–	97.6	–	–	–	–	–	–	98.8	–	–	–	–	–

		Non-robust test, T=1000							Robust test, T=1000							Robust test, T=1500						
N=5	$a\backslash b$	0	0.90	0.94	0.95	0.97	0.98	0.99	0	0.90	0.94	0.95	0.97	0.98	0.99	0	0.90	0.94	0.95	0.97	0.98	0.99
	0	5.2	–	–	–	–	–	–	6.2	–	–	–	–	–	–	7.0	–	–	–	–	–	–
	0.005	–	9.2	9.2	5.6	8.8	10.2	6.0	–	17.8	15.0	12.0	12.0	11.0	5.8	–	23.8	18.8	22.0	19.4	17.4	10.2
	0.01	–	12.0	11.2	11.2	10.8	11.4	–	–	35.8	30.4	27.6	28.2	27.2	–	–	50.4	51.8	47.2	48.4	44.4	–
	0.02	–	23.0	31.8	25.0	27.2	–	–	–	67.6	70.2	67.0	72.0	–	–	–	83.4	85.6	88.2	89.2	–	–
	0.03	–	50.0	47.0	52.0	–	–	–	–	86.2	88.6	89.8	–	–	–	–	95.0	95.0	96.6	–	–	–
	0.05	–	73.6	84.2	–	–	–	–	–	98.0	98.6	–	–	–	–	–	99.4	99.2	–	–	–	–
	0.09	–	96.0	–	–	–	–	–	–	99.4	–	–	–	–	–	–	99.9	–	–	–	–	–

		Non-robust test, T=1000							Robust test, T=1000							Robust test, T=1500						
N=10	$a\backslash b$	0	0.90	0.94	0.95	0.97	0.98	0.99	0	0.90	0.94	0.95	0.97	0.98	0.99	0	0.90	0.94	0.95	0.97	0.98	0.99
	0	8.6	–	–	–	–	–	–	7.4	–	–	–	–	–	–	7.4	–	–	–	–	–	–
	0.005	–	29.0	30.8	30.6	35.6	31.6	33.0	–	20.2	18.6	18.6	13.6	11.8	9.0	–	28.8	27.0	25.4	23.8	20.0	17.0
	0.01	–	53.2	61.2	55.2	56.0	63.4	–	–	40.4	42.8	43.2	36.6	31.6	–	–	61.8	59.0	60.0	61.8	64.2	–
	0.02	–	88.2	88.6	89.8	93.2	–	–	–	84.6	85.2	86.0	89.6	–	–	–	95.4	95.6	96.2	98.0	–	–
	0.03	–	96.4	97.6	97.4	–	–	–	–	97.2	97.8	97.4	–	–	–	–	98.4	98.2	99.6	–	–	–
	0.05	–	99.6	99.8	–	–	–	–	–	99.4	99.8	–	–	–	–	–	99.6	99.8	–	–	–	–
	0.09	–	100.0	–	–	–	–	–	–	100.0	–	–	–	–	–	–	99.8	–	–	–	–	–

		Non-robust test, T=1000							Robust test, T=1000							Robust test, T=1500						
N=30	$a\backslash b$	0	0.90	0.94	0.95	0.97	0.98	0.99	0	0.90	0.94	0.95	0.97	0.98	0.99	0	0.90	0.94	0.95	0.97	0.98	0.99
	0	31.6	–	–	–	–	–	–	8.6	–	–	–	–	–	–	9.2	–	–	–	–	–	–
	0.005	–	98.4	97.4	97.6	95.0	96.2	98.4	–	24.8	21.0	24.4	20.4	14.2	9.4	–	37.8	34.8	34.0	26.4	30.2	26.6
	0.01	–	99.6	99.4	99.4	99.2	99.6	–	–	62.6	64.0	60.0	56.8	56.8	–	–	80.6	83.8	82.4	83.2	86.0	–
	0.02	–	100.0	100.0	100.0	100.0	–	–	–	99.0	99.2	98.0	98.4	–	–	–	99.4	99.4	99.0	98.8	–	–
	0.03	–	100.0	100.0	100.0	–	–	–	–	99.6	99.2	99.6	–	–	–	–	99.8	99.8	99.4	–	–	–
	0.05	–	100.0	100.0	–	–	–	–	–	100.0	99.8	–	–	–	–	–	100.0	100.0	–	–	–	–
	0.09	–	100.0	–	–	–	–	–	–	100.0	–	–	–	–	–	–	100.0	–	–	–	–	–

Table 3: Test of constant loadings: percentage of rejections at the 5% confidence level for the robust test based on the artificial regression of U_t (see Section 3.3). The DGP is the Scalar DPC with the ARCH parameter of loading dynamic (a) reported in the rows and the GARCH parameter (b) in the columns. On simulated series we fit the Scalar DPC (left panels) and the Full DPC (right panels). In all cases the simulated series have length $T = 1000$.

		Scalar DPC						Full DPC					
N=2	$a\backslash b$	0.90	0.94	0.95	0.97	0.98	0.99	0.90	0.94	0.95	0.97	0.98	0.99
	0.005	7.6	7.2	6	8	8.6	8	6	7.2	6.6	7.2	7.2	7.8
	0.01	5.4	5.4	8.8	7.4	7.8	–	4.4	6	6.8	6.2	8.2	–
	0.02	5	5.4	5.8	5.8	–	–	4.6	4.4	6	6.2	–	–
	0.03	6.6	5.8	3.6	–	–	–	4.8	5.6	4.2	–	–	–
	0.05	4.6	6.6	–	–	–	–	4.2	6.2	–	–	–	–
	0.09	8.6	–	–	–	–	–	7.6	–	–	–	–	–

		Scalar DPC						Full DPC					
N=3	$a\backslash b$	0.90	0.94	0.95	0.97	0.98	0.99	0.90	0.94	0.95	0.97	0.98	0.99
	0.005	4	6.6	6.8	8.6	12.4	11.2	4.6	6.6	8.6	8.8	10.8	10
	0.01	7.4	5.6	6.4	9.6	10.8	–	6.2	6.4	5.6	9	12.6	–
	0.02	3.8	4.4	6.8	6.6	–	–	4.6	5.8	6.8	7.2	–	–
	0.03	6	3.2	5.6	–	–	–	4.6	2.2	5.6	–	–	–
	0.05	4.8	6	–	–	–	–	5.4	5.2	–	–	–	–
	0.09	11.6	–	–	–	–	–	9.4	–	–	–	–	–

		Scalar DPC						Full DPC					
N=5	$a\backslash b$	0.90	0.94	0.95	0.97	0.98	0.99	0.90	0.94	0.95	0.97	0.98	0.99
	0.005	7	10	8.4	12	12.2	9.8	8	10	9.2	12.8	16	12.2
	0.01	5	6.8	7.6	9.4	8.8	–	8.2	9.4	10.6	11	13.4	–
	0.02	6	6.6	5.4	5.2	–	–	7	7.2	6.4	6.4	–	–
	0.03	4.6	6.2	4.4	–	–	–	5.8	5.2	4.8	–	–	–
	0.05	9.2	6.2	–	–	–	–	7.8	5.4	–	–	–	–
	0.09	15.4	–	–	–	–	–	10	–	–	–	–	–

Test of constant loadings (continued): percentage of rejections at the 5% confidence level for the robust test based on the artificial regression of U_t (see Section 3.3). The DGP is the Scalar DPC with the ARCH parameter of loading dynamic (a) reported in the rows and the GARCH parameter (b) in the columns. On simulated series we fit the Scalar DPC (left panels) and the Full DPC (right panels). In all cases the simulated series have length $T = 1000$.

		Scalar DPC						Full DPC					
N=10	$a\backslash b$	0.90	0.94	0.95	0.97	0.98	0.99	0.90	0.94	0.95	0.97	0.98	0.99
	0.005	7.2	7.6	10.8	14.8	13.6	10.4	10	13.4	15	15.8	18.4	20.4
	0.01	9.8	5	7.8	7	7.8	–	6.8	10.8	10.6	12.6	16.4	–
	0.02	6.2	5.8	4.8	6.8	–	–	5.4	6.8	5.8	6	–	–
	0.03	6.8	5.4	7.4	–	–	–	5.2	4.2	5.6	–	–	–
	0.05	11.2	11.8	–	–	–	–	6.6	6.2	–	–	–	–
	0.09	34.6	–	–	–	–	–	12.6	–	–	–	–	–

		Scalar DPC					
N=30	$a\backslash b$	0.90	0.94	0.95	0.97	0.98	0.99
	0.005	5.0	6.4	6.2	6.2	8.4	25.2
	0.01	6.4	5.8	6.2	6.6	10.0	–
	0.02	7.6	6.0	5.6	14.4	–	–
	0.03	8.8	12.6	12.6	–	–	–
	0.05	17.4	54.0	–	–	–	–
	0.09	86.8	–	–	–	–	–

Table 4: Test of constant loadings: percentage of rejections at the 5% confidence level for the robust test based on the artificial regression of U_t (see Section 3.3). The DGP is the Full DPC with the persistence of components reported in the first row. On simulated series we fit the Scalar DPC (upper panel) and the Full DPC (lower panel). In all cases the simulated series have length $T = 1000$.

Fitted model	DGP Persistence $\backslash N$	2	3	5	10	30
	0.90	9.2	9.4	10.4	12.8	16.0
Scalar DPC	0.95	11.4	17.2	14.8	18.4	30.0
	0.99	10.2	16.6	34.6	52.8	90.4
	0.9	3.6	3.8	3.8	6	22.8
Full DPC	0.95	7	5	10	9.4	33.3
	0.99	11.2	8.4	16.4	12.6	55.9

Table 5: Simulation results

	OGARCH	SBEKK	DPC	CCC	DCC	CDCC
Constant	**5.21**	5.63	5.22	5.37	5.23	5.32
Sine	12.72	8.66	**6.68**	8.54	8.73	8.93
Fast Sine	12.98	13.30	**9.68**	13.30	14.34	14.50
Step	13.45	7.30	**5.99**	7.39	7.70	7.87
Ramp	13.23	9.95	**8.01**	10.10	10.18	10.37

The table reports the variance-covariance MSE. Smallest values in each row are in boldface.

Table 6: 10-assets case.

	OGARCH	SBEKK	DPC	DPC_r	DPC_f	DPC_s	DPC*	DPC^*_r	DPC^*_f	CCC	DCC	cDCC
OGARCH		-2	4	5	-1	4	4	4	-1	1	2	3
SBEKK	2		4	5	-1	5	2	3	-2	2	4	4
DPC	-4	-4		1	-4	1	1	2	-2	-2	-1	-1
DPC_r	-5	-5	-1		-4	0	0	0	-3	-2	-1	-1
DPC_f	1	1	4	4		2	2	3	0	2	2	2
DPC_s	-4	-5	-1	0	-2		0	0	-2	-3	0	0
DPC*	-4	-2	-1	0	-2	0		0	-2	-2	-1	-1
DPC^*_r	-4	-3	-2	0	-3	0	0		-3	-2	-1	-1
DPC^*_f	1	2	2	3	0	2	2	3		2	2	2
CCC	-1	-2	2	2	-2	3	2	2	-2		6	6
DCC	-2	-4	1	1	-2	0	1	1	-2	-6		2
cDCC	-3	-4	1	1	-2	0	1	1	-2	-6	-2	
	-23	-28	13	22	-23	17	15	19	-21	-16	10	15

The table reports a summary of the pairwise comparison results for the 10-dimensional case. Baseline results are included in the web appendix. The (i, j)-th entry is the score of model j against model i, computed as the number of winning comparisons (significance in favour of model j) minus the number of loosing comparisons (significance in favour of model i).

Table 7: 30-assets case.

	OGARCH	SBEKK	DPC	DPC_r	DPC_f	DPC_s	DPC*	DPC^*_r	DPC^*_f	CCC	DCC	cDCC
OGARCH		-2	3	5	-1	2	3	4	-1	-1	0	0
SBEKK	2		6	6	2	7	6	6	2	4	5	5
DPC	-3	-6		2	-4	1	3	3	-2	-3	-2	-2
DPC_r	-5	-6	-2		-4	-1	0	1	-2	-3	-2	-2
DPC_f	1	-2	4	4		1	2	2	0	0	1	1
DPC_s	-2	-7	-1	1	-1		0	1	-1	-2	-2	-2
DPC*	-3	-6	-3	0	-2	0		3	-2	-3	-2	-2
DPC^*_r	-4	-6	-3	-1	-2	-1	-3		-2	-3	-3	-3
DPC^*_f	1	-2	2	2	0	1	2	2		0	1	1
CCC	1	-4	3	3	0	2	3	3	0		8	8
DCC	0	-5	2	2	-1	2	2	3	-1	-8		5
cDCC	0	-5	2	2	-1	2	2	3	-1	-8	-5	
	-12	-51	13	26	-14	16	20	31	-10	-27	-1	9

The table reports a summary of the pairwise comparison results for the 30-dimensional case. Baseline results are included in the web appendix. The (i, j)-th entry is the score of model j against model i, computed as the number of winning comparisons (significance in favour of model j) minus the number of loosing comparisons (significance in favour of model i).

D Proofs

Proof of proposition 2.2.1 Applying the properties of the trace operator, the orthonormality of L_t, and recalling that $E[u_t u_t'] = E[D_t] = D$ and $E[y_t y_t'] = E[E_{t-1}[y_t y_t']] = E[H_t]$, we can write $\text{trace}(\bar{S}) = E[\text{trace}(y_t y_t')] = E[\text{trace}(H_t)] = E[\text{trace}(L_t D_t L_t')] = E[\text{trace}(D_t L_t' L_t)] = E[\text{trace}(D_t)] = \text{trace}(D) = \text{trace}(DL'L) = \text{trace}(LDL') = \text{trace}(S)$. Therefore, equation (7) holds, ensuring that the loading process is identified.

Proof of proposition 2.6.1 We prove the stationarity of $\bar{u}_t$ (see eq. (21-22)). Since $y_t = L\bar{u}_t$, the proof implies the stationarity of y_t. By recursively substituting backward for the lagged $\bar{u}_t$'s, and applying $\bar{u}_t = \bar{L}_t D_t^{1/2} z_t$, we can write $\bar{Q}_t$ as

$$\bar{Q}_t = C + \sum_{n=1}^{\infty} A\{B^{n-1}\}\bar{L}_{t-n} D_{t-n}^{1/2} z_{t-n} z_{t-n}' D_{t-n}^{1/2} \bar{L}_{t-n}' \{B^{n-1}\}' A', \tag{34}$$

where $B^0 = I_N$ and

$$C = \sum_{n=1}^{\infty} \{B^{n-1}\}(S - ASA' - BSB')\{B^{n-1}\}'. \tag{35}$$

Assumptions i-iii) imply that that the vector $[d_{1,t}, \ldots, d_{N,t}]'$ is strictly stationary (Nelson, 1990). Convergence of $\bar{Q}_t$, is ensured by the assumption that $0 \leqslant b_i < 1$, for $i = 1, 2, \ldots, n$, the strict stationarity of $D_t = \text{diag}(d_{1,t}, \ldots, d_{N,t})$, and the fact that L_t is bound. The matrix $\bar{Q}_t$ is therefore a measurable function of $\{z_t\}_{t=-\infty,+\infty}$. Since $\bar{Q}_t$ is a time invariant function of $\{z_t\}_{t=-\infty,+\infty}$, the matrix, $\bar{L}_t$ is a measurable function of $\{z_t\}_{t=-\infty,+\infty}$. Since D_t is a time invariant measurable function of $\{z_t\}_{t=-\infty,+\infty}$, the CCM of $\bar{u}_t$, that is, $\bar{H}_t = \bar{L}_t D_t \bar{L}_t'$, is a time invariant measurable function of $\{z_t\}_{t=-\infty,+\infty}$. Since $\{z_t\}_{t=-\infty,+\infty}$ is iid, using the ergodicity criterion from Corollary 1.4.2 in Krengel (1985),the strict stationarity of $\bar{u}_t$ is equivalent to the condition

$$\text{trace}(\bar{H}_t \bar{H}_t') < \infty \text{ a.s.},$$

where

$$\text{trace}(\bar{H}_t \bar{H}_t') = \text{trace}(\bar{H}_t^2) = \text{trace}(\bar{L}_t D_t' \bar{L}_t' \bar{L}_t D_t' \bar{L}_t') = \text{trace}(\bar{L}_t D_t^2 \bar{L}_t') = \text{trace}(D_t^2 \bar{L}_t' \bar{L}_t) = \sum_{i=1}^{N} d_{i,t}^2.$$

From Nelson (1990) $d_{i,t} < \infty$ a.s. and, therefore, $\sum_{i=1}^{N} d_{i,t}^2 < \infty$ a.s., which proves that $\bar{u}_t$ is strictly stationary. Since we assumed $\alpha_i + \beta_i < 1$, for $i = 1, 2, \ldots, N$, it holds that $E[\text{trace}(\bar{H}_t)] = E[\text{trace}(D_t)] = E[\sum_{i=1}^{n} d_{i,t}] = \sum_{i=1}^{n} d_i < 0$. The second moment of $\bar{u}_t$ is finite and, therefore, $\bar{u}_t$ is weakly stationary. ▫

Proof of proposition 2.6.2 Recall that $\bar{S} = E[y_t y_t'] = E[L\bar{u}_t \bar{u}_t' L'] = LE[\bar{u}_t \bar{u}_t']L'$. Therefore, in order to prove that the columns of L are the eigenvectors of $\bar{S}$, it suffices to prove that the second moment of $\bar{u}_t$ is diagonal. The proof of the proposition is based on symmetry arguments. In stating the proposition we will make use of the set of the $N \times N$ diagonal matrices with diagonal elements in $\{1, -1\}$, which will be denoted as $\{P_j\}_{j=1,2,\ldots,J}$. This is the set of the so-called signature matrices.[32] We first prove some properties of the signature matrices (LEMMA D.1); we then prove

[32]There are $J \equiv \sum_{m=0}^{N} \binom{N}{m}$ such matrices. Geometrically, a signature matrix represents a reflection in each of the axes corresponding to the negative diagonal elements.

that Proposition 2.6.2 holds under two additional assumptions (LEMMA D.2); we finally prove that the two additional assumptions hold (LEMMA D.3), which completes the proof of the proposition.

Lemma D.1. Properties of the signature matrices.

1. *The product $P_j X$ changes the sign of the rows of X indexed by the diagonal positions of the negative elements of P_j;*
2. *the product $P_j X P_j$ changes the sign of the off diagonal elements of the rows and columns of X indexed by the diagonal positions of the negative elements of P_j;*
3. *$P_j P_j = I$;*
4. *$P_j X P_j$ has the same diagonal elements as X;*
5. *if V is diagonal, $P_j V = V P_j$, and $P_j V P_j = V$;*
6. *the matrix $J^{-1} \sum_{j=1}^{J} P_j X P_j$ is diagonal with the same diagonal elements as X.*

Proof As for points 1-5, the proof is immediate. As for the proof of point 6, without loss of generality, let us arrange the set $\{P_j\}_{j=1,2,\dots,J}$ such that, for $j \in \{2,4,6,\dots,J\}$, it holds that $P_j = -P_{j-1}$ (J is always an even number). By Lemma D.1 - point 2 and point 4, for $j \in \{2,4,6,\dots,J\}$ it holds that $P_j X P_j + P_{j-1} X P_{j-1} = 2V$, where V is diagonal with the same diagonal elements as X. Therefore, $\sum_{j=1}^{J} P_j X P_j = \sum_{j=2,4,6,\dots,J}^{J} (P_j X P_j + P_{j-1} X P_{j-1}) = (J/2)2V = JV$. ▫

For arbitrarily fixed $\{\zeta_t\}$ in the support of $\{z_t\}$, set $\{z_t^{(j)}\} = \{P_j \zeta_t\}$ and denote as $\{\bar{u}_t^{(j)}, \bar{Q}_t^{(j)}, \bar{L}_t^{(j)}, D_t^{(j)}\}$ the sequence $\{\bar{u}_t, \bar{Q}_t, \bar{L}_t, D_t\}$ generated by $\{z_t^{(j)}\}$, $j = 1,2,\dots,J$. By the independence assumptions and the symmetry of the distribution of $z_{i,t}$, for arbitrarily fixed $\{\zeta_t\}$, we have that the sequences $\{z_t^{(j)}\}$, for $j = 1,2,\dots,J$, are equiprobable. The DGP of $\{\bar{u}_t, \bar{Q}_t, \bar{L}_t, D_t\}$ can thus be written as follows:

I. draw $\{\zeta_t\}$ from the support of $\{z_t\}$;

II. draw j from the uniform distribution on $\{1,2,\dots,J\}$;

IV. set $\{\bar{u}_t, \bar{Q}_t, \bar{L}_t, D_t\} = \{\bar{u}_t^{(j)}, \bar{Q}_t^{(j)}, \bar{L}_t^{(j)}, D_t^{(j)}\}$.

We now prove that Proposition 2.6.2 holds under two additional assumptions. We will then prove that the two additional assumptions are satisfied.

Lemma D.2. *Suppose that, for arbitrarily fixed $\{\zeta_t\}$ in the support of $\{z_t\}$, it holds that*

$$D_t^{(j)} = D_t^{(1)}, \tag{36}$$

and

$$\bar{L}_t^{(j)} = P_j \bar{L}_t^{(1)} P_j, \tag{37}$$

for $j \in \{1,2,\dots,J\}$. Then, $E[\bar{u}_t \bar{u}_t']$ is diagonal.

Proof. The unconditional covariance matrix of $\bar{u}_t$ for fixed $\{\zeta_t\}$ can be written as

$$E[\bar{u}_t \bar{u}_t' | \{\zeta_t\}] = \frac{1}{J} \sum_{j=1}^{J} \bar{u}_t^{(j)} \bar{u}_t^{(j)\prime} = \frac{1}{J} \sum_{j=1}^{J} \bar{L}_t^{(j)} \{D_t^{(j)}\}^{1/2} z_t^{(j)} z_t^{(j)\prime} \{D_t^{(j)}\}^{1/2} \bar{L}_t^{(j)\prime} =$$

$$= \frac{1}{J}\sum_{j=1}^{J}\{P_j\bar{L}_t^{(1)}P_j\}\{D_t^{(1)}\}^{1/2}P_j z_t^{(1)} z_t^{(1)\prime} P_j\{D_t^{(1)}\}^{1/2}\{P_j\bar{L}_t^{(1)}P_j\}' =$$

$$= \frac{1}{J}\sum_{j=1}^{J}\{P_j\bar{L}_t^{(1)}\}\{P_j D_t^{(1)}P_j\}^{1/2} z_t^{(1)} z_t^{(1)\prime}\{P_j D_t^{(1)}P_j\}^{1/2}\{\bar{L}_t^{(1)}P_j\}' =$$

$$= \frac{1}{J}\sum_{j=1}^{J} P_j\bar{L}_t^{(1)}\{D_t^{(1)}\}^{1/2} z_t^{(1)} z_t^{(1)\prime}\{D_t^{(1)}\}^{1/2}\bar{L}_t^{(1)\prime}P_j, = \frac{1}{J}\sum_{j=1}^{J} P_j X P_j,$$

where we applied eq. (36-37), Lemma D.1 - point 5, and $X = \bar{L}_t^{(1)}\{D_t^{(1)}\}^{1/2} z_t^{(1)} z_t^{(1)\prime}\{D_t^{(1)}\}^{1/2}\bar{L}_t^{(1)\prime}$. From Lemma D.1 - point 6, it follows that $E[\bar{u}_t\bar{u}_t'|\{\zeta_t\}]$ is diagonal; therefore, the matrix $E[\bar{u}_t\bar{u}_t'] = E[E[\bar{u}_t\bar{u}_t'|\{\zeta_t\}]]$ is diagonal. ▫.

Lemma D.3. *For arbitrarily fixed $\{\zeta_t\}$ in the support of $\{z_t\}$, equations (36-37) are satisfied.*

Proof The proof of (36) follows from the fact that $d_{i,t}$ does not depend on the sign of $z_{i,t}$, for $i = 1, 2, \ldots, N$. As for the proof of (37), by backward substitutions let us rewrite $\bar{Q}_t^{(1)}$ as in (34). Then, let us pre- and post-multiply the right hand side and the left hand side of the resulting equation of $\bar{Q}_t^{(1)}$ by P_j. Noting that

$$P_j A\bar{L}_t^{(1)}\{D_t^{(1)}\}^{1/2}\, z_t^{(1)} =$$

$$= AP_j\bar{L}_t^{(1)}\{P_j\{D_t^{(1)}\}^{1/2}P_j\}\, z_t^{(1)} = A\{P_j\bar{L}_t^{(1)}P_j\}\{D_t^{(1)}\}^{1/2}\{P_j z_t^{(1)}\} = A\{P_j\bar{L}_t^{(1)}P_j\}\{D_t^{(j)}\}^{1/2} z_t^{(j)},$$

where we applied eq. (36), lemma D.1 - points 5-6 (recall that A is diagonal), we can write

$$\tilde{Q}_t = C + \sum_{m=1}^{\infty} B^{m-1}A\tilde{L}_{t-m}\{D_t^{(j)}\}^{1/2} z_t^{(j)} z_t^{(j)\prime}\{D_t^{(j)}\}^{1/2}\tilde{L}_{t-m}' AB^{m-1}, \tag{38}$$

where

$$\tilde{Q}_t = P_j\bar{Q}_t^{(1)}P_j, \tag{39}$$

and

$$\tilde{L}_t = P_j\bar{L}_t^{(1)}P_j. \tag{40}$$

If we prove that $\tilde{L}_t$ in (38) is the eigenvector matrix of $\tilde{Q}_t$, computed under the identification conditions in Assumption 2.7.1, we prove that $\tilde{Q}_t$ is the loading driving process generated by $\{z_t^{(j)}\}$, and, therefore, that (37) holds. Applying Lemma D.1 - point 5, we can write

$$\tilde{Q}_t = P_j\bar{Q}_t^{(1)}P_j = P_j\bar{L}_t^{(1)}G_t^{(1)}\bar{L}_t^{(1)\prime}P_j = \tag{41}$$

$$= \{P_j\bar{L}_t^{(1)}\}P_j G_t^{(1)}P_j\{\bar{L}_t^{(1)}P_j\}' = \{P_j\bar{L}_t^{(1)}P_j\}G_t^{(1)}\{P_j\bar{L}_t^{(1)}P_j\}' = \tilde{L}_t G_t^{(1)}\tilde{L}_t',$$

where $G_t^{(1)}$ is the diagonal matrix of the decreasing eigenvalues of $\bar{Q}_t^{(1)}$, and $\tilde{L}_t$ is clearly orthonormal. Since $\bar{L}_t^{(1)}$ has positive diagonal elements, also $\tilde{L}_t$ has positive diagonal elements (see Lemma D.1 - point 4). Since $G_t^{(1)}$ is diagonal with decreasing diagonal elements, $\tilde{L}_t$ is the eigenvector matrix of $\tilde{Q}_t$ computed under the identification conditions in Assumption 2.7.1. ▫

A Additional material to the attention of referees and for web appendix

A.1 Additional Figures

Figure 10: Log-price difference of the DJIA constituents from 01/02/2001 - 02/01/2014.

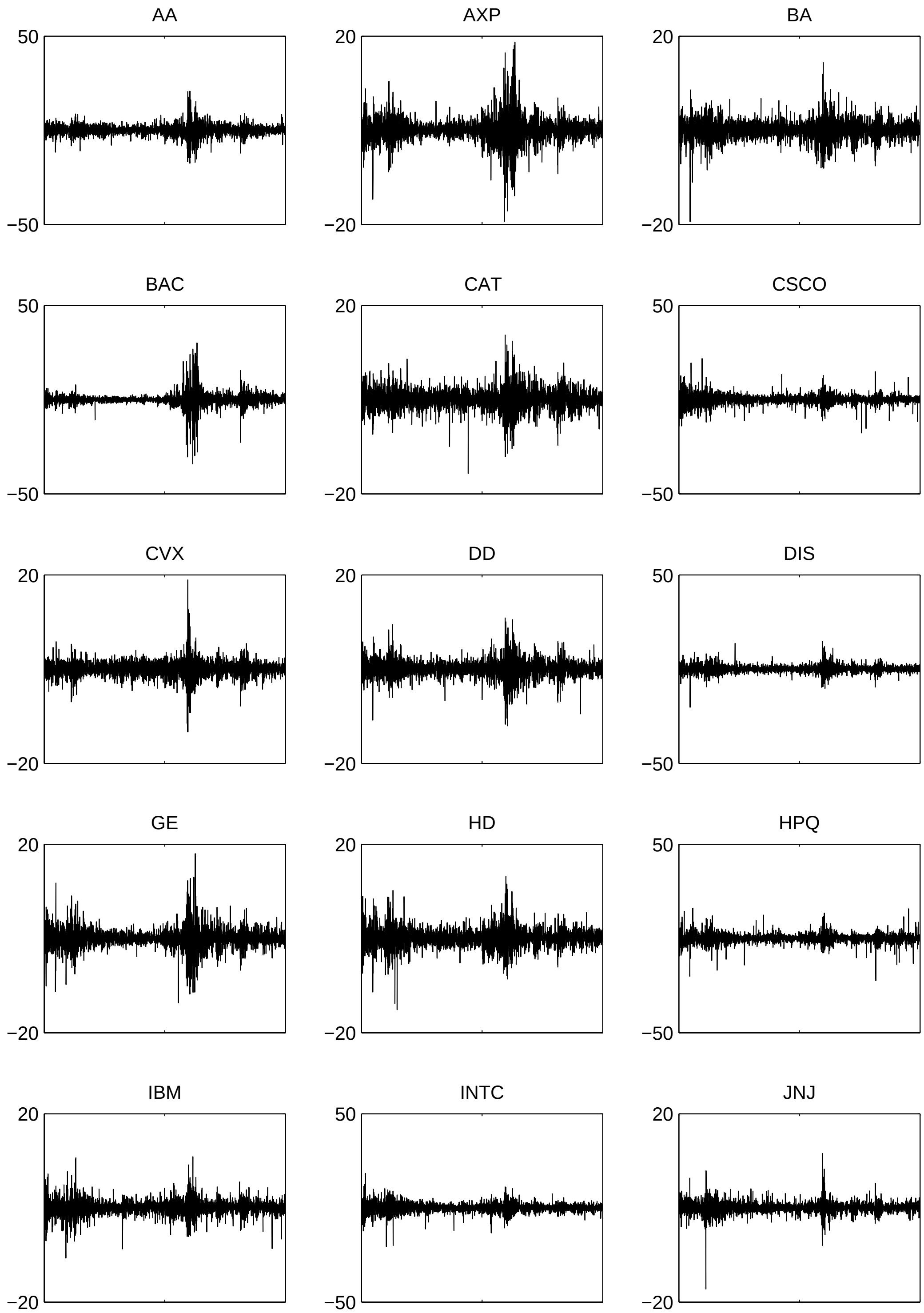


Figure 11: Log-price difference of the DJIA constituents from 01/02/2001 - 02/01/2014.

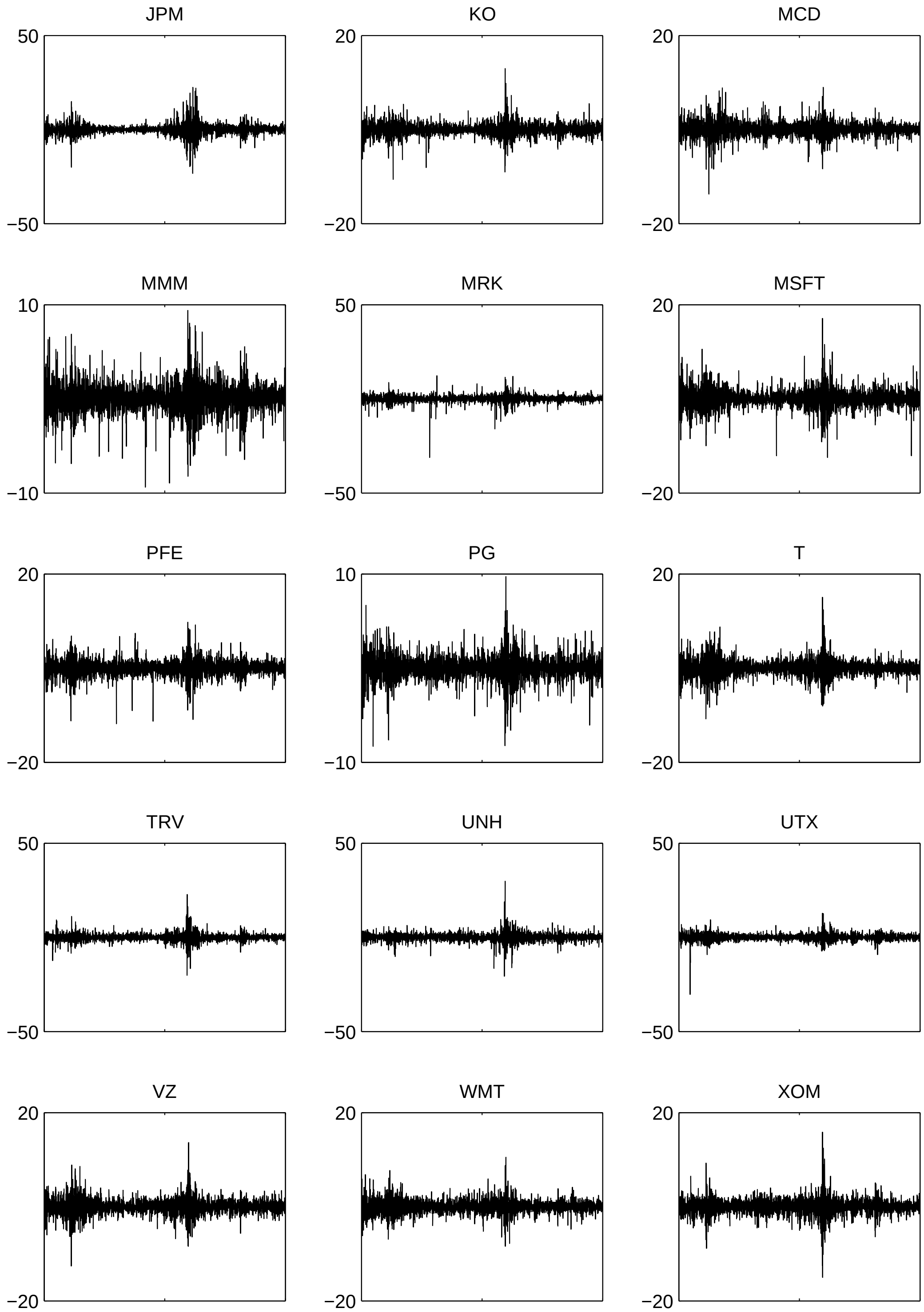


Figure 12: 10-assets dataset. Plots of the average rolling window estimated ARCH parameters (α_i), GARCH parameters (β_i), and persistence parameters ($\alpha_i + \beta_i$), of the DPC and OGARCH components conditional variances.

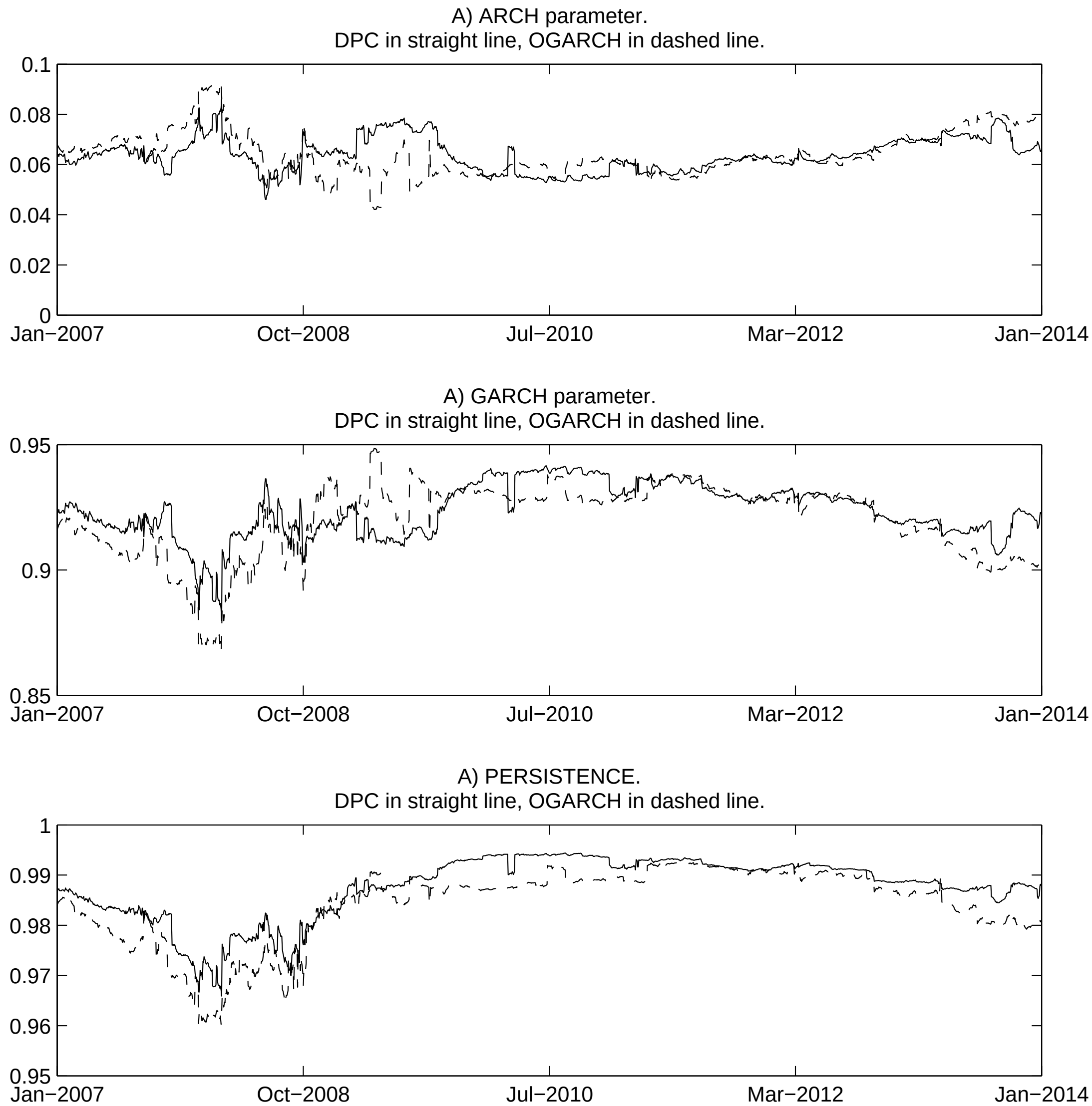
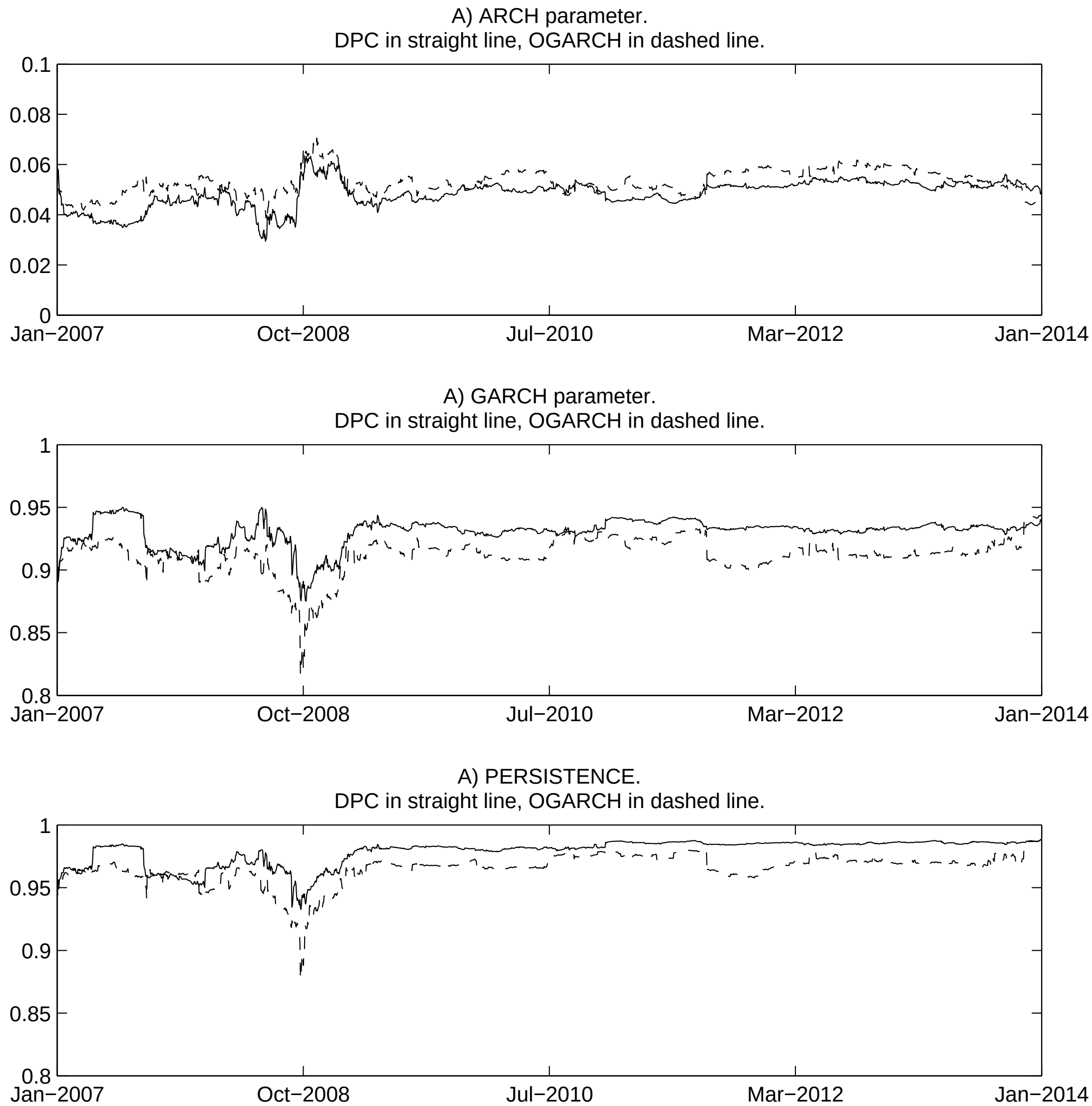


Figure 13: 30-assets dataset. Plots of the average rolling window estimated ARCH parameters (α_i), GARCH parameters (β_i), and persistence parameters ($\alpha_i + \beta_i$), of the DPC and OGARCH components conditional variances.



A.2 Simulations for loading dynamic and GARCH parameters

The following tables reports bias and mean squared errors for the dynamic loading parameters in the Scalar DPC model and for the GARCH dynamic parameters associated with the first conditional component. The statistics have been computed over 500 replications using Gaussian innovations in the DPC data generating process. True parameters are reported in the first row. Parameters for the intercepts and for the additional conditional components have not been reported to limit the number of tables but are available upon request.

Table 8: DPC loading dynamic parameters

N/T		$\alpha = 0.005 - \beta = 0.90$			$\alpha = 0.005 - \beta = 0.94$			$\alpha = 0.005 - \beta = 0.95$			$\alpha = 0.005 - \beta = 0.97$			$\alpha = 0.005 - \beta = 0.98$			$\alpha = 0.005 - \beta = 0.99$		
		500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500
Bias																			
2	α	0.053	0.052	0.053	0.051	0.053	0.053	0.054	0.052	0.052	0.052	0.053	0.052	0.051	0.052	0.052	0.052	0.053	0.052
	β	-0.037	-0.016	-0.013	-0.072	-0.058	-0.053	-0.087	-0.066	-0.062	-0.101	-0.085	-0.081	-0.113	-0.093	-0.089	-0.139	-0.106	-0.101
3	α	0.052	0.052	0.053	0.051	0.051	0.052	0.050	0.051	0.052	0.051	0.051	0.051	0.049	0.050	0.051	0.049	0.050	0.049
	β	-0.027	-0.016	-0.013	-0.066	-0.053	-0.051	-0.072	-0.062	-0.060	-0.094	-0.082	-0.078	-0.103	-0.090	-0.087	-0.114	-0.099	-0.094
5	α	0.043	0.044	0.043	0.042	0.042	0.042	0.041	0.042	0.042	0.036	0.036	0.037	0.031	0.031	0.031	0.026	0.026	0.027
	β	0.011	0.020	0.024	-0.031	-0.022	-0.018	-0.042	-0.031	-0.030	-0.069	-0.054	-0.050	-0.080	-0.059	-0.057	-0.132	-0.066	-0.060
10	α	0.027	0.027	0.027	0.026	0.026	0.026	0.025	0.026	0.026	0.024	0.025	0.025	0.023	0.023	0.023	0.017	0.018	0.017
	β	0.022	0.033	0.038	-0.016	-0.003	0.000	-0.025	-0.013	-0.009	-0.043	-0.030	-0.026	-0.051	-0.038	-0.034	-0.070	-0.042	-0.037
30	α	0.005	0.007	0.008	0.004	0.006	0.007	0.004	0.006	0.006	0.003	0.005	0.005	0.002	0.004	0.004	-0.001	0.002	0.003
	β	0.011	0.037	0.044	-0.020	0.003	0.009	-0.028	-0.004	0.002	-0.043	-0.017	-0.011	-0.107	-0.021	-0.015	-0.326	-0.030	-0.014
MSE																			
2	α	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
	β	0.009	0.001	0.001	0.001	0.004	0.003	0.014	0.005	0.004	0.014	0.008	0.007	0.019	0.009	0.008	0.038	0.012	0.011
3	α	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
	β	0.003	0.001	0.000	0.006	0.003	0.003	0.007	0.004	0.004	0.001	0.007	0.006	0.014	0.009	0.008	0.018	0.010	0.009
5	α	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	β	0.001	0.001	0.001	0.002	0.001	0.001	0.003	0.001	0.001	0.009	0.003	0.003	0.013	0.004	0.004	0.064	0.006	0.004
10	α	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.000
	β	0.001	0.001	0.002	0.001	0.000	0.000	0.001	0.000	0.000	0.002	0.001	0.001	0.003	0.002	0.001	0.013	0.002	0.001
30	α	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	β	0.000	0.002	0.002	0.001	0.000	0.000	0.001	0.000	0.000	0.006	0.000	0.000	0.063	0.000	0.000	0.279	0.001	0.000

Table 9: DPC loading dynamic parameters (continued)

		$\alpha = 0.01 - \beta = 0.90$			$\alpha = 0.01 - \beta = 0.94$			$\alpha = 0.01 - \beta = 0.95$			$\alpha = 0.01 - \beta = 0.97$			$\alpha = 0.01 - \beta = 0.98$			$\alpha = 0.02 - \beta = 0.90$		
N/T		500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500
Bias																			
2	α	0.049	0.048	0.048	0.050	0.048	0.049	0.049	0.048	0.048	0.048	0.048	0.048	0.048	0.047	0.049	0.041	0.041	0.041
	β	-0.039	-0.017	-0.012	-0.074	-0.057	-0.055	-0.083	-0.067	-0.062	-0.104	-0.083	-0.079	-0.128	-0.093	-0.090	-0.036	-0.021	-0.017
3	α	0.047	0.047	0.048	0.047	0.047	0.047	0.046	0.047	0.046	0.044	0.045	0.045	0.043	0.043	0.044	0.039	0.038	0.039
	β	-0.027	-0.014	-0.013	-0.066	-0.053	-0.050	-0.073	-0.060	-0.057	-0.087	-0.077	-0.073	-0.096	-0.081	-0.078	-0.026	-0.015	-0.012
5	α	0.036	0.036	0.036	0.029	0.031	0.031	0.028	0.028	0.029	0.023	0.024	0.024	0.020	0.022	0.022	0.021	0.022	0.022
	β	0.003	0.013	0.016	-0.033	-0.024	-0.023	-0.043	-0.031	-0.030	-0.060	-0.044	-0.041	-0.084	-0.047	-0.044	0.001	0.011	0.015
10	α	0.021	0.022	0.022	0.020	0.021	0.021	0.019	0.019	0.020	0.015	0.015	0.015	0.011	0.012	0.012	0.012	0.013	0.012
	β	0.021	0.034	0.037	-0.014	-0.002	0.001	-0.021	-0.012	-0.008	-0.034	-0.024	-0.020	-0.046	-0.025	-0.021	0.018	0.029	0.033
30	α	0.001	0.003	0.004	0.000	0.002	0.003	0.000	0.002	0.002	-0.002	0.000	0.001	-0.003	0.000	0.000	-0.005	-0.003	-0.003
	β	0.010	0.032	0.039	-0.016	0.004	0.009	-0.021	-0.002	0.003	-0.060	-0.009	-0.005	-0.148	-0.009	-0.006	0.006	0.024	0.030
MSE																			
2	α	0.003	0.002	0.002	0.003	0.002	0.003	0.003	0.002	0.002	0.003	0.002	0.002	0.003	0.002	0.002	0.002	0.002	0.002
	β	0.009	0.001	0.001	0.012	0.004	0.004	0.011	0.006	0.004	0.017	0.008	0.007	0.035	0.011	0.009	0.005	0.001	0.001
3	α	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
	β	0.002	0.001	0.000	0.006	0.003	0.003	0.007	0.004	0.004	0.010	0.006	0.006	0.013	0.007	0.006	0.002	0.001	0.000
5	α	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	β	0.001	0.001	0.001	0.002	0.001	0.001	0.003	0.001	0.001	0.008	0.002	0.002	0.029	0.002	0.002	0.001	0.000	0.000
10	α	0.000	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	β	0.001	0.001	0.002	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.001	0.000	0.008	0.001	0.000	0.001	0.001	0.001
30	α	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	β	0.000	0.001	0.002	0.000	0.000	0.000	0.001	0.000	0.000	0.034	0.000	0.000	0.121	0.000	0.000	0.000	0.001	0.001

Table 10: DPC loading dynamic parameters (continued)

		$\alpha = 0.02 - \beta = 0.94$			$\alpha = 0.02 - \beta = 0.95$			$\alpha = 0.02 - \beta = 0.97$			$\alpha = 0.03 - \beta = 0.90$			$\alpha = 0.03 - \beta = 0.94$			$\alpha = 0.03 - \beta = 0.95$		
N/T		500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500
Bias																			
2	α	0.040	0.040	0.041	0.040	0.040	0.040	0.039	0.040	0.039	0.033	0.033	0.034	0.034	0.033	0.033	0.031	0.032	0.032
	β	-0.068	-0.055	-0.053	-0.079	-0.064	-0.060	-0.096	-0.076	-0.074	-0.038	-0.023	-0.019	-0.071	-0.055	-0.051	-0.077	-0.059	-0.057
3	α	0.036	0.037	0.037	0.035	0.036	0.036	0.034	0.034	0.034	0.028	0.030	0.030	0.026	0.027	0.028	0.026	0.026	0.027
	β	-0.055	-0.048	-0.044	-0.063	-0.054	-0.051	-0.071	-0.060	-0.057	-0.020	-0.015	-0.012	-0.048	-0.040	-0.038	-0.054	-0.042	-0.041
5	α	0.017	0.018	0.018	0.016	0.017	0.018	0.014	0.014	0.015	0.012	0.014	0.014	0.011	0.011	0.012	0.010	0.011	0.011
	β	-0.025	-0.020	-0.016	-0.032	-0.024	-0.023	-0.039	-0.028	-0.025	0.003	0.010	0.012	-0.021	-0.013	-0.012	-0.023	-0.017	-0.015
10	α	0.007	0.008	0.008	0.006	0.007	0.007	0.003	0.004	0.005	0.002	0.003	0.003	-0.001	0.000	0.001	-0.001	0.000	0.000
	β	-0.009	0.001	0.004	-0.013	-0.004	-0.001	-0.019	-0.009	-0.007	0.019	0.028	0.032	-0.002	0.004	0.006	-0.007	0.001	0.002
30	α	-0.006	-0.004	-0.003	-0.006	-0.004	-0.004	-0.007	-0.005	-0.004	-0.010	-0.008	-0.008	-0.011	-0.009	-0.008	-0.011	-0.009	-0.008
	β	-0.009	0.005	0.009	-0.011	0.002	0.005	-0.013	-0.002	0.001	0.005	0.021	0.026	-0.005	0.006	0.008	-0.005	0.004	0.006
MSE																			
2	α	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	β	0.007	0.004	0.003	0.010	0.005	0.004	0.016	0.006	0.006	0.005	0.001	0.001	0.009	0.004	0.003	0.014	0.004	0.004
3	α	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	β	0.004	0.003	0.002	0.005	0.003	0.003	0.006	0.004	0.003	0.001	0.001	0.000	0.003	0.002	0.002	0.004	0.002	0.002
5	α	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	β	0.001	0.001	0.000	0.001	0.001	0.001	0.002	0.001	0.001	0.000	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000
10	α	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	β	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000
30	α	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	β	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000

Table 11: DPC loading dynamic parameters (continued)

		$\alpha = 0.05 - \beta = 0.90$			$\alpha = 0.05 - \beta = 0.94$			$\alpha = 0.09 - \beta = 0.90$		
N/T		500	1000	1500	500	1000	1500	500	1000	1500
Bias										
2	α	0.017	0.019	0.018	0.016	0.016	0.016	-0.010	-0.009	-0.009
	β	-0.034	-0.024	-0.020	-0.005	-0.035	-0.032	-0.009	-0.002	0.001
3	α	0.014	0.015	0.016	0.011	0.012	0.012	-0.009	-0.008	-0.008
	β	-0.018	-0.011	-0.010	-0.031	-0.024	-0.022	-0.004	0.000	0.002
5	α	0.003	0.003	0.004	0.000	0.000	0.001	-0.017	-0.016	-0.015
	β	0.002	0.008	0.009	-0.010	-0.005	-0.004	0.009	0.012	0.011
10	α	-0.011	-0.010	-0.009	-0.011	-0.010	-0.011	-0.033	-0.031	-0.031
	β	0.019	0.025	0.027	0.003	0.008	0.010	0.024	0.027	0.028
30	α	-0.020	-0.018	-0.017	-0.023	-0.021	-0.020	-0.049	-0.046	-0.045
	β	0.007	0.017	0.021	0.003	0.011	0.013	0.022	0.031	0.032
MSE										
2	α	0.001	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000
	β	0.005	0.001	0.001	0.007	0.002	0.001	0.001	0.000	0.000
3	α	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	β	0.001	0.000	0.000	0.001	0.001	0.001	0.000	0.000	0.000
5	α	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	β	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	α	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001
	β	0.001	0.001	0.001	0.000	0.000	0.000	0.001	0.001	0.001
30	α	0.000	0.000	0.000	0.001	0.000	0.000	0.002	0.002	0.002
	β	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001

Table 12: DPC first conditional component GARCH dynamic parameters

		$\alpha = 0.005 - \beta = 0.90$			$\alpha = 0.005 - \beta = 0.94$			$\alpha = 0.005 - \beta = 0.95$			$\alpha = 0.005 - \beta = 0.97$			$\alpha = 0.005 - \beta = 0.98$			$\alpha = 0.005 - \beta = 0.99$		
N/T		500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500
Bias																			
2	α	0.004	0.000	0.001	0.003	0.001	-0.001	0.005	0.001	0.000	0.006	0.002	0.000	0.001	0.001	-0.001	0.003	0.003	0.000
	β	-0.098	-0.024	-0.010	-0.120	-0.036	-0.009	-0.090	-0.030	-0.011	-0.123	-0.028	-0.014	-0.107	-0.027	-0.008	-0.098	-0.045	-0.009
3	α	0.006	0.002	0.000	0.004	0.000	0.000	0.003	0.000	0.000	0.004	0.003	0.000	0.003	0.000	0.000	0.003	0.001	0.000
	β	-0.119	-0.031	-0.012	-0.117	-0.033	-0.011	-0.123	-0.030	-0.010	-0.111	-0.028	-0.010	-0.118	-0.025	-0.010	-0.111	-0.028	-0.012
5	α	0.003	0.002	0.000	0.003	0.001	0.000	0.004	0.001	0.000	0.002	0.000	0.001	0.004	-0.001	0.000	0.004	0.002	0.001
	β	-0.085	-0.028	-0.008	-0.089	-0.027	-0.009	-0.110	-0.025	-0.012	-0.110	-0.021	-0.012	-0.087	-0.018	-0.011	-0.101	-0.021	-0.013
10	α	0.004	0.001	0.002	0.005	0.000	0.001	0.004	0.000	0.000	0.000	0.001	0.002	0.001	0.003	0.001	0.003	0.000	0.000
	β	-0.102	-0.021	-0.013	-0.110	-0.025	-0.012	-0.106	-0.031	-0.010	-0.119	-0.020	-0.013	-0.104	-0.022	-0.012	-0.094	-0.025	-0.010
30	α	0.002	0.002	-0.001	0.003	0.000	0.001	0.004	0.002	0.000	0.003	0.001	0.001	0.007	0.002	0.002	0.005	0.002	0.001
	β	-0.103	-0.031	-0.009	-0.110	-0.029	-0.012	-0.110	-0.033	-0.012	-0.108	-0.023	-0.011	-0.102	-0.027	-0.013	-0.115	-0.035	-0.012
MSE																			
2	α	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.001	0.000
	β	0.059	0.011	0.002	0.079	0.015	0.001	0.053	0.014	0.004	0.080	0.013	0.006	0.071	0.013	0.002	0.063	0.023	0.001
3	α	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000
	β	0.075	0.012	0.003	0.078	0.015	0.003	0.080	0.017	0.003	0.068	0.010	0.003	0.073	0.012	0.003	0.073	0.014	0.003
5	α	0.001	0.000	0.000	0.001	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000
	β	0.053	0.012	0.003	0.054	0.011	0.003	0.071	0.009	0.003	0.074	0.009	0.003	0.052	0.008	0.003	0.060	0.006	0.005
10	α	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000
	β	0.061	0.008	0.004	0.066	0.011	0.002	0.068	0.017	0.002	0.087	0.007	0.002	0.072	0.007	0.003	0.058	0.011	0.003
30	α	0.001	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.001	0.000	0.001	0.000	0.000
	β	0.066	0.014	0.002	0.069	0.015	0.003	0.070	0.017	0.003	0.071	0.008	0.001	0.062	0.012	0.002	0.071	0.016	0.002

Table 13: DPC first conditional component GARCH dynamic parameters (continued)

		$\alpha = 0.005 - \beta = 0.90$			$\alpha = 0.005 - \beta = 0.94$			$\alpha = 0.005 - \beta = 0.95$			$\alpha = 0.005 - \beta = 0.97$			$\alpha = 0.005 - \beta = 0.98$			$\alpha = 0.005 - \beta = 0.99$		
N/T		500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500
Bias																			
2	α	0.006	-0.001	-0.001	0.002	0.000	0.001	0.005	0.001	0.001	0.003	-0.001	0.000	0.006	0.000	0.001	0.002	0.001	0.000
	β	-0.117	-0.026	-0.005	-0.097	-0.025	-0.014	-0.106	-0.036	-0.009	-0.094	-0.020	-0.007	-0.114	-0.040	-0.014	-0.112	-0.022	-0.010
3	α	0.003	0.001	0.000	0.003	0.003	0.000	0.007	0.000	-0.001	0.004	0.001	0.000	0.004	0.001	-0.001	0.003	0.000	0.001
	β	-0.110	-0.027	-0.011	-0.123	-0.029	-0.008	-0.096	-0.024	-0.008	-0.116	-0.025	-0.011	-0.085	-0.024	-0.008	-0.101	-0.017	-0.012
5	α	0.004	0.000	0.000	0.002	-0.001	0.001	0.007	0.000	-0.001	0.002	0.001	-0.001	0.003	0.002	0.001	0.002	0.001	0.001
	β	-0.114	-0.024	-0.012	-0.114	-0.034	-0.013	-0.099	-0.017	-0.013	-0.099	-0.030	-0.007	-0.099	-0.029	-0.011	-0.110	-0.018	-0.010
10	α	0.003	0.001	0.001	0.003	0.002	0.001	0.004	0.001	0.000	0.005	0.003	0.000	0.004	0.002	0.000	0.004	0.000	0.001
	β	-0.096	-0.022	-0.010	-0.103	-0.033	-0.013	-0.101	-0.029	-0.012	-0.110	-0.031	-0.012	-0.089	-0.030	-0.009	-0.097	-0.030	-0.013
30	α	0.002	0.001	0.000	0.005	0.002	0.000	0.004	0.001	0.000	0.005	0.001	0.000	0.005	0.001	0.000	0.006	0.002	0.001
	β	-0.104	-0.026	-0.010	-0.113	-0.024	-0.011	-0.109	-0.029	-0.014	-0.108	-0.030	-0.014	-0.113	-0.022	-0.011	-0.113	-0.033	-0.014
MSE																			
2	α	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000
	β	0.075	0.014	0.001	0.062	0.011	0.002	0.065	0.019	0.002	0.056	0.010	0.002	0.071	0.022	0.004	0.074	0.007	0.002
3	α	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000
	β	0.071	0.013	0.003	0.080	0.012	0.001	0.058	0.012	0.001	0.076	0.010	0.003	0.049	0.010	0.002	0.063	0.005	0.002
5	α	0.001	0.000	0.000	0.001	0.000	0.000	0.003	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000
	β	0.073	0.012	0.003	0.073	0.022	0.002	0.060	0.005	0.006	0.062	0.014	0.003	0.063	0.013	0.002	0.072	0.005	0.002
10	α	0.001	0.000	0.000	0.001	0.001	0.000	0.001	0.000	0.000	0.001	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.000
	β	0.059	0.008	0.003	0.067	0.016	0.005	0.064	0.015	0.003	0.067	0.014	0.003	0.056	0.012	0.002	0.060	0.015	0.003
30	α	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000
	β	0.068	0.011	0.003	0.074	0.008	0.004	0.069	0.013	0.005	0.069	0.016	0.004	0.071	0.008	0.002	0.072	0.015	0.003

Table 14: DPC first conditional component GARCH dynamic parameters (continued)

		$\alpha = 0.005 - \beta = 0.90$			$\alpha = 0.005 - \beta = 0.94$			$\alpha = 0.005 - \beta = 0.95$			$\alpha = 0.005 - \beta = 0.97$			$\alpha = 0.005 - \beta = 0.98$			$\alpha = 0.005 - \beta = 0.99$		
N/T		500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500	500	1000	1500
Bias																			
2	α	0.003	0.000	0.001	0.005	0.001	-0.001	0.003	0.002	-0.001	0.002	0.000	0.000	0.002	0.002	-0.001	0.002	0.001	-0.001
	β	-0.114	-0.022	-0.012	-0.103	-0.031	-0.011	-0.104	-0.024	-0.009	-0.102	-0.019	-0.012	-0.107	-0.023	-0.010	-0.102	-0.025	-0.010
3	α	0.003	0.002	0.000	0.001	0.001	0.000	0.004	0.002	0.001	0.006	0.001	0.001	0.002	0.000	-0.001	0.003	-0.001	-0.001
	β	-0.101	-0.026	-0.007	-0.089	-0.025	-0.011	-0.094	-0.030	-0.014	-0.110	-0.019	-0.015	-0.094	-0.028	-0.011	-0.119	-0.022	-0.008
5	α	0.002	0.001	0.000	0.001	0.001	0.000	0.005	0.002	0.000	0.003	0.002	0.001	0.000	-0.001	0.000	0.004	-0.001	0.000
	β	-0.103	-0.025	-0.008	-0.109	-0.022	-0.014	-0.114	-0.024	-0.012	-0.111	-0.035	-0.010	-0.109	-0.032	-0.010	-0.110	-0.023	-0.012
10	α	0.003	0.001	0.000	0.002	0.001	-0.001	0.005	0.000	0.000	0.005	0.000	0.001	0.002	0.001	0.000	0.003	0.000	0.000
	β	-0.111	-0.034	-0.014	-0.095	-0.031	-0.008	-0.116	-0.021	-0.010	-0.107	-0.039	-0.015	-0.098	-0.025	-0.010	-0.111	-0.022	-0.011
30	α	0.002	0.003	0.001	0.005	0.001	-0.001	0.002	0.000	0.001	0.005	0.000	-0.001	0.002	0.002	0.000	0.003	0.001	0.000
	β	-0.107	-0.025	-0.010	-0.102	-0.033	-0.012	-0.100	-0.031	-0.015	-0.096	-0.028	-0.009	-0.104	-0.029	-0.014	-0.127	-0.029	-0.013
MSE																			
2	α	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.001	0.000
	β	0.077	0.007	0.002	0.063	0.014	0.006	0.064	0.010	0.003	0.067	0.005	0.004	0.070	0.008	0.003	0.067	0.009	0.003
3	α	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000
	β	0.065	0.009	0.001	0.057	0.011	0.002	0.057	0.012	0.004	0.072	0.005	0.005	0.059	0.012	0.003	0.077	0.011	0.001
5	α	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000
	β	0.065	0.009	0.002	0.069	0.009	0.005	0.075	0.008	0.004	0.073	0.017	0.002	0.071	0.018	0.003	0.073	0.010	0.002
10	α	0.001	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000
	β	0.073	0.018	0.004	0.061	0.015	0.001	0.076	0.010	0.001	0.067	0.020	0.006	0.064	0.010	0.001	0.073	0.008	0.002
30	α	0.001	0.000	0.000	0.001	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000
	β	0.070	0.008	0.003	0.065	0.016	0.004	0.067	0.015	0.003	0.053	0.013	0.002	0.067	0.011	0.003	0.084	0.011	0.005

Table 15: DPC first conditional component GARCH dynamic parameters (continued)

		$\alpha = 0.05 - \beta = 0.90$			$\alpha = 0.05 - \beta = 0.94$			$\alpha = 0.09 - \beta = 0.90$		
N/T		500	1000	1500	500	1000	1500	500	1000	1500
Bias										
2	α	0.001	0.000	0.000	0.003	-0.002	-0.002	-0.004	-0.006	-0.008
	β	-0.105	-0.022	-0.012	-0.107	-0.023	-0.012	-0.065	-0.005	0.006
3	α	0.001	0.001	0.001	0.001	-0.003	-0.003	-0.004	-0.010	-0.011
	β	-0.119	-0.020	-0.011	-0.076	-0.024	-0.005	-0.049	0.011	0.018
5	α	0.003	-0.001	-0.001	0.002	-0.002	-0.004	-0.007	-0.011	-0.011
	β	-0.098	-0.039	-0.008	-0.104	-0.025	-0.008	-0.048	0.011	0.019
10	α	0.002	0.001	0.001	-0.001	-0.003	-0.005	-0.009	-0.011	-0.012
	β	-0.110	-0.026	-0.014	-0.094	-0.013	-0.005	-0.028	0.022	0.029
30	α	0.002	0.001	-0.001	-0.001	-0.003	-0.005	-0.007	-0.015	-0.015
	β	-0.120	-0.032	-0.010	-0.087	-0.012	-0.002	0.012	0.042	0.045
MSE										
2	α	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000
	β	0.067	0.009	0.004	0.069	0.013	0.006	0.043	0.007	0.001
3	α	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.000	0.000
	β	0.079	0.005	0.002	0.044	0.012	0.001	0.034	0.004	0.001
5	α	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.001	0.000
	β	0.063	0.021	0.003	0.064	0.011	0.002	0.039	0.005	0.002
10	α	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.001	0.000
	β	0.069	0.009	0.004	0.060	0.005	0.002	0.029	0.002	0.002
30	α	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.001	0.001
	β	0.080	0.015	0.003	0.057	0.004	0.001	0.007	0.003	0.003

A.3 Pairwise comparisons: 10-dimensional dataset

Tables (16)-(25) report the pairwise comparison test statistics for the 10-dimensional dataset computed under the losses defined in Section 4.4. The test-statistics in boldface (italic) identify 5% significant cases in favour of the model in column (row). By construction the (i, j)-th entry of each table is equal to minus the (j, i)-th entry, where $i \neq j$. For a given a column in one table, if the number of test-statistics in boldface is greater (smaller) than the number of test-statistics in italic, there is evidence that, for the considered loss function, the pair model/estimator in column performs better than the other pairs model/estimator. See the legend in Section 4.2 for the model/estimator acronyms in the first row.

Table 16: 10-assets case: GR MSE Loss.

	OGARCH	SBEKK	DPC	DPC_r	DPC_f	DPC_s	DPC*	DPC^*_r	DPC^*_f	CCC	DCC	cDCC
OGARCH		-1.36	1.72	1.67	1.45	0.36	1.64	1.67	1.42	0.59	1.05	1.10
SBEKK	1.36		**2.22**	**2.20**	**2.16**	**2.85**	1.98	2.01	1.98	**3.38**	**3.51**	**3.50**
DPC	-1.72	*-2.22*		*-2.14*	*-2.16*	-1.03	-0.34	-0.35	-0.62	-0.30	0.20	0.27
DPC_r	-1.67	*-2.20*	**2.14**		-1.87	-0.99	-0.24	-0.24	-0.52	-0.27	0.24	0.31
DPC_f	-1.45	*-2.16*	**2.16**	1.87		-0.82	0.17	0.23	-0.11	-0.14	0.43	0.50
DPC_s	-0.36	*-2.85*	1.03	0.99	0.82		0.67	0.70	0.60	0.65	1.79	1.95
DPC*	-1.64	-1.98	0.34	0.24	-0.17	-0.67		0.26	-0.97	-0.18	0.29	0.35
DPC^*_r	-1.67	-2.01	0.35	0.24	-0.23	-0.70	-0.26		-1.11	-0.19	0.29	0.35
DPC^*_f	-1.42	-1.98	0.62	0.52	0.11	-0.60	0.97	1.11		-0.10	0.40	0.46
CCC	-0.59	*-3.38*	0.30	0.27	0.14	-0.65	0.18	0.19	0.10		1.69	1.62
DCC	-1.05	*-3.51*	-0.20	-0.24	-0.43	-1.79	-0.29	-0.29	-0.40	-1.69		1.06
cDCC	-1.10	*-3.50*	-0.27	-0.31	-0.50	-1.95	-0.35	-0.35	-0.46	-1.62	-1.06	

Table 17: 10-assets case: SR MSE Loss.

	OGARCH	SBEKK	DPC	DPC_r	DPC_f	DPC_s	DPC*	DPC^*_r	DPC^*_f	CCC	DCC	cDCC
OGARCH		-1.08	**2.53**	**2.21**	**2.77**	1.50	**2.12**	2.03	**2.69**	-0.44	-0.37	-0.37
SBEKK	1.08		1.72	1.65	**2.68**	1.97	1.63	1.59	**2.49**	1.00	1.02	1.02
DPC	*-2.53*	-1.72		-1.92	**2.48**	-0.73	0.31	-0.11	**2.32**	*-3.84*	*-3.76*	*-3.76*
DPC_r	*-2.21*	-1.65	1.92		**2.66**	-0.36	0.99	0.68	**2.48**	*-3.34*	*-3.25*	*-3.25*
DPC_f	*-2.77*	*-2.68*	*-2.48*	*-2.66*		*-2.76*	*-2.64*	*-2.70*	-0.40	*-4.42*	*-4.36*	*-4.36*
DPC_s	-1.50	-1.97	0.73	0.36	**2.76**		0.67	0.52	**2.47**	*-2.53*	*-2.40*	*-2.40*
DPC*	*-2.12*	-1.63	-0.31	-0.99	**2.64**	-0.67		-1.34	**2.59**	*-3.37*	*-3.32*	*-3.32*
DPC^*_r	-2.03	-1.59	0.11	-0.68	**2.70**	-0.52	1.34		**2.63**	*-3.13*	*-3.07*	*-3.07*
DPC^*_f	*-2.69*	*-2.49*	*-2.32*	*-2.48*	0.40	*-2.47*	*-2.59*	*-2.63*		*-4.07*	*-4.03*	*-4.03*
CCC	0.44	-1.00	**3.84**	**3.34**	**4.42**	**2.53**	**3.37**	**3.13**	**4.07**		**3.59**	**3.50**
DCC	0.37	-1.02	**3.76**	**3.25**	**4.36**	**2.40**	**3.32**	**3.07**	**4.03**	*-3.59*		1.68
cDCC	0.37	-1.02	**3.76**	**3.25**	**4.36**	**2.40**	**3.32**	**3.07**	**4.03**	*-3.50*	-1.68	

Table 18: 10-assets case: CR MSE Loss.

	OGARCH	SBEKK	DPC	DPC_r	DPC_f	DPC_s	DPC*	DPC^*_r	DPC^*_f	CCC	DCC	cDCC
OGARCH		*-2.36*	-1.46	-1.46	-1.58	-1.60	-1.31	-1.31	-1.38	-0.69	-0.74	-0.75
SBEKK	**2.36**		**2.06**	**2.06**	2.00	**2.73**	1.87	1.90	1.87	1.47	1.71	1.77
DPC	1.46	*-2.06*		0.02	*-2.92*	-0.88	-0.21	-0.17	-0.33	-0.27	-0.25	-0.23
DPC_r	1.46	*-2.06*	-0.02		*-3.12*	-0.88	-0.21	-0.17	-0.33	-0.27	-0.25	-0.23
DPC_f	1.58	-2.00	**2.92**	**3.12**		-0.75	0.08	0.16	-0.02	-0.21	-0.18	-0.16
DPC_s	1.60	*-2.73*	0.88	0.88	0.75		0.61	0.65	0.59	0.10	0.22	0.26
DPC*	1.31	-1.87	0.21	0.21	-0.08	-0.61		0.54	-0.42	-0.23	-0.20	-0.18
DPC^*_r	1.31	-1.90	0.17	0.17	-0.16	-0.65	-0.54		-0.81	-0.25	-0.21	-0.20
DPC^*_f	1.38	-1.87	0.33	0.33	0.02	-0.59	0.42	0.81		-0.22	-0.18	-0.16
CCC	0.69	-1.47	0.27	0.27	0.21	-0.10	0.23	0.25	0.22		0.29	0.31
DCC	0.74	-1.71	0.25	0.25	0.18	-0.22	0.20	0.21	0.18	-0.29		0.38
cDCC	0.75	-1.77	0.23	0.23	0.16	-0.26	0.18	0.20	0.16	-0.31	-0.38	

Table 19: 10-assets case: PDen Loss.

	OGARCH	SBEKK	DPC	DPC$_r$	DPC$_f$	DPC$_s$	DPC*	DPC$^*_r$	DPC$^*_f$	CCC	DCC	cDCC
OGARCH		0.90	**5.71**	**5.93**	*-3.85*	**5.38**	**5.91**	**5.98**	*-3.88*	**3.00**	**4.39**	**4.53**
SBEKK	-0.90		**3.16**	**3.43**	*-4.45*	**2.97**	**3.21**	**3.32**	*-4.49*	1.74	**2.77**	**2.86**
DPC	*-5.71*	*-3.16*		1.14	*-4.54*	-0.01	1.17	1.29	*-4.58*	-0.33	1.34	1.47
DPC$_r$	*-5.93*	*-3.43*	-1.14		*-4.57*	-1.06	0.08	0.55	*-4.61*	-0.58	1.10	1.24
DPC$_f$	**3.85**	**4.45**	**4.54**	**4.57**		**4.42**	**4.52**	**4.53**	0.70	**4.06**	**4.34**	**4.36**
DPC$_s$	*-5.38*	*-2.97*	0.01	1.06	*-4.42*		1.18	1.95	*-4.45*	-0.35	1.48	1.63
DPC*	*-5.91*	*-3.21*	-1.17	-0.08	*-4.52*	-1.18		0.49	*-4.56*	-0.61	1.12	1.26
DPC$^*_r$	*-5.98*	*-3.32*	-1.29	-0.55	*-4.53*	-1.95	-0.49		*-4.57*	-0.69	1.07	1.21
DPC$^*_f$	**3.88**	**4.49**	**4.58**	**4.61**	-0.70	**4.45**	**4.56**	**4.57**		**4.09**	**4.38**	**4.40**
CCC	*-3.00*	-1.74	0.33	0.58	*-4.06*	0.35	0.61	0.69	*-4.09*		**5.55**	**5.34**
DCC	*-4.39*	*-2.77*	-1.34	-1.10	*-4.34*	-1.48	-1.12	-1.07	*-4.38*	*-5.55*		1.86
cDCC	*-4.53*	*-2.86*	-1.47	-1.24	*-4.36*	-1.63	-1.26	-1.21	*-4.40*	*-5.34*	-1.86	

Table 20: 10-assets case: EQW MSE Loss.

	OGARCH	SBEKK	DPC	DPC$_r$	DPC$_f$	DPC$_s$	DPC*	DPC$^*_r$	DPC$^*_f$	CCC	DCC	cDCC
OGARCH		-1.49	0.32	0.31	0.31	-0.60	0.28	0.30	0.39	-0.41	-0.07	-0.02
SBEKK	1.49		1.91	1.91	1.92	**2.36**	1.83	1.84	1.86	**2.22**	**2.75**	**2.79**
DPC	-0.32	-1.91		*-2.63*	-0.40	-1.12	-0.01	0.03	0.20	-0.61	-0.23	-0.17
DPC$_r$	-0.31	-1.91	**2.63**		0.35	-1.11	0.01	0.06	0.22	-0.61	-0.22	-0.16
DPC$_f$	-0.31	-1.92	0.40	-0.35		-1.12		0.05	0.21	-0.61	-0.22	-0.17
DPC$_s$	0.60	*-2.36*	1.12	1.11	1.12		0.92	0.95	0.99	-0.07	0.70	0.84
DPC*	-0.28	-1.83	0.01	-0.01		-0.92		0.38	1.12	-0.62	-0.23	-0.17
DPC$^*_r$	-0.30	-1.84	-0.03	-0.06	-0.05	-0.95	-0.38		0.99	-0.63	-0.24	-0.18
DPC$^*_f$	-0.39	-1.86	-0.20	-0.22	-0.21	-0.99	-1.12	-0.99		-0.66	-0.28	-0.22
CCC	0.41	*-2.22*	0.61	0.61	0.61	0.07	0.62	0.63	0.66		1.57	1.52
DCC	0.07	*-2.75*	0.23	0.22	0.22	-0.70	0.23	0.24	0.28	-1.57		1.14
cDCC	0.02	*-2.79*	0.17	0.16	0.17	-0.84	0.17	0.18	0.22	-1.52	-1.14	

Table 21: 10-assets case: MMV MSE Loss.

	OGARCH	SBEKK	DPC	DPC$_r$	DPC$_f$	DPC$_s$	DPC*	DPC$^*_r$	DPC$^*_f$	CCC	DCC	cDCC
OGARCH		1.68	**2.13**	**2.04**	1.15	**2.10**	**2.08**	1.97	1.13	-0.95	-0.31	-0.12
SBEKK	-1.68		-0.94	-0.87	-1.91	-0.95	-0.87	-0.82	-1.87	-1.28	-1.26	-1.25
DPC	*-2.13*	0.94		1.10	-0.16	0.41	1.11	1.11	-0.17	-1.34	-1.35	-1.33
DPC$_r$	*-2.04*	0.87	-1.10		-0.51	-1.18	-0.51	1.08	-0.50	-1.34	-1.35	-1.34
DPC$_f$	-1.15	1.91	0.16	0.51		0.23	0.45	0.75	-0.22	-1.09	-0.93	-0.89
DPC$_s$	*-2.10*	0.95	-0.41	1.18	-0.23		1.26	1.21	-0.24	-1.34	-1.36	-1.34
DPC*	*-2.08*	0.87	-1.11	0.51	-0.45	-1.26		0.98	-0.45	-1.35	-1.36	-1.35
DPC$^*_r$	-1.97	0.82	-1.11	-1.08	-0.75	-1.21	-0.98		-0.74	-1.34	-1.35	-1.33
DPC$^*_f$	-1.13	1.87	0.17	0.50	0.22	0.24	0.45	0.74		-1.09	-0.92	-0.88
CCC	0.95	1.28	1.34	1.34	1.09	1.34	1.35	1.34	1.09		1.32	1.33
DCC	0.31	1.26	1.35	1.35	0.93	1.36	1.36	1.35	0.92	-1.32		1.46
cDCC	0.12	1.25	1.33	1.34	0.89	1.34	1.35	1.33	0.88	-1.33	-1.46	

Table 22: 10-assets case: HDG MSE Loss.

	OGARCH	SBEKK	DPC	DPC$_r$	DPC$_f$	DPC$_s$	DPC*	DPC$^*_r$	DPC$^*_f$	CCC	DCC	cDCC
OGARCH		1.46	1.30	1.41	0.90	1.42	1.37	1.41	0.91	0.67	1.14	1.22
SBEKK	-1.46		-1.97	-0.51	*-2.57*	-0.41	-1.16	-0.59	*-2.56*	*-2.60*	-1.63	-1.47
DPC	-1.30	1.97		**2.29**	-1.73	**2.39**	1.77	**2.43**	-1.72	*-2.18*	-1.07	-0.87
DPC$_r$	-1.41	0.51	*-2.29*		*-2.23*	0.13	-1.05	-0.18	*-2.21*	*-2.37*	-1.43	-1.26
DPC$_f$	-0.90	**2.57**	1.73	**2.23**		**2.10**	1.96	**2.27**	0.14	-0.80	0.29	0.47
DPC$_s$	-1.42	0.41	*-2.39*	-0.13	*-2.10*		-1.54	-0.23	*-2.09*	*-2.65*	-1.70	-1.51
DPC*	-1.37	1.16	-1.77	1.05	-1.96	1.54		1.32	-1.95	*-2.45*	-1.40	-1.20
DPC$^*_r$	-1.41	0.59	*-2.43*	0.18	*-2.27*	0.23	-1.32		*-2.26*	*-2.41*	-1.45	-1.28
DPC$^*_f$	-0.91	**2.56**	1.72	**2.21**	-0.14	**2.09**	1.95	**2.26**		-0.82	0.29	0.47
CCC	-0.67	**2.60**	**2.18**	**2.37**	0.80	**2.65**	**2.45**	**2.41**	0.82		**3.27**	**3.36**
DCC	-1.14	1.63	1.07	1.43	-0.29	1.70	1.40	1.45	-0.29	*-3.27*		**2.93**
cDCC	-1.22	1.47	0.87	1.26	-0.47	1.51	1.20	1.28	-0.47	*-3.36*	*-2.93*	

Table 23: 10-assets case: EQW PDen Loss.

	OGARCH	SBEKK	DPC	DPC_r	DPC_f	DPC_s	DPC*	DPC^*_r	DPC^*_f	CCC	DCC	cDCC
OGARCH		*-4.01*	0.52	0.50	0.57	-1.50	0.52	0.58	0.71	-1.02	-0.38	-0.32
SBEKK	**4.01**		**4.14**	**4.14**	**4.17**	**4.80**	**3.94**	**3.94**	**4.07**	**5.05**	**5.21**	**5.24**
DPC	-0.52	*-4.14*		-1.62	1.25	*-2.16*	0.42	0.55	0.91	-1.29	-0.74	-0.68
DPC_r	-0.50	*-4.14*	1.62		1.52	*-2.15*	0.44	0.57	0.93	-1.28	-0.73	-0.67
DPC_f	-0.57	*-4.17*	-1.25	-1.52		*-2.24*	0.34	0.46	0.82	-1.33	-0.79	-0.73
DPC_s	1.50	*-4.80*	**2.16**	**2.15**	**2.24**		**2.07**	**2.06**	**2.27**	0.49	**2.45**	**2.67**
DPC*	-0.52	*-3.94*	-0.42	-0.44	-0.34	*-2.07*		0.32	0.65	-1.32	-0.83	-0.78
DPC^*_r	-0.58	*-3.94*	-0.55	-0.57	-0.46	*-2.06*	-0.32		0.48	-1.32	-0.84	-0.79
DPC^*_f	-0.71	*-4.07*	-0.91	-0.93	-0.82	*-2.27*	-0.65	-0.48		-1.44	-0.97	-0.92
CCC	1.02	*-5.05*	1.29	1.28	1.33	-0.49	1.32	1.32	1.44		**2.46**	**2.46**
DCC	0.38	*-5.21*	0.74	0.73	0.79	*-2.45*	0.83	0.84	0.97	*-2.46*		1.07
cDCC	0.32	*-5.24*	0.68	0.67	0.73	*-2.67*	0.78	0.79	0.92	*-2.46*	-1.07	

Table 24: 10-assets case: MMV PDen Loss.

	OGARCH	SBEKK	DPC	DPC_r	DPC_f	DPC_s	DPC*	DPC^*_r	DPC^*_f	CCC	DCC	cDCC
OGARCH		1.44	**3.02**	**3.53**	-1.41	**3.05**	**3.38**	**3.67**	-1.37	1.28	**2.20**	**2.22**
SBEKK	-1.44		1.58	**2.14**	*-2.71*	2.01	1.92	**2.23**	*-2.68*	0.08	1.25	1.26
DPC	*-3.02*	-1.58		**2.04**	*-3.01*	0.16	1.34	1.92	*-2.98*	-1.15	-0.21	-0.21
DPC_r	*-3.53*	*-2.14*	*-2.04*		*-3.12*	-1.21	-0.56	0.99	*-3.10*	-1.54	-0.69	-0.68
DPC_f	1.41	**2.71**	**3.01**	**3.12**		**2.94**	**3.04**	**3.11**	0.85	**2.52**	**2.90**	**2.88**
DPC_s	*-3.05*	-2.01	-0.16	1.21	*-2.94*		0.80	1.78	*-2.93*	-1.26	-0.30	-0.29
DPC*	*-3.38*	-1.92	-1.34	0.56	*-3.04*	-0.80		1.78	*-3.03*	-1.44	-0.58	-0.57
DPC^*_r	*-3.67*	*-2.23*	-1.92	-0.99	*-3.11*	-1.78	-1.78		*-3.10*	-1.69	-0.87	-0.87
DPC^*_f	1.37	**2.68**	**2.98**	**3.10**	-0.85	**2.93**	**3.03**	**3.10**		**2.50**	**2.88**	**2.87**
CCC	-1.28	-0.08	1.15	1.54	*-2.52*	1.26	1.44	1.69	*-2.50*		**3.38**	**3.22**
DCC	*-2.20*	-1.25	0.21	0.69	*-2.90*	0.30	0.58	0.87	*-2.88*	*-3.38*		0.15
cDCC	*-2.22*	-1.26	0.21	0.68	*-2.88*	0.29	0.57	0.87	*-2.87*	*-3.22*	-0.15	

Table 25: 10-assets case: HDG PDen Loss.

	OGARCH	SBEKK	DPC	DPC_r	DPC_f	DPC_s	DPC*	DPC^*_r	DPC^*_f	CCC	DCC	cDCC
OGARCH		1.86	1.13	**2.06**	*-3.24*	**2.59**	2.03	**2.26**	*-3.15*	0.49	1.95	**2.07**
SBEKK	-1.86		-1.14	-0.15	*-3.90*	0.87	0.02	0.29	*-3.82*	-1.34	0.24	0.35
DPC	-1.13	1.14		**3.26**	*-3.41*	**3.80**	**3.26**	**3.49**	*-3.32*	-0.45	1.27	1.41
DPC_r	*-2.06*	0.15	*-3.26*		*-3.53*	**2.68**	0.68	1.87	*-3.45*	-1.32	0.40	0.53
DPC_f	**3.24**	**3.90**	**3.41**	**3.53**		**3.68**	**3.55**	**3.61**	1.38	**3.04**	**3.44**	**3.47**
DPC_s	*-2.59*	-0.87	*-3.80*	*-2.68*	*-3.68*		*-2.73*	*-2.10*	*-3.60*	*-2.31*	-0.64	-0.51
DPC*	-2.03	-0.02	*-3.26*	-0.68	*-3.55*	**2.73**		1.74	*-3.47*	-1.46	0.23	0.36
DPC^*_r	*-2.26*	-0.29	*-3.49*	-1.87	*-3.61*	**2.10**	-1.74		*-3.53*	-1.69	-0.03	0.09
DPC^*_f	**3.15**	**3.82**	**3.32**	**3.45**	-1.38	**3.60**	**3.47**	**3.53**		**2.96**	**3.37**	**3.39**
CCC	-0.49	1.34	0.45	1.32	*-3.04*	**2.31**	1.46	1.69	*-2.96*		**5.04**	**5.26**
DCC	-1.95	-0.24	-1.27	-0.40	*-3.44*	0.64	-0.23	0.03	*-3.37*	*-5.04*		**2.45**
cDCC	*-2.07*	-0.35	-1.41	-0.53	*-3.47*	0.51	-0.36	-0.09	*-3.39*	*-5.26*	*-2.45*	

A.4 Pairwise comparisons: 30-dimensional dataset

Tables (26)-(35) report the pairwise comparison test statistics for the 10-dimensional dataset computed under the losses defined in Section 4.4. The test-statistics in boldface (italic) identify 5% significant cases in favour of the model in column (row). By construction the (i, j)-th entry of each table is equal to minus the (j, i)-th entry, where $i \neq j$. For a given a column in one table, if the number of test-statistics in boldface is greater (smaller) than the number of test-statistics in italic, there is evidence that, for the considered loss function, the pair model/estimator in column performs better than the other pairs model/estimator. See the legend in Section 4.2 for the model/estimator acronyms in the first row.

Table 26: 30-assets case: GR MSE Loss.

	OGARCH	SBEKK	DPC	DPC_r	DPC_f	DPC_s	DPC*	DPC^*_r	DPC^*_f	CCC	DCC	cDCC
OGARCH		-1.96	**2.21**	**2.22**	1.98	-0.54	1.04	1.23	1.91	-0.70	-0.45	-0.42
SBEKK	1.96		**2.28**	**2.28**	**2.27**	**2.59**	**2.33**	**2.33**	**2.30**	**3.59**	**3.44**	**3.43**
DPC	*-2.21*	*-2.28*		-0.23	*-2.60*	-1.40	-0.28	-0.17	-1.06	-1.21	-1.01	-0.99
DPC_r	*-2.22*	*-2.28*	0.23		*-2.68*	-1.40	-0.28	-0.17	-1.06	-1.21	-1.01	-0.99
DPC_f	-1.98	*-2.27*	**2.60**	**2.68**		-1.34	-0.10	0.03	-0.17	-1.17	-0.97	-0.95
DPC_s	0.54	*-2.59*	1.40	1.40	1.34		1.14	1.21	1.36	-0.75	-0.23	-0.17
DPC*	-1.04	*-2.33*	0.28	0.28	0.10	-1.14		0.94	0.08	-1.19	-0.97	-0.95
DPC^*_r	-1.23	*-2.33*	0.17	0.17	-0.03	-1.21	-0.94		-0.07	-1.22	-1.01	-0.98
DPC^*_f	-1.91	*-2.30*	1.06	1.06	0.17	-1.36	-0.08	0.07		-1.19	-0.98	-0.96
CCC	0.70	*-3.59*	1.21	1.21	1.17	0.75	1.19	1.22	1.19		**2.40**	**2.41**
DCC	0.45	*-3.44*	1.01	1.01	0.97	0.23	0.97	1.01	0.98	*-2.40*		**2.37**
cDCC	0.42	*-3.43*	0.99	0.99	0.95	0.17	0.95	0.98	0.96	*-2.41*	*-2.37*	

Table 27: 30-assets case: SR MSE Loss.

	OGARCH	SBEKK	DPC	DPC_r	DPC_f	DPC_s	DPC*	DPC^*_r	DPC^*_f	CCC	DCC	cDCC
OGARCH		-1.45	1.83	**2.15**	**3.33**	-0.70	1.93	**2.33**	**3.23**	*-2.30*	*-2.26*	*-2.26*
SBEKK	1.45		1.57	1.58	**2.10**	1.55	1.57	1.60	**2.08**	1.25	1.26	1.26
DPC	-1.83	-1.57		1.46	**3.37**	-1.28	0.86	1.64	**3.24**	*-3.31*	*-3.29*	*-3.29*
DPC_r	*-2.15*	-1.58	-1.46		**3.30**	-1.37	0.42	1.23	**3.17**	*-3.35*	*-3.33*	*-3.33*
DPC_f	*-3.33*	*-2.10*	*-3.37*	*-3.30*		*-3.52*	*-3.24*	*-3.14*	*-3.31*	*-5.25*	*-5.23*	*-5.23*
DPC_s	0.70	-1.55	1.28	1.37	**3.52**		1.26	1.39	**3.50**	*-2.43*	*-2.34*	*-2.34*
DPC*	-1.93	-1.57	-0.86	-0.42	**3.24**	-1.26		**2.98**	**3.08**	*-3.10*	*-3.08*	*-3.08*
DPC^*_r	*-2.33*	-1.60	-1.64	-1.23	**3.14**	-1.39	*-2.98*		**2.98**	*-3.25*	*-3.22*	*-3.23*
DPC^*_f	*-3.23*	*-2.08*	*-3.24*	*-3.17*	**3.31**	*-3.50*	*-3.08*	*-2.98*		*-5.23*	*-5.20*	*-5.21*
CCC	**2.30**	-1.25	**3.31**	**3.35**	**5.25**	**2.43**	**3.10**	**3.25**	**5.23**		**4.18**	**3.99**
DCC	**2.26**	-1.26	**3.29**	**3.33**	**5.23**	**2.34**	**3.08**	**3.22**	**5.20**	*-4.18*		1.15
cDCC	**2.26**	-1.26	**3.29**	**3.33**	**5.23**	**2.34**	**3.08**	**3.23**	**5.21**	*-3.99*	-1.15	

Table 28: 30-assets case: CR MSE Loss.

	OGARCH	SBEKK	DPC	DPC_r	DPC_f	DPC_s	DPC*	DPC^*_r	DPC^*_f	CCC	DCC	cDCC
OGARCH		*-2.38*	-1.14	-1.13	-1.26	-1.72	-0.88	-0.81	-1.25	-0.54	-0.67	-0.67
SBEKK	**2.38**		**2.19**	**2.19**	**2.18**	**2.49**	**2.23**	**2.24**	**2.20**	**2.06**	**2.16**	**2.16**
DPC	1.14	*-2.19*		1.87	*-4.40*	-1.34	-0.26	-0.14	-0.67	-0.32	-0.43	-0.43
DPC_r	1.13	*-2.19*	-1.87		*-4.61*	-1.34	-0.27	-0.15	-0.72	-0.33	-0.44	-0.43
DPC_f	1.26	*-2.18*	**4.40**	**4.61**		-1.30	-0.17	-0.04	-0.12	-0.29	-0.40	-0.39
DPC_s	1.72	*-2.49*	1.34	1.34	1.30		1.09	1.16	1.32	0.49	0.50	0.51
DPC*	0.88	*-2.23*	0.26	0.27	0.17	-1.09		0.98	0.18	-0.27	-0.38	-0.38
DPC^*_r	0.81	*-2.24*	0.14	0.15	0.04	-1.16	-0.98		0.03	-0.32	-0.44	-0.44
DPC^*_f	1.25	*-2.20*	0.67	0.72	0.12	-1.32	-0.18	-0.03		-0.29	-0.40	-0.40
CCC	0.54	*-2.06*	0.32	0.33	0.29	-0.49	0.27	0.32	0.29		-0.39	-0.31
DCC	0.67	*-2.16*	0.43	0.44	0.40	-0.50	0.38	0.44	0.40	0.39		0.74
cDCC	0.67	*-2.16*	0.43	0.43	0.39	-0.51	0.38	0.44	0.40	0.31	-0.74	

Table 29: 30-assets case: PDen Loss.

	OGARCH	SBEKK	DPC	DPC$_r$	DPC$_f$	DPC$_s$	DPC*	DPC$^*_r$	DPC$^*_f$	CCC	DCC	cDCC
OGARCH		-1.52	**6.22**	**6.35**	*-5.02*	**5.99**	**6.46**	**6.48**	*-5.00*	**3.02**	**3.73**	**3.79**
SBEKK	1.52		**4.07**	**4.15**	*-6.36*	**4.08**	**4.21**	**4.27**	*-6.34*	**2.86**	**3.31**	**3.34**
DPC	*-6.22*	*-4.07*		**3.37**	*-5.89*	**2.33**	**2.68**	**3.79**	*-5.87*	-0.03	1.06	1.15
DPC$_r$	*-6.35*	*-4.15*	*-3.37*		*-5.87*	-0.20	-1.09	0.91	*-5.85*	-0.71	0.43	0.52
DPC$_f$	**5.02**	**6.36**	**5.89**	**5.87**		**5.73**	**5.88**	**5.89**	**2.52**	**4.98**	**5.22**	**5.24**
DPC$_s$	*-5.99*	*-4.08*	*-2.33*	0.20	*-5.73*		-0.70	1.12	*-5.72*	-0.67	0.47	0.56
DPC*	*-6.46*	*-4.21*	*-2.68*	1.09	*-5.88*	0.70		**2.54**	*-5.86*	-0.51	0.57	0.65
DPC$^*_r$	*-6.48*	*-4.27*	*-3.79*	-0.91	*-5.89*	-1.12	*-2.54*		*-5.87*	-0.81	0.32	0.41
DPC$^*_f$	**5.00**	**6.34**	**5.87**	**5.85**	*-2.52*	**5.72**	**5.86**	**5.87**		**4.97**	**5.21**	**5.23**
CCC	*-3.02*	*-2.86*	0.03	0.71	*-4.98*	0.67	0.51	0.81	*-4.97*		**7.61**	**7.55**
DCC	*-3.73*	*-3.31*	-1.06	-0.43	*-5.22*	-0.47	-0.57	-0.32	*-5.21*	*-7.61*		**3.36**
cDCC	*-3.79*	*-3.34*	-1.15	-0.52	*-5.24*	-0.56	-0.65	-0.41	*-5.23*	*-7.55*	*-3.36*	

Table 30: 30-assets case: EQW MSE Loss.

	OGARCH	SBEKK	DPC	DPC$_r$	DPC$_f$	DPC$_s$	DPC*	DPC$^*_r$	DPC$^*_f$	CCC	DCC	cDCC
OGARCH		-1.84	0.05	0.05	0.05	-1.11	0.04	0.16	0.08	-1.32	-1.18	-1.17
SBEKK	1.84		1.96	1.96	1.97	**2.21**	2.01	2.02	1.99	**2.64**	**2.62**	**2.63**
DPC	-0.05	-1.96		0.93	-0.09	-1.34	0.02	0.15	0.15	-1.45	-1.32	-1.31
DPC$_r$	-0.05	-1.96	-0.93		-0.24	-1.34	0.02	0.15	0.14	-1.45	-1.32	-1.31
DPC$_f$	-0.05	-1.97	0.09	0.24		-1.34	0.02	0.16	0.16	-1.45	-1.32	-1.31
DPC$_s$	1.11	*-2.21*	1.34	1.34	1.34		1.20	1.26	1.38	-1.34	-1.03	-0.99
DPC*	-0.04	-2.01	-0.02	-0.02	-0.02	-1.20		0.76		-1.52	-1.39	-1.37
DPC$^*_r$	-0.16	-2.02	-0.15	-0.15	-0.16	-1.26	-0.76		-0.15	-1.54	-1.42	-1.40
DPC$^*_f$	-0.08	-1.99	-0.15	-0.14	-0.16	-1.38		0.15		-1.49	-1.36	-1.35
CCC	1.32	*-2.64*	1.45	1.45	1.45	1.34	1.52	1.54	1.49		**2.32**	**2.33**
DCC	1.18	*-2.62*	1.32	1.32	1.32	1.03	1.39	1.42	1.36	*-2.32*		**2.36**
cDCC	1.17	*-2.63*	1.31	1.31	1.31	0.99	1.37	1.40	1.35	*-2.33*	*-2.36*	

Table 31: 30-assets case: MMV MSE Loss.

	OGARCH	SBEKK	DPC	DPC$_r$	DPC$_f$	DPC$_s$	DPC*	DPC$^*_r$	DPC$^*_f$	CCC	DCC	cDCC
OGARCH		0.81	1.16	1.79	0.60	1.12	1.65	1.80	0.59	-1.28	-1.23	-1.22
SBEKK	-0.81		0.26	0.51	0.02	0.43	0.86	1.04	0.01	-1.45	-1.45	-1.45
DPC	-1.16	-0.26		0.61	-0.10	0.03	1.82	1.65	-0.11	-1.41	-1.40	-1.40
DPC$_r$	-1.79	-0.51	-0.61		-0.23	-0.32	1.04	1.42	-0.23	-1.42	-1.42	-1.42
DPC$_f$	-0.60	-0.02	0.10	0.23		0.09	0.38	0.44	-0.77	-1.21	-1.15	-1.15
DPC$_s$	-1.12	-0.43	-0.03	0.32	-0.09		0.94	1.05	-0.09	-1.47	-1.49	-1.48
DPC*	-1.65	-0.86	-1.82	-1.04	-0.38	-0.94		0.60	-0.39	-1.45	-1.45	-1.45
DPC$^*_r$	-1.80	-1.04	-1.65	-1.42	-0.44	-1.05	-0.60		-0.45	-1.45	-1.45	-1.45
DPC$^*_f$	-0.59	-0.01	0.11	0.23	0.77	0.09	0.39	0.45		-1.21	-1.15	-1.14
CCC	1.28	1.45	1.41	1.42	1.21	1.47	1.45	1.45	1.21		1.41	1.42
DCC	1.23	1.45	1.40	1.42	1.15	1.49	1.45	1.45	1.15	-1.41		1.55
cDCC	1.22	1.45	1.40	1.42	1.15	1.48	1.45	1.45	1.14	-1.42	-1.55	

Table 32: 30-assets case: HDG MSE Loss.

	OGARCH	SBEKK	DPC	DPC$_r$	DPC$_f$	DPC$_s$	DPC*	DPC$^*_r$	DPC$^*_f$	CCC	DCC	cDCC
OGARCH		0.85	1.06	1.36	0.73	1.08	1.28	1.31	0.69	-0.30	-0.16	-0.12
SBEKK	-0.85		0.45	1.52	-0.34	0.68	0.60	0.91	-0.40	-0.92	-0.74	-0.70
DPC	-1.06	-0.45		1.41	-0.55	0.18	0.38	0.84	-0.60	-0.94	-0.76	-0.72
DPC$_r$	-1.36	-1.52	-1.41		-1.49	-1.13	-1.17	-0.78	-1.55	-1.10	-0.93	-0.89
DPC$_f$	-0.73	0.34	0.55	1.49		0.60	0.78	0.91	-1.13	-0.69	-0.53	-0.49
DPC$_s$	-1.08	-0.68	-0.18	1.13	-0.60		0.17	0.68	-0.66	-0.99	-0.81	-0.77
DPC*	-1.28	-0.60	-0.38	1.17	-0.78	-0.17		1.26	-0.84	-0.92	-0.76	-0.72
DPC$^*_r$	-1.31	-0.91	-0.84	0.78	-0.91	-0.68	-1.26		-0.96	-0.98	-0.82	-0.78
DPC$^*_f$	-0.69	0.40	0.60	1.55	1.13	0.66	0.84	0.96		-0.67	-0.51	-0.47
CCC	0.30	0.92	0.94	1.10	0.69	0.99	0.92	0.98	0.67		**2.16**	**2.42**
DCC	0.16	0.74	0.76	0.93	0.53	0.81	0.76	0.82	0.51	*-2.16*		**2.64**
cDCC	0.12	0.70	0.72	0.89	0.49	0.77	0.72	0.78	0.47	*-2.42*	*-2.64*	

Table 33: 30-assets case: EQW PDen Loss.

	OGARCH	SBEKK	DPC	DPC$_r$	DPC$_f$	DPC$_s$	DPC*	DPC$^*_r$	DPC$^*_f$	CCC	DCC	cDCC
OGARCH		*-4.59*	1.15	1.14	1.14	*-2.14*	1.15	1.51	1.50	*-2.08*	-1.76	-1.75
SBEKK	**4.59**		**4.68**	**4.68**	**4.69**	**5.39**	**4.69**	**4.75**	**4.70**	**5.75**	**5.76**	**5.77**
DPC	-1.15	*-4.68*		-0.51	0.37	*-2.31*	0.90	1.35	0.36	*-2.20*	-1.89	-1.89
DPC$_r$	-1.14	*-4.68*	0.51		0.44	*-2.31*	0.90	1.36	0.36	*-2.20*	-1.89	-1.89
DPC$_f$	-1.14	*-4.69*	-0.37	-0.44		*-2.33*	0.89	1.36	0.29	*-2.21*	-1.90	-1.90
DPC$_s$	**2.14**	*-5.39*	**2.31**	**2.31**	**2.33**		**2.34**	**2.52**	**2.34**	-0.98	0.18	0.21
DPC*	-1.15	*-4.69*	-0.90	-0.90	-0.89	*-2.34*		1.10	-0.76	*-2.28*	-1.99	-1.99
DPC$^*_r$	-1.51	*-4.75*	-1.35	-1.36	-1.36	*-2.52*	-1.10		-1.19	*-2.40*	*-2.12*	*-2.13*
DPC$^*_f$	-1.50	*-4.70*	-0.36	-0.36	-0.29	*-2.34*	0.76	1.19		*-2.23*	-1.93	-1.93
CCC	**2.08**	*-5.75*	**2.20**	**2.20**	**2.21**	0.98	**2.28**	**2.40**	**2.23**		**3.92**	**3.68**
DCC	1.76	*-5.76*	1.89	1.89	1.90	-0.18	1.99	**2.12**	1.93	*-3.92*		0.53
cDCC	1.75	*-5.77*	1.89	1.89	1.90	-0.21	1.99	**2.13**	1.93	*-3.68*	-0.53	

Table 34: 30-assets case: MMV PDen Loss.

	OGARCH	SBEKK	DPC	DPC$_r$	DPC$_f$	DPC$_s$	DPC*	DPC$^*_r$	DPC$^*_f$	CCC	DCC	cDCC
OGARCH		-0.58	**4.21**	**5.02**	-1.22	**3.81**	**5.06**	**5.24**	-1.19	-2.00	-1.60	-1.61
SBEKK	0.58		**2.31**	**2.45**	-0.57	**2.97**	**2.53**	**2.63**	-0.54	*-2.49*	-1.69	-1.70
DPC	*-4.21*	*-2.31*		1.64	*-3.23*	0.73	**2.19**	**2.32**	*-3.19*	*-3.15*	*-2.98*	*-3.00*
DPC$_r$	*-5.02*	*-2.45*	-1.64		*-3.34*	0.08	1.29	**2.28**	*-3.30*	*-3.25*	*-3.10*	*-3.12*
DPC$_f$	1.22	0.57	**3.23**	**3.34**		**3.33**	**3.37**	**3.45**	0.87	-1.40	-0.78	-0.79
DPC$_s$	*-3.81*	*-2.97*	-0.73	-0.08	*-3.33*		0.30	0.57	*-3.30*	*-3.48*	*-3.36*	*-3.38*
DPC*	*-5.06*	*-2.53*	*-2.19*	-1.29	*-3.37*	-0.30		1.04	*-3.33*	*-3.30*	*-3.16*	*-3.18*
DPC$^*_r$	*-5.24*	*-2.63*	*-2.32*	*-2.28*	*-3.45*	-0.57	-1.04		*-3.41*	*-3.37*	*-3.25*	*-3.27*
DPC$^*_f$	1.19	0.54	**3.19**	**3.30**	-0.87	**3.30**	**3.33**	**3.41**		-1.42	-0.80	-0.81
CCC	2.00	**2.49**	**3.15**	**3.25**	1.40	**3.48**	**3.30**	**3.37**	1.42		**3.50**	**3.39**
DCC	1.60	1.69	**2.98**	**3.10**	0.78	**3.36**	**3.16**	**3.25**	0.80	*-3.50*		-0.54
cDCC	1.61	1.70	**3.00**	**3.12**	0.79	**3.38**	**3.18**	**3.27**	0.81	*-3.39*	0.54	

Table 35: 30-assets case: HDG PDen Loss.

	OGARCH	SBEKK	DPC	DPC$_r$	DPC$_f$	DPC$_s$	DPC*	DPC$^*_r$	DPC$^*_f$	CCC	DCC	cDCC
OGARCH		-0.11	1.64	**2.26**	*-2.94*	**2.59**	**2.28**	**2.56**	*-2.90*	1.52	1.96	2.00
SBEKK	0.11		**2.04**	**2.67**	*-3.93*	**2.88**	**2.54**	**2.88**	*-3.87*	1.51	1.95	1.99
DPC	-1.64	*-2.04*		**2.52**	*-3.73*	**2.91**	**2.20**	**2.70**	*-3.68*	0.57	1.13	1.19
DPC$_r$	*-2.26*	*-2.67*	*-2.52*		*-3.95*	1.38	-0.60	0.81	*-3.90*	0.08	0.64	0.70
DPC$_f$	**2.94**	**3.93**	**3.73**	**3.95**		**3.90**	**3.84**	**3.94**	1.39	**2.95**	**3.19**	**3.21**
DPC$_s$	*-2.59*	*-2.88*	*-2.91*	-1.38	*-3.90*		*-2.22*	-1.07	*-3.85*	-0.16	0.42	0.48
DPC*	*-2.28*	*-2.54*	*-2.20*	0.60	*-3.84*	**2.22**		**2.17**	*-3.79*	0.17	0.75	0.81
DPC$^*_r$	*-2.56*	*-2.88*	*-2.70*	-0.81	*-3.94*	1.07	*-2.17*		*-3.89*	-0.04	0.54	0.60
DPC$^*_f$	**2.90**	**3.87**	**3.68**	**3.90**	-1.39	**3.85**	**3.79**	**3.89**		**2.91**	**3.15**	**3.17**
CCC	-1.52	-1.51	-0.57	-0.08	*-2.95*	0.16	-0.17	0.04	*-2.91*		**3.62**	**3.91**
DCC	-1.96	-1.95	-1.13	-0.64	*-3.19*	-0.42	-0.75	-0.54	*-3.15*	*-3.62*		**3.26**
cDCC	-2.00	-1.99	-1.19	-0.70	*-3.21*	-0.48	-0.81	-0.60	*-3.17*	*-3.91*	*-3.26*	