EFREM CASTELNUOVO

University of Padova, CESifo, and CAMA

GIOVANNI PELLEGRINO

University of Padova

LAUST L. SÆRKJÆR

Aarhus University

MONETARY POLICY SHOCKS AND NARRATIVE RESTRICTIONS: RULES MATTER

November 2025

Marco Fanno Working Papers – 328

DIPARTIMENTO DI SCIENZE ECONOMICHE E AZIENDALI 'MARCO FANNO'

UNIVERSITÀ DEGLI STUDI DI PADOVA

Monetary Policy Shocks and Narrative Restrictions: Rules Matter*

Efrem Castelnuovo[1], Giovanni Pellegrino[2], and Laust L. Særkjær[3]

[1]University of Padova, CESifo, and CAMA

[2]University of Padova

[3]Aarhus University

November 2025

Abstract

Imposing restrictions on policy rule coefficients in vector autoregressive (VAR) models enhances the identification of monetary policy shocks obtained with sign and narrative restrictions. Monte Carlo simulations and empirical analyses for the United States and the Euro area support this result. For the U.S., adding policy coefficient restrictions yields a larger and more precise short-run output response and more stable Phillips multiplier estimates. Heterogeneity in output responses reflects variation in systematic policy reactions to output. In the Euro area, policy coefficient restrictions sharpen the identification of corporate bond spread responses to monetary policy shocks.

Keywords: Monetary policy shocks, narrative restrictions, policy coefficient restrictions, vector autoregressive models, Monte Carlo simulations, DSGE models.

JEL codes: C32, E32, E52.

*We thank Guido Ascari, Isaac Baley, Andrea Bastianin, Fabio Canova, Francesco Furlanetto, Eric Leeper, Mirela Miescu, Tommaso Monacelli, Francesca Monti, Lorenzo Mori, Ivan Petrella, Mikkel Plagborg-Møller, Giorgio Primiceri, Francesco Ravazzolo, Matthew Read, Daniele Siena, Tommaso Tornese, and audiences at presentations at the 2024 ASSET conference (Venice), the 2025 IAAE conference (Turin), the 2025 Lancaster Workshop on Empirical and Theoretical Macroeconomics, the 2025 PRIN workshop on "Inflation: Causes, Consequences, and Policy Implications" (Milan), and the University of Milan for useful comments and suggestions. Financial support by the European Union - NextGenerationEU (PRIN 2022 on "Inflation: Causes, Consequences, and Policy Implications", CUP: C53D23002650006, project code: 2022A9PAT3) is gratefully acknowledged. Authors' accounts: efrem.castelnuovo@unipd.it, giovanni.pellegrino@unipd.it, lsaerkjaer@econ.au.dk.

1 Introduction

Motivation. How to identify the business cycle effects of monetary policy shocks? This question has often been addressed by estimating vector autoregressive models and imposing identification restrictions to recover the structural impulse response of real activity to an unexpected policy rate change. The seminal contribution by Sims (1980) popularized zero restrictions that imply a recursive representation of the economy characterized by "slow-moving" variables (variables that are assumed to not respond to policy shocks within a given period, e.g., inflation and output) and "fast-moving" variables (that are allowed to react on impact, i.e., asset prices). While convenient, this representation has been criticized because of its frequent association with "puzzling" responses of key variables (e.g., the increase in inflation following an unexpected policy tightening, see Eichenbaum (1992)) and its inconsistency with the micro-founded frameworks that do not feature such timing restrictions (e.g., Woodford (2003), Galí (2015)).

Faust (1998), Canova and de Nicoló (2002), Canova and Pina (2005), and Uhlig (2005) propose to depart from zero restrictions and impose instead uncontroversial sign restrictions on relevant conditional moments (e.g., impulse responses) that provide enough information to pin down the macroeconomic response to monetary policy shocks. Unfortunately, however, sign restrictions have been shown to fail to correctly identify the real effects of monetary policy shocks in the context of Monte Carlo exercises with reasonably calibrated data-generating processes such as small and medium-scale dynamic stochastic general equilibrium frameworks (Wolf (2020, 2023)). Two effective refinements of impulse responses-related sign restrictions have recently been put forth. On the one hand, Antolín-Díaz and Rubio-Ramírez (2018) propose to fix this issue by adding narrative sign restrictions to sharpen the identification of the macroeconomic effects of monetary policy shocks otherwise estimated with sign restriction only. Narrative sign restrictions are constraints imposed on the sign of monetary policy shocks and their contribution to the unexpected change – or the historical decomposition – of the policy rate in selected dates that characterize the history of the US economy, e.g., the large monetary policy tightening implemented by the Federal Reserve in October 1979. Working with Uhlig's (2005) dataset, Antolín-Díaz and Rubio-Ramírez (2018) show that narrative sign restrictions lead to a larger and more precise estimation of the output effects of monetary policy shocks. On the other hand, Arias, Caldara, and Rubio-Ramírez (2019) propose to work with policy coefficient restrictions, which are restrictions on the systematic contemporaneous response of the policy rate to selected macroeconomic indicators - e.g., inflation and output - on top of sign restrictions on impulse responses. This is done to ensure that the set-identified models (also) meet economists' beliefs on the systematic monetary policy conduct by the Federal Reserve (see e.g. Taylor (1993)). Similarly to Antolín-Díaz and Rubio-Ramírez (2018), Arias, Caldara, and Rubio-Ramírez (2019) show that policy coefficient restrictions lead to a sharper estimation of the real effects of monetary policy shocks. While being potentially complementary, narrative restrictions and policy coefficient restrictions have *de facto* been considered as substitutes by the profession.

Contribution and findings. Our contribution to the existing literature is threefold.

First, we make extensive use of Monte Carlo simulations to compare the relative ability of sign, narrative, and policy coefficient restrictions to pin down the true response of output to a monetary policy shock.[1] Our main Monte Carlo-related findings are the following: (i) narrative restrictions substantially improve the performance of the estimator of the impulse response of output to a monetary policy shock with respect to an approach solely based on sign restrictions; (ii) policy coefficient restrictions work in favor of further sharpening the estimated response of output when added to narrative and sign restrictions. This second result points to the complementary (as opposed to substitutability) of narrative and policy coefficient restrictions. To the best of our knowledge, our contribution is the first to assess narrative restrictions à la Antolín-Díaz and Rubio-Ramírez (2018) with a Monte Carlo exercise. Moreover, we complement the analysis of Wolf (2020), who document the good performance of policy coefficient restrictions à la Arias, Caldara, and Rubio-Ramírez (2019) in a Monte Carlo setting, by showing that such restrictions contribute to sharpen the identification of monetary policy shocks even when narrative restrictions are already in place.

Our second contribution is empirical. Working with sign and narrative restrictions, we first replicate the empirical findings put forth by Antolín-Díaz and Rubio-Ramírez (2018), who show that the wide model and statistical uncertainty surrounding the response of output documented in Uhlig (2005) on the basis of sign restrictions only is substantially reduced when adding narrative restrictions. Then, we document the marginal contribution of policy coefficient restrictions à la Arias, Caldara, and Rubio-Ramírez (2019), which we add to those imposed by Antolín-Díaz and Rubio-Ramírez (2018). We find policy coefficient restrictions to: (i) shift the median response of output downward, e.g., real GDP's peak response within a year after a 25 basis points unexpected jump in the policy rate is estimated to be about -0.1% vs. the -0.06% one gets when working without imposing policy coefficient restrictions; (ii) reduce the uncertainty surrounding the short-run response of real activity. The model and statistical uncertainty surrounding the response of

[1] In this study, we will consider the narrative restrictions à la Antolín-Díaz and Rubio-Ramírez (2018), which are restrictions on the sign or the contribution of a given shock to the unexpected change – or historical decomposition – of a given variable in a given date. An alternative way to impose narrative restrictions is the "narrative-proxy" one, which casts the information about sign of the shock in selected dates as a discrete-valued proxy that enables the researcher to achieve point-identification. For a comparison between these two approaches, see Giacomini, Kitagawa, and Read (2022) and Plagborg-Møller (2022).

output points to a significantly negative response of real activity already after six months, while the same response becomes significant only after one year when not considering policy coefficients restrictions; (iii) imply a counterfactual path of the federal funds rate driven only by monetary policy shocks that is comparable to that one can obtain by alternatively imposing a narrative sign restriction for the Volcker disinflation episode in October 1979 as done by Antolín-Díaz and Rubio-Ramírez (2018).

The output response to a monetary policy shock differs markedly across the three identification strategies under consideration because each strategy implies a distinct estimate of the systematic monetary policy response to output fluctuations. We demonstrate this by formally deriving the mapping between the systematic policy response to output and the contemporaneous output response to a monetary policy shock. The intuition behind the link between the systematic policy response to output and the response of output to an unexpected change in the policy rate can be understood as follows. Consider, for simplicity, a bivariate framework including only output and the policy rate. In the data we use, the correlation between the (VAR residual) components of output and the federal funds rate is positive. Uhlig's (2005) identification strategy implies a *negative* systematic response of the policy rate to output fluctuations. This means that output shocks generate a negative correlation between the policy rate and output, which monetary policy shocks must offset to reproduce the positive correlation observed in the data. Consequently, the impulse response of output to a monetary policy shock must be *positive*. Imposing narrative restrictions à la Antolín-Díaz and Rubio-Ramírez (2018) substantially improves the situation, as the estimated systematic policy response to output becomes close to zero. Still, monetary policy shocks need to account for the positive correlation between output and the policy rate observed in the data. When policy coefficient restrictions are finally added, the picture improves substantially: a *positive* policy reaction to output helps reproduce the positive output–policy rate correlation in the data and, in turn, "opens the door" to the possibility of a *negative* conditional correlation between these two variables generated by monetary policy shocks.

The implications of these three different identification strategies go beyond the output effects of unexpected monetary policy interventions. We show that the estimates of the Phillips multiplier - computed as the ratio between the average response of inflation and that of real activity as in Barnichon and Mesters (2021) - can vary from negative (with sign restrictions on impulse responses only) to unstable (adding narrative restrictions) to stable and precisely estimated (with the combination of narrative and policy coefficient restrictions). Hence, the combination of narrative and policy coefficient restrictions is useful to obtain more precise and robust estimates of the inflation-output policy trade-off.

Third, we show that the combination of narrative and policy coefficient restrictions importantly sharpens the macroeconomic effects of monetary policy shocks even when working with Euro area data. As a reference, we consider the contribution by Badinger and Shiman (2023). They identify monetary policy shocks by working with narrative restrictions related to four dates (October and November of the years of 2008 and 2011) that saw financial markets surprised the most by policy decisions by the European Central Bank. Their VAR points to textbook effects of an unexpected increase in the policy rate. However, their monthly VAR analysis points to an uncertain contemporaneous response of the corporate bond spread after a policy rate hike. Differently, when combining their narrative sign restrictions with a policy coefficient restriction on the systematic policy response to the corporate bond spread (a restriction that requires a policy easing after an increase in credit stress), we document a largely positive, more precisely estimated contemporaneous increase in such a spread, which is more in line with a rapid response of financial markets to monetary policy impulses.

Structure of the paper. Section 2 draws connections with the literature. Section 3 conducts the Monte Carlo experiment to document the ability of sign, narrative, and policy coefficient restrictions to recover the true response of output to a monetary policy shock. Section 4 deals with the empirical exercise based on US data and provides some intuition on why policy coefficient restrictions are relevant in this context. Section 5 focuses on the Euro area data analysis. Section 6 concludes.

2 Connections with the literature

Uhlig (2005) proposes to work with restrictions imposed on the signs of the impulse responses of selected variables in his VAR to identify a monetary policy shock without appealing to zero restrictions. He finds extremely uncertain output effects of monetary policy shocks. Antolín-Díaz and Rubio-Ramírez (2018) and Arias, Caldara, and Rubio-Ramírez (2019) propose cleverly designed refinements to the sign restriction-only approach by Uhlig (2005) by putting forth the idea of working with narrative restrictions (the former paper) and policy coefficient restrictions (the latter).[2] Andrade, Ferroni, and Melosi (2023) employ inequality restrictions on higher-order moments of the distribution of structural shocks (such as skewness) to sharpen the identification of monetary policy shocks that are otherwise identified using standard sign restrictions on impulse responses. In a simulated setting, they show that these higher-order restrictions successfully

[2] In the context of the analysis of the oil market, Kilian and Murphy (2012) also stress the importance of combining sign restrictions on both impulse responses and structural parameters, with the latter restrictions being justified by the idea of obtaining elasticities in line with conventional wisdom.

recover the true response of real activity to monetary policy shocks. Using data for the United States and the Euro area, they document a large and significant output response when imposing higher-order moment restrictions in the former case, and significant effects following shocks to government bond spreads in the latter. Differently, we combine already existing identification strategies and demonstrate that narrative and policy coefficient restrictions are not mutually exclusive but, in fact, should be used jointly to achieve a more robust identification of monetary policy shocks.

Empirically, our exercise conducted with the dataset provided by Uhlig (2005) confirms that the output response to a monetary policy shock is intimately related to the way in which an econometrician implicitly or explicitly models the systematic component of monetary policy. This point confirms previous intuitions and applications linking unexpected and systematic policy components (see Leeper, Sims, and Zha (1996), Leeper and Zha (2003), Sims and Zha (2006), Caldara and Kamps (2017), Caldara and Herbst (2019), and Arias, Caldara, and Rubio-Ramírez (2019)). Wolf (2020, 2023) offers simulation-based evidence in favor of policy coefficient restrictions as a more effective way to recover the true impulse responses to a monetary policy shock than zero restrictions or sign restrictions à la Uhlig (2005). Our paper complements his contributions by showing that: (i) narrative restrictions (not dealt with by Wolf (2020, 2023)) add substantial identification power to standard sign restrictions; (ii) policy coefficient restrictions can add relevant information to sign and narrative restrictions for the identification of the real effects of monetary policy shocks; (iii) policy coefficient restrictions can importantly affect empirical analysis conducted with US and Euro area data.

A different strand of the literature has developed instruments to pin down exogenous changes in monetary policy and their macroeconomic effects (see, among others, Romer and Romer (2004), Faust, Swanson, and Wright (2004), Gürkaynak, Sack, and Swanson (2005), Stock and Watson (2012), Gertler and Karadi (2015), Swanson (2021), Jarociński and Karadi (2020), Miranda-Agrippino and Ricco (2021), Swanson and Jayawickrema (2023), Bauer and Swanson (2023), Swanson (2023), Aruoba and Drechsel (2024)). Braun and Brüggemann (2023) propose combining sign restrictions with instruments to sharpen the identification of macroeconomic shocks (among which, monetary policy shocks). In the presence of a valid instrument, proxy-SVARs represent powerful tools to establish empirical facts on the macroeconomic impact of monetary policy shocks (Wolf (2020, 2023)). We see our combination of sign, narrative, and policy coefficient restrictions as an interesting alternative because: (i) it does not depend on the way a proxy is constructed, which is important because inference conducted with different instruments may lead to heterogeneous indications on the role of monetary policy shocks (Brennan, Jacobson, Matthes, and Walker (2024)); (ii) it naturally deals with the separation of monetary policy shocks and information shocks via the imposition of a negative conditional correlation between the policy rate and inflation to a monetary policy shock that is associated with the former but not with the latter (see, e.g., Jarociński and Karadi (2020), Miranda-Agrippino and Ricco (2021)). Finally, Carriero, Marcellino, and Tornese (2024) propose a novel algorithm to combine different types of restrictions (sign, narrative) and instruments for the identification of monetary policy shocks while accounting also for heteroskedasticity. With respect to them, we: (i) conduct a Monte Carlo exercise with a medium-scale model (the Smets and Wouters (2007) one) to assess different combinations of set identification restrictions with a focus on the marginal contribution of policy coefficient restrictions when an econometrician adds them to sign and narrative constraints; (ii) work with US data and show that policy coefficient restrictions à la Arias, Caldara, and Rubio-Ramírez (2019) induce a larger and more precise response of output to a monetary policy shock when added to those employed by Antolín-Díaz and Rubio-Ramírez (2018); (iii) show that policy coefficient restrictions, combined with narrative restrictions, imply an estimate of the Phillips multiplier à la Barnichon and Mesters (2021) that is more stable and precise over horizons; (iv) conduct an empirical exercise with Euro area data and show that policy coefficient restrictions can help an econometrician better estimate the response of the corporate bond spread to a monetary policy shock.

3 Monte Carlo simulations

We conduct Monte Carlo simulations to assess the relative ability of sign, narrative, and policy coefficient restrictions to pin down the true response of output to a monetary policy shock. Following Wolf (2020), we work with the Smets and Wouters (2007) model as our data-generating process. We simulate 100 datasets of 200 observations (50 years of quarterly data, which approximates the post-WWII sample often used in empirical analysis) and 100 datasets of 500 observations (to simulate a longer sample available to the econometrician when working with monthly data). The simulated data we consider are: real output y, inflation π, nominal interest rate r, hours worked h, and investment inv. Wolf (2020) considers these five variables in his Monte Carlo exercise and shows that a VAR model can, in principle, recover the true response of output to a monetary policy shock (even if the Smets and Wouters (2007) model is non-invertible).[3] Differently from Wolf (2020), who works "in population" (i.e., he considers a large

[3]Wolf (2020) finds a $R^2 = 0.93$ when regressing the series of the true monetary policy shocks on current values and infinitely many lags of the five selected macro variables in the VAR. We compute the R^2 for each one of our 100 simulations (both in the case of 200 observations and in the one of 500) and verify that, on average, its value is in line with the one documented by Wolf (2020). Note that our regressions feature only four lags. This is consistent with Wolf (2020) (Figure B.1 of his Appendix), who shows that a VAR that is parsimonious in terms

sample of 5,000 observations), we consider finite samples to understand how a small number of narrative sign restrictions can sharpen the identification of monetary policy shocks. For each simulated sample, we estimate a VAR with four lags and identify the output effects of monetary policy shocks with one of the strategies presented below.

VAR. Following Antolín-Díaz and Rubio-Ramírez (2018) and Arias, Caldara, and Rubio-Ramírez (2019), we consider the following structural VAR representation:

$$\mathbf{y}_t'\mathbf{A}_0 = \sum_{l=1}^{p} \mathbf{y}_{t-l}'\mathbf{A}_l + \varepsilon_t' \tag{1}$$

In this representation, $\mathbf{y}_t$ is an $n \times 1$ vector of endogenous variables (in our case, $\mathbf{y}_t = [i_t, y_t, \pi_t, h_t, inv_t]'$), ε_t is an $n \times 1$ vector of structural shocks, $\mathbf{A}_0$ is an invertible $n \times n$ matrix of structural parameters, $\mathbf{A}_l$ is a matrix collecting the coefficients associated with lag l of the VAR, p is the number of lags of the VAR (for us $p = 4$), and t is a time-index ranging from 1 to T, where T is the size of the considered sample. The vector ε_t, conditional on past information and initial conditions, is Gaussian with mean zero and covariance matrix $\mathbf{I}_n$ (the $n \times n$ identity matrix).

Post-multiplying the SVAR in equation (1) by $\mathbf{A}_0^{-1}$, one can move to the reduced form representation:

$$\mathbf{y}_t' = \sum_{l=1}^{p} \mathbf{y}_{t-l}'\mathbf{B}_l + \mathbf{u}_t' \tag{2}$$

where $\mathbf{B}_l = \mathbf{A}_l\ \mathbf{A}_0^{-1}$, $\mathbf{u}_t' = \varepsilon_t'\mathbf{A}_0^{-1}$, and $\mathbb{E}(\mathbf{u}_t\mathbf{u}_t') = \mathbf{\Sigma} = (\mathbf{A}_0\mathbf{A}_0')^{-1} = (\mathbf{A}_0\mathbf{Q}\mathbf{Q}'\mathbf{A}_0')^{-1}$, where $\mathbf{Q}$ is a conformable orthonormal matrix that rotates the $\mathbf{A}_0$ one.

As is well known, there are uncountable rotations of the $\mathbf{A}_0$ matrix all equally consistent with the data but that offer different interpretations of the economy (e.g., of the macroeconomic effects of a monetary policy shock). We then exclude rotations (interpretations) that are economically unpalatable by imposing a set of identifying restrictions - presented below - that also include constraints on the systematic monetary policy response to macroeconomic fluctuations. Without loss of generality, we place the policy rate on top of the vector $\mathbf{y}_t$. The monetary policy equation reads as follows:

$$\mathbf{y}_t'\mathbf{a}_{0,1} = \sum_{l=1}^{p} \mathbf{y}_{t-l}'\mathbf{a}_{l,1} + \varepsilon_{1t} \tag{3}$$

Focusing on the contemporaneous responses of the central bank to macroeconomic fluctuations, the systematic component of monetary policy reads:

$$i_t = \psi_y y_t + \psi_\pi \pi_t + \psi_h h_t + \psi_{inv} inv_t + \sigma_{MP}\varepsilon_t^{MP}. \tag{4}$$

of dynamic structure is sufficient to get a high R^2 in this type of regression.

We now present and discuss our identification strategy.

Identification restrictions. The different identification restrictions we deal with in the Monte Carlo study are:

- Sign restrictions (SR). As in Uhlig (2005) and Wolf (2020), we impose the following sign restrictions: after a monetary policy shock, the interest rate is non-negative for 6 periods, and inflation is non-positive for 6 periods.[4]

- Narrative sign restrictions (NSR). In line with Antolín-Díaz and Rubio-Ramírez (2018), we require that, on selected dates, the monetary policy shock is the overwhelming driver of the unexpected change in the interest rate. In particular, we assume the econometrician works with either 1, 3, or 5 dates randomly drawn from all the dates that satisfy the narrative sign restrictions in the simulated sample at hand.[5] At these dates, the monetary policy shock can be either positive or negative given that no-asymmetric effect is associated with monetary policy shocks in the Smets and Wouters (2007) model.[6]

- Policy coefficient restrictions (PCR). As in Arias, Caldara, and Rubio-Ramírez (2019), we require that the contemporaneous systematic response of the policy rate to output and inflation movements in eq. (4) is positive, i.e., $\psi_y > 0$ and $\psi_\pi > 0$.[7]

We consider three different sets of identification restrictions: (i) sign restrictions only; (ii) narrative sign restrictions on top of sign restrictions; (iii) policy coefficient restrictions on top of sign

[4] In his empirical application, Uhlig (2005) also imposes sign restrictions on commodity prices and nonborrowed reserves. These variables are not present in the Smets and Wouters (2007) model. We will impose those restrictions in the empirical application in Section 4.

[5] We define the contribution of shock j to the unexpected change in variable i in period t as $H^i_j(t) = IRF^i_j(0)\varepsilon_{j,t}$, where $IRF^i_j(0)$ is the impulse response of variable i to shock j at horizon $h = 0$. As such, the restriction that shock j is the overwhelming driver of variable i in period t reads $|H^i_j(t)| \geq \sum_{j' \neq j} |H^i_{j'}(t)|$. This is the strong NSR considered in Antolín-Díaz and Rubio-Ramírez (2018), whereas the weak one would have required that the shock j is the single largest contributor of variable i (i.e., $|H^i_j(t)| \geq \max_{j' \neq j} |H^i_{j'}(t)|$). For our application, $j = 1$ corresponds to the monetary policy shock and $i = 1$ to the policy rate.

[6] We randomly draw the dates that the econometrician works with when imposing narrative restrictions to be in line with the assumption implicit in the functional form of the likelihood function used by Antolín-Díaz and Rubio-Ramírez (2018) and Giacomini, Kitagawa, and Read (2021) that the narrative signals arrive randomly over time to the econometrician, i.e., in a way that does not depend on the shock magnitude in addition to the restrictions being correct (see Plagborg-Møller (2022)). We also performed a Monte Carlo exercise with narrative restrictions imposed on the date where the overwhelming driver restriction is most clearly satisfied, i.e., such that $\max \left(|H^i_j(t)| - \sum_{j' \neq j} |H^i_{j'}(t)| \right)$. Our results are very similar to our baseline ones.

[7] We leave unrestricted the systematic response to hours worked and investment, i.e., ψ_h and ψ_{inv}, as Arias, Caldara, and Rubio-Ramírez (2019) do for commodity prices in their application. Arias, Caldara, and Rubio-Ramírez (2019) also impose a zero contemporaneous response of the policy rate to total reserves and nonborrowed reserves. These variables, however, are not present in the Smets and Wouters (2007) model. We will impose those restrictions in the empirical application in Section 4.

and narrative restrictions. This "gradually incremental" approach enables us to appreciate the marginal contribution of narrative restrictions à la Antolín-Díaz and Rubio-Ramírez (2018) with respect to the sign-restriction only proposal by Uhlig (2005), and that of policy coefficient restrictions à la Arias, Caldara, and Rubio-Ramírez (2019) vs. the narrative restrictions approach à la Antolín-Díaz and Rubio-Ramírez (2018), which works with narrative restrictions on top of sign restrictions.

Findings. Figure 1 summarizes the findings from our Monte Carlo experiment as regards the impulse response of output, a key variable often used to assess the findings of several forms of sign restrictions identification (see, e.g., Uhlig (2005), Antolín-Díaz and Rubio-Ramírez (2018), Arias, Caldara, and Rubio-Ramírez (2019)). The Figure puts in comparison the true IRFs to a 25 basis points monetary policy shock in the Smets and Wouters (2007) DSGE model with the *median* estimated IRFs and 68% credible sets across the 100 datasets resulting from two different identification schemes: (i) sign restrictions only as in Uhlig (2005); (ii) sign and narrative restrictions as in Antolín-Díaz and Rubio-Ramírez (2018).[8] Overall, while the imposition of sign restrictions alone implies an estimated response of output that is very uncertain in terms of size and even sign, adding narrative restrictions leads to a sharper estimation of the real effects of monetary policy shocks. In particular: (i) the statistical and model uncertainty surrounding the output response is narrower; (ii) the evidence of negative response of output is much clearer, above all from roughly 3 periods after the shock onward; (iii) the median response of output is much closer to the true one when narrative restrictions belong to the set of identification assumptions the econometrician works with; (iv) the more narrative sign restrictions are imposed and the larger the sample, the more precisely the output effects of monetary policy shocks are estimated.

Figure 2 puts in evidence the marginal contribution brought along by policy coefficient restrictions when added to the sign and narrative ones. Again, we focus on the response of output to a monetary policy shock. Two results emerge from this Figure: (i) working with PCR on top of NSR & SR allows to better estimate the impulse responses in the short- to medium-run, as the estimated median responses get closer to the "true" ones from the DSGE model; (ii) the addition of PCR on top of NSR & SR also allows to sharpen the estimation of the impulse responses

[8] We conduct our estimations by working with the Bayesian algorithm proposed by Antolín-Díaz and Rubio-Ramírez (2018) (which is an adaptation of algorithms previously proposed by Rubio-Ramírez, Waggoner, and Zha (2010) and Arias, Rubio-Ramírez, and Waggoner (2018)), which we modified to allow for the imposition of policy coefficient restrictions à la Arias, Caldara, and Rubio-Ramírez (2019). As suggested by Giacomini, Kitagawa, and Read (2021), we skip the implementation of the importance sampling to draw from the uniform-normal inverse-Wishart posterior using the unconditional likelihood to construct the posterior. We do the same for the empirical analysis in Sections 4 and 5, noting that for our applications our empirical findings are not sensitive to allowing for the resampling step of the algorithm.

as the credible sets around the median responses tightens. In particular, when considering the case with 200 observations and one or three NSRs, imposing them substantially augments the precision of the estimated response of output.[9]

Wrapping up, our Monte Carlo exercise: (i) points to a large marginal contribution of narrative sign restrictions when added to a sign-restriction only identification scheme, i.e., the median response of output is substantially closer to the true one, and much more precisely estimated, than the one obtained with sign restrictions only; (ii) suggests that policy coefficient restrictions can further improve the correctness and precision of the estimated output response when added to sign and narrative restrictions. These two findings justify the combined use of sign, narrative, and policy coefficient restrictions to estimate the real effects of monetary policy shocks.

4 Empirical analysis

This Section applies a combination of sign, narrative, and policy coefficient restrictions to the standard dataset used by Uhlig (2005), Antolín-Díaz and Rubio-Ramírez (2018), and Arias, Caldara, and Rubio-Ramírez (2019). This "all in" combination, which is supported by our Monte Carlo simulation, is, to our knowledge, new to the literature.

We estimate a VAR similar to that in equation (1). Following Uhlig (2005), Antolín-Díaz and Rubio-Ramírez (2018), and Arias, Caldara, and Rubio-Ramírez (2019), the VAR model we work with includes six variables: real output y, the GDP deflator p, a commodity price index p_c, total reserves tr, non-borrowed reserves nbr, and the federal funds rate i, i.e., $\mathbf{y}_t = [i_t, y_t, p_t, p_{c,t}, tr_t, nbr_t]'$. The investigated sample period is January 1965-November 2007. All series are expressed in logarithms, except for the federal funds rate, which enters the vector in levels. Observations are monthly. The VAR features 12 lags and no constant or deterministic trends.[10] As in the previous section, we consider three different sets of identification restrictions, following closely the restrictions in the empirical analyses by Uhlig (2005), Antolín-Díaz and Rubio-Ramírez (2018), and Arias, Caldara, and Rubio-Ramírez (2019): (i) sign restrictions only; (ii) narrative sign restrictions on top of sign restrictions; (iii) policy coefficient restrictions on top of sign and narrative sign restrictions. As already shown by Antolín-Díaz and Rubio-

[9] In the Appendix, we document that in the case of a strong signal for monetary policy shocks, i.e., in the case in which their standard deviation in the Smets and Wouters (2007) is multiplied by 10, then all the different identification schemes considered in this Section can recover quire accurately and without significant bias – although with different degrees of precision – the true impulse response functions to a monetary policy shock in our Monte Carlo setup with small samples and VAR(4) specifications (Figure A.10).

[10] Further details on the modeling choices are offered by Uhlig (2005), Antolín-Díaz and Rubio-Ramírez (2018), and Arias, Caldara, and Rubio-Ramírez (2019).

Ramírez (2018) (with the same dataset), the addition of their narrative restrictions substantially improves the estimation of the output effects of monetary policy shocks with respect to sign restrictions only. Very much in line with our results in the Monte Carlo exercise, we focus on the marginal contribution of policy coefficient restrictions when added to sign and narrative restrictions, something not ascertained yet in the literature.

Sign and narrative sign restrictions. The baseline identification, i.e., the one based on sign restrictions on impulse responses only, is identical to that in Uhlig (2005), i.e., a contractionary monetary policy shock increases the federal funds rate and reduces the GDP deflator, the commodity price index, and non-borrowed reserves for periods 0 to 5 months. Antolín-Díaz and Rubio-Ramírez (2018) add to Uhlig's restrictions two narrative sign constraints that regard the monetary policy shock materialized in October 1979: (i) it must be of positive value; (ii) it must be the overwhelming driver of the unexpected movement in the federal funds rate, i.e., the absolute value of the contribution of monetary policy shocks to the unexpected movement in the federal funds rate must be larger than the sum of the absolute value of the contributions of all other structural shocks.[11] As pointed out by Antolín-Díaz and Rubio-Ramírez (2018), while sign restrictions on the impulse responses truncate the support of the prior distribution of the structural parameters, narrative sign restrictions instead truncate the support of the likelihood function. Hence, Antolín-Díaz and Rubio-Ramírez (2018) modify the Bayesian methods in Rubio-Ramírez, Waggoner, and Zha (2010) and Arias, Rubio-Ramírez, and Waggoner (2018) for the case of narrative sign restrictions.

Our Appendix documents the impact of narrative sign restrictions by replicating the empirical evidence in Antolín-Díaz and Rubio-Ramírez (2018). We stress here that their NSRs have a substantial impact on the response of output to a monetary policy shock, which becomes clearly negative and is much more precisely estimated than the one obtained with restrictions on impulse response functions only. In line with our Monte Carlo evidence, these empirical findings point to the relevance of NSRs for the identification of monetary policy shocks.

Policy coefficient restrictions: Marginal contribution. Arias, Caldara, and Rubio-Ramírez (2019) propose to restrict the systematic contemporaneous monetary policy component of the federal funds rate VAR equation, which for the Uhlig (2005) VAR reads as follows:

$$i_t = \psi_y y_t + \psi_p p_t + \psi_{p_c} p_{c,t} + \psi_{tr} tr_t + \psi_{nbr} nbr_t + \sigma_{MP} \varepsilon_t^{MP}. \tag{5}$$

[11] The two narrative restrictions described in the text are restrictions 4 and 5 in the Antolín-Díaz and Rubio-Ramírez (2018) paper, page 2824. A further exercise conducted by Antolín-Díaz and Rubio-Ramírez (2018) imposes a variety of other restrictions regarding other events/months in the investigated sample (restrictions 6 and 7, page 2827 of their paper). As documented by Antolín-Díaz and Rubio-Ramírez (2018), the results they obtain are very similar to those obtained by imposing restrictions 4 and 5.

They require that: (i) the federal funds rate reacts contemporaneously only to output, prices, and commodity prices; (ii) the contemporaneous reaction of the federal funds rate to output and prices is positive. The first set of restrictions, which they term "Restriction 1", implies a zero contemporaneous response of the federal funds rate to total and nonborrowed reserves, i.e., $\psi_{tr} = 0$ and $\psi_{nbr} = 0$, while the second set of restrictions, which they term "Restriction 2", implies a sign restriction on the policy response to output and inflation, i.e., $\psi_y > 0$ and $\psi_p > 0$. The response to commodity prices ψ_{p_c} is left unrestricted. When imposing these restrictions (in particular, those on the sign of the policy coefficients responsible for the systematic monetary policy response to output and prices), the VAR points to a negative response of output to a contractionary monetary policy shock with high posterior probability.[12] In controlled environments, Wolf (2020, 2023) documents the ability of policy coefficient restrictions to recover the true output effects of monetary policy shocks.

What are the implications of imposing these policy coefficient restrictions on top of the traditional sign restrictions and the narrative sign restriction of the October 1979 event? We answer this question by comparing the estimates of the macroeconomic effects of monetary policy shocks on the basis of the sign and narrative restrictions used in Antolín-Díaz and Rubio-Ramírez (2018) with those one gets when adding to their set of restrictions the ones (restrictions 1 and 2 as described above) on the policy coefficients as in Arias, Caldara, and Rubio-Ramírez (2019). Figure 3 offers this comparison focusing on the output effects of a contractionary monetary policy shock (whereas the Appendix reports the full set of results). The short-run negative response of output becomes larger and less uncertain, with the 68% credible set not including the zero line already 5 months after the shock vs. after 16 months when policy coefficient restrictions are not imposed. Our Appendix shows that this result is driven by the positive sign imposed on the policy coefficient responsible for the systematic policy response to output.[13] Figure 4 slices the output responses plotted in Figure 3 across selected horizons - in particular, it considers horizons 0, 6, 12, and 18.[14] Most models selected by sign, narrative, and policy coefficient restrictions combined point to a negative response of output and the 68% credible sets are entirely characterized by negative responses of output. Moreover, the shares of retained models associated with a

[12] Our Appendix documents the replicability of this result via an extension of the codes by Antolín-Díaz and Rubio-Ramírez (2018) to also allow for sign restrictions on the policy coefficients (the codes by Antolín-Díaz and Rubio-Ramírez (2018) already allow to impose zero coefficient restrictions).

[13] The Appendix also documents that our results hold true if we consider alternative events to the October 1979 event used in the baseline analysis of Antolín-Díaz and Rubio-Ramírez (2018) such as February 1994. Antolín-Díaz and Rubio-Ramírez (2018) stress that the February 1994 event is of particular interest because the historical record identifies a major contractionary monetary policy shock, but output accelerated during 1994.

[14] More precisely, Figure 4 plots distributions conditional on all retained models per each selected horizon, where 68% credible sets are identified by dashed vertical bars. Our Appendix shows that our results hold also when considering a wider set of horizons.

negative response of output are larger when imposing the policy coefficient restrictions (e.g., such a share reads 67% for the on-impact response of output when we (also) impose the policy coefficient restrictions vs. 35% when we do not impose them). The comparison between these distributions and those associated with the Antolín-Díaz and Rubio-Ramírez (2018) points to the informativeness of policy coefficient restrictions for the identification of the real effects of monetary policy disturbances.

Table 1 shows the consequences for the estimates of the policy coefficients of the various identification strategies analyzed here. The first set of coefficients are those implied by Uhlig's (2005) restrictions - this is the message first proposed by Arias, Caldara, and Rubio-Ramírez (2019), i.e., sign restrictions imposed only on impulse responses retain a substantial number of models that are at odds with the conventional view on the systematic monetary policy response to macroeconomic fluctuations. The second set of estimated policy coefficients is the one implied by Antolín-Díaz and Rubio-Ramírez (2018). Interestingly, the imposition of the narrative restrictions on the contribution of the policy shock in October 1979, which are designed to "get the behavior of the shock right" in that particular month and sharpen the identification of the role of monetary policy shocks in general, also lead to an improvement of the description of Federal Reserve's systematic monetary policy as well. The posterior median of the systematic response to output turns negative, a sign in line with conventional wisdom. However, the 68% and 95% credible sets feature negative realizations of ψ_y. Here is where policy coefficient restrictions à la Arias, Caldara, and Rubio-Ramírez (2019) kick in. By imposing them, we - by construction - fix the above-discussed issue. In doing so, we obtain a sharper identification of the real activity effects of a monetary policy shock.

Policy coefficient restrictions: Historical implications. What are the implications of imposing policy coefficient restrictions for the estimated monetary policy shocks and their contributions? We know that a tight link exists between the systematic monetary policy representation and policy coefficients (see Leeper and Zha (2003), Sims and Zha (2006), Caldara and Kamps (2017), Caldara and Herbst (2019), and Arias, Caldara, and Rubio-Ramírez (2019); for a similar concept concerning fiscal policy, see Caldara and Kamps (2017)). Does this imply that the imposition of policy coefficient restrictions on a version of our VAR that abstracts from narrative restrictions can return a representation of monetary policy shocks in line with the extant narrative evidence? While policy coefficient restrictions are meant to imply a systematic monetary policy response to macroeconomic fluctuations in line with conventional wisdom along the entire sample, we find that such restrictions are actually effective also when it comes to pinning down the contribution of monetary policy shocks in specific historical events. Following Antolín-Díaz and Rubio-Ramírez (2018), we focus on October 1979, a month when Volcker is understood to have implemented a large, recessionary monetary policy shock. Figure 5 (top panels) replicates the evidence proposed by Antolín-Díaz and Rubio-Ramírez (2018) (Figure 5, p. 2824). The top-left panel shows that imposing a narrative sign restriction to induce a large contribution of the monetary policy shock to the unexpected change of the policy rate implies a more condensed distribution of the policy shocks with a clearly positive support. The right-hand top panel shows instead the counterfactual policy rate path conditional on monetary policy shocks only: when the narrative sign restriction is employed, the path of the policy rate is substantially closer to the actual one. Interestingly, the bottom panels of our Figure 5 shows that imposing the policy coefficient restrictions proposed in Arias, Caldara, and Rubio-Ramírez (2019) on top of Uhlig's (2005) ones - in this case, without narrative restrictions - generates a relatively similar distribution (although less narrow) of monetary policy shocks and a similar counterfactual policy rate path. Hence, while policy coefficient restrictions are not explicitly designed to target any particular historical event, their implications also align the distribution and effects of monetary policy shocks on output during the Volcker disinflation to conventional wisdom. Figure 6 extends this intuition to seven dates different from October 1979 selected by Antolín-Díaz and Rubio-Ramírez (2018) as candidates for the imposition of narrative sign restrictions for the identification of the output effects of monetary policy shocks. We comment on two findings. First, the narrative sign restrictions about October 1979 are informative not only as far as the realization of the monetary policy shock in October 1979 is concerned, but also for the other selected dates. Intuitively, conditional on the data description offered by the VAR, when "educating" the realization of a policy shock in a given month we are requiring realizations of other shocks to adjust to obey the constraint represented by the data we are modeling. Second, when imposing policy coefficient restrictions only, we find that not only the density of the realization of the monetary policy shocks does shrink and tilt rightward in October 1979 as already documented, but also that the densities in all other selected dates do so. Again, our intuition is that putting an extra structure on the systematic behavior of the Federal Reserve naturally implies a more plausible set of realizations of monetary policy shocks (via the estimated VAR structure) in the crucial events of the US monetary policy history.[15]

Reconciling heterogeneous estimates of the output effects of monetary policy shocks: The role of policy coefficients. Our empirical evidence points to different output effects of monetary policy shocks identified with our three different identification strategies. Cru-

[15] In the Appendix, we demonstrate that, in our empirical application, narrative and policy coefficient restrictions function as complements, rather than substitutes. Specifically, adopting a symmetric approach with respect to our baseline analysis—starting from SR and PCR, as in Arias, Caldara, and Rubio-Ramírez (2019), and subsequently incorporating NSR à la Antolín-Díaz and Rubio-Ramírez (2018)—enables a more precise identification of the effects of monetary policy shocks, consistently with our baseline findings.

cially, these differences are driven by the different estimated values of the policy coefficients across the three different identification strategies at hand. We formally show the mapping between policy coefficients and impulse responses by working with a simplified version of the interest rate equation of our VAR that assumes that policymakers systematically respond to output only. (As shown in our Appendix, our results are robust to admitting a systematic response to output and the price level.)

Following Caldara and Kamps (2017), we focus on the mapping between reduced-form residuals and policy shocks, which is as informative about the role played by the systematic response to output for the output reaction to monetary policy shocks. Then, we have:

$$u_{i,t} = \psi_y u_{y,t} + \sigma_{MP} \varepsilon_t^{MP} \tag{6}$$

Conditional on this policy rule, we show in our Appendix that one can derive the following expression for the on-impact impulse response of output to a monetary policy shock (normalized by the size of the shock):

$$\frac{IRF(0)_{y,i}}{\sigma_{MP}} = \frac{\sigma_{i,y} - \psi_y \sigma_y^2}{\psi_y^2 \sigma_y^2 + \sigma_i^2 - 2\psi_y \sigma_{i,y}} \tag{7}$$

In the data, $\sigma_{i,y} = 0.035$, i.e., the correlation between output and the policy rate in the data is positive. How can the VAR reconstruct such a correlation? The answer to this question is clearly affected by the estimate of the policy coefficient ψ_y.[16]

Suppose that $\psi_y = 0$. Then, the policy rate is exogenous in this simple model, and the only shock able to generate a positive correlation between output and the policy rate is the monetary policy shock. Hence, the impulse response of output to an unexpected monetary policy tightening will be bound to be positive as well. Assume now that $\psi_y < 0$. Then, once again, the impulse response of output will necessarily be positive, because both the numerator and the denominator of $\frac{IRF(0)_{y,i}}{\sigma_{MP}}$ are positive. If, instead, $\psi_y > 0$, then an "output shock" will be able to contribute to reconstruct $\sigma_{i,y} > 0$, and this will "leave room" to monetary policy shocks to generate $IRF(0)_{y,i} < 0$.

Figure 7 depicts the standardized impact response of output to a monetary policy shock ($IRF(0)_{y,i}/\sigma_{MP}$) as a function of ψ_y under a simple rule $i_t = \psi_y y_t + \sigma_{MP}\varepsilon_t^{MP}$, alongside the median values of ψ_y retrieved from Table 1.[17] Several findings are worth noting. First, $\psi_y > 0$

[16] The assumption that the variance-covariance matrix of the reduced-form residuals is positive definite ensures that the denominator of equation 7 is strictly larger than zero. Hence only the numerator contributes to the sign of the impulse response function.

[17] Our Appendix documents the robustness of our findings, particularly as regards the threshold value of ψ_y that implies a negative impact response, to also considering a systematic policy response to prices.

decreases the probability of a positive impact impulse response. However, depending on its data-driven magnitude, it can imply either a positive or a negative response. Second, narrative sign restrictions, as in Antolín-Díaz and Rubio-Ramírez (2018), prove highly effective in mitigating the large positive and counterintuitive impulse response associated with Uhlig (2005)'s identification. This is achieved by indirectly enforcing $\psi_y > 0$ without explicitly imposing it. Third, explicitly imposing $\psi_y > 0$ in addition to narrative sign restrictions allows for the estimation of a negative impact impulse response, which is consistent with our findings.

Phillips multiplier under different identification strategies. One of the most scrutinized statistics in monetary macroeconomics is the Phillips multiplier, i.e., the statistic that characterizes the inflation-real activity trade-off faced by a central bank. Barnichon and Mesters (2021) propose inference on the Phillips multiplier based on an instrumental variable regression of cumulative inflation on cumulative real activity (unemployment, in their case) using monetary shocks as instruments. Fundamentally, the approach requires computing the responses of inflation and real activity to an exogenous change in the policy rate. As stressed by Barnichon and Mesters (2021), there are multiple advantages related to the estimation of the Phillips multiplier via the combination of impulse responses of inflation and real activity. First, one avoids endogeneity issues that would otherwise arise when estimating a Phillips curve. Second, the multiplier characterizes the monetary policy trade-off nonparametrically, with the advantage of being robust to misspecification of the Phillips curve. Third, a correct assessment of the trade-off via the estimation of structural equations would also imply the specification and estimation of the IS curve, which is crucial for the monetary policy transmission. Hence, misspecification issues would get compounded because of the need to specify also such a demand schedule. Hence, following Barnichon and Mesters (2021), we circumvent these three issues by estimating the Phillips multiplier conditional on the three different identification strategies we focus on in this paper.

Formally, we measure the inflation-real activity trade-off via the following definition of the Phillips multiplier:

$$\mathcal{P}_h = \frac{\mathcal{R}_h^p}{\mathcal{R}_h^y} \tag{8}$$

where $\mathcal{R}_j^\pi$ and $\mathcal{R}_j^y$ are the causal impulse responses of the price level and output at horizon h to an unexpected increase in the policy rate. Intuitively, since both variables are trending and enter the VAR in logarithms, the expression above captures the (percent) deviation of the price level with respect to its trend that the central bank could obtain by inducing a *temporary* reduction in output of 1% (again, with respect to its trend) via a monetary policy shock.[18] Notice that a positive (negative) number is associated with a reduction (increase) of the price level with respect to its trend, coupled with output below (above) its trend.

Figure 8 (first row) displays our estimates of the Phillips multiplier. Sign restrictions only (left panel) point to negative values of the inflation-output Phillips multiplier. This result, which is mostly due to the mass of positive output responses to an unexpected increase in the policy rate, is associated with a negative sacrifice ratio, and it is clearly unpalatable from an economic standpoint. Adding narrative restrictions à la Antolín-Díaz and Rubio-Ramírez (2018) (central panel) lead to a dramatic improvement of the picture, with positive 2-year and 3-year ahead Phillips multipliers equal to 0.78 and 0.61, respectively. However, the 6-month and 1-year-ahead estimates are either close to zero (the latter reads 0.19) or surrounded by sizeable uncertainty (the former). Once again, these issues are due to the uncertain response of output in the short run. Sharpening estimates by adding policy coefficient restrictions (right panel) effectively works in favor of fixing these issues, in that: (i) the Phillips multiplier is stable across horizons; (ii) it is positive for $h = 6$, 12, 24, 36; (iii) it is precisely estimated. Interestingly, the multiplier associated with our "combo" of restrictions is 0.64 for the three-year horizon, which is in line with the estimate one can obtain at a 3-year horizon without policy coefficient restrictions. However, adding policy coefficient restrictions improves the precision of the estimate of the Phillips multiplier at that horizon too.

5 Empirical analysis: Euro area data

Are our empirical results on the marginal contribution of policy coefficient restrictions (with respect to narrative sign restrictions) limited to the US economy? We address this question by revisiting some recent findings by Badinger and Shiman (2023). Their aim is to identify the effects of conventional monetary policy shocks in a standard small-scale VAR that features indicators

[18] The Phillips multiplier is conceptually connected to the (inverse of) the sacrifice ratio, which is typically computed by assuming a *permanent* reduction of inflation of 1% and assessing the real activity-related costs of this reduction (see, e.g., Ball (2014)). Barnichon and Mesters (2021) compute the Phillips multiplier as follows: $\mathcal{P}_h = \frac{\frac{1}{h}\sum_{j=0}^{h}\mathcal{R}_j^{\pi}}{\frac{1}{h}\sum_{j=0}^{h}\mathcal{R}_j^{u}}$, where $\mathcal{R}_j^{\pi}$ and $\mathcal{R}_j^{u}$ capture the responses of inflation and unemployment (as opposed to prices and output) at horizon j. A link between their definition of the Phillips multiplier and ours exists if one: (i) considers a form of Okun's law connecting unemployment u_t and output growth Δy_t, and (ii) works out the following expression: $\mathcal{P}_h = \frac{\mathcal{R}_h^{p}}{\mathcal{R}_h^{y}} = \frac{\frac{1}{h}p_h}{\frac{1}{h}y_h} = \frac{\frac{1}{h}\sum_{j=0}^{h}\pi_j}{\frac{1}{h}\sum_{j=0}^{h}\Delta y_j}$, with p_h and y_h respectively being the impulse responses of the log-price level and log-output at horizon h, $\pi_j \equiv p_j - p_{j-1}, \Delta y_j \equiv y_j - y_{j-1}$ and $p_{-1} = y_{-1} = 0$, these last "initial conditions" representing the pre-shock values of the impulse responses of the price level and output in deviation with respect to their trends.

of real activity as well as a financial spread, i.e., the corporate bond spread. They propose to identify monetary policy shocks by imposing restrictions related to four events identified with a narrative analysis. The dates of these events are October and November 2008 and 2011, which are months in which financial markets were most surprised by interest rate announcements by the European Central Bank, as reflected by changes in overnight interest rates swaps. Based on financial markets' reactions in these dates, Badinger and Shiman (2023) impose signs on the interest rate shocks associated with these dates consistent with the assumption of expansive shocks in October 2008 and November 2011, and restrictive ones in November 2008 and October 2011. These (narrative) sign restrictions are combined with a magnitude restriction, i.e., the requirement that the unexpected change associated to the policy rate cut in November 2011 was predominantly driven by the monetary policy shock.[19] This restriction is motivated by the observation that the money market rate moved only on the day the lowered policy rates came into effect, i.e., it is unlikely other macroeconomic shocks played a role in that month. This parsimonious set of narrative restrictions leads the VAR to associate strong macroeconomic effects to a monetary policy shock. In particular, such a shock is followed by an increase in short and long-term rates, an appreciation of the US dollar, and a drop in prices and output. Their VAR also predicts an increase in the corporate bond spread (CBS), which however does not materialize on impact. The contemporaneous response of CBS to the monetary policy shock is what we now turn our attention to.

Figure 7 plots the density of the on-impact response of the CBS to a monetary policy shock identified by the VAR by Badinger and Shiman (2023), whose impulse responses we perfectly replicate. While such a spread is typically thought of as a fast-moving variable, the VAR associates a substantial uncertainty to the on-impact reaction to an unexpected interest rate hike. How so? Table 2 documents the coefficients associated with the VAR policy function in Badinger and Shiman (2023). As we can see, their identification strategy is able to correctly pin down the systematic positive response of the ECB to output and inflation, at least as far as the posterior medians are concerned. However, it also features a zero response to CBS, with credible intervals that contain positive realizations. Differently, the attention paid to financial indicators by central banks when monitoring the extant economic conditions makes a *negative* response to movements in proxies for the financial cycle likely (see, e.g., the analysis with US data by Caldara and Herbst (2019)) and, under certain conditions, optimal (Cúrdia and Woodford (2010)). Such a negative response can be rationalized by the recessionary and deflationary effects triggered by

[19] Using the notation of footnote 5, the magnitude restriction used by Badinger and Shiman (2023) requires $|H_j^i(t)| > 0.5 \cdot |u_t^i|$, where $H_j^i(t)$ is the contribution of shock j to the unexpected change in variable i in period t and u_t^i is the forecast error in period t on variable i.

a corporate bond spread shock (see, e.g., Gilchrist and Zakrajek (2012) for evidence based on US data, and Gilchrist and Mojon (2018) for similar evidence with Euro area data), which call for the implementation of a monetary policy easing to return inflation to the target. In the raw data, the correlation between the Eonia rate and the CBS is 0.33. Leveraging on the analysis developed in Section 4, a possibility is that part of the rotations of the set-identification strategy proposed by Badinger and Shiman (2023) match the likelihood via *positive* values of the policy coefficient ψ_{CBS}, which mechanically generate a positive correlation between the policy rate and the spread. Imposing $\psi_{CBS} < 0$ – on top of $\psi_p > 0$ and $\psi_y > 0$ – as we do avoids this economically implausible representation of the systematic policy conduct by the ECB while retaining the informativeness of the narrative restrictions cleverly proposed by Badinger and Shiman (2023). Figure 7 overplots the so-obtained density of the contemporaneous response of CBS to a policy rate shock. First, the posterior median is clearly positive (0.48%, vs. -0.06% in the absence of policy coefficient restrictions). Second, 93% of the realizations of the spread are positive (vs. just 47%) in the benchmark model.[20]

What about the response of the other modeled variables? Figure 8 plots the dynamic responses of all modeled variables to a monetary policy shock. Policy coefficient restrictions lead to stronger responses of M1 and the EUR/USD exchange rate and, in the short-run, also of CBS. Moreover, the credible sets surrounding all variables are tighter. This evidence speaks in favor of the relevant sharpening of the identification of monetary policy shocks one can obtain by applying policy coefficient restrictions also to Euro area data and for analysis concerned with the interaction between monetary policy and financial conditions.

6 Conclusions

This paper combines narrative restrictions à la Antolín-Díaz and Rubio-Ramírez (2018) with restrictions on policy coefficients as those proposed by Arias, Caldara, and Rubio-Ramírez (2019). The idea is to exploit the positive aspects of both strategies (punctual characterization of monetary policy shocks in specific dates the former set of restrictions, sensible description of the systematic policy conduct the latter one) while limiting their possible weaknesses (misrepresentation of the systematic policy conduct the former, possible misattribution of changes in the policy rate to shocks other than policy ones in key dates the latter).

Our three main findings are the following.

First, Monte Carlo simulations conducted with the workhorse macroeconomic framework

[20] Our result on the contemporaneous response of the CBS crucially hinges on the restriction $\psi_{CBS} < 0$ that we impose.

proposed by Smets and Wouters (2007) point to a quantitatively important marginal contribution by policy coefficient restrictions when added to narrative ones for the correct estimation of the output effects of monetary policy shocks.

Second, we show that imposing policy coefficient restrictions materially alters the estimated response of output to a monetary policy shock when combined with narrative sign restrictions in an empirical application using a standard U.S. dataset. Specifically, these restrictions yield a larger and more precisely estimated short-run response of output to a monetary policy shock. As discussed in the paper, the reason is that policy coefficient restrictions prevent the VAR from replicating correlations in the data with policy coefficients that make the description of systematic monetary policy implausible. In doing so, policy coefficient restrictions assign more weight to monetary policy shocks (vs. systematic monetary policy) to replicate the data. Conditional on the data at hand, we also show that policy coefficient restrictions alone are actually able to pin down the contribution of monetary policy shocks in key dates (e.g., October 1979) to the unexpected change of the policy rate. We also formally show that there exists a mapping between the systematic policy response to output and the contemporaneous output effects of a monetary policy shock that can reconcile the disparate estimates of such effects obtained with different identification strategies. Moreover, we show that different restrictions may lead to different assessments of the inflation-real activity policy trade-off, with the combination of sign, narrative, and policy coefficient restrictions providing a more stable and statistically precise quantification of the Phillips multiplier.

Third, working with Euro area data, we show that policy coefficient restrictions can be fruitfully added to narrative restrictions to identify the contemporaneous response of financial indicators (specifically, the corporate bond spread) to a monetary policy shock. Assuming a policy easing as a response to a jump in the corporate bond spread tilts rightward the empirical density of the contemporaneous responses of such a spread to the monetary policy shock and makes it narrower. Moreover, it sharpens the credible sets of the impulse responses of all variables in the VAR.

We conclude with two considerations. First, our investigation suggests to impose policy coefficient restrictions on top of other identification constraints whenever such restrictions are economically sensible and uncontroversial. Second, our simulation results do confirm that narrative sign restrictions are powerful and work in favor of sharpening the identification of monetary policy shocks. If a researcher can rely on unquestionable narrative evidence that can be translate into narrative restrictions, she should implement them as well. Hence, we argue that we should view narrative and policy coefficient restrictions as "complements", not "substitutes".

References

Andrade, P., F. Ferroni, and L. Melosi (2023): "Higher-Order Moment Inequality Restrictions for SVARs," Federal Reserve Bank of Chicago Working Paper No. 2023-28.

Antolín-Díaz, J., and J. F. Rubio-Ramírez (2018): "Narrative Sign Restrictions," *American Economic Review*, 108(10), 2802–2829.

Arias, J. E., D. Caldara, and J. Rubio-Ramírez (2019): "The Systematic Component of Monetary Policy in SVARs: An Agnostic Identification Procedure," *Journal of Monetary Economics*, 101, 1–13.

Arias, J. E., J. F. Rubio-Ramírez, and D. F. Waggoner (2018): "Inference Based on SVARs Identified with Sign and Zero Restrictions: Theory and Applications," *Econometrica*, 86(2), 685–720.

Aruoba, S., and T. Drechsel (2024): "Identifying Monetary Policy Shocks: A Natural Language Approach," mimeo, available at https://aruoba.econ.umd.edu/.

Badinger, H., and S. Shiman (2023): "Measuring Monetary Policy in the Euro Area Using SVARs with Residual Restrictions," *American Economic Journal: Macroeconomics*, 15(2), 279–305.

Ball, L. (2014): "The Case for a Long-Run Inflation Target of Four Percent," International Monetary Fund Working Paper No. 14/92.

Barnichon, R., and G. Mesters (2021): "The Phillips multiplier," *Journal of Monetary Economics*, 117, 689–705.

Bauer, M. D., and E. T. Swanson (2023): "An Alternative Explanation for the "Fed Information Effect"," *American Economic Review*, 113(3), 664–700.

Braun, R., and R. Brüggemann (2023): "Identification of SVAR Models by Combining Sign Restrictions With External Instruments," *Journal of Business and Economic Statistics*, 41(4), 1077–1089.

Brennan, C. M., M. M. Jacobson, C. Matthes, and T. B. Walker (2024): "Monetary Policy Shocks: Data or Methods?," Federal Reserve Board and Indiana University, mimeo.

Caldara, D., and E. Herbst (2019): "Monetary Policy, Real Activity, and Credit Spreads: Evidence from Bayesian Proxy SVAR," *American Economic Journal: Macroeconomics*, 11(1), 157–192.

Caldara, D., and C. Kamps (2017): "The Analytics of SVARs: A Unified Framework to Measure Fiscal Multipliers," *Review of Economic Studies*, 84, 1015–1040.

Canova, F., and G. de Nicoló (2002): "Monetary Disturbances Matter for Business Fluctuations in the G-7," *Journal of Monetary Economics*, 49, 1131–1159.

Canova, F., and J. P. Pina (2005): "What VAR Tell Us About DSGE Models?," in C. Diebolt. C. and Kyrtsou (eds): New Trends In Macroeconomics, Springer Verlag, New York, NY, 89-124.

Carriero, A., M. Marcellino, and T. Tornese (2024): "Blended identification in structural VARs," *Journal of Monetary Economics*, 146(103581).

Cúrdia, V., and M. Woodford (2010): “Credit Spreads and Monetary Policy,” *Journal of Money, Credit and Banking*, 42, 3–35.

Eichenbaum, M. (1992): “Comment on "Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy" by C.A. Sims,” *European Economic Review*, 36(5), 1001–1011.

Faust, J. (1998): “The robustness of identified VAR conclusions about money,” *Carnegie Rochester Conference Series on Public Policy*, 49, 207–244.

Faust, J., E. T. Swanson, and J. H. Wright (2004): “Identifying VARs based on high frequency futures data,” *Journal of Monetary Economics*, 51, 1107–1131.

Galí, J. (2015): *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications - Second Edition*. Princeton University Press. Princeton, New Jersey.

Gertler, M., and P. Karadi (2015): “Monetary Policy Surprises, Credit Costs, and Economic Activity,” *American Economic Journal: Macroeconomics*, 7(1), 44–76.

Giacomini, R., T. Kitagawa, and M. Read (2021): “Identification and Inference Under Narrative Restrictions,” available at https://arxiv.org/abs/2102.06456.

——— (2022): “Narrative Restrictions and Proxies,” *Journal of Business Economic Statistics*, 40(4), 1415–1425.

Gilchrist, S., and B. Mojon (2018): “Credit Risk in the Euro Area,” *Economic Journal*, 128, 118–158.

Gilchrist, S., and E. Zakrajek (2012): “Credit Spreads and Business Cycle Fluctuations,” *American Economic Review*, 102(4), 1692–1720.

Gürkaynak, R. S., B. Sack, and E. Swanson (2005): “The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models,” *American Economic Review*, 95(1), 425–36.

Jarociński, M., and P. Karadi (2020): “Deconstructing Monetary Policy Surprises: The Role of Information Shocks,” *American Economic Journal: Macroeconomics*, 12(2), 1–43.

Kilian, L., and D. Murphy (2012): “Why Agnostic Sign Restrictions Are Not Enough: Understanding the Dynamics of Oil Market VAR Models,” *Journal of the European Economic Association*, 10(5), 1166–1188.

Leeper, E. M., C. A. Sims, and T. Zha (1996): “What Does Monetary Policy Do?,” *Brookings Papers on Economic Activity*, 2, 1–63.

Leeper, E. M., and T. Zha (2003): “2003,” *Modest Policy Interventions*, 50(8), 1673–1700.

Miranda-Agrippino, S., and G. Ricco (2021): “The Transmission of Monetary Policy Shocks,” *American Economic Journal: Macroeconomics*, 13(3), 74–107.

Paustian, M. (2007): “Assessing Sign Restrictions,” *B.E. Journal of Macroeconomics*, 7:1 (Topics), Article 23, 1–31.

Plagborg-Møller, M. (2022): “Discussion of Giacomini, Kitagawa, and Read’s "Narrative Restrictions and Proxies",” *Journal of Business and Economic Statistics*, 40(4)(1434-1437).

Romer, C., and D. Romer (2004): “A New Measure of Monetary Policy Shocks: Derivation and Implications,” *American Economic Review*, 94(4), 1055–1084.

Rubio-Ramírez, J. F., D. F. Waggoner, and T. Zha (2010): “Structural Vector Autoregressions: Theory of Identification and Algorithms for Inference,” *Review of Economic Studies*, 77, 665–696.

Sims, C. (1980): “Macroeconomics and Reality,” *Econometrica*, 48(1), 1–48.

Sims, C., and T. Zha (2006): “Does monetary policy generate recessions?,” *Macroeconomic Dynamics*, 10, 231–272.

Smets, F., and R. Wouters (2007): “Shocks and Frictions in US Business Cycle: A Bayesian DSGE Approach,” *American Economic Review*, 97(3), 586–606.

Stock, J. H., and M. W. Watson (2012): “Disentangling the Channels of the 2007-2009 Recession,” *Brookings Papers on Economic Activity*, Spring, 81–135.

Swanson, E. T. (2021): “Measuring the Effects of Federal Reserve Forward Guidance and Asset Purchases on Financial Markets,” *Journal of Monetary Economics*, 118, 32–53.

——— (2023): “The Macroeconomic Effects of the Federal Reserve’s Conventional and Unconventional Monetary Policies,” NBER Working Paper No. 31603.

Swanson, E. T., and V. Jayawickrema (2023): “Speeches by the Fed Chair Are More Important than FOMC Announcements: An Improved High-Frequency Measure of U.S. Monetary Policy Shocks,” University of California at Irvine, mimeo.

Taylor, J. (1993): “Discretion versus Policy Rules in Practice,” *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214.

Uhlig, H. (2005): “What Are the Effects of Monetary Policy? Results from an Agnostic Identification Procedure,” *Journal of Monetary Economics*, 52, 381–419.

——— (2017): “Shocks, Sign Restrictions, and Identification,” in Bo Honoré, Ariel Pakes, Monika Piazzesi, and Larry Samuelson (eds): Advances in Economics and Econometrics, Vol. 1 and 2: Eleventh World Congress, Chapter 4, Cambridge University Press, Cambridge.

Wolf, C. K. (2020): “SVAR (Mis-)Identification and the Real Effects of Monetary Policy,” *American Economic Journal: Macroeconomics*, 12(4), 1–32.

——— (2023): “Fiscal Stimulus and the Systematic Response of Monetary Policy,” *American Economic Association Papers and Proceedings*, 113, 388–393.

Woodford, M. (2003): “Optimal Interest-Rate Smoothing,” *Review of Economic Studies*, 70, 861–886.

Tables

Sign restrictions of Uhlig (2005)					
Coefficient	ψ_y	ψ_p	ψ_{pc}	ψ_{tr}	ψ_{nbr}
Median	-0.37	1.93	0.11	0.10	0.03
68% Prob. Interval	[-2.45; 0.82]	[-0.06; 6.15]	[0.00; 0.36]	[-0.44; 0.66]	[-0.41; 0.65]
95% Prob. Interval	[-14.18; 14.49]	[-26.66; 30.33]	[-1.50; 1.79]	[-3.59; 3.79]	[-4.25; 4.39]
Sign and narrative restrictions of Antolín-Díaz and Rubio-Ramírez (2018)					
Coefficient	ψ_y	ψ_p	ψ_{pc}	ψ_{tr}	ψ_{nbr}
Median	0.03	1.33	0.05	-0.02	0.05
68% Prob. Interval	[-0.34; 0.38]	[0.66; 1.98]	[0.02; 0.09]	[-0.20; 0.11]	[-0.07; 0.22]
95% Prob. Interval	[-0.70; 0.73]	[0.28; 2.68]	[-0.00; 0.14]	[-0.40; 0.18]	[-0.14; 0.41]
Policy coefficient restrictions of Arias et. al (2019) and sign & narrative restrictions of ADRR					
Coefficient	ψ_y	ψ_p	ψ_{pc}	ψ_{tr}	ψ_{nbr}
Median	0.24	1.25	0.05	0	0
68% Prob. Interval	[0.07; 0.50]	[0.73; 1.88]	[0.02; 0.10]		
95% Prob. Interval	[0.00; 1.08]	[0.01; 3.32]	[-0.02; 0.19]		

Table 1: **Contemporaneous coefficients in the monetary policy equations under different identification strategies: US data**. The entries in the table denote the posterior median estimates of the contemporaneous coefficients in the monetary equation under several identification schemes (ADRR stands for the identification scheme adopted in Antolín-Díaz and Rubio-Ramírez (2018) to identify monetary policy shocks, i.e., their Narrative Sign Restrictions 4 and 5 on top of the Uhlig (2005) sign restrictions). The 68 and 95% equal-tailed posterior probability intervals are reported in brackets. See Arias, Caldara, and Rubio-Ramírez (2019) for details.

Badinger & Schiman (2023)					
Coefficient	ψ_y	ψ_p	ψ_{CBS}	ψ_{M1}	$\psi_{EUR/USD}$
Median	0.10	0.36	0.00	0.13	0.01
68% Prob. Interval	[0.03; 0.22]	[-0.07; 0.78]	[-0.06; 0.08]	[0.03; 0.26]	[-0.02; 0.05]
95% Prob. Interval	[-0.08; 0.44]	[-1.29; 2.16]	[-0.14; 0.28]	[-0.21; 0.51]	[-0.08; 0.20]
Badinger & Schiman (2023) and PCR					
Coefficient	ψ_y	ψ_p	ψ_{CBS}	ψ_{M1}	$\psi_{EUR/USD}$
Median	0.08	0.36	-0.04	0.12	0.02
68% Prob. Interval	[0.02; 0.17]	[0.17; 0.74]	[-0.08; -0.01]	[0.05; 0.24]	[-0.01; 0.06]
95% Prob. Interval	[0.00; 0.35]	[0.05; 1.13]	[-0.17; 0.00]	[-0.04; 0.60]	[-0.03; 0.17]

Table 2: **Contemporaneous coefficients in the monetary policy equations under different identification strategies: Euro area data**. The entries in the Table denote the posterior median estimates of the contemporaneous coefficients in the monetary equation under several identification schemes. The 68 and 95% equal-tailed posterior probability intervals are reported in brackets. See Arias, Caldara, and Rubio-Ramírez (2019) for details.

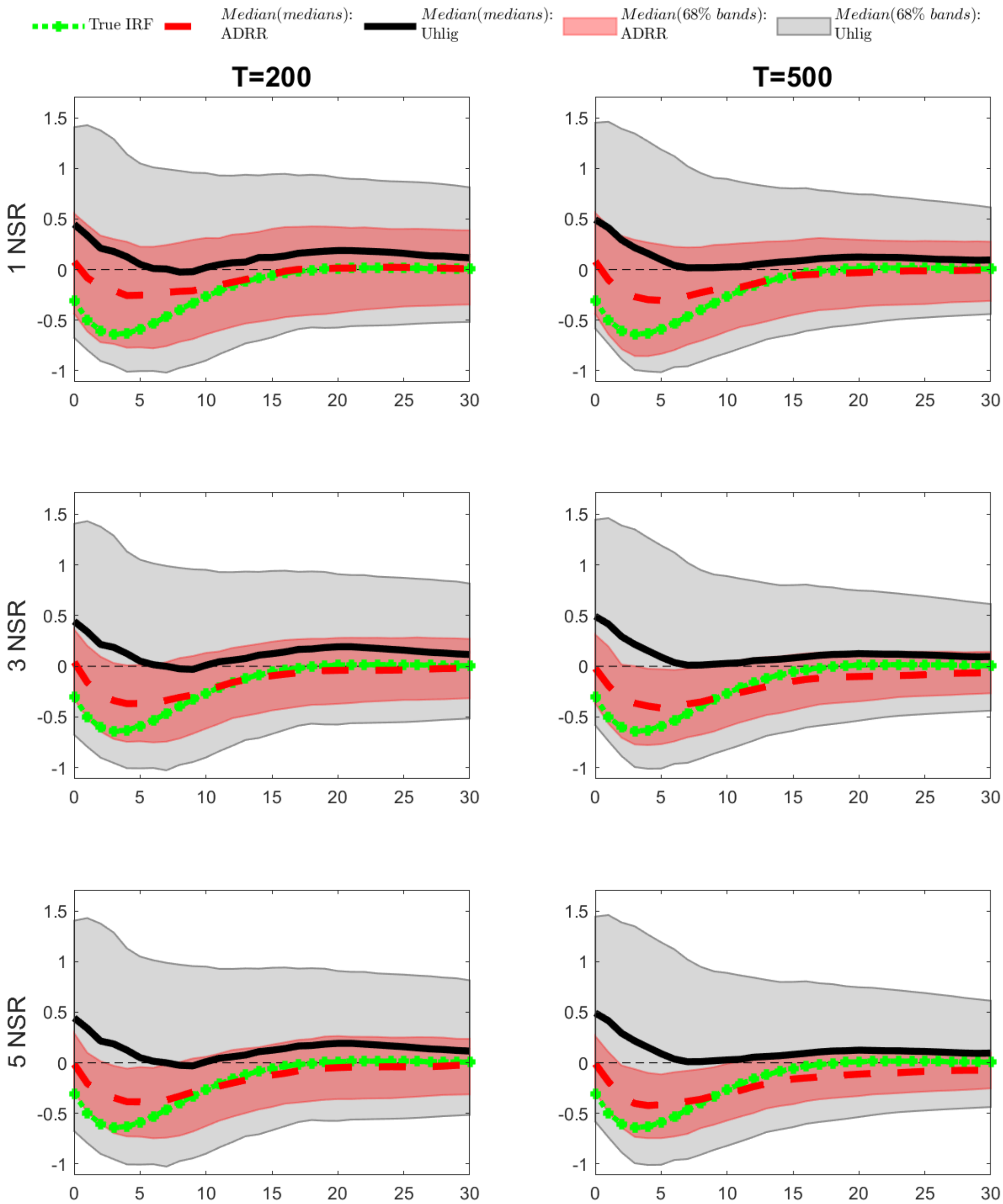

Figure 1: **Monte Carlo simulations: Output response to a monetary policy shock under either (Uhlig's) SR or SR & NSR à la ADRR.** The starred black line represents the output response to a monetary policy shock in the Smets and Wouters (2007) DSGE model, which has been used to simulate 100 datasets of 200 observations (left panels) and 100 datasets of 500 observations (right panels). The red (grey) shaded area represents the *median* of the 68 percent credible sets for the output IRF across the 100 different datasets and the red dashed (grey solid) lines are the *median* of the median IRFs of output across the 100 datasets when using SR & NSR à la ADRR (only SR). SR stands for sign restriction as in Uhlig (2005). The first, second, and third rows show results when using one, three, or five dates, respectively. The monetary policy shock has been normalized to have an impact of 25 basis points on the federal funds rate.

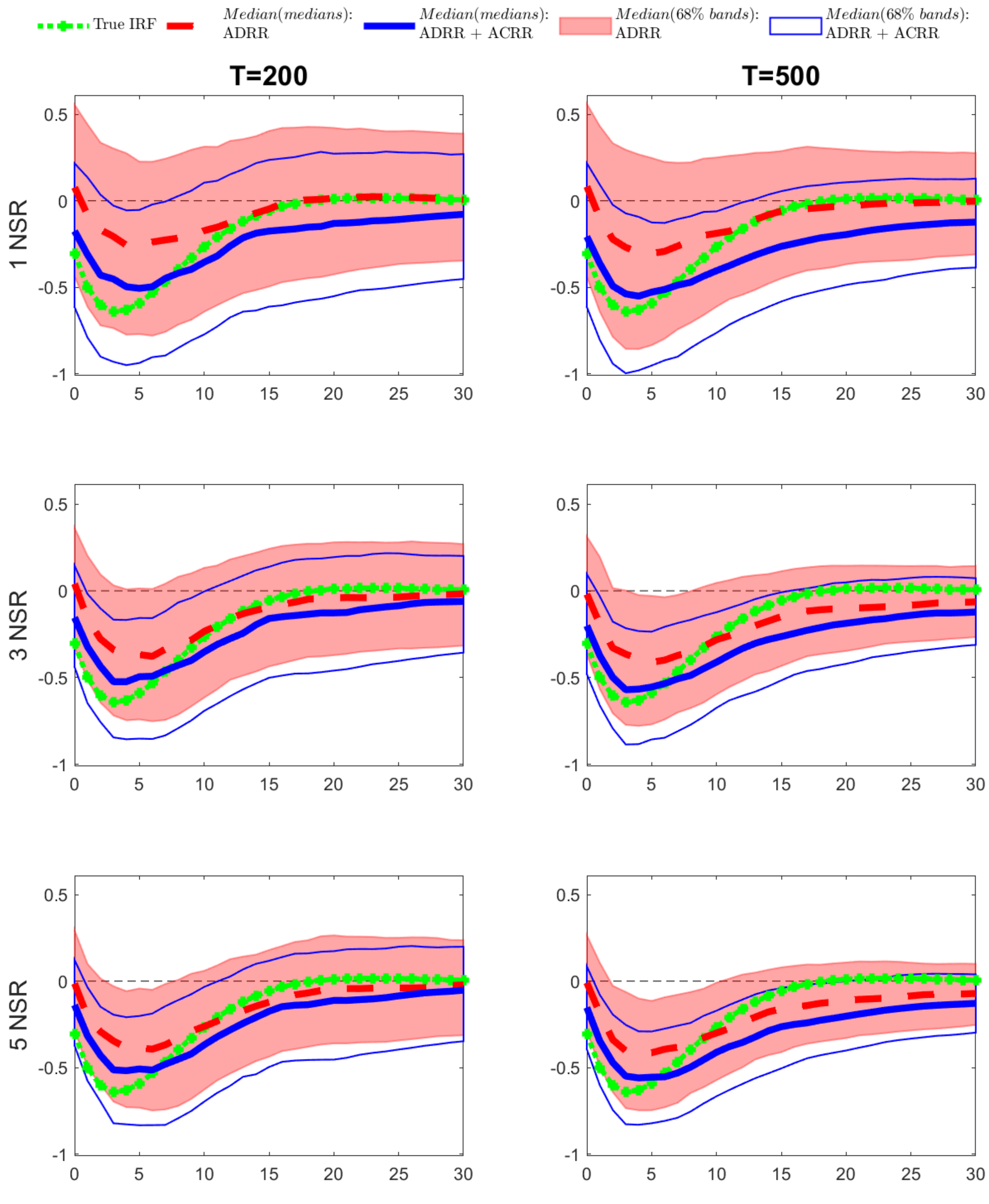

Figure 2: **Monte Carlo simulations: Output response to a monetary policy shock under either SR & NSR à la ADRR or SR & NSR à la ADRR *plus* PCR à la ACRR.** The starred black line represents the output response to a monetary policy shock in the Smets and Wouters (2007) DSGE model, which has been used to simulate 100 datasets of 200 observations (left panels) and 100 datasets of 500 observations (right panels). The red shaded (blue contoured) area represents the *median* of the 68 percent credible sets for the output IRF across the 100 different datasets and the red dashed (blue solid) lines are the *median* of the median IRFs of output across the 100 datasets when using SR & NSR à la ADRR (Arias, Caldara, and Rubio-Ramírez (2019) Restrictions 1 and 2 in addition to SR & NSR à la ADRR ones). The first, second, and third rows show results when using one, three, or five dates, respectively. The monetary policy shock has been normalized to have an impact of 25 basis points on the federal funds rate.

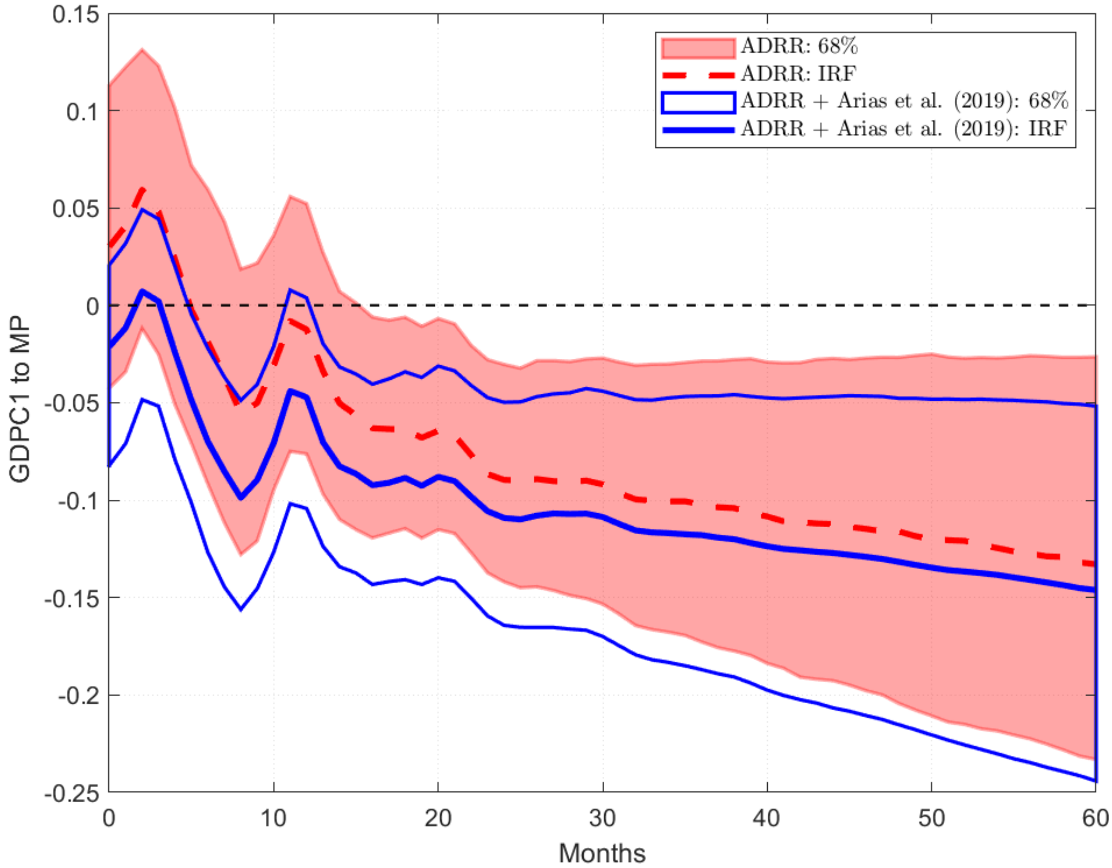

Figure 3: **Output response to a monetary policy shock: Adding policy coefficient restrictions à la Arias, Caldara, and Rubio-Ramírez (2019) to ADRR restrictions.** The red shaded (blue contoured) area represents the 68 percent credible sets for the output IRF and the red dashed (blue solid) lines are the median IRF of output when using the ADRR restrictions (Arias, Caldara, and Rubio-Ramírez (2019) Restrictions 1 and 2 in addition to ADRR ones). The monetary policy shock has been normalized to have an impact of 25 basis points on the federal funds rate.

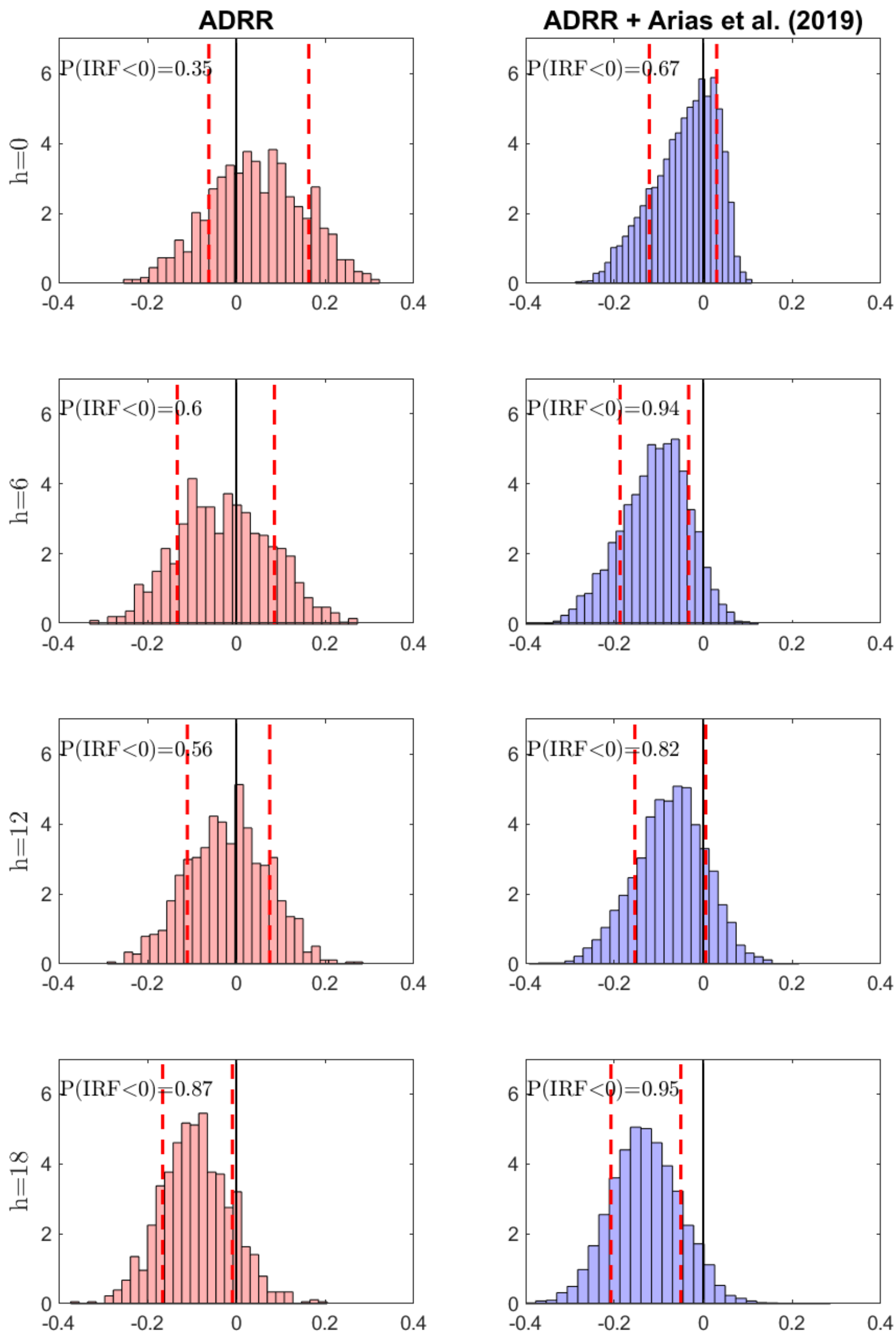

Figure 4: **Output response to a monetary policy shock: Distribution at several horizons when adding policy coefficient restrictions à la Arias, Caldara, and Rubio-Ramírez (2019) to ADRR restrictions.** Left (Right) panels: The red (blue) histogram represents the distributions of the output IRF for all retained models per each selected horizon, $h = 0, 6, 12, 18$, when using the ADRR restrictions (Arias, Caldara, and Rubio-Ramírez (2019) Restrictions 1 and 2 in addition to ADRR ones). The dashed red vertical lines indicate the corresponding 68% credible sets. The number in the top right panel is the share of retained models that implies a negative response of output per each selected horizon. The monetary policy shock has been normalized to have an impact of 25 basis points on the federal funds rate.

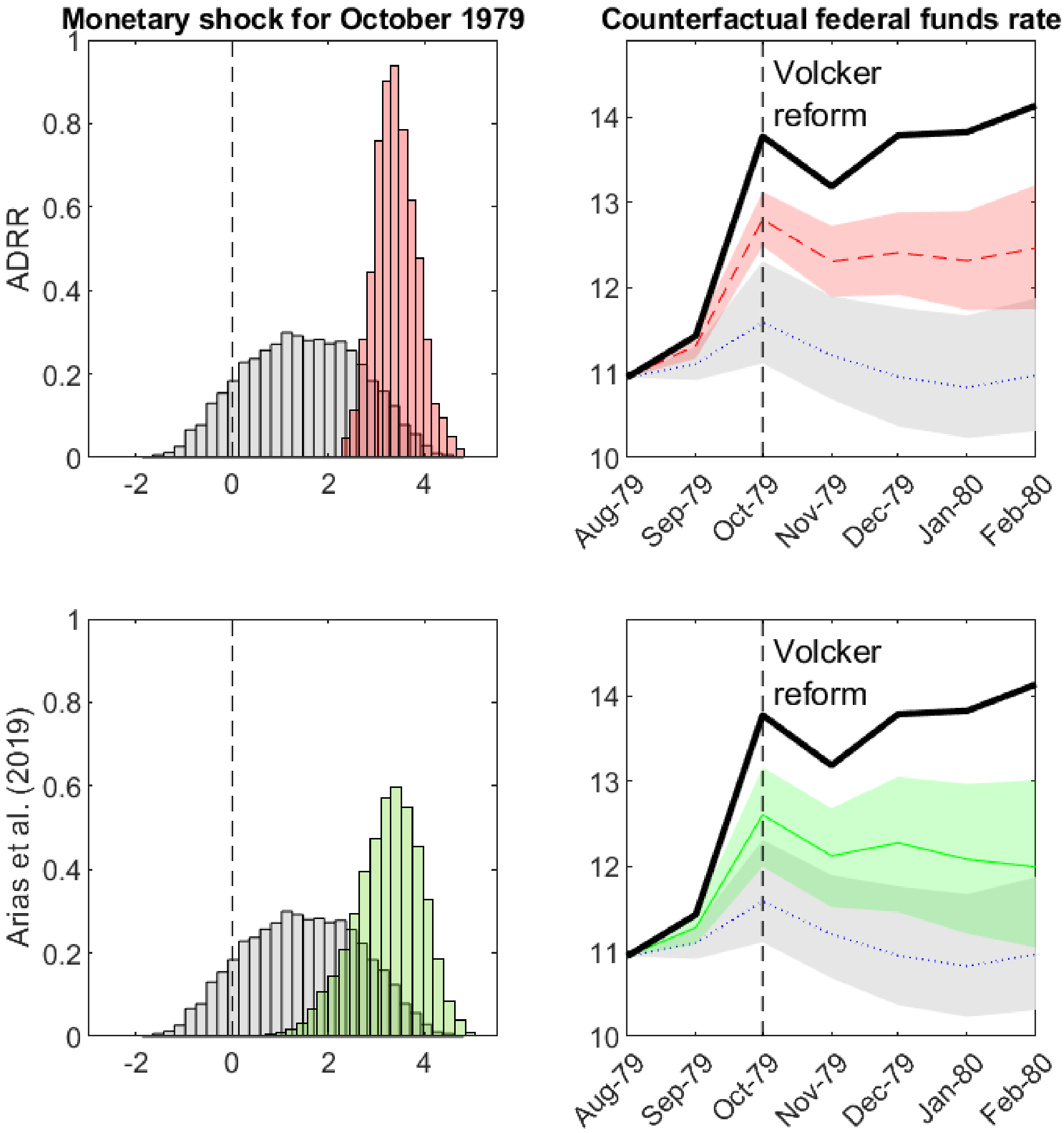

Figure 5: **Results around October 1979 either with narrative restrictions à la ADRR or with policy coefficient restrictions à la Arias, Caldara, and Rubio-Ramírez (2019).** Left panels: The grey histogram plots the posterior distribution of the monetary policy shock for October 1979 using Uhlig (2005) identification. The red (green) histogram plots the same distribution after incorporating Narrative Restrictions 4 and 5 in ADRR (Restrictions 1 and 2 in Arias, Caldara, and Rubio-Ramírez (2019)). Right panels: the solid black thick line represents the actual federal funds rate and the dotted line is the median of the counterfactual federal funds rate resulting from excluding all non-monetary structural shocks using the baseline identification. The grey bands represent 68 percent credible sets around the median. The dashed red (solid green) thin line and red (green) shaded area plot the same result after incorporating Narrative Restrictions 4 and 5 in ADRR (Restrictions 1 and 2 in Arias, Caldara, and Rubio-Ramírez (2019)).

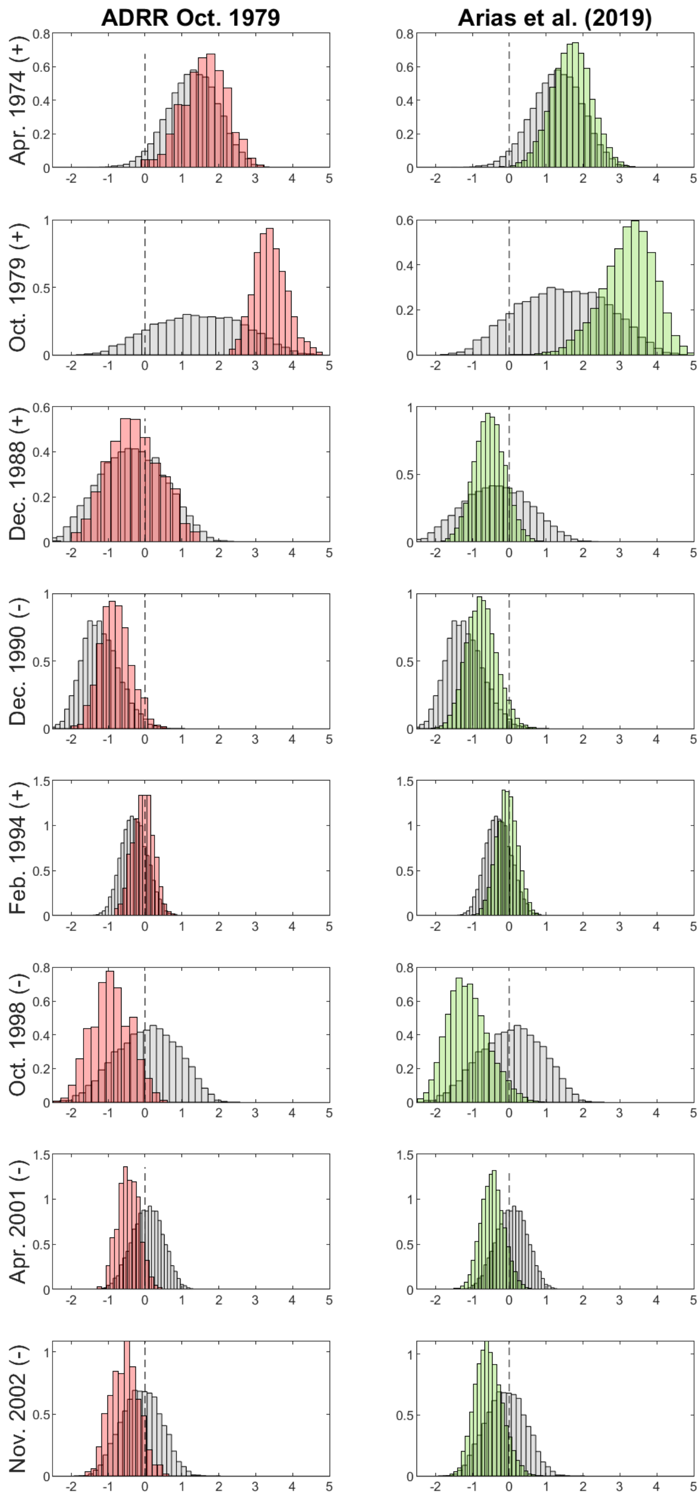

Figure 6: **Results around selected dates conditional on the imposition of the October 1979 narrative restrictions à la ADRR or with policy coefficient restrictions à la Arias, Caldara, and Rubio-Ramírez (2019).** The grey histogram plots the posterior distribution of the monetary policy shock for the selected dates considered by ADRR using Uhlig (2005) identification. Left panels: The red histogram plots the same distribution after incorporating Narrative Restrictions 4 and 5 in ADRR). Right panels: The green histogram plots the same distribution after incorporating the policy coefficient restrictions 1 and 2 in Arias, Caldara, and Rubio-Ramírez (2019).

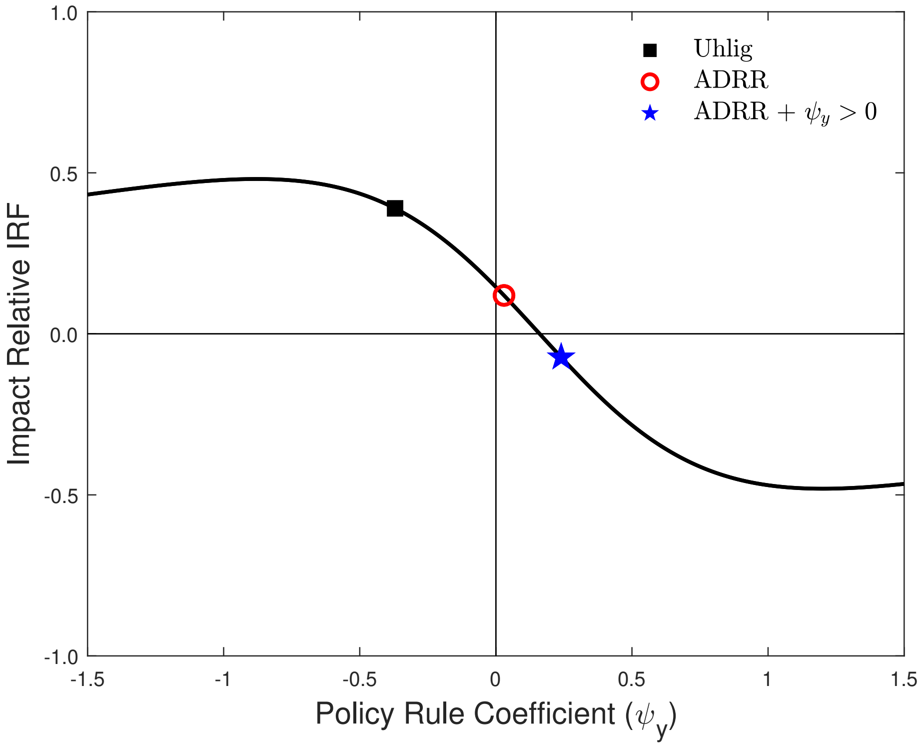

Figure 7: **Impact relative impulse response as a function of the policy coefficient ψ_y (simple monetary rule).** The solid line plots the relationship between the relative impulse response to a monetary policy shock ($IRF(0)_{y,i}/\sigma_{MP}$) and the systematic response of monetary policy to output under the simple monetary rule $i_t = \psi_y y_t + \sigma_{MP}\varepsilon_t^{MP}$. The line is plot on the basis of equation 11 in Caldara and Kamps (2017). More details are provided in the Appendix.

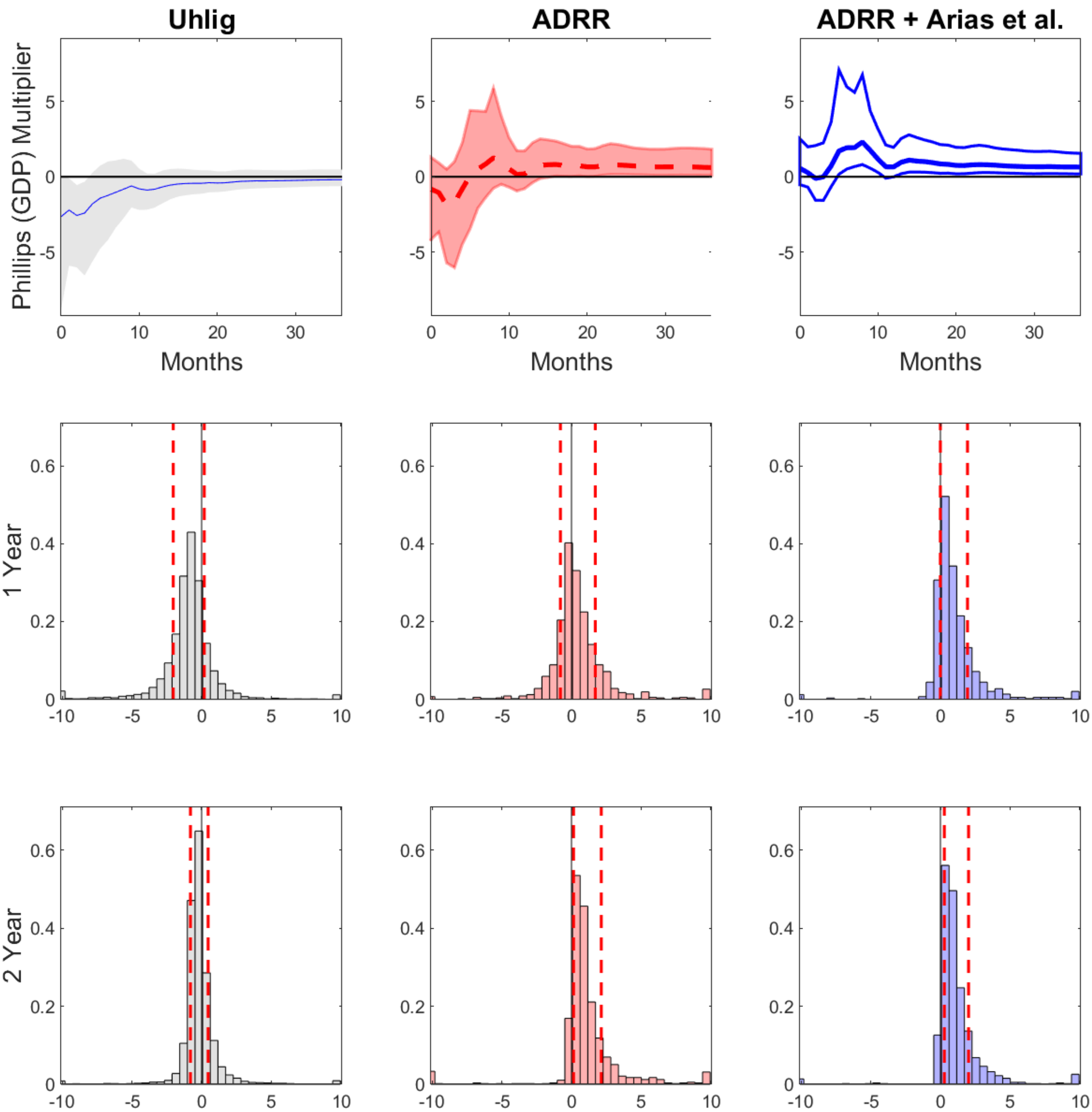

Figure 8: **Estimates of the Phillips multiplier across three different identification strategies.** Left/center/right columns: Estimates of the Phillips multiplier over horizons (top panel) and conditional on 1-year and 2-year horizons (middle and bottom panels) obtained with sign restrictions on impulse responses only/sign and narrative sign restrictions/sign, narrative, and policy coefficient restrictions.

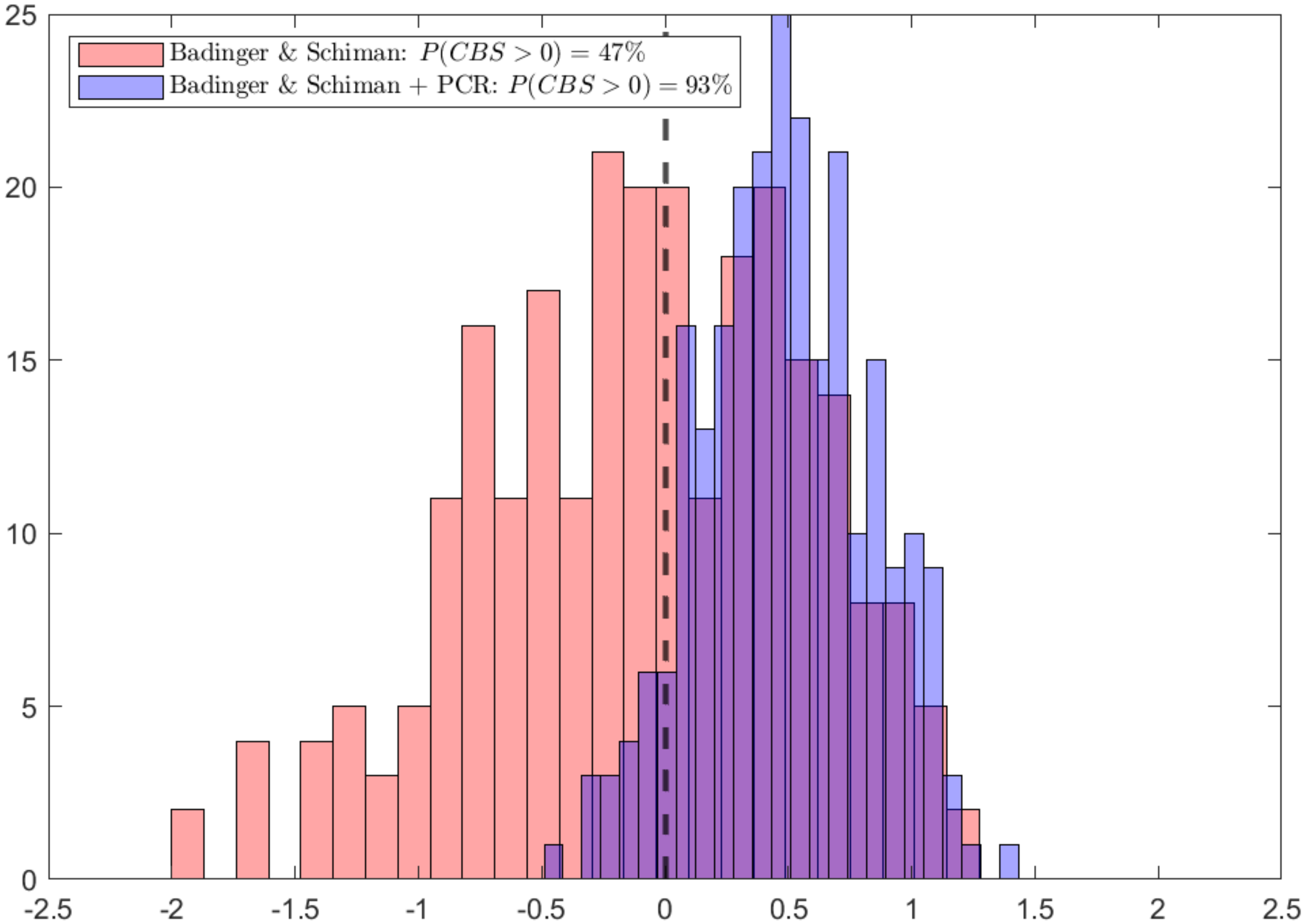

Figure 9: **Contemporaneous response of the corporate bond spread to a monetary policy shock: Role of policy coefficient restrictions.** Histograms plot the posterior distribution of the contemporaneous response of the corporate bond spread (CBS) to a 25 basis points unexpected hike in the Eonia rate under two different identification strategies. Red histogram: Identification achieved via the narrative restrictions by Badinger and Shiman (2023). Blue histogram: Identification achieved by adding policy coefficient restrictions on the systematic response to inflation, output, and the corporate bond spread to the narrative restrictions by Badinger and Shiman (2023).

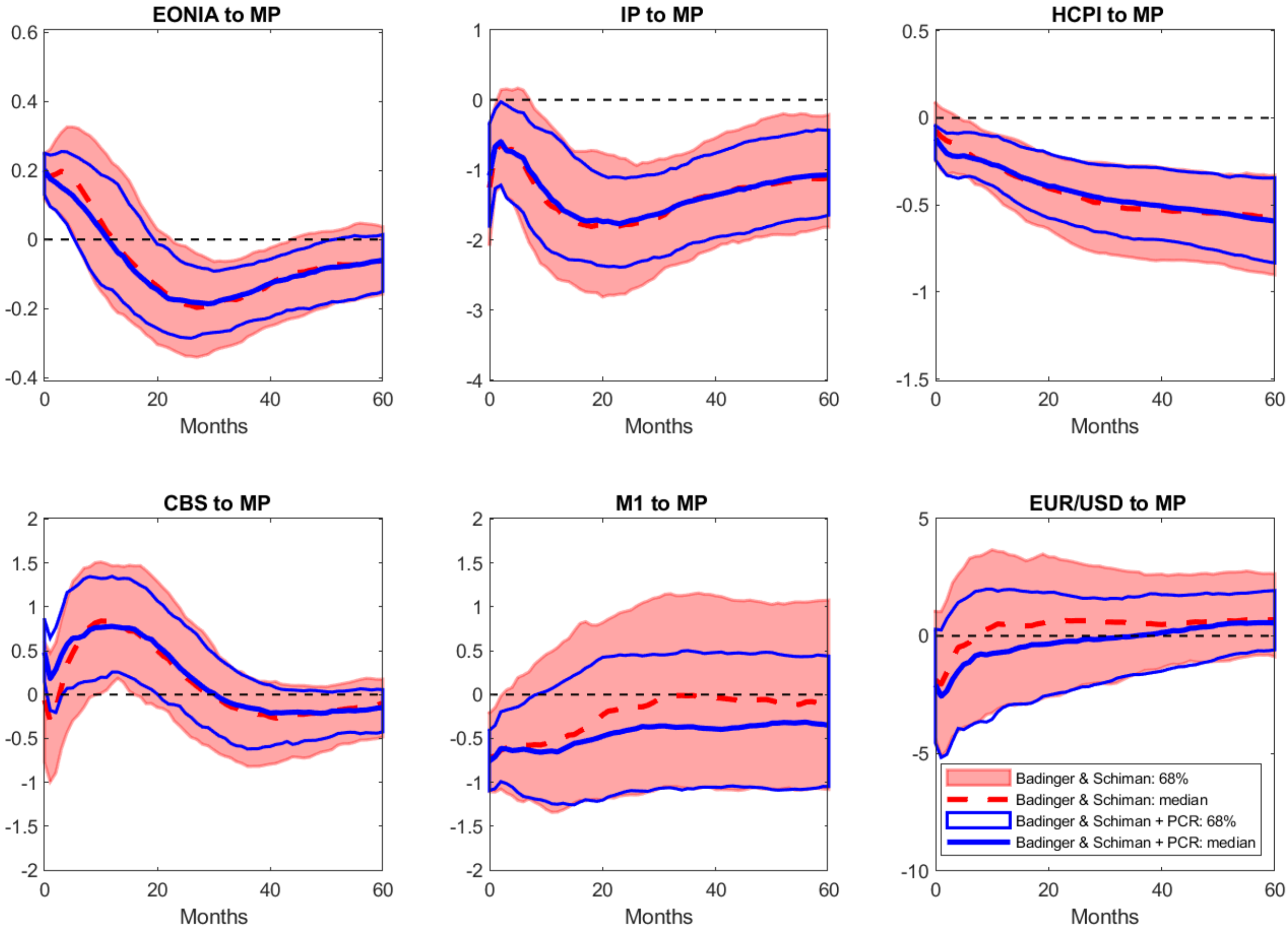

Figure 10: **Dynamic macroeconomic responses to a monetary policy shock: Role of policy coefficient restrictions.** Responses to a 25 basis points unexpected hike in the Eonia rate under two different identification strategies. Red dashed responses (reddish areas): Posterior medians (68% credible sets) obtained with the identification via narrative restrictions proposed by Badinger and Shiman (2023). Blue solid middle lines (blue external lines): Posterior medians (68% credible sets) obtained by adding policy coefficient restrictions on the systematic response to inflation, output, and the corporate bond spread (CBS) to the narrative restrictions by Badinger and Shiman (2023).

Appendix of the paper "Monetary Policy Shocks and Narrative Restrictions: Rules Matter" by Efrem Castelnuovo, Giovanni Pellegrino, and Laust L. Særkjær

This Appendix contains additional material with respect to the contents of our paper. It features:

A1) The proof of the expression mapping the policy response to output to the impulse response function of output to a monetary policy shock;

A2) A generalization of the previous expression that considers the possibility of a systematic response of the policy rate to prices;

A3) Various additional empirical findings obtained with the U.S. dataset;

A4) Additional results based on our Monte Carlo simulations.

A1: Proof of the expression mapping the policy response to output to the impulse response of output to a monetary policy shock

We derive expression (7) in the paper, which maps the policy response to output to the on-impact output effects of a monetary policy shock. As in Caldara and Kamps (2017), we focus on the simple monetary policy rule considering only a policy response to output, i.e., $i_t = \psi_y y_t + \sigma_{MP}\varepsilon_t^{MP}$, where $\varepsilon_t^{MP} \sim \mathcal{N}(0,1)$.[21] Following Caldara and Kamps (2017), we focus on the mapping between reduced-form residuals and policy shocks, which is as informative about the role played by the systematic response to output for the output reaction to monetary policy shocks. Then, we have:

$$u_{i,t} = \psi_y u_{y,t} + \sigma_{MP}\varepsilon_t^{MP},$$

from which we can rewrite the monetary policy shocks in terms of reduced-form residuals as:

$$\varepsilon_t^{MP} = \frac{u_{i,t} - \psi_y u_{y,t}}{\sigma_{MP}}. \tag{A.1}$$

[21] A generalization of the expression derived in this Section that also admits a policy response to inflation is offered in the following Section of this Appendix.

The on-impact impulse response of output (y) to a monetary policy shock affecting the interest rate (i), $IRF(0)_{y,i}$, can be computed by projecting the structural shocks (ε_t^{MP}) on the reduced-form residuals from the output equation ($u_{y,t}$), yielding:

$$IRF(0)_{y,i} = \frac{\mathbb{E}(\varepsilon_t^{MP} u_{y,t})}{\mathbb{E}((\varepsilon_t^{MP})^2)} = \mathbb{E}(\varepsilon_t^{MP} u_{y,t}) = \frac{\mathbb{E}((u_{i,t} - \psi_y u_{y,t}) u_{y,t})}{\sigma_{MP}} = \frac{\sigma_{i,y} - \psi_y \sigma_y^2}{\sigma_{MP}},$$

where $\sigma_{i,y} \equiv \mathbb{E}(u_{i,t} u_{y,t})$ denotes the covariance between the reduced-form residuals of the equations for output and the interest rate and $\sigma_y^2 \equiv \mathbb{E}((u_{y,t})^2)$ denotes the variance of the residuals of the equation for output.

Since in the paper we are interested in the on-impact impulse response normalized by the size of the shock, we have:

$$M(0;\psi_y)_{y,i} = \frac{IRF(0)_{y,i}}{\sigma_{MP}} = \frac{\sigma_{i,y} - \psi_y \sigma_y^2}{\sigma_{MP}^2},$$

To recover an expression for σ_{MP}^2, we operate with squares and expected values on both sides of (A.1):

$$\mathbb{E}((\varepsilon_t^{MP})^2) = \frac{\mathbb{E}((u_{i,t} - \psi_y u_{y,t})^2)}{\sigma_{MP}^2},$$

from which we can obtain:

$$\sigma_{MP}^2 = \sigma_i^2 + \psi_y^2 \sigma_y^2 - 2\psi_y \sigma_{i,y} > 0,$$

where we have used $\mathbb{E}((\varepsilon_t^{MP})^2) = 1$ and where $\sigma_i^2 \equiv \mathbb{E}((u_{i,t})^2)$ denotes the variance of the residuals of the interest rate equation in the VAR. The expression is always non-negative as it is the result of a square. Plugging this expression in the one for $M(0;\psi_y)_{y,i}$ derived above, we obtain the on-impact impulse response of output to a monetary policy shock, which reads:

$$M(0;\psi_y)_{y,i} = \frac{IRF(0)_{y,i}}{\sigma_{MP}} = \frac{\sigma_{i,y} - \psi_y \sigma_y^2}{\sigma_i^2 + \psi_y^2 \sigma_y^2 - 2\psi_y \sigma_{i,y}},$$

which corresponds to the expression (7) in the paper. The discussion on the link between the sign of the on impact response of output to a monetary policy shock and the policy coefficient ψ_y is offered in the paper.

A2: Generalization of the previous expression that admits a systematic policy response to prices

Figure A.1 generalizes the results in the paper on the relationship between the standardized impact response of output to a monetary policy shock as a function of ψ_y to the case of a policy rule that also considers the systematic response to inflation, i.e., $i_t = \psi_y y_t + \psi_p p_t + \sigma_{MP}\varepsilon_t^{MP}$. From Caldara and Kamps (2017, eqt. 21) we have[22]:

$$\frac{IRF(0)_{y,i}}{\sigma_{MP}} = \frac{\sigma_{i,y} - \psi_y \sigma_y^2 - \psi_p \sigma_{y,p}}{\psi_y^2 \sigma_y^2 + \sigma_i^2 - 2\psi_y \sigma_{i,y} + \psi_p^2 \sigma_p^2 + -2\psi_p \sigma_{i,p} + 2\psi_y \psi_p \sigma_{y,p}} \tag{A.2}$$

where $\sigma_{j,k}$ denotes the covariance between the reduced-form residuals of the equation for the variable j and those for the variable k and where σ_j^2 denotes the variance of the residuals for the variable j.

Figure A.1 plots three lines, each one corresponding to a different value of ψ_p, which we take for illustrative purposes from the estimated ψ_p posterior mode values in Table 1 associated to Uhlig(2005)'s identification, Antolín-Díaz and Rubio-Ramírez's (2018) identification, and our identification that adds policy coefficient restrictions (PCR) à la Arias, Caldara, and Rubio-Ramírez (2019) on top of Antolín-Díaz and Rubio-Ramírez's (2018) restrictions.

One key result emerges from the figure: considering a policy rule that also considers the systematic response to inflation, ψ_p, does not significantly affect the on-impact response of output to a monetary policy shock, particularly as regards the threshold value of ψ_y that implies a negative impact response. This finding provides support to our focus in the paper on the role of the systematic policy response to output, ψ_y, and is in line with the results in Figure A.6 suggesting that the restriction $\psi_y > 0$ is the main driver of a more precisely estimated short-run negative response of output.[23]

A3: Empirical analysis with US data: Additional evidence

- Figure A.2 replicates the key findings documented in Figure 6 in Antolín-Díaz and Rubio-Ramírez (2018). Even though we skip the resampling step in the authors's algorithm following the suggestion in Giacomini, Kitagawa, and Read (2022), we still find findings virtually unchanged with respect to the ones in Antolín-Díaz and Rubio-Ramírez (2018). The light-shaded area represents the 68 percent (point-wise) highest probability density credible sets for the IRFs and the dotted lines are the median IRFs using the baseline

[22] The formula we use in the paper under the simple rule $i_t = \psi_y y_t + \sigma_{MP}\varepsilon_t^{MP}$ can be recovered by imposing $\psi_p = 0$ in the following formula (and corresponds to equation 11 in Caldara and Kamps (2017)). The assumption that the variance-covariance matrix of the reduced-form residuals is positive definite ensures that the denominator of equation A.2 is strictly larger than zero. Hence only the numerator contributes to the sign of the impulse response function. For details, please refer to the proof earlier in this Appendix.

[23] ψ_p does not significantly affect the sign of on-impact response since $\sigma_{y,p}$, the covariance between the residuals referring to the output and inflation equations, is estimated to be a very small number (the posterior mode is 0.0001). This means that, given that the denominator of equation A.2 is always strictly positive, only ψ_y positive and big enough can imply a negative sign of the on-impact response of output. $\sigma_{i,y}$ and σ_y^2 are estimated to be equal to 0.035 and 0.22, respectively, by their posterior mode.

identification. These impulse responses replicate those produced by Uhlig (2005) (see his Figure 6, p. 397). Notice in particular that the response of output is quantitatively very uncertain and points to a *positive* response, which is against conventional wisdom. Differently, the darker shaded areas and dashed lines are those obtained by adding the narrative sign restrictions to the set of identifying restrictions. One can easily appreciate the dramatic difference in terms of evidence, in particular as far as the output effects of monetary policy shocks are concerned, i.e., contractionary monetary policy shocks are in this case recessionary.

- Figure A.3 replicates the results referring to the 68% credible sets in panel b of Figure 4 on page 9 in Arias, Caldara, and Rubio-Ramírez (2019) via a modification of the codes by Antolín-Díaz and Rubio-Ramírez (2018) and using the latter authors' dataset.

- Figure A.4 complements Figure 3 in the main paper and shows the full set of results obtained when adding Restrictions 1 and 2 in Arias, Caldara, and Rubio-Ramírez (2019) to the Antolín-Díaz and Rubio-Ramírez (2018) restrictions. On top of estimating more sharply the output effects to monetary policy shocks, the estimated responses of both measures of reserves are also much more precisely estimated.

- Figure A.5 documents that our main results in Figure 3 are virtually identical when we use the original algorithm in Antolín-Díaz and Rubio-Ramírez (2018) codes rather than our baseline algorithm that excludes the resampling step for the narrative sign restrictions as suggested by Giacomini, Kitagawa, and Read (2021).[24]

- Figure A.6 investigates the drivers of the sharpening of the output credible set we obtain after imposing PCR as in Arias, Caldara, and Rubio-Ramírez (2019) to the Antolín-Díaz and Rubio-Ramírez (2018) restrictions. We consider different combinations of the PCR in Arias, Caldara, and Rubio-Ramírez (2019): $\psi_{tr} = \psi_{nbr} = 0$; $\psi_y > 0$; $\psi_p > 0$. The results show that our main result on the less uncertain response of output in the short run is driven by the positive sign imposed on the policy coefficient responsible for the systematic policy response to output ψ_y (see the first panel to the left in the bottom row and compare it with the top panels).

[24] Giacomini, Kitagawa, and Read (2021) argue that the Markov Chain resampling step of the algorithm proposed by Antolín-Díaz and Rubio-Ramírez (2018) can be problematic because, for some types of narrative restrictions (such as historical decomposition restrictions), a component of the prior is updated only in the direction that makes the narrative restrictions unlikely to hold ex ante. Giacomini, Kitagawa, and Read (2021) suggest Bayesian researchers to base the analysis on the unconditional likelihood, rather than the conditional likelihood used by Antolín-Díaz and Rubio-Ramírez (2018) and amend the algorithm by the latter authors by dropping their resampling step.

- Figure A.7 shows that the addiction of PCR as in Arias, Caldara, and Rubio-Ramírez (2019) imply less uncertain output responses – i.e., narrower credible sets – also when considering an alternative event to the October 1979 one that we have used in our main analysis. In particular, our results are robust to the use of February 1994 as an alternative event where a monetary policy shock is imposed to be both positive and the overwhelming driver of the unexpected movement in the federal funds rate as done in Antolín-Díaz and Rubio-Ramírez (2018) (see Restrictions 8 and 9 and Figure C.3 in their Appendix).[25]

- Figure A.8 complements Figure 4 by plotting the distribution of the impulse response function to a monetary policy shock for more horizons, i.e., $0, 3, 6, 9, 12, 15, 18$. The findings show that, consistently with our conclusion in the paper, adding the PCR to Antolín-Díaz and Rubio-Ramírez (2018) restrictions shifts the IRF distribution to the negative domain, permitting to achieve a sharper identification for the effects of monetary policy shocks in the US.

- Figure A.9 illustrates that, in our empirical application on U.S. data, narrative and policy coefficient restrictions function as complements, rather than substitutes. Specifically, adopting a symmetric approach with respect to our baseline analysis—starting from SR and PCR, as in Arias, Caldara, and Rubio-Ramírez (2019), and subsequently incorporating NSR à la Antolín-Díaz and Rubio-Ramírez (2018)—allows for a more precise identification of the effects of monetary policy shocks, consistent with our baseline findings. Nonetheless, our baseline approach, which includes PCR on top of SR and NSR, proves particularly effective in obtaining more a precise short-run estimate of the IRF.

A4: Monte Carlo simulations: Additional results

- Figure A.10 adds extra findings related to the Monte Carlo experiment in the paper. The Figure documents that in the case of a strong signal for monetary policy shocks, i.e., in the case in which their standard deviation in the Smets and Wouters (2007) is multiplied by 10, all the different identification schemes considered in the Monte Carlo section of the paper can recover quite precisely and without significant bias – although with different degrees of precision – the true impulse response functions to a monetary policy shock. This exercise shows that our Monte Carlo setup with small samples and VAR(4) specifications

[25] We get similar findings if we use either December 1988 or April 2001 as alternative events as done in Antolín-Díaz and Rubio-Ramírez (2018) (see page 2827). For the sake of brevity we do not report these results, which are available upon request.

can recover the true response of monetary policy shocks in a favorable case (see, e.g., Paustian (2007)).

Figures

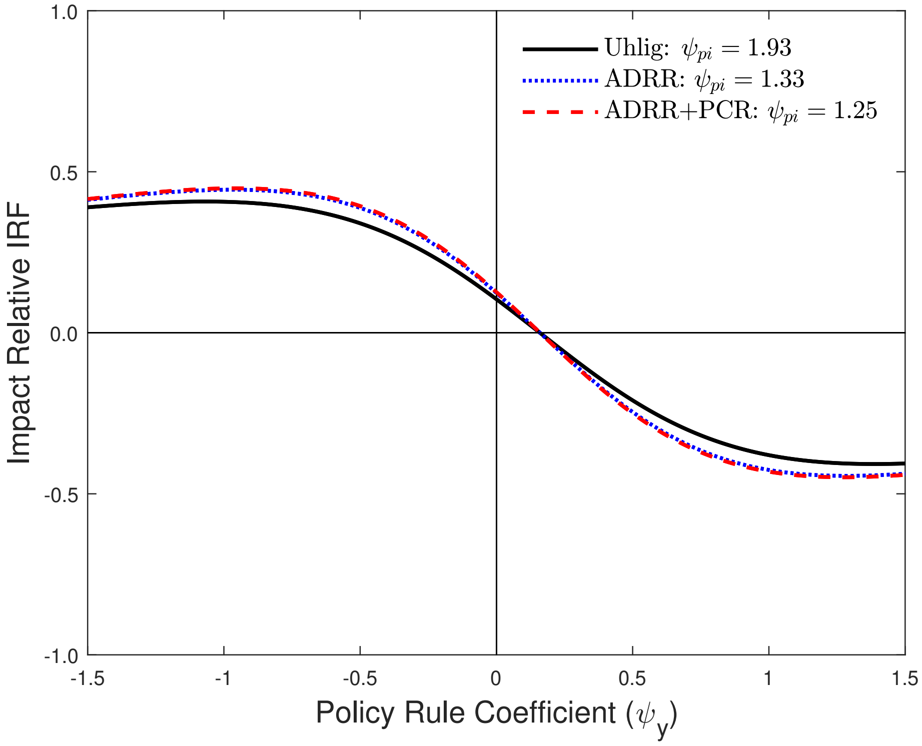

Figure A.1: **Impact relative impulse response as a function of the policy coefficient ψ_y.** The lines plot the relationship between the relative impulse response to a monetary policy shock $(IRF(0)_{y,i}/\sigma_{MP})$ and the systematic response of monetary policy to output ψ_y under the monetary policy rule $i_t = \psi_y y_t + \psi_p p_t + \sigma_{MP}\varepsilon_t^{MP}$ for three different values of ψ_p taken from the posterior mode estimates in Table 1, corresponding to Uhlig(2005)'s identification, Antolín-Díaz and Rubio-Ramírez's (2018) identification, and our identification that adds policy coefficient restrictions (PCR) à la Arias, Caldara, and Rubio-Ramírez (2019) on top of Antolín-Díaz and Rubio-Ramírez's (2018) restrictions. The lines are plot on the basis of equation 21 in Caldara and Kamps (2017).

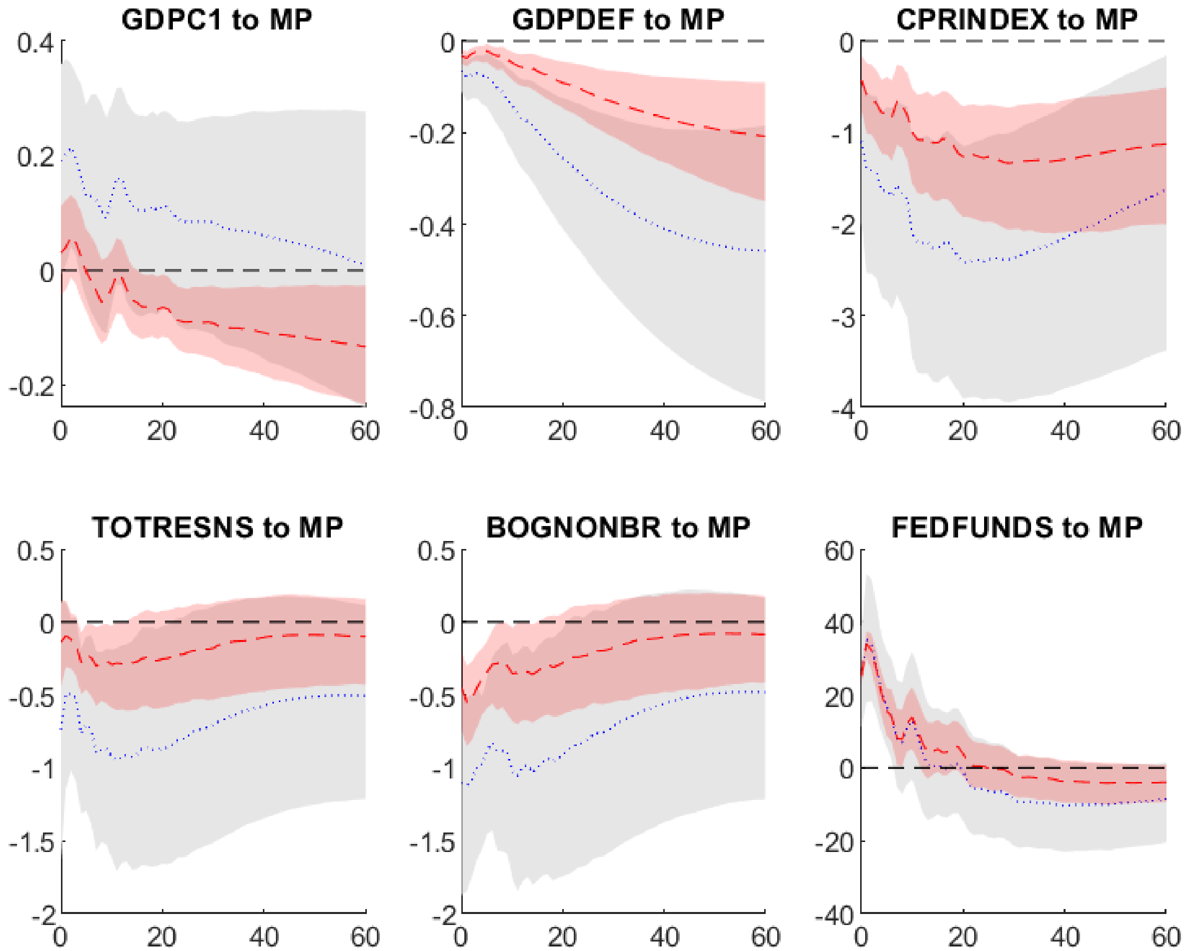

Figure A.2: **IRF to a monetary policy shock: Replication of Antolín-Díaz and Rubio-Ramírez (2018) (ADRR).** The grey (light) shaded area represents the 68 percent (point-wise) credible sets for the IRFs and the dotted lines are the median IRFs using the Uhlig (2005) identification restrictions. The red (darker) shaded areas and dashed lines display the equivalent quantities for the models that additionally satisfy Narrative Sign Restrictions 4 and 5 in ADRR. The monetary policy shock has been normalized to have an impact of 25 basis points on the federal funds rate.

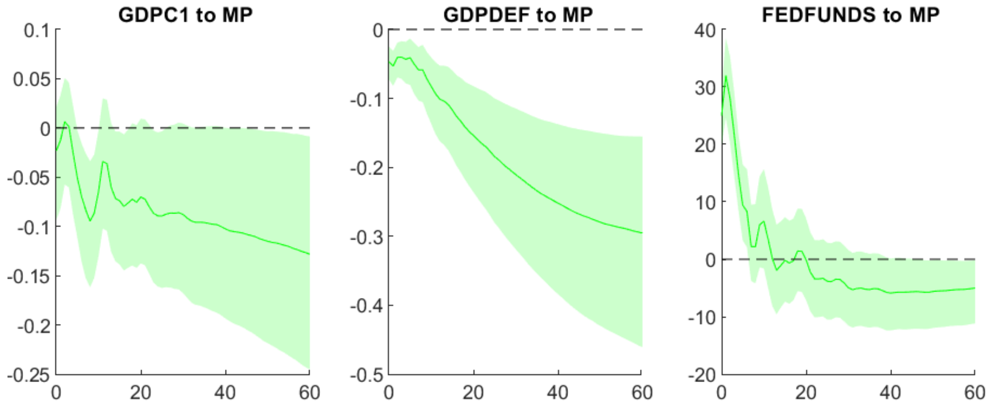

Figure A.3: **IRF to a monetary policy shock: Replication of Arias, Caldara, and Rubio-Ramírez (2019).** The shaded area represents the 68 percent credible sets for the IRFs and the solid lines are the median IRFs when using Arias, Caldara, and Rubio-Ramírez (2019) Restrictions 1 and 2 in addition to ADRR ones (Restriction 3 in Arias, Caldara, and Rubio-Ramírez (2019) – i.e., Uhlig (2005) sign restrictions – is already incorporated in ADRR restrictions). The monetary policy shock has been normalized to have an impact of 25 basis points on the federal funds rate.

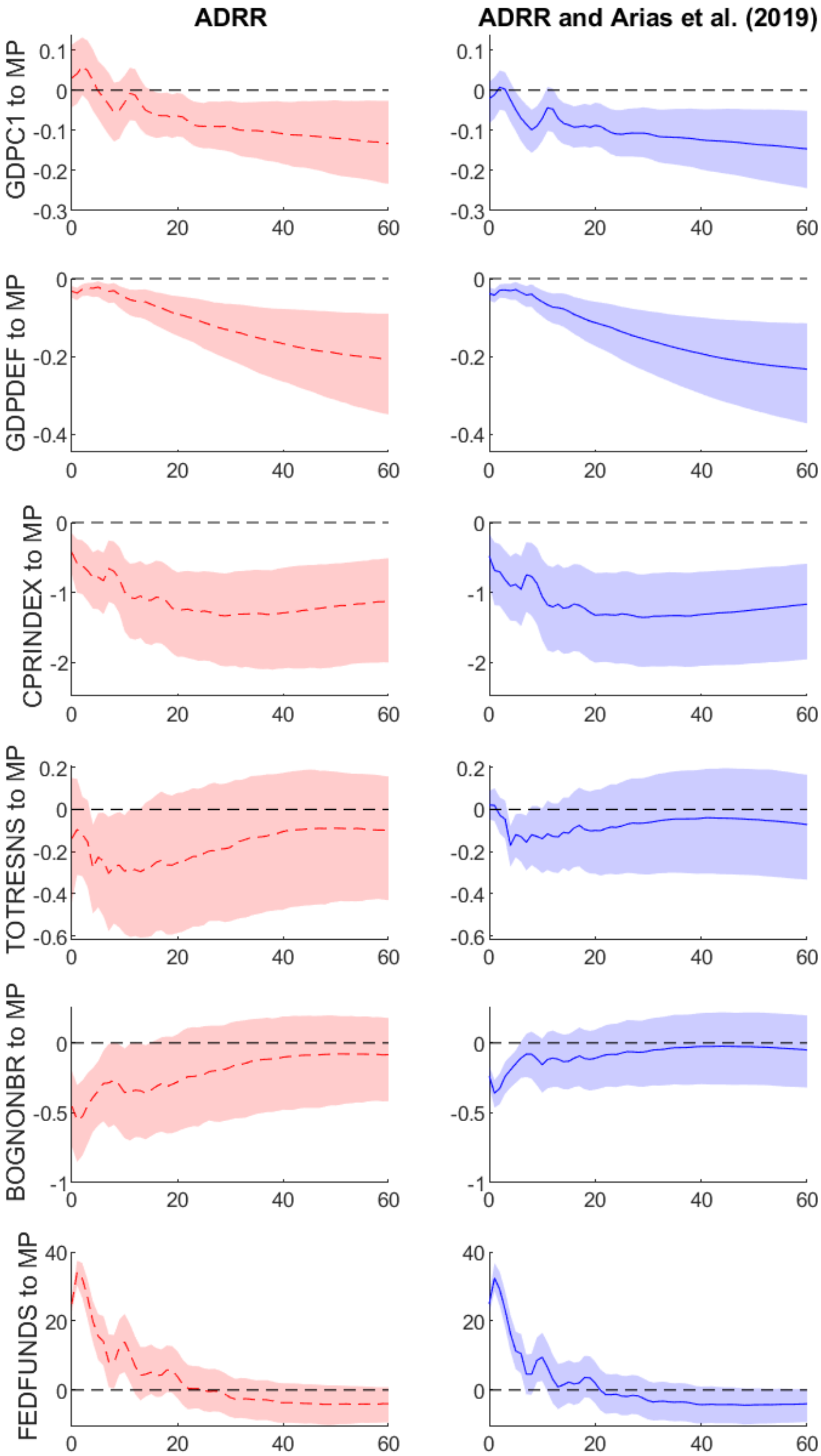

Figure A.4: **Adding policy coefficient restrictions à la Arias, Caldara, and Rubio-Ramírez (2019) to ADRR restrictions.** Left (Right) panels: The red (blue) shaded area represents the 68 percent credible sets for the IRFs and the dashed red (blue) lines are the median IRFs using the ADRR restrictions (Arias, Caldara, and Rubio-Ramírez (2019) Restrictions 1 and 2 in addition to ADRR ones) when we exclude the resampling step in the ADRR algorithm as suggested by Giacomini, Kitagawa, and Read (2021). The monetary policy shock has been normalized to have an impact of 25 basis points on the federal funds rate.

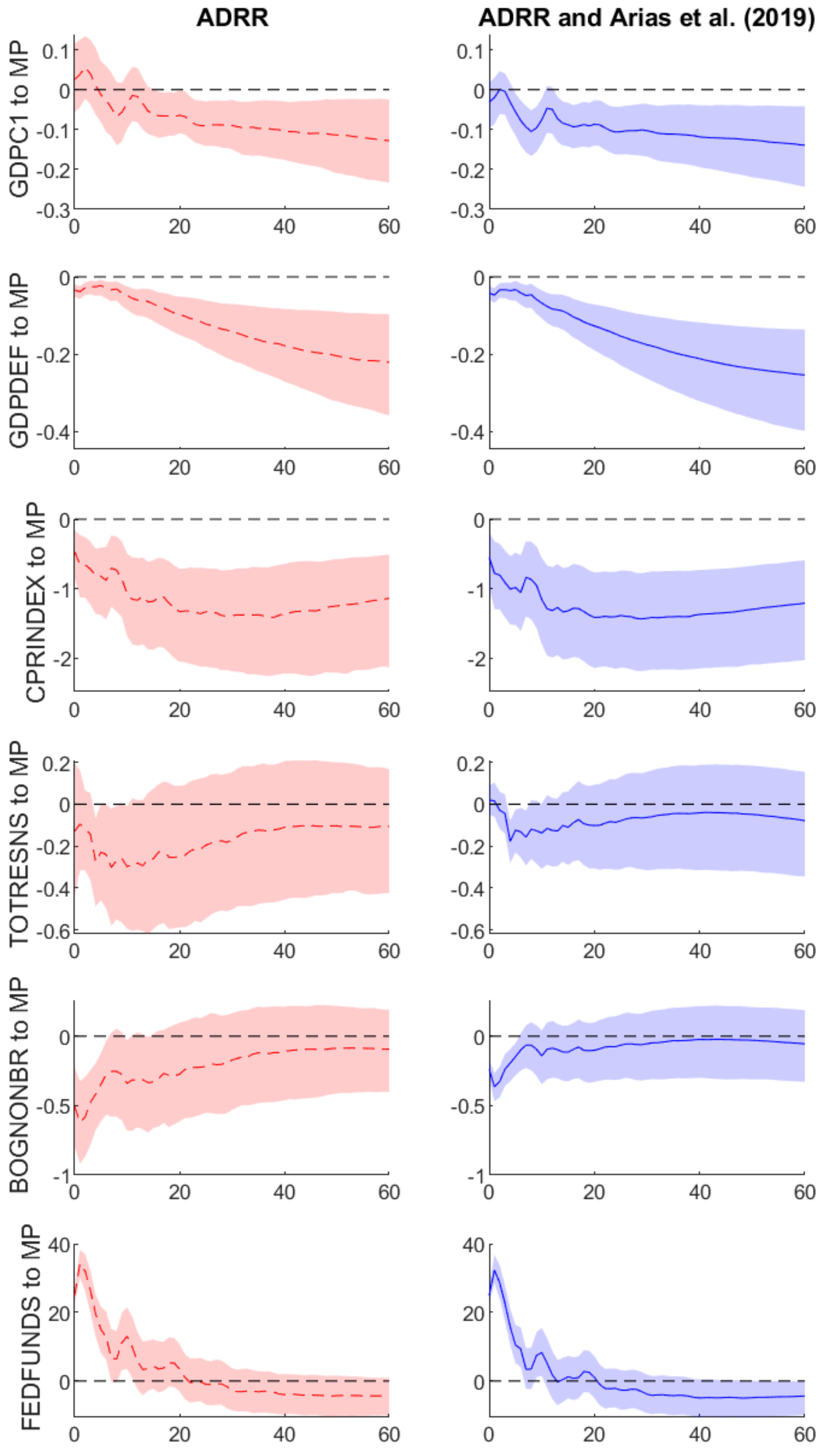

Figure A.5: **IRF to a monetary policy shock: Adding policy coefficient restrictions à la Arias, Caldara, and Rubio-Ramírez (2019) to ADRR restrictions (when also using the resampling step in the ADRR algorithm).** Left (Right) panels: The red (blue) shaded area represents the 68 percent credible sets for the IRFs and the dashed red (blue) lines are the median IRFs when using the ADRR restrictions (Arias, Caldara, and Rubio-Ramírez (2019) Restrictions 1 and 2 in addition to ADRR ones). The monetary policy shock has been normalized to have an impact of 25 basis points on the federal funds rate.

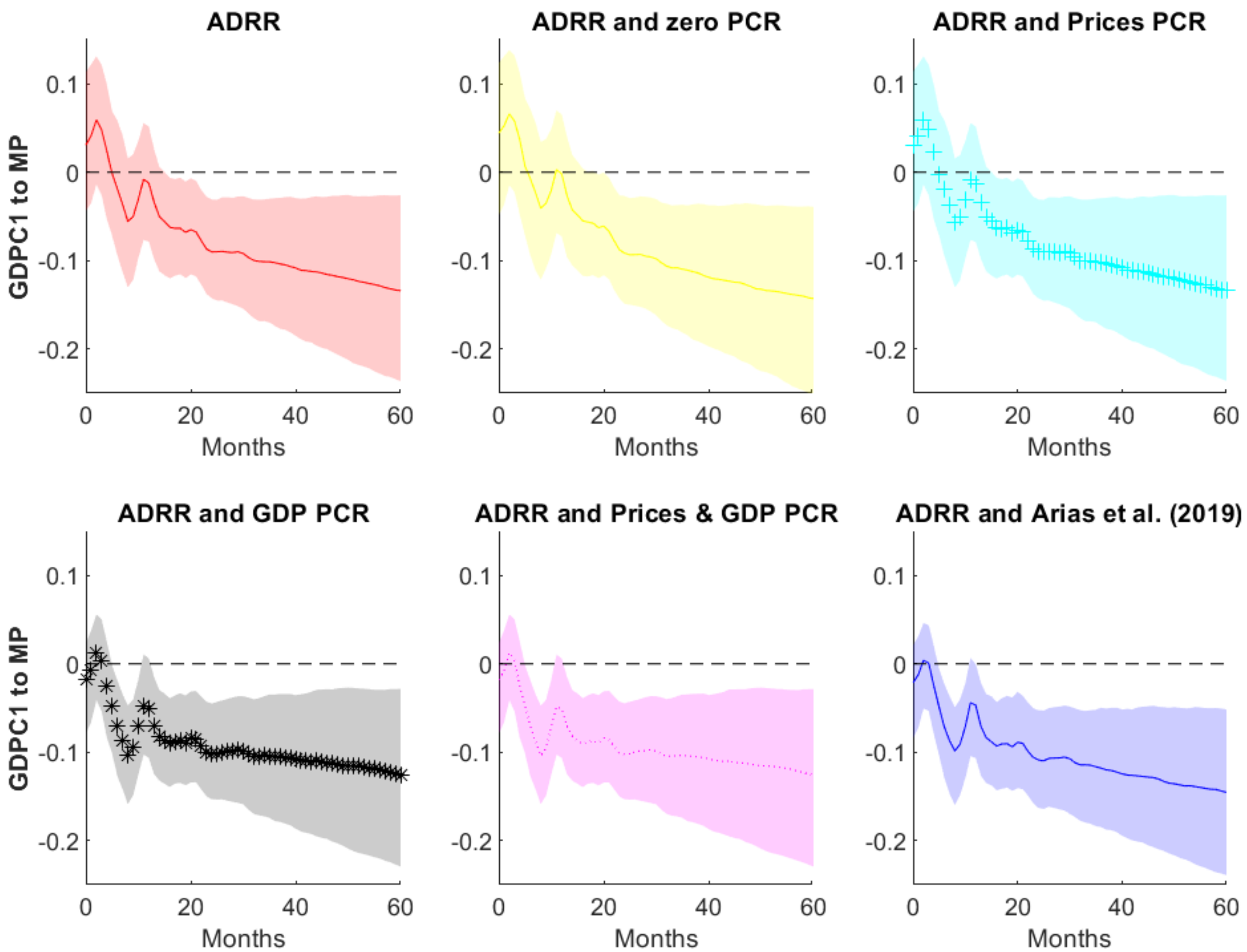

Figure A.6: **Output response to a monetary policy shock: Drivers of the narrower credible set when adding policy coefficients à la Arias, Caldara, and Rubio-Ramírez (2019) to the ADRR restrictions.** The shaded areas represent the 68 percent credible sets for the IRFs and the lines are the median IRFs for alternative identification schemes. From the top left panel to the bottom right panel the different identification schemes are in order: i) ADRR (i.e., Narrative Restrictions 4 and 5 in Antolín-Díaz and Rubio-Ramírez (2018) in addition to Uhlig (2017) sign restrictions); ii) Arias, Caldara, and Rubio-Ramírez (2019) Restriction 1 – i.e., zero policy coefficient restrictions for total reserves and non-borrowed reserves – in addition to ADRR ones; iii) positive policy coefficient for prices in addition to ADRR restrictions; iv) positive policy coefficient for GDP in addition to ADRR restrictions; v) Arias, Caldara, and Rubio-Ramírez (2019) Restriction 2 – i.e., positive policy coefficients for prices and GDP – in addition to ADRR restrictions; vi) Arias, Caldara, and Rubio-Ramírez (2019) Restrictions 1 and 2 in addition to ADRR restrictions. The monetary policy shock has been normalized to have an impact of 25 basis points on the federal funds rate.

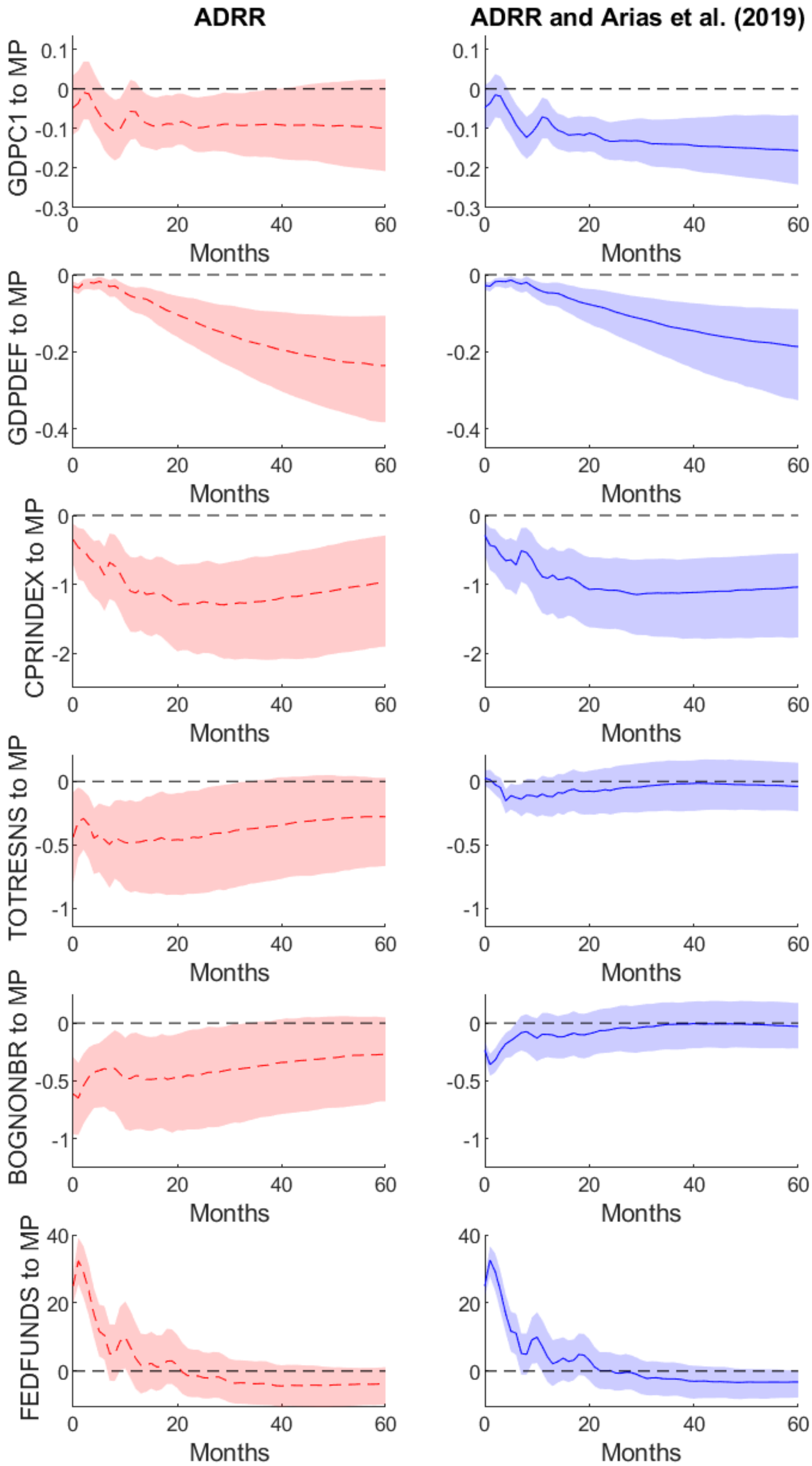

Figure A.7: **Adding policy coefficient restrictions à la Arias, Caldara, and Rubio-Ramírez (2019) to AD-RR restrictions: February 1994 as alternative event.** Left (Right) panels: The red (blue) shaded area represents the 68 percent credible sets for the IRFs and the dashed red (blue) lines are the median IRFs using the ADRR alternative Narrative Sign Restrictions 8 and 9 on top of Uhlig (2005) sign restrictions (Arias, Caldara, and Rubio-Ramírez (2019) Restrictions 1 and 2 in addition to ADRR alternative ones). The monetary policy shock has been normalized to have an impact of 25 basis points on the federal funds rate.

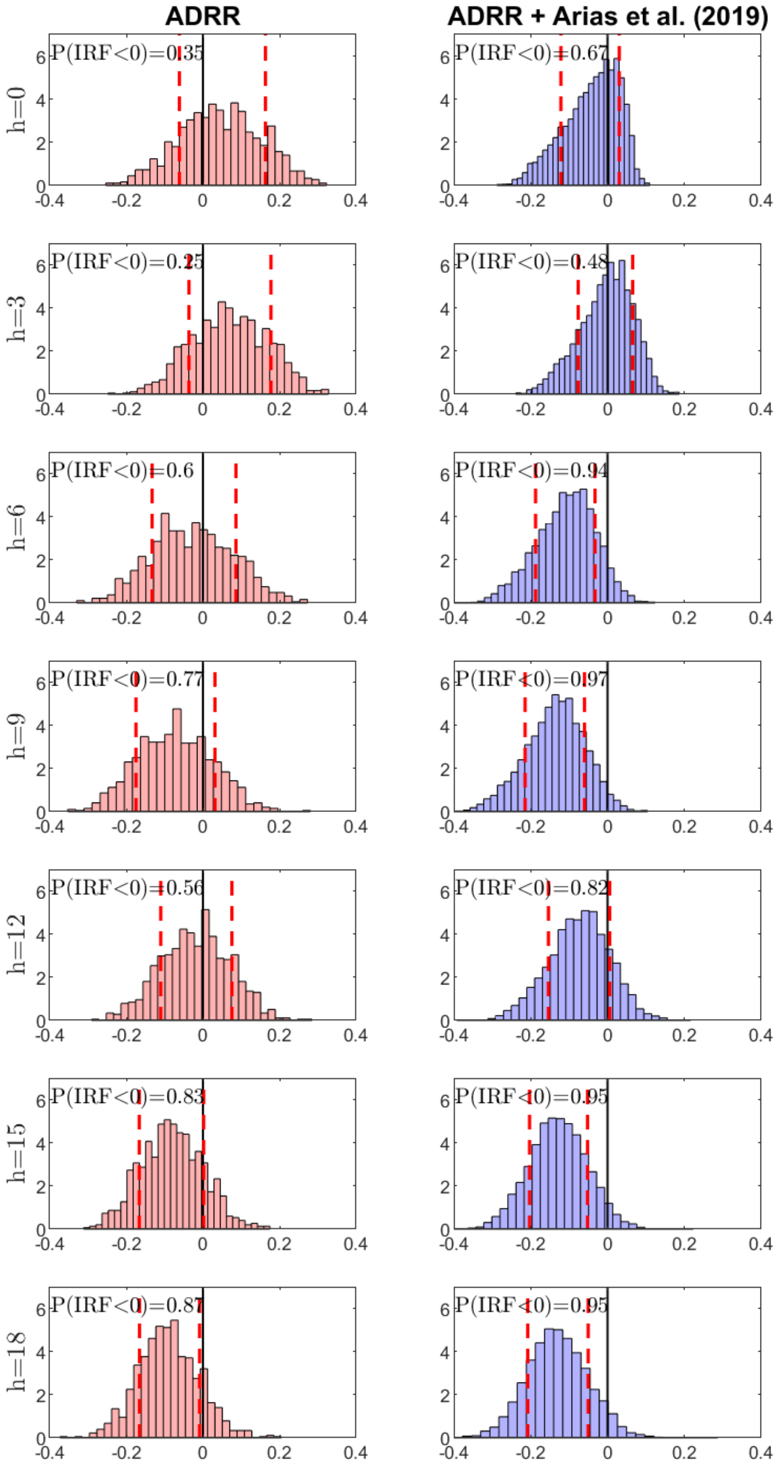

Figure A.8: **Output response to a monetary policy shock: Distribution at several horizons when adding policy coefficient restrictions à la Arias, Caldara, and Rubio-Ramírez (2019) to ADRR restrictions.** Left (Right) panels: The red (blue) histogram represents the distributions of the output IRF for all retained models per each selected horizon, $h = 0, 3, 6, 9, 12, 15, 18$, when using the ADRR restrictions (Arias, Caldara, and Rubio-Ramírez (2019) Restrictions 1 and 2 in addition to ADRR ones). The dashed red vertical lines indicate the corresponding 68% credible sets. The number in the top right panel is the share of retained models that implies a negative response of output per each selected horizon. The monetary policy shock has been normalized to have an impact of 25 basis points on the federal funds rate.

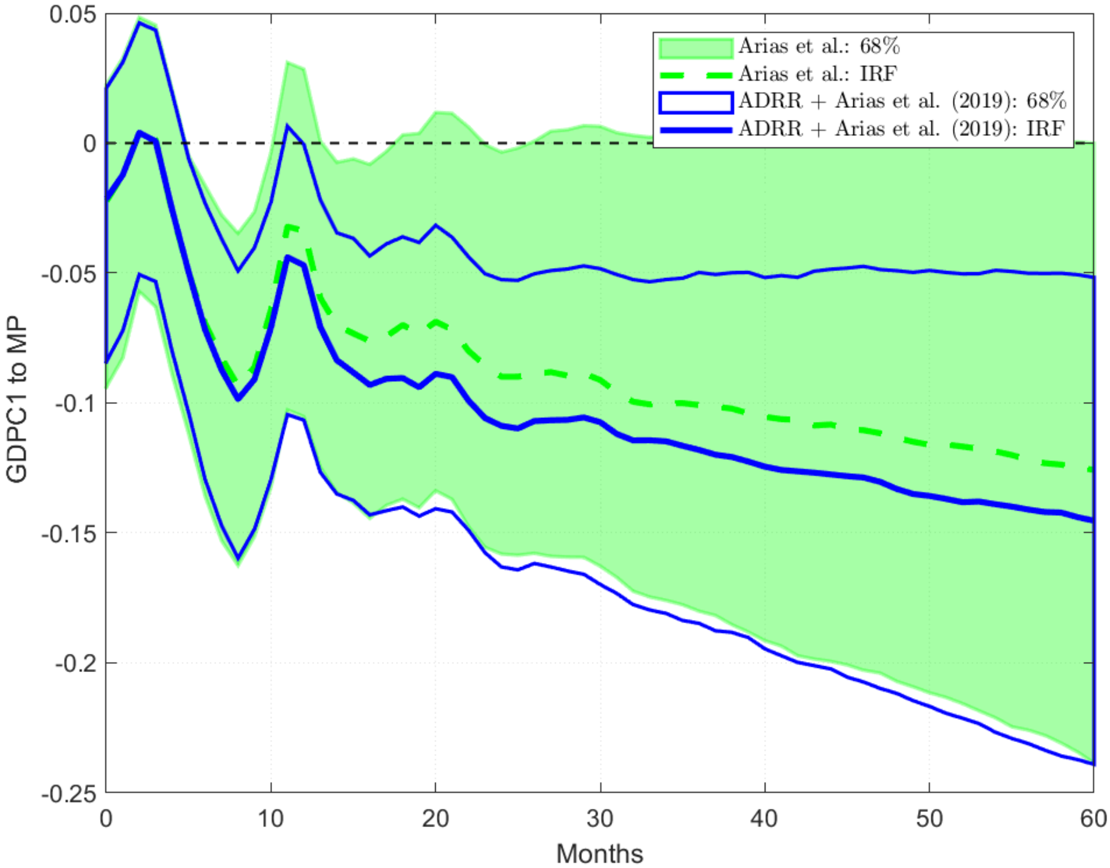

Figure A.9: **Output response to a monetary policy shock: Adding narrative sign restrictions à la Antolín-Díaz and Rubio-Ramírez (2018) to ACRR restrictions.** The green shaded (blue contoured) area represents the 68 percent credible sets for the output IRF and the green dashed (blue solid) lines are the median IRF of output when using the ACRR restrictions (Antolín-Díaz and Rubio-Ramírez (2018) Restrictions 4 and 5 in addition to ACRR ones). The monetary policy shock has been normalized to have an impact of 25 basis points on the federal funds rate.

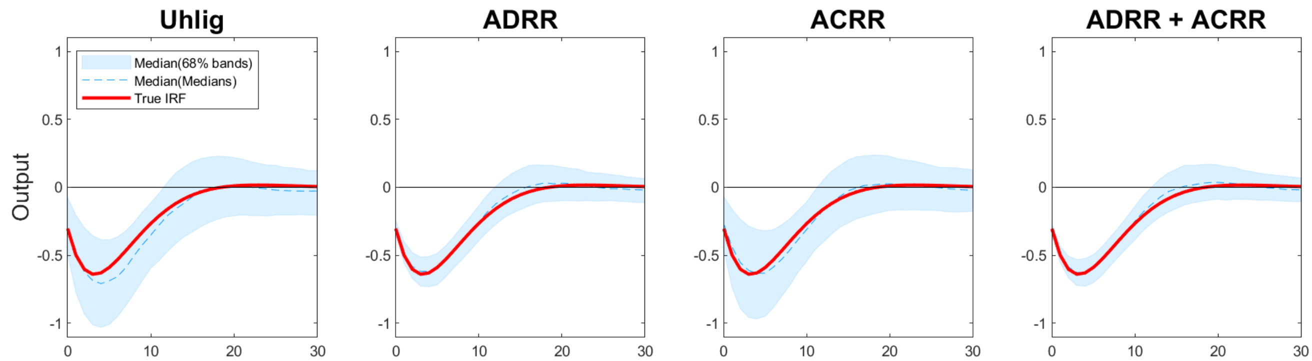

Figure A.10: **Monte Carlo simulations with strong signal: Output response to a monetary policy shock under either SR & NSR à la ADRR or SR & NSR à la ADRR *plus* PCR à la ACRR.** The solid red line represents the output response to a monetary policy shock in the Smets and Wouters (2007) DSGE model, which has been used to simulate 100 datasets of 200 observations. The standard deviation of the monetary policy shock in the Smets and Wouters (2007) model has been multiplied by 10 in order to assess the accuracy of sign restrictions identification in a favorable case (see, e.g., Paustian (2007)). The light blue shaded area represents the *median* of the 68 percent credible sets for the output IRF across the 100 different datasets and the light blue dashed lines are the *median* of the median IRFs of output across the 100 datasets when using different identification schemes. The first, second, third, and fourth columns show results from different identification schemes: SR à la Uhlig (2005) and Wolf (2020); SR & NSR à la ADRR; PCR only à la ACCR (Arias, Caldara, and Rubio-Ramírez (2019) Restrictions 1 and 2); PCR à la ACCR on top of SR & NSR à la ADRR, respectively. The monetary policy shock has been normalized to have an impact of 25 basis points on the federal funds rate. More details are available in Section 3.